Online Appendix for "Investment in Schooling and the Marriage Market"

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1 Existence and Uniqueness of Equilibrium

Substitute $z_{11}-V_1$ for U_1 and $z_{22}-V_2$ for U_2 in equation (21), and define $\Psi(V_1, V_2)$ as

$$\begin{aligned}\Psi(V_1,V_2) \quad &\equiv \quad F(V_1)+\int\limits_{V_1}^{V_2} G(R^m+V_2-\theta)f(\theta)d\theta \\ &\qquad -F(z_{11}-V_1)-\int\limits_{z_{11}-V_1}^{z_{22}-V_2} G(R^w+z_{22}-V_2-\theta)f(\theta)d\theta\ .\end{aligned} \tag{A1}$$

Note, first, that

$$\Psi(0,0)=F(0)-F(z_{11})-\int\limits_{z_{11}}^{z_{22}} G(R^w+z_{22}-\theta)f(\theta)d\theta<0 \tag{A2}$$

and that

$$\Psi\left(z_{11},z_{22}\right)\equiv F\left(z_{11}\right)-F\left(0\right)+\int\limits_{z_{11}}^{z_{22}} G(R^m+z_{22}-\theta)f(\theta)d\theta>0\ , \tag{A3}$$

since $z_{11}>0$ implies that $F\left(z_{11}\right)-F\left(0\right)>0$. By continuity, we conclude that there exists a set of couples (V_1,V_2) for which $\Psi\left(V_1,V_2\right)=0$.

In addition, we have

$$\begin{aligned}\frac{\partial\Psi(V_1,V_2)}{\partial V_1} &= f(V_1)[1-G(R^m+V_2-V_1)] \\ &\qquad +f(z_{11}-V_1)[1-G(R^w+z_{22}-z_{11}-(V_2-V_1)]>0\end{aligned} \tag{A4}$$

and

$$\frac{\partial\Psi(V_1,V_2)}{\partial V_2} = G(R^m)f(V_2) + G(R^w)f(z_{22}-V_2)]$$
$$+\int_{V_1}^{V_2} g(R^m+V_2-\theta)f(\theta)d\theta + \int_{U_1}^{U_2} g(R^w+U_2-\theta)f(\theta)d\theta > 0\ . \tag{A5}$$

By the implicit function theorem, $\Psi(V_1,V_2)=0$ defines V_2 as a differentiable, decreasing function of V_1 over some open set in $\mathbb{R}$. Equivalently, the locus $\Psi(V_1,V_2)=0$ defines a smooth, decreasing curve in the (V_1,V_2) plane.

Using (22), define $\Omega(V_1,\,V_2)$ as

$$\begin{aligned}\Omega(V_1,V_2) \quad &\equiv \quad F(V_1)\left[1-G(R^m+V_2-V_1)\right] \\ &\quad -F(z_{11}-V_1)\left[1-G(R^w-z_{11}+V_1+z_{22}-V_2)\right].\end{aligned} \tag{A6}$$

Note that Ω is continuously differentiable, increasing in V_1 and decreasing in V_2. Moreover,

$$\begin{aligned}\lim_{V_1\to\infty}\Omega(V_1,V_2) \quad &= \quad 1, \\ \lim_{V_2\to\infty}\Omega(V_1,V_2) \quad &= \quad -F(z_{11}-V_1)<0.\end{aligned} \tag{A7}$$

By continuity, there exists a locus on which $\Omega(V_1,V_2)=0$; by the implicit function theorem, it is a smooth, increasing curve in the (V_1,V_2) plane. In addition,

$$\Omega(V_1,V_2) = A(V_1,V_2-V_1)\,, \tag{A8}$$

where

$$A(V,X) = F(V)\left[1-G(R^m+X)\right] - F(z_{11}-V)\left[1-G(R^w-z_{11}+z_{22}-X)\right]. \tag{A9}$$

Since

$$\frac{\partial A(V,X)}{\partial V} = f(V)\left[1-G(R^m+X)\right]+f(z_{11}-V)\left[1-G(R^w-z_{11}+z_{22}-X)\right] > 0 \tag{A10}$$

and

$$\frac{\partial A(V,X)}{\partial X} = -F(V)\,g(R^m+X) - F(z_{11}-V)\,g(R^w-z_{11}+z_{22}-X) < 0, \tag{A11}$$

the equation $A(V,X)=0$ defines X as some increasing function ϕ of V. Therefore,

$$\Omega(V_1,V_2) = A(V_1,V_2-V_1) = 0 \tag{A12}$$

gives

$$V_2 = V_1 + \phi(V_1), \tag{A13}$$

where $\phi'(V) > 0$. Thus in the (V_1,V_2) plane, the slope of the $\Omega(V_1,V_2) = 0$ curve is always more than 1. In particular, the curve must intersect the decreasing curve $\Psi(V_1,V_2)=0$, and this intersection (V_1^*,V_2^*) is unique.

Finally, stability requires that

$$U_1 + V_2 \geq z_{21} \qquad \text{and} \qquad U_2 + V_1 \geq z_{12} \tag{A14}$$

which implies that, at any stable match, we have

$$z_{21} - z_{11} \leq V_2 - V_1 \leq z_{22} - z_{12}, \tag{A15}$$

and

$$z_{12} - z_{11} \leq U_2 - U_1 \leq z_{22} - z_{21}. \tag{A16}$$

Three cases are thus possible:

1. If $z_{21} - z_{11} \leq V_2^* - V_1^* \leq z_{22} - z_{12}$, then (V_1^*, V_2^*) is the unique equilibrium (see Figure A.1). Indeed, it is the only equilibrium with perfectly assortative matching. Moreover, a point such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{21} - z_{11} \tag{A17}$$

cannot be an equilibrium, because at that point $\Omega(V_1, V_2) > 0$, which contradicts the fact that the number of educated men should exceed that of educated women for such an equilibrium to exist. Similarly, a point such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{22} - z_{12} \tag{A18}$$

cannot be an equilibrium, because at that point $\Omega(V_1, V_2) < 0$, which contradicts the fact that the number of educated women should exceed that of educated men for such an equilibrium to exist.

2. If $z_{21} - z_{11} > V_2^* - V_1^*$, then the unique equilibrium (see Figure A.2) is such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{21} - z_{11}\ . \tag{A19}$$

Indeed, a perfectly assortative matching equilibrium is not possible because the only possible candidate, (V_1^*, V_2^*), violates the condition $z_{21} - z_{11} \leq V_2^* - V_1^* \leq z_{22} - z_{12}$. And a point such that

$$\Psi(V_1, V_2) = 0 \quad \text{and} \quad V_2 - V_1 = z_{22} - z_{12} \tag{A20}$$

cannot be an equilibrium, because at that point $\Omega(V_1, V_2) < 0$ which contradicts the fact that the number of educated women should exceed that of educated men for such an equilibrium to exist.

3. Finally, if $V_2^* - V_1^* > z_{22} - z_{12}$, then the unique equilibrium (see Figure A.3) is such that

$$\Psi\left(V_1, V_2\right) = 0 \quad \text{and} \quad V_2 - V_1 = z_{22} - z_{12} \ . \tag{A21}$$

Indeed, a perfectly assortative matching equilibrium is not possible because the only possible candidate, (V_1^*, V_2^*), violates the condition $z_{21} - z_{11} \leq V_2^* - V_1^* \leq z_{22} - z_{12}$. And a point such that

$$\Psi\left(V_1, V_2\right) = 0 \quad \text{and} \quad V_2 - V_1 = z_{21} - z_{11} \tag{A22}$$

cannot be an equilibrium, because at that point $\Omega\left(V_1, V_2\right) > 0$ which contradicts the fact that the number of educated men should exceed that of educated women for such an equilibrium to exist.

2 A Numerical Example

Suppose that μ and θ are uniformly and independently distributed. Although wages vary across the two regimes, we assume that in both regimes, educated women are more productive in the market and uneducated women are more productive at home. We further assume that in both regimes, men earn more than women with the same schooling level but educated women earn more than uneducated men. Finally, in both regimes, women have a higher market return from schooling. The transition from the old regime to the new regime is characterized by three features: (i) productivity at home is higher and women are required to work less at home; (ii) men and women obtain higher market returns from schooling; and (iii) couples move from a traditional mode to an efficient one in which the high-wage spouse works in the market.

All the above economic changes raise the gains from marriage and would cause higher marriage rates. To calibrate the model, we assume that the variance in the preference for marriage rises over time which, other things being the same, reduces the propensity to marry. We thus assume that in both periods μ is distributed over the interval $[-4, 4]$, while θ is distributed over the intervals $[-4, 4]$ and $[-8, 8]$ in the old and the new regimes, respectively. It is important to note that the shift in the distribution of θ has *no* impact on the equilibrium surplus shares, which are our main concern. However, it changes the proportion of individuals who invest and marry given these shares.

Table A.1 reflects these assumptions.

Table A.1: Parameters in the old and the new regimes

Parameter	Old Regime	New Regime
Wage of uneducated men	$w_1^m = 2$	$w_1^m = 2.375$
Wage of uneducated women	$w_1^w = 1.2$	$w_1^w = 1.425$
Wage of educated men	$w_2^m = 3$	$w_2^m = 4.0$
Wage of educated women	$w_2^w = 2.4$	$w_2^w = 3.2$
Wage difference among the uneducated	$d_1 = .6$	$d_1 = .6$
Wage difference among the uneducated	$d_2 = .8$	$d_2 = .8$
Market return to schooling, men	$R^m = .25$	$R^m = 1.18$
Market return to schooling, men	$R^w = .72$	$R^w = 1.54$
Work requirements	$\tau = .8$	$\tau = .3$
Productivity at home	$\gamma = 2$	$\gamma = 2.5$
Distribution of tastes for schooling	$[-4, 4]$	$[-4, 4]$
Distribution of tastes for marriage	$[-4, 4]$	$[-8, 8]$
Norms	Wife at home	Efficient

The marriage market implications of these changes are summarized in Tables A.2-A.4 below.

Table A.2: Impact of parameter changes on marital surplus

Old regime

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 1.72$
Educ. husband	$z_{21} = 3.25$	$z_{22} = 2.76$

New Regime

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 3.90$
Educ. husband	$z_{21} = 3.75$	$z_{22} = 5.66$

A decrease in the amount of time worked at home, raises the contribution of an educated woman to the material surplus and lowers the contribution of an uneducated woman. Therefore, in the old regime with $\tau = .8$, the material surplus *declines* with the education of the wife when the husband is uneducated, while in the new regime with $\tau = .3$, it rises. This happens because educated women are more productive in the market than uneducated women but, by assumption, equally productive at home. In the old regime, if an educated wife would marry an uneducated man (which does not happen in equilibrium) she would be assigned to household work even though she has a higher wage than her husband. In the new regime, couples act efficiently, household roles are reversed and educated women *do* marry uneducated men. Note that for couples among whom both husband and wife are uneducated, the wife continues to work at home in the new regime, because she has the lower wage. The parameters are chosen in such a way that technology has no impact on the marital surplus of such couples. In the new regime, uneducated women work less time at home but their productivity at home is higher as well as the wage that they obtain from work.

Table A.3: Impact of parameter changes on the equilibrium shares

Old regime

	Uneducated	Educated
Men	$V_1 = .76$	$V_2 = 1.68$
Women	$U_1 = 1.57$	$U_2 = 1.09$

New Regime

	Uneducated	Educated
Men	$V_1 = 1.13$	$V_2 = 2.88$
Women	$U_1 = 1.20$	$U_2 = 2.78$

Compared with the old regime, educated women receive a higher share of the marital surplus in the new regime, while uneducated women receive a lower share. These changes reflect the higher (lower) contributions to marriage of educated (uneducated) women. The marital surplus shares of both educated and uneducated men rise as a consequence of the rising productivity of their wives.

The implied returns from schooling within marriage in the old regime are

$$U_2 - U_1 = 1.09 - 1.57 = z_{22} - z_{21} = 2.76 - 3.25 = -.49 \ ,$$

$$V_2 - V_1 = 1.68 - .76 = z_{21} - z_{11} = 3.25 - 2.33 = .92 \ .$$

That is, men receive the lower bound on their return from schooling within marriage while women receive the upper bound on their return from schooling. This pattern is reversed in the new regime:

$$U_2 - U_1 = 2.78 - 1.20 = z_{12} - z_{11} = 3.90 - 2.33 = 1.58,$$

$$V_2 - V_1 = 2.88 - 1.13 = z_{22} - z_{12} = 5.66 - 3.90 = 1.75,$$

where women receive their lower bound and men receive their upper bound. Both men and women receive a higher return from schooling within marriage in the new regime, reflecting the increased efficiency although the rise for women is much sharper.

Table A.4: Impact of parameter changes on the investment and marriage rates*

Old Regime

	Married	Unmarried	All
Educ.	.452, .335	.153, .215	.606, .550
Uned.	.211, .323	.183, .122	.394, .450
All	.662, .666	.334, .334	1

New Regime

	Married	Unmarried	All
Educ.	.577, .590	.207, .226	.784, .816
Uned.	.077, .063	.139, .121	.216, .184
All	.653, .653	.347, .347	1

$*$ First and second entries in each cell refer to men and women resp.

In the old regime, more men invest in schooling than women and some educated men marry down to match with uneducated women. This pattern is reversed in the new regime and women invest in schooling more than men and some educated women marry down to join uneducated men. That is, women increase their investment in schooling more than men. Although market returns have risen for both men and women, the returns for schooling within marriage have risen substantially more for women. The basic reason for that is the release of married women from the obligation to spend most of their time at home, due to the reduction in the time requirement of child care and the change in norms that allow educated women who are married to uneducated men to enter the labor market. Uneducated men gain a higher share in the surplus in all marriages because of their new opportunity to marry educated women, while uneducated women lose part of their share in the marital surplus in all marriages because they no longer marry educated men. Notice that the proportion of educated women who remain single declines from $.215/.550 = .39$ to $226/.816 = 0.28$ in the new regime. In contrast, the proportion of educated men who marry remains roughly the same, $.153/.606 = 0.28$ and $207/.784 = 0.26$ in the old and new regimes, respectively. This gender difference arises because, under the old regime, women were penalized in marriage by being forced to work at home.

We can use these examples to discuss the impact of norms. To begin with, suppose that in the old regime couples acted efficiently and, if the wife was more educated than her husband, she went to work full time and the husband engaged in child care. Comparing Tables A.2 and A.5, we see that the impact of such a change on the surplus matrix is only through the rise in z_{12}. Because women receive lower wages than men at all levels of schooling, the household division of labor is not affected by the norms for couples with identically educated spouses; for all such couples, the husband works in the market and the wife takes care of the child. However, the norm does affect the division of labor for couples among whom the wife has a higher education level than her husband. This is due to our assumptions that educated women have a higher wage than uneducated men in the labor market and their market wage exceeds their productivity at home. In contrast to the case in which the mother always works at home, we see in Table A.5 that the education levels now become substitutes, namely $z_{11} + z_{22} < z_{12} + z_{21}$, implying that we can no longer assume that there will be some educated men married to educated women and some uneducated men married to uneducated women. More specifically, an educated woman contributes more to an uneducated man than she does to an educated man (i.e. $z_{12} - z_{11} > z_{22} - z_{21}$) so that uneducated men can bid away the educated women from educated men. Thus changes in norms can influence the patterns of assortative mating.

Table A.5: Impact of norms on material surplus

Old regime, efficient

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 2.40$
Educ. husband	$z_{21} = 3.25$	$z_{22} = 2.76$

New Regime with norms

	Uned. wife	Educ. wife
Uned. husband	$z_{11} = 2.33$	$z_{12} = 3.23$
Educ. husband	$z_{21} = 3.75$	$z_{22} = 5.66$

Consider, next, the possibility that the norms persist also in the new regime and the mother must work at home even if she is more educated than her husband. Again, the norm bites only in those marriages in which the wife is more educated than the husband. In the new regime, positive assortative mating persists independently of the norms. However, the mixing equilibrium in which some educated women marry uneducated men is replaced by strict assortative mating in which educated men marry only educated women and uneducated men marry only uneducated women. Thus, again, norms can have a qualitative impact on the type of equilibrium that emerges.

The new marriage and investment patterns are presented in the lower panel of Table A.6. The main difference is that educated women are less likely to marry when the norms require them to work at home, where they are relatively less efficient.

Table A.6: Impact of norms on investment and marriage rates (new regime)*

Efficient work pattern

	Married	Unmarried	All
Educ.	.577, .589	.207, .126	.784, .816
Uned.	.077, .063	.139, .121	.216, .184
All	.653, .653	.347, .347	1

Wife work pattern

	Married	Unmarried	All
Educ.	.583, .583	.207, .227	.790, .810
Uned.	.070, .070	.140, .120	.210, .190
All	.653, .653	.347, .347	1

∗ The first and second entry in each cell refer to men and women resp.

Consider, finally, the impact on the shares in the material surplus when norms are replaced by an efficient allocation in the new regime (see Table A.7). The removal of social norms that the wife must work at home benefits uneducated men and harms uneducated women. This example illustrates the differences between the predictions of general equilibrium models with frictionless matching, like the one we present here, and partial equilibrium models that rely on bargaining. The latter would predict that no woman would lose from the removal of norms that forces women in general to stay at home and take care of the child, but as this example demonstrates, the market equilibrium can change and uneducated women are hurt because they can no longer marry with educated men.

Table A.7: Impact of norms on the equilibrium shares in the new regime

Efficient pattern of work

	Uneducated	Educated
Men	$V_1 = 1.13$	$V_2 = 2.89$
Women	$U_1 = 1.20$	$U_2 = 2.78$

Wife always works at home

	Uneducated	Educated
Men	$V_1 = 1.06$	$V_2 = 2.89$
Women	$U_1 = 1.28$	$U_2 = 2.77$

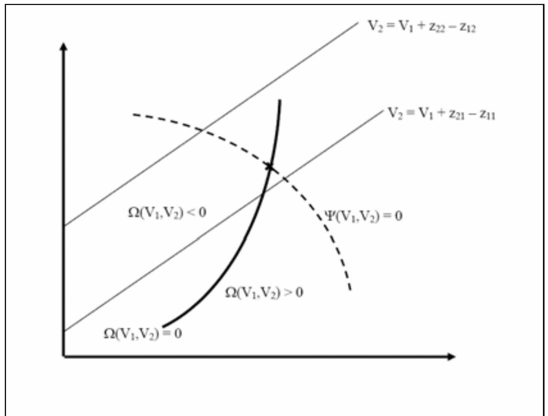

Figure A1

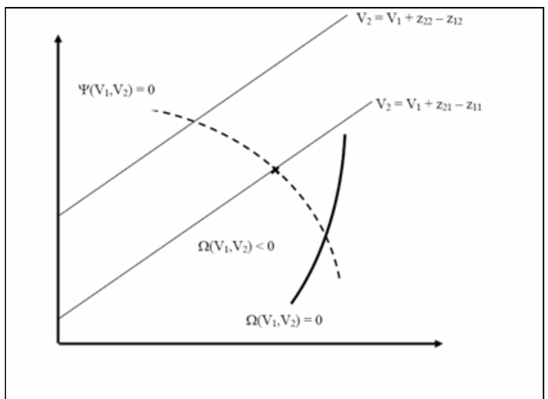

Figure A2

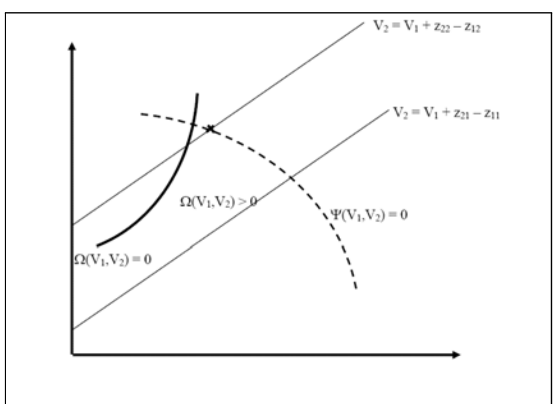

Figure A3

Figure A. 4: Equilibrium lines of the old regime

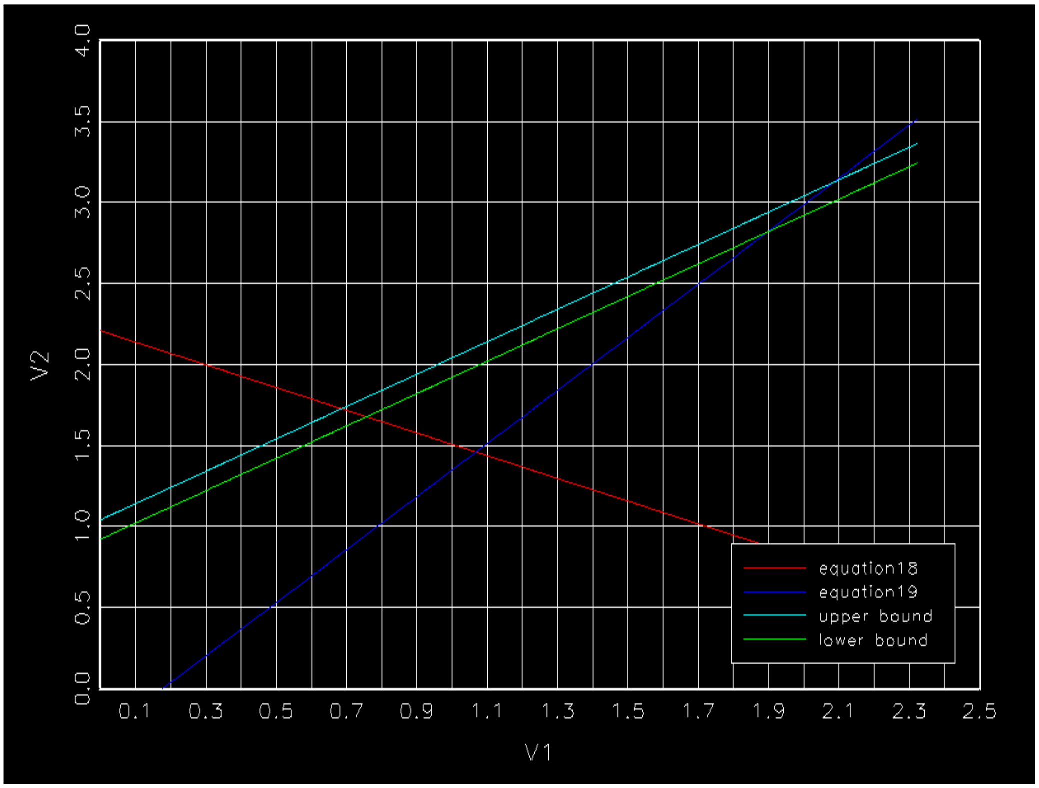

Figure A. 5: Equilibrium lines of the new regime