

Online Appendix: Inventories, Lumpy Trade, and Large Devaluations

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This appendix presents additional empirical, theoretical, and quantitative results. First, we show that lumpiness in the trade data is not driven by seasonalities and is pervasive across many goods. Next, we present a simplified model that yields closed form results for pricing and pass-through. Third, we show that the relatively high short-run response of trade in devaluations is even sharper in US exports to these countries. Finally, we report the sensitivity of our results to an alternative calibration with low markups and a different structure of fixed transactions costs.

I Lumpiness of International Transactions

The high level of concentration in the import data does not appear to be driven by seasonalities, as Table 4B shows. The top half of the table reproduces the HH index and fraction of trade numbers from Table 4, where the fractions are the fraction of trade in a given year. The numbers in the bottom half reproduce the analogous numbers for the fraction of trade in a given month (e.g., December) across years in the data. For these numbers, trade is normalized by annual trade to prevent concentrations from developing by secular changes in trade.¹ The numbers show that, except for Mexico, there is even more concentration within a given month, but across years. The numbers are not strictly comparable, however, since the bottom row shows that there are fewer years when

¹Shares for month i in year j are defined as follows:

$$\tilde{v}_{i,j} = \text{value}_{i,j} / \left(\sum_{i=1}^{12} \text{value}_{i,j} \right) \quad \tilde{s}_{i,j} = \tilde{v}_{i,j} / \left(\sum_{j=1990}^{2004} \tilde{v}_{i,j} \right)$$

and the HH index is computed: $\widetilde{HH}_i = \sum_{j=1990}^{2004} \tilde{s}_{i,j}^2$

a good is imported than months in a year. Nevertheless, the HH numbers greatly exceed $1/(\text{total number of years traded})$, so there is still a great deal of concentration. Hence, lumpiness does not appear to be a result of seasonalities in which goods are traded only in certain months every year, but consistently each year.

Table 4C presents lumpiness statistics by end-use categories (for Argentina). It shows that lumpiness is also not driven by one particular type of good but is pervasive across different types of goods. There is some variation, with food being the most lumpy ($HH = 0.53$) and automobiles and automotive parts being the least lumpy ($HH = 0.35$), but even these numbers are similar to the overall number ($HH = 0.40$). The fraction of trade accounted for by the top one, three, and five months is also similar across end-use categories.

II Simplified Model with Analytical Solution

We show analytically² that a model with shipping lags and inventory generates incomplete pass-through following shocks to the cost of inputs. We simplify the problem by assuming that there is no fixed cost, $f = 0$ and no idiosyncratic demand shock, $\nu = 0$. Below, we substitute in $p = q^{-\frac{1}{\theta}}$, and write the firm's problem recursively.

$$\begin{aligned}
 V(s; \omega) &= \max_{q, i} \left(q^{\frac{\theta-1}{\theta}} - \omega i \right) + \beta V((1-\delta)(s-q+i)) \\
 st \quad &: \\
 q &\leq s \\
 0 &\leq i
 \end{aligned}$$

Reformulating the problem with multipliers, the first order conditions are:

$$\begin{aligned}
 V(s; \omega) &= \max_{q, i} \left(q^{\frac{\theta-1}{\theta}} - \omega i \right) + \beta V((1-\delta)(s-q+i)) + \lambda [s-q] + \mu [i] \\
 q &: \frac{\theta-1}{\theta} q^{-\frac{1}{\theta}} = \beta (1-\delta) \frac{dV}{ds}(s') + \lambda \\
 i &: \omega - \mu = \beta (1-\delta) \frac{dV}{ds}(s') \\
 s &: \frac{dV}{ds}(s) = \beta (1-\delta) \frac{dV}{ds}(s') + \lambda
 \end{aligned}$$

²These notes build on an analytical exercise suggested by Ariel Burstein in a discussion of our paper.

A. Steady State

We start by characterizing the steady state. Let s^* be the steady state value of s . In the steady state:

$$\begin{aligned}\lambda^* &= [1 - \beta(1 - \delta)] \frac{dV}{ds}(s^*), \\ \frac{dV}{ds}(s^*) &= \frac{\theta - 1}{\theta} q^{*-1/\theta}, \\ \omega - \mu^* &= \beta(1 - \delta) \frac{dV}{ds}(s^*).\end{aligned}$$

It is straightforward to prove that at $\lambda > 0$ and $s^* = q^*$ by assuming the contrary. If $\lambda^* = 0$, then $\frac{dV}{ds}(s^*) = 0$, and q^* is infinite but $\mu^* = \omega > 0$, which would then imply a contradiction that $i^* = 0$. Thus, in the steady:

$$s^* = q^* = i^*/(1 - \delta).$$

Substituting in and combining first-order conditions, we can solve for the steady state sales and price as:

$$\begin{aligned}s^* &= \left(\frac{\theta}{\theta - 1} \omega \right)^{-\theta} [\beta(1 - \delta)]^\theta \\ p^* &= \left(\frac{\theta}{\theta - 1} \frac{\omega}{\beta(1 - \delta)} \right)\end{aligned}$$

The price equation shows that steady state prices are a gross markup $\frac{\theta}{\theta - 1}$ over marginal cost $\frac{\omega}{\beta(1 - \delta)}$, and marginal cost is increasing in the replacement price of goods, depreciation, and the time cost of goods ($1/\beta$). Note that the latter two will be larger, the longer the shipping delay/period is. Similarly, steady state inventories are decreasing in the markup and marginal cost.

B. A General Solution

Given this general problem, we claim that the pricing function is

$$p = \begin{cases} s^{-1/\theta} & s \leq s_{a,1} \\ \frac{\theta}{\theta - 1} \beta^{n-1} (1 - \delta)^{n-1} \omega & s \in [s_{a,n}, s_{b,n}] \quad \text{for } n > 1 \\ \beta^n (1 - \delta)^n \left[\sum_{j=0}^n \frac{[\beta^j (1 - \delta)^j]^{-\theta}}{(1 - \delta)^{n-j}} \right]^{\frac{1}{\theta}} s^{-1/\theta} & \text{for } s \in [s_{b,n}, s_{a,n+1}] \end{cases}$$

where

$$s_{a,n} = \left[\frac{\theta}{\theta-1} \omega \right]^{-\theta} \left[\sum_{j=0}^n \frac{[\beta^j (1-\delta)^j]^{-\theta}}{(1-\delta)^{n-j}} \right]$$

$$s_{b,n} = \left[\frac{\theta}{\theta-1} \omega \right]^{-\theta} \left\{ \sum_{j=0}^n \frac{(\beta(1-\delta))^{(j-1)\theta}}{(1-\delta)^{n-j}} \right\}$$

Here n indicates the number of periods in which inventories will reach steady state. That is, the $s < s_{a,1}$ region indicates a stockout region, and inventories will be at their steady state in the next period (the steady state itself s^* is contained in this region.) In the $(s_{b,n}, s_{a,n+1})$ regions, inventories will eventually be stocked out in $n-1$ periods, while in the $(s_{a,n}, s_{b,n})$ regions, positive inventories will be carried forward from $n-1$ to n . In both cases, new shipments will first be ordered in period $n-1$ and arrive in period n .

C. Comparative Statics and Transition Dynamics

To examine the solution of this problem, it is useful to rewrite prices and inventory relative to their steady state level: i.e., define $\hat{s} = s/s^* = s \left(\frac{\theta}{\theta-1} \omega \right)^\theta [\beta(1-\delta)]^{-\theta}$ and $\hat{p} = p/p^* = p \left(\frac{\theta}{\theta-1} \frac{\omega}{\beta(1-\delta)} \right)^{-1}$, which lead to pricing rules of the following form

$$\hat{p} = \begin{cases} \hat{s}^{-\frac{1}{\theta}} & \hat{s} \leq \hat{s}_{a,1} \\ [\beta(1-\delta)]^n & \hat{s} \in [\hat{s}_{a,n}, \hat{s}_{b,n}] \quad \text{for } n > 1 \\ [\beta(1-\delta)]^n \left[\sum_{j=0}^n \frac{[\beta(1-\delta)]^{-j\theta}}{(1-\delta)^{n-j}} \right]^{\frac{1}{\theta}} \hat{s}^{-\frac{1}{\theta}} & \hat{s} \in [\hat{s}_{b,n}, \hat{s}_{a,n+1}] \end{cases}$$

where

$$\hat{s}_{a,n} = \left[\sum_{j=0}^n \frac{[\beta(1-\delta)]^{-(j+1)\theta}}{(1-\delta)^{n-j}} \right]$$

$$\hat{s}_{b,n} = \left\{ \sum_{j=0}^n \frac{[\beta(1-\delta)]^{(j-2)\theta}}{(1-\delta)^{n-j}} \right\}$$

By normalizing the price and inventory by their steady state levels, we can easily trace out the impact of a permanent change to the marginal cost of inputs by transforming the current inventory level into the current inventory steady state sales ratio (\hat{s}) at the new marginal cost ω . Given the pricing policy and the law of motion of inventory holdings

$$\hat{s}' = (1-\delta) \left(\hat{s} - [\hat{p}(\hat{s})]^{-\theta} \right)$$

one can solve for the path of prices. Along the transition, prices will increase gradually as the stock of inventory converges to the steady state level.

Consider the effect of a permanent increase in ω on \hat{p} and \hat{s} . An increase in ω will increase the steady state price by the same percentage (complete pass-through), and the sales level will fall with an elasticity of θ . In the transition to the new steady state, prices will be below the steady state level and sales will exceed the steady state level. To trace out the transition, we normalize the current inventory holdings by the lower steady state sales rate, $\hat{s} = s/s^* = s \left(\frac{\theta}{\theta-1} \omega \right)^\theta [\beta(1-\delta)]^{-\theta}$. Assuming the model is in steady state prior to the shock, given a higher ω , the inventory-sales ratio will be greater than one and fall into one of the regions described above. If the increase in ω is not too large so that the inventory-sales ratio rises but remains $\hat{s} \leq \hat{s}_{a,1}$, then the firm will raise its price proportionally by less than the increase in costs and there is incomplete pass-through for one period. Pass-through will be smaller the larger is \hat{s} . If the increase in ω is larger, so that $\hat{s} \in (\hat{s}_{a,1}, \hat{s}_{b,1})$, then the firm will raise its price but lower its markup to $\beta(1-\delta)$ in the first period. In the next period, the firm will raise its price and pass-through will be complete. For cost increases that push $\hat{s} > \hat{s}_{b,1}$, the normalized price is weakly decreasing in \hat{s} and so there is even less pass-through initially. These inventory levels will require more than one period to converge to the new steady state and so the price charged in the first period will depend on the entire path of prices, particularly whether in the period prior to convergence, the inventory holdings are in the stockout region. For $\hat{s} \in (\hat{s}_{b,n}, \hat{s}_{a,n+1})$, the firm's inventory holdings will eventually be in the stockout region while for $\hat{s} \in (\hat{s}_{a,n}, \hat{s}_{b,n})$, the firm will not have an inventory level that is in the stockout region.

We can also consider the effect of a permanent decrease in β (capturing an increase in interest rates, for example). An increase in δ has a similar effect. Similar to an increase in ω , an increase in β increases the cost of selling goods, leading to a rise in the long-run price and a lowering of steady state sales. Since we have included the interest costs in the firms' cost, this implies no long-run change in markups and complete pass-through.³ The decrease in β has a similar effect on \hat{s} as an increase in ω , and indeed for $\hat{s} < \hat{s}_{a,1}$, the effect is identical. However, by changing the carrying costs of inventories, a decline in β alters the incentive to carry inventory across periods. Thus, for a decrease in β that leads $\hat{s} > \hat{s}_{a,1}$, markups will be lower. In particular, following a change in β that leads the firm to not order for n periods pass-through will be lower initially than for a change in ω that leads a firm not to order for n periods (assuming both shocks lead to the same \hat{s} and that in the period prior to converging to the steady state the firm places an order). Additionally,

³If one were to treat β as part of the markup, then the level of markups would vary with β .

a decrease in β also increases the cutoffs for each region, since it is now more costly for the firm to delay selling its products and so a firm will generally converge to the steady state faster following a change in β than a change in ω . Finally, one can see that a decrease in β and increase in ω that we model compound each other because they are effectively multiplicative.

Given these policy rules, one can also calculate the change in markups. For instance, consider a shock such that it takes five periods to converge to the steady state, and further suppose without loss of generality that the shock puts the firm on the flat portion of the pricing function. In this case, by taking the log of the normalized pricing function, the firm will lower its markup by

$$\Delta\mu \approx -n(r + \delta)$$

where we have used $r \approx -\ln \beta$, $\delta \approx -\ln(1 - \delta)$ for values close to one. Using the calibrated values from the model, such a shock would generate a cut in the markup of approximately 15 percent initially. Then over the next five months, the firm would increase its markup by 3.0 percent per period. Holding the time it takes to converge constant, it is clear that the decline in markups will be initially larger if the depreciation rate or discount rate ($1/\beta$) is larger, but that markups will converge faster.

D. Solving for the General Pricing Formulas and Cutoffs

We start by solving for the cutoffs for $s_{a,n}$ and $s_{b,n}$. A couple of things should be noticed. First, in general, if the firm starts out with $s \in (s_{b,n}, s_{a,n+1})$, then in the penultimate period before converging ($n - 1$), the firm will come into the period with a stock, call it $s_{-1} \in (s^*, s_{a,1})$, and so the price will be determined strictly by $p_{-1} = (s_{-1})^{-\frac{1}{\theta}}$. We can use this price to solve for the price along the transition. Second, if the firm starts out with $s \in (s_{a,n}, s_{b,n})$, then in the penultimate period it will charge $p_{-1} = \frac{\theta}{\theta-1}\omega$. Now, we can determine $s_{a,n}$ and $s_{b,n}$ as the minimum and maximum stocks such that the firm optimally charges $p_{-1} = \frac{\theta}{\theta-1}\omega$.

Solving for $s_{b,n}$

Suppose that the firm has s such that it will not order for two periods, $i = 0, i' = 0$. In this case, the current stock has to last for three periods,

$$s_{b,2} = \frac{q''}{(1 - \delta)^2} + \frac{q'}{(1 - \delta)} + q.$$

Now, in the penultimate period, the firm will charge $p' = \frac{\theta}{\theta-1}\omega$ and in the last period, it will charge $p'' = \frac{\theta}{\theta-1}\frac{1}{\beta(1-\delta)}\omega$. Using the Euler Equation, $p = \beta(1-\delta)p'$. Substituting this in

$$\begin{aligned} s_{b,2} &= \frac{\left[\frac{\theta}{\theta-1}\frac{1}{\beta(1-\delta)}\omega\right]^{-\theta}}{(1-\delta)^2} + \frac{\left[\frac{\theta}{\theta-1}\omega\right]^{-\theta}}{(1-\delta)} + \left[\frac{\theta}{\theta-1}\beta(1-\delta)\omega\right]^{-\theta}, \\ &= \left[\frac{\theta}{\theta-1}\omega\right]^{-\theta} \left\{ \frac{\left[\frac{1}{\beta(1-\delta)}\right]^{-\theta}}{(1-\delta)^2} + \frac{[1]^{-\theta}}{(1-\delta)} + [\beta(1-\delta)]^{-\theta} \right\}, \\ &= \left[\frac{\theta}{\theta-1}\omega\right]^{-\theta} \left\{ \frac{[\beta(1-\delta)]^\theta}{(1-\delta)^2} + \frac{[1]^{-\theta}}{(1-\delta)} + [\beta(1-\delta)]^{-\theta} \right\}. \end{aligned}$$

Following the pattern yields the general rule, when it takes n periods to converge to the new steady state,

$$s_{b,n} = \left[\frac{\theta}{\theta-1}\omega\right]^{-\theta} \left\{ \sum_{j=0}^n \frac{(\beta(1-\delta))^{(j-1)\theta}}{(1-\delta)^{n-j}} \right\}.$$

Solving for $s_{a,n}$ Suppose the firm has slightly more than $s_{b,2}$, so that $i = 0$, and $p' = \frac{p}{\beta(1-\delta)}$ but $i' > 0$. This means that today and tomorrow, the firm will sell more than the unconstrained optimal, i.e., $\{q, q', q^*\}$, where $q > q' > q^*$. Since we know that $q' = \left[\frac{\theta}{\theta-1}\omega\right]^{-\theta}$ since $p' = \frac{\theta}{\theta-1}\omega$ and so $p = \beta(1-\delta)\frac{\theta}{\theta-1}\omega$.

What is the maximum s such that the firm is indifferent to ordering in the second period ($i = 0, i' = 0, q > q' > q^*$)? If in the second period, the firm sells only $s_{a,1}$, then

$$q + \frac{s_a}{(1-\delta)} = s$$

Recall that in period 2, the firm will charge a price of $\frac{\theta}{\theta-1}\omega$, and that its price in the first period has to be lower $\beta(1-\delta)p'$ so,

$$\begin{aligned} s_{a,2} &= q + \frac{s_a}{(1-\delta)} = [\beta(1-\delta)p']^{-\theta} + \frac{(p')^{-\theta}}{(1-\delta)} = p'^{-\theta} \left[[\beta(1-\delta)]^{-\theta} + \frac{1}{(1-\delta)} \right], \\ &= \left[\frac{\theta}{\theta-1}\omega\right]^{-\theta} \left[[\beta(1-\delta)]^{-\theta} + \frac{1}{(1-\delta)} \right]. \end{aligned}$$

Now let's suppose it takes three periods:

$$s_{a,3} = \frac{s_a}{(1-\delta)^2} + \frac{q'}{(1-\delta)} + q = \frac{p''^{-\theta}}{(1-\delta)^2} + \frac{(\beta(1-\delta)p'')^{-\theta}}{(1-\delta)} + (\beta^2(1-\delta)^2 p'')^{-\theta},$$

$$\begin{aligned}
&= p''^{-\theta} \left[\frac{1}{(1-\delta)^2} + \frac{(\beta(1-\delta))^{-\theta}}{(1-\delta)} + (\beta^2(1-\delta)^2)^{-\theta} \right], \\
&= \left[\frac{\theta}{\theta-1} \omega \right]^{-\theta} \left(\frac{1}{(1-\delta)^2} + \frac{\beta^{-\theta}(1-\delta)^{-\theta}}{1-\delta} + \beta^{-2\theta}(1-\delta)^{-2\theta} \right).
\end{aligned}$$

The general rule can be derived as:

$$s_{a,n} = \left[\frac{\theta}{\theta-1} \omega \right]^{-\theta} \left[\sum_{j=0}^n \frac{[\beta^j(1-\delta)^j]^{-\theta}}{(1-\delta)^{n-j}} \right].$$

Solving for p on a Path That Leads to a Stockout Given the cutoffs $s_{a,n}$ and $s_{b,n}$, what is the pricing rule if $s \in (s_{b,1}, s_{a,2})$? To find this, we need to figure out how much the firm will sell in the penultimate period.

Consider a firm with enough inventory for one period. The firm will charge $p = \beta(1-\delta)p'$ and will initially need

$$s = q + \frac{q'}{1-\delta} = p^{-\theta} + \frac{p'^{-\theta}}{1-\delta} = p'^{-\theta} \left\{ [\beta(1-\delta)]^{-\theta} + \frac{1}{1-\delta} \right\},$$

which can be rearranged

$$p' = \left(\frac{s}{\left\{ [\beta(1-\delta)]^{-\theta} + \frac{1}{1-\delta} \right\}} \right)^{-\frac{1}{\theta}}.$$

and so

$$p = \beta(1-\delta)p' = \left(\frac{\beta^{-\theta}(1-\delta)^{-\theta}}{[\beta(1-\delta)]^{-\theta} + \frac{1}{1-\delta}} \right)^{-\frac{1}{\theta}} s^{-\frac{1}{\theta}} = \left(\frac{1-\delta}{1-\delta + \beta^{-\theta}(1-\delta)^{-\theta}} \right)^{-\frac{1}{\theta}} s^{-\frac{1}{\theta}}.$$

Next, suppose it takes 3 periods to work off the inventory, i.e $s \in (s_{b,2}, s_{a,3})$

$$\begin{aligned}
s &= p''^{-\theta} \left[\frac{1}{(1-\delta)^2} + \frac{(\beta(1-\delta))^{-\theta}}{(1-\delta)} + (\beta^2(1-\delta)^2)^{-\theta} \right], \\
s &= p''^{-\theta} \left(\frac{1}{(1-\delta)^2} + \beta^{-\theta}(1-\delta)^{-\theta-1} + \beta^{-2\theta}(1-\delta)^{-2\theta} \right), \\
p'' &= s^{-\frac{1}{\theta}} \left(\frac{1}{(1-\delta)^2} + \beta^{-\theta}(1-\delta)^{-\theta-1} + \beta^{-2\theta}(1-\delta)^{-2\theta} \right)^{\frac{1}{\theta}}.
\end{aligned}$$

To convert this into the current period, note that $p = \beta^2(1-\delta)^2 p''$ so

$$p = s^{-\frac{1}{\theta}} \beta^2(1-\delta)^2 \left(\frac{1}{(1-\delta)^2} + \beta^{-\theta}(1-\delta)^{-\theta-1} + \beta^{-2\theta}(1-\delta)^{-2\theta} \right)^{\frac{1}{\theta}},$$

More generally, if it takes n periods, $s \in (s_{b,n-1}, s_{a,n})$

$$s = p_n^{-\theta} \left[\sum_{j=0}^n \frac{[\beta^j (1-\delta)^j]^{-\theta}}{(1-\delta)^{n-j}} \right],$$

where p_n is the n period ahead price. Recall that $\frac{p}{\beta^n(1-\delta)^n} = p_n$, then

$$\begin{aligned} s &= p_n^{-\theta} \left[\sum_{j=0}^n \frac{[\beta^j (1-\delta)^j]^{-\theta}}{(1-\delta)^{n-j}} \right], \\ p_n &= s^{-\frac{1}{\theta}} \left[\sum_{j=0}^n \frac{[\beta^j (1-\delta)^j]^{-\theta}}{(1-\delta)^{n-j}} \right]^{\frac{1}{\theta}}, \end{aligned}$$

and recall that $p = \beta^n (1-\delta)^n p_n$ then

$$p = \beta^n (1-\delta)^n \left[\sum_{j=0}^n \frac{[\beta^j (1-\delta)^j]^{-\theta}}{(1-\delta)^{n-j}} \right]^{\frac{1}{\theta}} s^{-\frac{1}{\theta}}.$$

III Price Elasticity Based on US Imports

We now report the elasticity of trade using just US exports to each developing country. The US data are ideal since they measures goods as they are exported from the US rather than when these goods arrive in the destination. We can also remove from the data shipments of aircrafts, which can have outsized effects on trade flows that are unrelated to relative price fluctuations. Figure 4b shows a similar pattern to Figure 4, with an even larger short-run response of trade flows in Argentina, Brazil, Mexico, and Russia.

IV Sensitivity

Here we explore the sensitivity of our results to the nature of fixed cost and the amount of competition among firms selling imported goods. Specifically, with regards to fixed costs, consistent with the absence of any scale effects in inventory holdings among Chilean plants, we follow Cooper and Haltiwanger (2006) and assume that this adjustment cost is an ‘‘opportunity cost,’’ that is, proportional to the firm’s revenue. The firm that imports loses a fraction, $(1-\lambda)$, of its revenue,

$p_j(\eta^t)q_j(\eta^t)$, where q is quantity sold by the firm.^{4,5} With regards to the nature of competition among importers, we now allow importers to compete more directly with one another by increasing the elasticity of substitution among imported varieties while maintaining the same substitutability between domestic and imported goods. In contrast to our previous calibration of the Armington elasticity of substitution, $\theta = 1.5$, which implied counterfactually high markups, we can generate more standard markups.

Rather than pay a fixed cost to order, we now require a firm to forego a fraction of its current revenue, $(1 - \lambda)p_j(\eta^t)q_j(\eta^t)$, whenever it chooses to import a positive quantity. The firm's problem can be concisely summarized by the following system of two functional Bellman equations. Let $V^a(s, \nu)$ denote the firm's value of adjusting its stock of inventory and $V^n(s, \nu)$ denote the value of inaction, as a function of its beginning-of-period stock of inventory and its demand shock. Let $V(s, \nu) = \max[V^a(s, \nu), V^n(s, \nu)]$ denote the firm's value. Then the firm's problem is:

$$(1) \quad \begin{aligned} V^a(s, \nu) &= \max_{p, i > 0} \lambda q(p, s, v)p - \omega i + \beta EV(s', \nu') \\ V^n(s, \nu) &= \max_p q(p, s, v)p + \beta EV(s', \nu') \end{aligned}$$

where

$$\begin{aligned} q(p, s, v) &= \min(e^v p^{-\theta}, s) \\ s' &= \begin{cases} (1 - \delta)[s - q(p, s, v) + i] & \text{if adjust} \\ (1 - \delta)[s - q(p, s, v)] & \text{if don't adjust} \end{cases} \end{aligned}$$

Modelling the cost of importing in this way implies slightly different policy rules than before. As before, for the same demand shock, firms with lower inventory holdings will be more likely to adjust. In contrast to before, now a firm with a high demand shock will have less incentive to import because it will forego more revenue. Similarly, a firm with a low demand shock will have more incentive to import, even if it has a relative high inventory. In terms of pricing, as before, when current inventory

⁴ Assuming a fixed cost that is independent of how much the firm sells, as in our benchmark calibration, implies that following an increase in the relative price of imports, ω , the costs of importing become relatively more important which tends to amplify the effect of the shock (by lowering trade volumes, the fraction of importing firms, and raising prices importers charge).

⁵ The assumption that fixed costs are proportional to measures of firm activity has often been used in earlier work, especially in environments in which shocks have permanent effects, since it is needed to ensure stationarity of decision rules. See, e.g., Danziger (1999) and Gertler and Leahy (2008).

holdings do not constrain current sales, the optimal price the firm charges is generally proportional to the firm's marginal valuation of an additional unit of inventories (which will, in this economy with inventory frictions, differ from the replacement cost ω). If the firm adjusts its inventory holding it charges

$$p = \frac{\theta}{\theta - 1} \frac{1}{\lambda} \beta(1 - \delta) EV_s(s', v'),$$

and if it does not, it charges

$$p = \frac{\theta}{\theta - 1} \beta(1 - \delta) EV_s(s', v').$$

In turn, the marginal value of inventories, V_s , decreases with the current stock of inventories. Ultimately, the value of the marginal unit of inventory is realized when the firm next adjusts inventory. At that time, it is either valued at ω , since it reduces needed inventory purchases, or it is sold in a stock-out situation, in which case it has a higher valuation. High inventory levels lower the probability that the marginal unit will be needed in a stock-out situation, and, in expectation, it shifts the next adjustment date into the future. Higher expected discounting and depreciation costs lower its expected value. Hence, both the marginal valuation and the price are falling in the stock of inventories.

To allow for more competition, and lower markups, we now assume that consumers have CES preferences over home, h , and imported, m , goods:

$$c = \left(h^{\frac{\theta-1}{\theta}} + \alpha m^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

where m is a composite good made up of a continuum of varieties of imports:

$$m = \left(\int_0^1 m_j^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}}$$

This choice of preferences allows us to maintain the empirically justified low Armington elasticity, by setting $\theta = 1.5$, but allows us to vary the markup that importers charge. In particular, we choose $\gamma = 4$, a number in the range of those estimated by David Hummels (1999), Michael Gallaway, Christine McDaniel and Sandra Rivera (2003), and Christian Broda and David Weinstein (2006), which corresponds to a frictionless markup of 33 percent. Given these preferences, consumers' demand for an importer's product is

$$m_j = \left(\frac{p_j}{P_m} \right)^{-\gamma} P_m^{-\theta}.$$

The column denoted low markup in Table 5B reports the result of a calibration in which the price elasticity is increased to $\theta = 4$. The high elasticity economy requires more volatile demand

shocks and larger (relative to median sales, but not relative to mean sales) fixed costs. With the high elasticity, the fixed cost per shipment is now 2.2 percent and the tariff equivalent of the frictions is approximately 15 percent. While the frictions are smaller than our benchmark case, the trade distortion is even larger because the price elasticity is higher. For instance, in this, calibration trade would be 60 percent lower than without these frictions while in the benchmark case, these frictions lower trade by about 30 percent.

In terms of dynamics, Figure 6B illustrates that the response of our low markup economy to a devaluation is very similar to that of our benchmark setup. When solving for the transition path to the new steady state, we require consistency of firm decision rules with the path for P_m to derive these decision rules.⁶

⁶This economy features strategic complementarities in firms' decision rules: the lower the prices charged by a firm's competitors, the lower a firm's sales, and thus the larger the inventory-holding costs. Thus, firms find it optimal to lower their prices. These complementarities turn out to be weak in the model, since the firm's problem is dynamic and current P_m have a smaller effect on the firm's decision rules than in a static economy.

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Table 4B: Concentrations Across Months (Within Years) vs. Across Years (Within Months)

	Argentina	Brazil	Korea	Mexico	Russia	Thailand
Within Year, Across Month						
Herfindahl-Hirschman index	0.40	0.37	0.28	0.21	0.45	0.35
fract. of ann. trade in top mo.	0.50	0.47	0.38	0.27	0.53	0.45
fract. of ann. trade in top 3 mos.	0.83	0.78	0.70	0.53	0.85	0.79
fract. of ann. trade in top 5 mos.	0.94	0.91	0.85	0.71	0.95	0.92
Across Year, Within Month						
Herfindahl-Hirschman index	0.50	0.44	0.34	0.17	0.75	0.46
fract. of trade in top mo.	0.60	0.54	0.45	0.26	0.80	0.56
fract. of trade in top 3 mos.	0.96	0.91	0.87	0.55	1.00	0.94
fract. of trade in top 5 mos.	1.00	1.00	0.99	0.76	1.00	1.00
median years traded	7	8	9	13	4	7

Table 4C: Lumpiness by End Use (Argentina)

Lumpiness Statistic	Overall	Food	Intermed. Goods	Capital Goods	Autos & Parts	Consumer Goods
fract. of mos. exported	0.27	0.14	0.33	0.23	0.42	0.33
fract. of mos. in year exported	0.47	0.33	0.48	0.37	0.65	0.51
Hirschman-Herfindahl index	0.40	0.51	0.38	0.51	0.31	0.39
fract. of ann. trade in top mo.	0.50	0.58	0.47	0.60	0.42	0.49
fract. of ann. trade in top 3 mos.	0.83	0.89	0.82	0.90	0.74	0.83
fract. of ann. trade in top 5 mos.	0.94	0.97	0.93	0.97	0.88	0.94
Fraction of Imports from U.S.	1.0	0.02	0.42	0.13	0.06	0.07

Notes: Lumpiness statistics reflect the country-specific, trade-weighted median good. Fractions of imports sum to only 0.70 across end uses shown. The remaining goods end uses were military (0.19), unclassified (0.10), and re-exports.

Table 5B: Moments and Parameters

Moments							
	Data	Benchmark	Domestic	No fixed cost	No lag	High deprec.	Low markup
used for calibration							
Herfindhal-Hirschmann ratio	0.44	0.44	0.23	0.14	0.32	0.33	0.44
inventory to annual purchases ratio	0.36	0.36	0.21	0.29	0.12	0.25	0.36
additional implications							
tariff equivalent of frictions	-	0.20	0.09	0.13	0.08	0.32	0.15
f (relative to mean shipment)		0.036	0.017	0	0.050	0.047	0.022
Parameters							
Calibrated							
f (fixed cost, rel. median revenue)		0.095	0.025	0	0.095	0.095	0.21
std. dev. of demand, σ		1.15	1.15	1.15	1.15	1.15	1.8
Assigned							
Period length		1 month	1/2 month	1 month	1 month	1 month	1 month
Shipping lag		1 month	1/2 month	1 month	0 months	1 month	1 month
Elasticity of demand for imports, θ		1.5	1.5	1.5	1.5	1.5	1.5
Elasticity of subs. across imported goods		-	-	-	-	-	4
Monthly discount factor, β		0.995	0.995	0.995	0.995	0.995	0.995
Monthly depreciation rate, δ		0.025	0.025	0.025	0.025	0.05	0.05
Parameters characterizing devaluation	Change in wholesale import price: $\Delta \log \omega = 0.50$ Interest rate change: $\beta = 0.70$ (annually) Change in consumption: $\Delta \log C = -0.15$ Local labor share: 25%						

Figure 4b: Median Import Price Elasticity (US data)

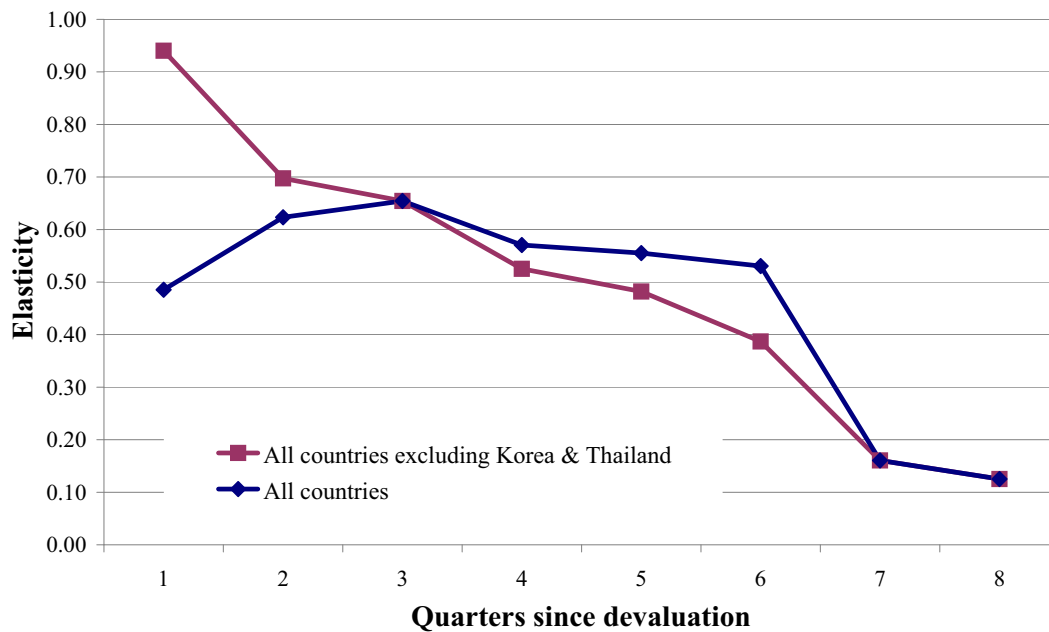


Figure 6b: Low markup economy

