Online Appendix
Married with children:
A collective labor supply model with detailed time use and intrahousehold expenditure information

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Appendix 1: Proof of Proposition 1

As explained in the main text, the proof of the identification result in Proposition 1 uses the fact that we can also represent the solution to the optimization programme (4) as stemming from a two-stage allocation process. The result is best explained by first focusing on the second stage. Therefore, in Subsection A we first consider this second stage. Subsection B then deals with the first stage of the allocation process.

A. Second stage of the allocation process

It is clear that the outputs u^k and u^p of the household production process are not observable. Still, the fact that we observe the inputs (i.e., (c^k, h_k^1, h_k^2) and (c^p, h_p^1, h_p^2)) as functions of $(w^1, w^2, y, \mathbf{z}, \mathbf{s})$ allows us to recover the functions u^k and u^p up to a strictly increasing transformation. This happens through the assumption of cost minimization in the production of the domestic goods ($j = k, p$):

$$(1) \quad \frac{\frac{\partial u^j\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right)}{\partial h_j^1}}{\frac{\partial u^j\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right)}{\partial c^j}} \equiv \phi_j^1\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right) = w^1; \quad \frac{\frac{\partial u^j\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right)}{\partial h_j^2}}{\frac{\partial u^j\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right)}{\partial c^j}} \equiv \phi_j^2\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right) = w^2.$$

Making use of Frobenius' theorem (see, e.g., Afriat, 1977), these systems of partial differential equations can be integrated to respectively $u^k\left(c^k, h_k^1, h_k^2; \mathbf{s}^k\right)$ and $u^p\left(c^p, h_p^1, h_p^2; \mathbf{s}^p\right)$ if the following Slutsky equations are satisfied:

(2)

$$\frac{\partial \phi_j^1\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right)}{\partial h_j^2} + \frac{\partial \phi_j^1\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right)}{\partial c^j}\phi_j^2 = \frac{\partial \phi_j^2\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right)}{\partial h_j^1} + \frac{\partial \phi_j^2\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right)}{\partial c^j}\phi_j^1.$$

Given the specification of the individual utility functions $u^i\left(c^i, l^i, u^k,\ u^p\right)$ $(i = 1, 2)$, the subutility functions u^k and u^p can be identified only up to a strictly increasing transformation. At this point, it is worth to note that the set of appropriate strictly increasing transformations is restricted due to the assumption that the subutility functions should be characterized by constant returns to scale. More precisely, let r_k and r_p be strictly increasing functions. Given our identification strategy, $r_k\left(u^k\right)$ and $r_p\left(u^p\right)$ should also be characterized by constant returns to scale.[1] Obviously, substituting u^k by $r_k\left(u^k\right)$ and u^p by $r_p\left(u^p\right)$, and the individual utility functions by $\widetilde{u}^i(c^i,\ l^i,\ u^k,\ u^p) = u^i(c^i,\ l^i,\ r_k^{-1}\left(u^k\right),\ r_p^{-1}\left(u^p\right))$ will give rise to the same observable choices. The chosen cardinalization of the household production technologies (that exhibit constant returns to scale) is thus a matter of normalization.

In what follows, we will assume that particular cardinalizations for the functions u^k and u^p have been identified in this way. In other words, we assume that $u^k(c^k,\ h_k^1,\ h_k^2;\ \mathbf{s}^k)$ and $u^p(c^p,\ h_p^1,\ h_p^2;\ \mathbf{s}^p)$ are known functions of respectively $(c^k,\ h_k^1,\ h_k^2)$ and $(c^p,\ h_p^1,\ h_p^2)$. The latter variables are themselves known functions of $(w^1,\ w^2,\ y,\ \mathbf{z},\ \mathbf{s})$, which is inherited by $u^k(c^k,\ h_k^1,\ h_k^2;\ \mathbf{s}^k)$ and $u^p(c^p,\ h_p^1,\ h_p^2;\ \mathbf{s}^p)$. This obtains the functions $u^k(w^1,\ w^2,\ y,\ \mathbf{z},\ \mathbf{s})$ and $u^p(w^1,\ w^2,\ y,\ \mathbf{z},\ \mathbf{s})$ (with a slight abuse of notation for the sake of simplicity).

To identify the allocation of the adult members' shares to own consumption and leisure, we hold the quantities of the domestic goods constant at the (arbitrary) levels $\overline{u}^k$ and $\overline{u}^p$. Let us denote a particular distribution factor in the vector $\mathbf{z}$ by z, while the other distribution factors in $\mathbf{z}$ are contained in the (possibly empty) vector $\mathbf{z}^-$. In a similar way, we denote a particular production shifter in the vector $\mathbf{s}$ by s and the remaining production shifters are captured by the (possibly empty) vector $\mathbf{s}^-$. Assuming that the matrix $\begin{bmatrix} \frac{\partial u^k(.)}{\partial z} & \frac{\partial u^k(.)}{\partial s} \\ \frac{\partial u^p(.)}{\partial z} & \frac{\partial u^p(.)}{\partial s} \end{bmatrix}$ is nonsingular in an appropriately defined subset of the domain of $u^k(w^1, w^2,\ y, \mathbf{z}, \mathbf{s})$ and $u^p(w^1, w^2,\ y, \mathbf{z}, \mathbf{s})$, we can make use of the implicit function theorem to express the distribution factor z and the production shifter s as functions of the observable exogenous variables w^1, w^2, y, $\mathbf{z}^-$ and $\mathbf{s}^-$ and the levels $\overline{u}^k$ and $\overline{u}^p$: $z = z\left(w^1, w^2, y, \overline{u}^k, \overline{u}^p, \mathbf{z}^-, \mathbf{s}^-\right)$ and $s = s\left(w^1, w^2, y, \overline{u}^k, \overline{u}^p, \mathbf{z}^-, \mathbf{s}^-\right)$. The role of the distribution factor z and the production shifter s becomes immediately clear: they serve to keep the output of the domestic goods constant while allowing variation in the individual wages and the nonlabor income. A related but distinct idea is used by BCM. Because we have more than one domestic good in our model, we need at least one production shifter in addition to a distribution factor to keep the output of the domestic goods constant at $\overline{u}^k$ and $\overline{u}^p$.[2]

[1] For example, the following strictly increasing transformations of u^k and u^p are allowed: $r_k\left(u^k\right) = \alpha^k u^k$ and $r_p\left(u^p\right) = \alpha^p u^p$ with α^k and α^p some strictly positive numbers.

[2] It is worth to indicate that having multiple distribution factors but no production shifter would not be sufficient here. For example, suppose we have two distribution factors, z_1 and z_2. Then, the corresponding matrix $\begin{bmatrix} \frac{\partial u^k(.)}{\partial z_1} & \frac{\partial u^k(.)}{\partial z_2} \\ \frac{\partial u^p(.)}{\partial z_1} & \frac{\partial u^p(.)}{\partial z_2} \end{bmatrix}$ is singular by construction, because any distribution

We define the adult members' conditional shares as follows ($i = 1, 2$):

$$\rho^i \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) = w^i l^i \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) + c^i \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) - w^i. \tag{3}$$

The shares $\rho^1 (w^1, w^2, y, \mathbf{z}, \mathbf{s})$ and $\rho^2 (w^1, w^2, y, \mathbf{z}, \mathbf{s})$ define the conditional sharing rule by distributing among the adult members the residual nonlabor income that is left over after purchasing the inputs in the household production process. We thus get:

$$\begin{aligned} &\rho^1 \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) + \rho^2 \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) = \\ &y - c^k \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) - w^1 h_k^1 \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) - w^2 h_k^2 \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) \\ &- c^p \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) - w^1 h_p^1 \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right) - w^2 h_p^2 \left(w^1, w^2, y, \mathbf{z}, \mathbf{s}\right). \end{aligned} \tag{4}$$

Let us introduce the following notation: $\rho (w^1, w^2, y, \mathbf{z}, \mathbf{s}) = \rho^1 (w^1, w^2, y, \mathbf{z}, \mathbf{s})$ and $\rho^2 (w^1, w^2, y, \mathbf{z}, \mathbf{s}) = y - c^k - w^1 h_k^1 - w^2 h_k^2 - c^p - w^1 h_p^1 - w^2 h_p^2 - \rho$. Given the above, $l^1 (w^1, w^2, y, \mathbf{z}, \mathbf{s})$, $l^2 (w^1, w^2, y, \mathbf{z}, \mathbf{s})$ and $\rho (w^1, w^2, y, \mathbf{z}, \mathbf{s})$ are functions of $(w^1, w^2, y,$ $z \left(w^1, w^2, y, \overline{u}^k, \overline{u}^p, \mathbf{z}^-, \mathbf{s}^-\right), s \left(w^1, w^2, y, \overline{u}^k, \overline{u}^p, \mathbf{z}^-, \mathbf{s}^-\right), \mathbf{z}^-, \mathbf{s}^-)$. However, because $\overline{u}^k$ and $\overline{u}^p$ are fixed, $l^1 (w^1, w^2, y, \mathbf{z}, \mathbf{s})$, $l^2 (w^1, w^2, y, \mathbf{z}, \mathbf{s})$ and $\rho (w^1, w^2, y, \mathbf{z}, \mathbf{s})$ solely depend on (w^1, w^2, y). Given cost minimization in the household production process and the abovementioned properties of the subutility functions $u^k \left(c^k, h_k^1, h_k^2; \mathbf{s}^k\right)$ and $u^p \left(c^p, h_p^1, h_p^2; \mathbf{s}^p\right)$, there will be unique values for the inputs in the household production process that generate the outputs $\overline{u}^k$ and $\overline{u}^p$. We denote these optimal input values by ($\overline{c}^k$, $\overline{h}_k^1$, $\overline{h}_k^2$, $\overline{c}^p$, $\overline{h}_p^1$, $\overline{h}_p^2$). Using $\overline{y} = y - \overline{c}^k - w^1 \overline{h}_k^1 - w^2 \overline{h}_k^2 - \overline{c}^p - w^1 \overline{h}_p^1 - w^2 \overline{h}_p^2$, we can then define the following individual maximization programmes for the second stage of the allocation problem:

$$\max_{c^1, l^1} u^1 \left(c^1, l^1, \overline{u}^k, \overline{u}^p\right) \tag{5}$$

subject to

$$c^1 + w^1 l^1 = w^1 + \rho,$$

and

$$\max_{c^2, l^2} u^2 \left(c^2, l^2, \overline{u}^k, \overline{u}^p\right) \tag{6}$$

subject to

$$c^2 + w^2 l^2 = w^2 + \overline{y} - \rho.$$

Chiappori (1988, 1992) proved that the observability of both members' individual labor supply functions allows us to recover the sharing rule up to a constant and the individual preferences up to a translation. A similar result applies to the above

factor impacts on household consumption only through the Pareto weight. We thank an anonymous referee for pointing this out.

setting with household production, provided that a distribution factor and a production shifter are available. The only difference between Chiappori's original setting and BCM's extension with household production is that the unidentified constant generally depends on $\overline{u}^k$ and $\overline{u}^p$. Importantly, we do not have such an unidentified constant in our case, which implies that the sharing rule and individual preferences are completely identified. The reason is that we observe c^1 and c^2 in our data set, which obtains two boundary conditions in the individual integrability problems.

Summarizing, for any given $\overline{u}^k$ and $\overline{u}^p$, every two 3-tuples (u^1, u^2, ρ) and $(\widehat{u}^1, \widehat{u}^2, \widehat{\rho})$ that generate the same solutions to programmes (5) and (6), for all (w^1, w^2, y), bear the following relations to each other:

$$\begin{aligned}
\widehat{u}^1\left(c^1, l^1, \overline{u}^k, \overline{u}^p\right) &= f^1\left(u^1\left(c^1, l^1, \overline{u}^k, \overline{u}^p\right), \overline{u}^k, \overline{u}^p\right) \\
\widehat{u}^2\left(c^2, l^2, \overline{u}^k, \overline{u}^p\right) &= f^2\left(u^2\left(c^2, l^2, \overline{u}^k, \overline{u}^p\right), \overline{u}^k, \overline{u}^p\right) \\
\widehat{\rho}\left(w^1, w^2, y\right) &= \rho\left(w^1, w^2, y\right),
\end{aligned}$$

where the functions f^1 and f^2 are increasing in respectively u^1 and u^2. Analogously, we get the following relations between the collective indirect utilities:

$$\begin{aligned}
\widehat{v}^1\left(w^1, \rho^1, \overline{u}^k, \overline{u}^p\right) &= f^1\left(v^1\left(w^1, \rho^1, \overline{u}^k, \overline{u}^p\right), \overline{u}^k, \overline{u}^p\right) \\
\widehat{v}^2\left(w^2, \rho^2, \overline{u}^k, \overline{u}^p\right) &= f^2\left(v^2\left(w^2, \rho^2, \overline{u}^k, \overline{u}^p\right), \overline{u}^k, \overline{u}^p\right).
\end{aligned} \tag{7}$$

Thus, also the collective indirect utilities can be identified up to a strictly increasing transformation that depends on the levels $\overline{u}^k$ and $\overline{u}^p$.

From now on, we suppose that (v^1, v^2, ρ) are known functions. We next focus on identifying the functions f^i $(i = 1, 2)$, which capture the trade-offs between private consumption and leisure (through ρ^i) on the one hand, and the domestic goods on the other.

B. First stage of the allocation process

If we interpret u^k and u^p as standard direct utility functions, we can define the following cost or expenditure functions $(j = k, p)$:

$$e^j\left(u^j, w^1, w^2\right) = \min_{c^j, h_j^1, h_j^1}\left[c^j + w^1 h_j^1 + w^2 h_j^2 | u^j\left(c^j, h_j^1, h_j^2; \mathbf{s}^j\right) = u^j\right] = x^j. \tag{8}$$

These functions give the minimal expenditures x^j on the inputs $\left(c^j, h_j^1, h_j^2\right)$ needed to produce a quantity u^j of the domestic good j. Since $u^k\left(c^k, h_k^1, h_k^2; \mathbf{s}^k\right)$ and $u^p(c^p, h_p^1, h_p^2; \mathbf{s}^p)$ represent technologies that exhibit constant returns to scale, the above cost functions will be of the form:

$$e^j(u^j, w^1, w^2) = g^j\left(w^1, w^2\right) u^j,$$

where g^j is a linearly homogeneous price index, which we will refer to as the price of the domestic good j in what follows.

We are now in a position to formulate the maximization programme associated with the first stage of the allocation process. This is achieved by substituting particular cardinalizations for the adult members' indirect utility functions in (7). The optimal choice of $\left(\rho^1, \rho^2, u^k, u^p\right)$ is a solution to the following maximization programme:

$$\max_{\rho^1,\rho^2,u^k,u^p} \lambda \widehat{v}^1\left(w^1, \rho^1, u^k, u^p\right) + (1-\lambda)\,\widehat{v}^2\left(w^2, \rho^2, u^k, u^p\right) \tag{9}$$

subject to $\rho^1 + \rho^2 + g^k\left(w^1, w^2\right) u^k + g^p\left(w^1, w^2\right) u^p = y$.

An interior solution to this programme satisfies the following first-order conditions (with $\mathcal{L}$ the Lagrangian function and δ the Lagrange multiplier associated with the budget constraint):

$$\frac{\partial \mathcal{L}}{\partial \rho^1} = \lambda \frac{\partial \widehat{v}^1}{\partial \rho^1} - \delta = 0$$

$$\frac{\partial \mathcal{L}}{\partial \rho^2} = (1-\lambda)\,\frac{\partial \widehat{v}^2}{\partial \rho^2} - \delta = 0$$

$$\frac{\partial \mathcal{L}}{\partial u^k} = \lambda \frac{\partial \widehat{v}^1\left(w^1, \rho^1, u^k, u^p\right)}{\partial u^k} + (1-\lambda)\,\frac{\partial \widehat{v}^2\left(w^2, \rho^2, u^k, u^p\right)}{\partial u^k} - \delta g^k\left(w^1, w^2\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial u^p} = \lambda \frac{\partial \widehat{v}^1\left(w^1, \rho^1, u^k, u^p\right)}{\partial u^p} + (1-\lambda)\,\frac{\partial \widehat{v}^2\left(w^2, \rho^2, u^k, u^p\right)}{\partial u^p} - \delta g^p\left(w^1, w^2\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \delta} = y - \rho^1 - \rho^2 - g^k\left(w^1, w^2\right) u^k - g^p\left(w^1, w^2\right) u^p = 0.$$

A few interesting relationships emerge when rewriting these first-order conditions. Firstly, we have

$$\lambda = \frac{\frac{\partial \widehat{v}^2\left(w^1,\rho^1,u^k,u^p\right)}{\partial \rho^2}}{\frac{\partial \widehat{v}^1\left(w^1,\rho^1,u^k,u^p\right)}{\partial \rho^1} + \frac{\partial \widehat{v}^2\left(w^2,\rho^2,u^k,u^p\right)}{\partial \rho^2}},$$

which shows that, if f^1 and f^2 are identified, one can recover the Pareto weight. Secondly, we obtain $(j = k, p)$:

$$\frac{\frac{\partial \widehat{v}^1\left(w^1,\rho^1,u^k,u^p\right)}{\partial u^j}}{\frac{\partial \widehat{v}^1\left(w^1,\rho^1,u^k,u^p\right)}{\partial \rho^1}} + \frac{\frac{\partial \widehat{v}^2\left(w^2,\rho^2,u^k,u^p\right)}{\partial u^j}}{\frac{\partial \widehat{v}^2\left(w^2,\rho^2,u^k,u^p\right)}{\partial \rho^2}} = g^j\left(w^1, w^2\right), \tag{10}$$

which are standard Bowen-Lindahl-Samuelson conditions for the optimal provision of public goods inside the household. The left-hand side of the above equation is the sum (over the two adult members) of the marginal rates of substitution between the domestic good j and the private good, while the right-hand side gives the price ratio for the two goods (with the price of the private good normalized to one).

Let us then consider what we can identify in the first stage. As a first step, we compute the partials of $\widehat{v}^i$ $(i = 1, 2; j = k, p)$ via equation (7), which gives (with obvious notation):

$$\begin{aligned} \frac{\partial \widehat{v}^i}{\partial \rho^i} &= \frac{\partial f^i}{\partial v^i}\frac{\partial v^i}{\partial \rho^i} \\ \frac{\partial \widehat{v}^i}{\partial u^j} &= \frac{\partial f^i}{\partial v^i}\frac{\partial v^i}{\partial u^j} + \frac{\partial f^i}{\partial u^j}. \end{aligned}$$

Substituting these partials in equation (10) obtains $(j = k, p)$:

$$\frac{1}{\frac{\partial v^1}{\partial \rho^1}}\frac{\frac{\partial f^1}{\partial u^j}}{\frac{\partial f^1}{\partial v^1}} + \frac{1}{\frac{\partial v^2}{\partial \rho^2}}\frac{\frac{\partial f^2}{\partial u^j}}{\frac{\partial f^2}{\partial v^2}} = g^j\left(w^1, w^2\right) - \left(\frac{\frac{\partial v^1}{\partial u^j}}{\frac{\partial v^1}{\partial \rho^1}} + \frac{\frac{\partial v^2}{\partial u^j}}{\frac{\partial v^2}{\partial \rho^2}}\right), \tag{11}$$

where the functions v^i $(i = 1, 2)$ and the price indices $g^j\left(w^1, w^2\right)$ are known given the identification results that we discussed above. Because we can identify utilities only up to a strictly increasing transformation, it directly follows that at best we can identify only the ratios $\frac{\partial f^i}{\partial u^j}/\frac{\partial f^i}{\partial v^i}$ $(i = 1, 2; j = k, p)$. Let $\Theta_j^i\left(v^i, u^j\right) = \frac{\partial f^i}{\partial u^j}/\frac{\partial f^i}{\partial v^i}$, which allows us to rewrite equation (11) $(j = k, p)$ as follows:

$$\frac{1}{\frac{\partial v^1}{\partial \rho^1}}\Theta_j^1\left(v^1, u^j\right) + \frac{1}{\frac{\partial v^2}{\partial \rho^2}}\Theta_j^2\left(v^2, u^j\right) = g^j\left(w^1, w^2\right) - \left(\frac{\frac{\partial v^1}{\partial u^j}}{\frac{\partial v^1}{\partial \rho^1}} + \frac{\frac{\partial v^2}{\partial u^j}}{\frac{\partial v^2}{\partial \rho^2}}\right). \tag{12}$$

Following a similar argument as BCM, we will now show that a solution to (12) is unique in a generic sense. Specifically, the model is identified unless the structural components of our model satisfy two partial differential equations that can be explicitly characterized.

To obtain these differential equations, we assume two different solutions $(\Theta_k^1\left(v^1, u^k\right),$ $\Theta_p^1\left(v^1, u^p\right), \Theta_k^2\left(v^2, u^k\right), \Theta_p^2\left(v^2, u^p\right))$ and $(\Theta_k^{1'}(v^1, u^k), \Theta_p^{1'}(v^1, u^p), \Theta_k^{2'}(v^2, u^k), \Theta_p^{2'}(v^2,$ $u^p))$ for (12). By construction, the differences $\psi_j^i\left(v^i, u^j\right) = \Theta_j^i\left(v^i, u^j\right) - \Theta_j^{i'}\left(v^i, u^j\right)$ $(i = 1, 2;\ j = k, p)$ must satisfy $(j = k, p)$:

$$\frac{1}{\frac{\partial v^1}{\partial \rho^1}}\psi_j^1\left(v^1, u^j\right) + \frac{1}{\frac{\partial v^2}{\partial \rho^2}}\psi_j^2\left(v^2, u^j\right) = 0.$$

Clearly, if $\psi_j^i\left(v^i, u^j\right) \neq 0$ then $\psi_j^{i'}\left(v^{i'}, u^j\right) \neq 0$ for $i \neq i'$. Using this, we can derive $(j = k, p)$:

$$\ln\left(\left|\psi_j^1\left(v^1, u^j\right)\right|\right) - \ln\left(\left|\psi_j^2\left(v^2, u^j\right)\right|\right) = \ln\left(\frac{\frac{\partial v^1}{\partial \rho^1}}{\frac{\partial v^2}{\partial \rho^2}}\right).$$

For general functions v^1, v^2 and ρ, these equality conditions will almost never be satisfied.[3] As a result, $\psi_j^i(v^i, u^j) = 0$ $(i = 1, 2; j = k, p)$ almost everywhere, implying that a solution $\Theta_k^1(v^1, u^k), \Theta_p^1(v^1, u^p), \Theta_k^2(v^2, u^k), \Theta_p^2(v^2, u^p)$ is generically unique. We can thus conclude that observing $(l^i,\, c^i,\, h_k^i,\, h_p^i,\, c^k,\, c^p)$ as functions of $(w^1, w^2, y, \mathbf{z}, \mathbf{s})$ generically identifies the individual preferences and the Pareto weights.

Appendix 2: Data

A. Time use categories

(1) paid work (excluding time spent on commuting); (2) commuting (for work or school); (3) domestic work (cleaning, dish washing, cooking, shopping, gardening, do-it-yourself, etc., but no tasks related to caring for children or other persons); (4) personal care (washing, dressing, eating, visits to the hairdresser and doctor, etc.); (5) activities with children (washing, dressing, playing, reading, visiting the doctor, etc.); (6) helping parents (administrative tasks, washing, dressing, visiting the doctor, etc.); (7) helping other family members (administrative tasks, washing, dressing, visiting the doctor, etc.); (8) helping other persons who are not family members (administrative tasks, washing, dressing, visiting the doctor, etc.); (9) leisure activities (watching TV, reading, sports, hobbies, visiting friends or family, travelling, going out, etc.); (10) schooling (day or evening education, vocational training, language training, etc.); (11) administrative tasks related to own household; (12) sleeping and relaxing (sleeping, thinking, meditating, etc.); (13) other activities not mentioned above.

B. Public expenditure categories

The public goods categories in the data are the following: (1) expenditures on mortgages (rent and payment); (2) rent without expenditures on utilities; (3) utilities (heating, electricity, water, telephone, internet, etc. but not insurances); (4) transportation costs (public transport, gasoline, etc., but no insurances or purchase of transportation means); (5) insurances (house, car, health, etc.); (6) child care (kindergarten, after school care, guest parent, home work supervision, etc.); (7) alimony and financial support for children not (or no longer) living at home; (8) expenditures to service debt (but no mortgages); (9) trips and holidays with (part of) the family (airplane tickets, hotel, restaurant, etc.); (10) expenditures related to cleaning the

[3] A specific case that meets these partial differential equations involves a structural consumption model where the Pareto weights do not depend on wages or the household's nonlabor income. It is well known that such a model implies that the household behaves as if it were a single decision maker, which makes identification of the individual preferences impossible. We refer to Chiappori and Ekeland (2009) for a detailed discussion on generic identification results like the one we obtain here.

house or gardening; (11) food and drinks consumed at home; and (12) other public expenditures not mentioned above. Recall that a follow-up question with respect to food and drinks consumed at home was added where all respondents had to indicate how much of these expenditures they personally consumed.

C. Private expenditure categories

The private goods categories in the data are the following: (1) food and drinks outside the home (restaurant, bar, company restaurant etc., but no expenditures consumed with (part of) the family; (2) cigarettes and other tobacco products; (3) clothing (clothing, shoes, jewelry, etc.); (4) personal care and services (hair care, body care, manicure, hair dresser etc., but no medical expenditures); (5) medical expenditures not covered by an insurance (medicines, doctor, dentist, hospital, maternity grant, spectacles, hearing device, etc.); (6) leisure activities (film, theater, hobbies, sports, photography, books, CDs, DVDs, expenditures related to traveling without the family, etc.); (7) schooling (courses, tuition fees, etc.); (8) gifts (to family members, friends, charity, etc.); and (9) other private expenditures not mentioned above.

Appendix 3: A basic collective model

In this appendix, we compare the estimation results for the 'main model' discussed in the main text to the ones for a more 'basic model' that does not account for domestic goods that are produced within the household. Specifically, we consider a model that was originally considered by Chiappori, Fortin and Lacroix (2002). This comparison should provide insight into the extent to which using such a basic model specification can affect the estimation results (and corresponding welfare analyses) for households where children are present.

A. Collective model without domestic goods

The basic collective model assumes that individuals in a couple divide their time only between leisure and market work. Further, the couple's income is spent on a Hicksian composite good that is privately consumed by the spouses. Finally, leisure and private consumption are assumed not to have any external effects inside the household (i.e., individuals have egoistic preferences).

Bringing this model to our data involves two main issues. Firstly, to keep the specification as close as possible to the one of the model we discuss in our main text, we again use the profitable approach proposed in the main text, which exploits the two-stage allocation representation of the collective model. We assume that the adult members' preferences regarding the second stage's leisure and own consumption allocation can be represented by the following indirect utility functions ($i = 1, 2$):

$$v^i\left(w^i,\rho^i\right)=\frac{\ln\left(w^i+\rho^i\right)-\ln a^i\left(w^i\right)}{\left(w^i\right)^{\beta^i}}, \tag{13}$$

where $\ln a^i\left(w^i\right)=\left(\alpha_1^i\left(\mathbf{d}^i\right)\right)\ln w^i$, with $\mathbf{d}^i$ a vector of individual taste shifters. Clearly, these preferences no longer depend on the level of the public goods. Roy's identity implies the following Marshallian demand for leisure and own consumption:

$$\begin{aligned}
l^i &= \left[\alpha_1^i\left(\mathbf{d}^i\right)+\beta^i\ln\left(\frac{w^i+\rho^i}{a^i\left(w^i\right)}\right)\right]\frac{\left(w^i+\rho^i\right)}{w^i}\\
c^i &= \left[1-\alpha_1^i\left(\mathbf{d}^i\right)-\beta^i\ln\left(\frac{w^i+\rho^i}{a^i\left(w^i\right)}\right)\right]\left(w^i+\rho^i\right).
\end{aligned}$$

Given the specification of the individual indirect utility functions, the first-stage maximization programme equals:

$$\begin{aligned}
&\max_{\rho^1,\rho^2}\lambda\left(w^1,w^2,y,\mathbf{z}\right)\left(\frac{\ln\left(w^1+\rho^1\right)-\ln a^1\left(w^1\right)}{\left(w^1\right)^{\beta^1}}\right)\\
&+\left(1-\lambda\left(w^1,w^2,y,\mathbf{z}\right)\right)\left(\frac{\ln\left(w^2+\rho^2\right)-\ln a^2\left(w^2\right)}{\left(w^2\right)^{\beta^2}}\right)
\end{aligned}$$

subject to

$$\rho^1+\rho^2=y.$$

Solving the first-stage maximization problem gives the following system of equations:

$$\begin{aligned}
\rho^1 &= \frac{w^1+w^2+y}{X\left(w^1,w^2,\lambda\right)}\frac{\lambda}{\left(w^1\right)^{\beta^1}}-w^1\\
\rho^2 &= \frac{w^1+w^2+y}{X\left(w^1,w^2,\lambda\right)}\frac{\left(1-\lambda\right)}{\left(w^2\right)^{\beta^2}}-w^2,
\end{aligned}$$

where $X\left(w^1,w^2,\lambda\right)=\frac{\lambda}{\left(w^1\right)^{\beta^1}}+\frac{(1-\lambda)}{\left(w^2\right)^{\beta^2}}$. We obtain the individuals' leisure and private consumption as functions of the exogenous variables by substituting the first stage functions in the second stage functions ($i=1,2$):

$$\begin{aligned}
l^i &= \left[\alpha_1^i\left(\mathbf{d}^i\right)+\beta^i\left(\ln\left(\frac{w^1+w^2+y}{X\left(w^1,w^2,\lambda\right)}\frac{\lambda^i}{\left(w^i\right)^{\beta^i}}\right)-A^i\ln w^i\right)\right]\times\frac{\left(\frac{w^1+w^2+y}{X\left(w^1,w^2,\lambda\right)}\frac{\lambda^i}{\left(w^i\right)^{\beta^i}}\right)}{w^i}\\
c^i &= \left[1-\alpha_1^i\left(\mathbf{d}^i\right)-\beta^i\left(\ln\left(\frac{w^1+w^2+y}{X\left(w^1,w^2,\lambda\right)}\frac{\lambda^i}{\left(w^i\right)^{\beta^i}}\right)-A^i\ln w^i\right)\right]\times\left(\frac{w^1+w^2+y}{X\left(w^1,w^2,\lambda\right)}\frac{\lambda^i}{\left(w^i\right)^{\beta^i}}\right),
\end{aligned}$$

where

$$\begin{aligned}
\lambda &= \lambda\left(w^1, w^2, y, \mathbf{z}\right) \\
&= \frac{\exp\left(\Lambda_1 + \Lambda_2 \frac{w^1}{w^2} + \Lambda_3 y + \Lambda_4' \mathbf{z}\right)}{1 + \exp\left(\Lambda_1 + \Lambda_2 \frac{w^1}{w^2} + \Lambda_3 y + \Lambda_4' \mathbf{z}\right)} \\
\lambda^1 &= \lambda\left(w^1, w^2, y, \mathbf{z}\right) \\
\lambda^2 &= 1 - \lambda\left(w^1, w^2, y, \mathbf{z}\right).
\end{aligned}$$

To account of the same explanatory variables as in our main model, we assume that $\alpha_1^i\left(\mathbf{d}^i\right) = \alpha_{10}^i + \alpha_{11}^i age^i + \alpha_{12}^i kids + \alpha_{13}^i meanagekids$. Observe that the system brought to the data now consists of four equations (instead of ten equations, as for our model in the main text).

The second issue concerns the definition of the individuals' leisure. In standard labor supply models such as the basic model under study, all time not spent on market labor is treated as leisure (possibly after subtracting some time needed for sleep and personal care). We follow the same route here and, thus, we add all time spent on home work to the (pure) leisure time we used in our main model.

B. Comparison of estimation results

Table 1 gives the estimation results for the basic model under consideration. Contrary to the results for our main model, leisure is now identified as a necessity for both husbands and wives. This change in the nature of leisure is directly related to the fact that leisure now also contains time spent on home work in addition to pure leisure. In our opinion, this remarkable result once more demonstrates that focusing on a simple dichotomization of time into leisure and market work can substantially bias the estimation results and, thus, also the associated welfare analyses. Next, we again find that an increase in the husband's relative wage has a positive impact on his Pareto weight, ceteris paribus.

As a final comparison with our main model, we also calculated labor supply elasticities defined at the sample median for the basic model under study. These elasticities are given in Table 2. The male own wage elasticity is positive and not too different from the one that we obtained for our main model. By contrast, the wife's own wage elasticity now turns out to be negative. We obtain a similar sign reversal for the male cross-wage elasticity, while the female cross-wage elasticity is fairly similar for the two models. Finally, the husband's nonlabor income elasticity is small and positive, while the wife's is small and negative. Similar to before, we can conclude from Table 2 that not accounting for public consumption and production for households with children may considerably impact on the conclusions that are drawn from the empirical analysis.

Table 1: Estimation results standard collective labor supply model

	Estimate	Std. error
Preference parameters		
α_{10}^1	0.899*	0.036
$\alpha_{11}^1[age^1/10]$	0.008	0.007
$\alpha_{12}^1[kids]$	-0.001	0.006
$\alpha_{13}^1[meanagekids]$	0.007	0.013
β^1	-0.054	0.036
α_{10}^2	0.900*	0.031
$\alpha_{11}^2[age^2/10]$	0.007	0.006
$\alpha_{12}^2[kids]$	0.001	0.005
$\alpha_{13}^2[meanagekids]$	0.014	0.011
β^2	-0.084*	0.043
Pareto weight parameters		
Λ_1	-0.841*	0.138
Λ_2 $[w^1/w^2]$	0.610*	0.037
Λ_3 $[y]$	-0.027	0.031
Λ_4 [husband's share in nonlabor income]	0.000	0.000
Λ_5 $[age^1/10 - age^2/10]$	-0.212	0.149

Note: Coefficient estimates were obtained by the feasible generalized nonlinear least squares estimator. An asterisk denotes significance at the 5 percent significance level. The expressions in brackets refer to the objects that are related to the respective parameters.

Table 2: Labor supply elasticities standard collective labor supply model

	Husband	Wife
Own wage elasticity	0.30	-2.08
Partner's wage elasticity	-0.24	2.21
Nonlabor income elasticity	0.01	-0.04

Note: Elasticities were calculated numerically for the sample median.