Appendix B:
NUMERICAL RESULTS

The theoretical analysis establishes that the expected profit of borrowers must be downward sloping for low and high interest rates. It also identifies two extreme cases in which the expected profit function does have two turning points. In order to understand how likely it is for such turning points to occur we also conducted a detailed numerical analysis.

The first step was to solve the model numerically and then examine the borrower payoff function and implied loan demand function. This program, **figs.m,** produces figures as the output.[1] Below we illustrate a case where there is a turning point.

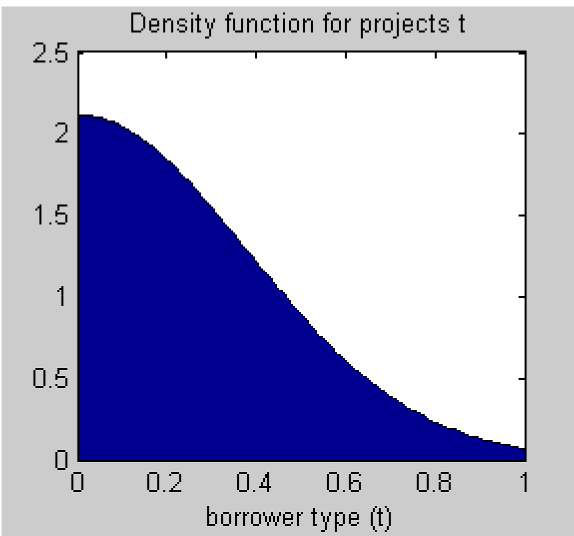

Fig. B.1: Density function of borrower types

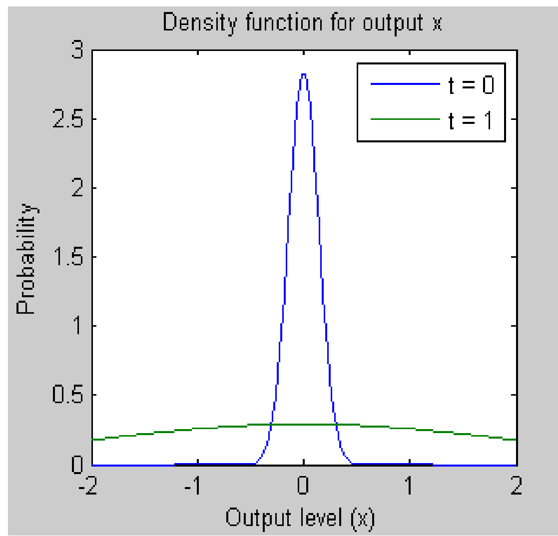

Fig. B.2: Extreme density functions for output

Fig. B.1 shows the truncated normal distribution for the type distribution. Since we focus on distributions for which the mode is in the interior of the support, this is the limiting case. Fig. B.2 shows the density functions for $y-\mu$, for the lowest and highest risks. The probability of bankruptcy is 0.12 for $t=0$ and 0.46 for $t=1$.

[1] This program **figs.m** and the ancillary sub-programs can all be downloaded from the web-site johngriley.com. The input vector (type distribution parameters, the output distribution parameters and the collateral) is all entered from a spreadsheet **grid.xls.**

Fig. B.3 shows the expected borrower return for different interests rates and the final figure shows the implied demand for loanable funds. Note that there can be no adverse selection for interest rates $R \le \mu = 2.0$. In this case there is a turning point at $R_1 = 2.0$ and a second turning point at $R_2 = 2.02$. The program also computes the smallest $R > R_2$ for which the expected borrower return is lower than at R_1. That is, it computes R_3 satisfying

(i) $\bar{U}(R) \ge \bar{U}(R_1)$, $R \in [R_2, R_3)$ and (ii) $\bar{U}(R_1) > \bar{U}(R_3)$.

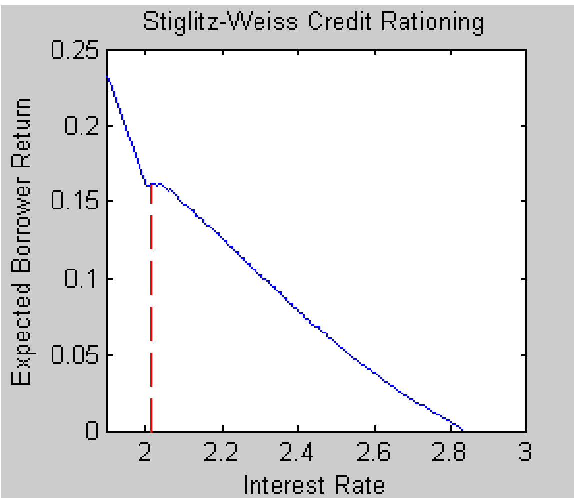

Fig.B.3: Mean borrower return $\bar{U}(R)$

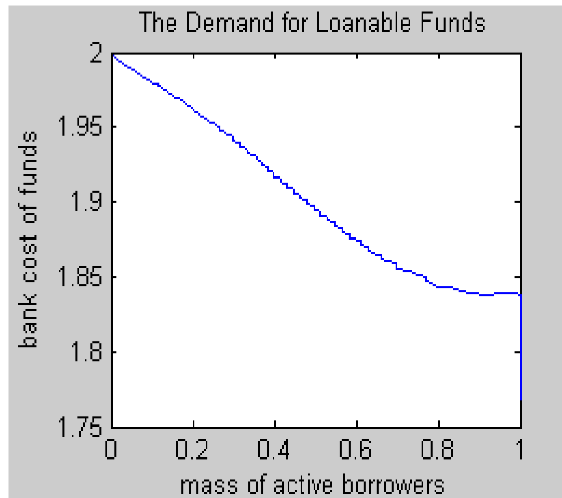

Fig. B.4: Implied demand for loanable funds

For the example illustrated in Fig. B.3, $R_3 = 2.05$. The fact that the turning point lies below $R = 2.02$ indicates that the non-monotonicity is an example of case (ii) where the tails of the output distribution are very thin for a mass of low risks. Since the interest rate at the turning point is very close to 2.0 (where demand is 1) the implied demand for loanable funds barely slopes upwards. It follows that there is only a small difference in the interest rates in a two interest rate equilibrium with rationing.

The second phase of the numerical analysis was to write a program that would systematically search over many possible parameter values.[2] The distribution of types has two parameters: the mode parameter $\bar{m}$ and spread parameter $\bar{s}$. For each "run" we selected 100

[2] This program **search.m** can also be downloaded from the web-site johngriley.com. Data is inputted from the spread-sheet **grid.xls**.

parameter pairs for the type distribution and matched them with a vector of parameters for the output distribution and preferences. The program **search.m** completes six such runs so that each table summarizes 600 data points. Initially we restricted attention to truncated normal distributions for which the mode parameter is in the support of the type distribution. For each mode parameter we chose a spread parameter such that

$$\text{PR}\{\text{tail}\} \equiv MIN(\text{PR}\{t \leq 0.1\}, PR\{t \geq 0.9\})$$

took on one of 10 values. These are indicated below.

MODE	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
PR{tail}	**0.001**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**

Table B.1: The mode of the type distribution and tail probabilities

That is, for each mode $\overline{m}$ in the top row of the table above, we chose a vector of values for $\overline{s}$ such that the tail probabilities are equal to the 10 values in the bottom row. In each run we therefore have 100 mode-spread pairs.

We then picked a value for the collateral and selected six different pairs of minimum and maximum spread parameters σ_0 and σ_1. Results are presented as in Table B.2 below. Rows 1 through 4 show the minimum and maximum spread parameters and the implied probability of bankruptcy for each of the 6 runs. The program solves for the expected lender revenue function $V(R)$ and searches for turning points. We find at most 2 turning points.

In each of the 60 cells inside the block with a heavy border we summarize results for the 10 different spread parameters and hence values of PR{tail} (see above.) Consider, for example, the top left cell inside this block. For six of the 10 values of PR{tail} the function $V(R)$ has a turning point. In the parentheses the first term is $Max\{R_2 - \mu\}$, the maximum distance between the maximizing value of R and μ over the 10 values of PR{tail}. The second term is

$Max\{R_3 - \mu\}$, where R_3 is defined above. Thus for the top left cell the largest value of $R_2 - \mu$ is 0.02 and the largest value of $R_3 - \mu$ is 0.08. An empty cell indicates no turning points.

Number of cases in which there is a turning point	**C=0.2**	σ_0	0.1	0.12	0.14	0.17	0.21	0.24
		σ_1	2	2	2	2	2	2
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1	0.023 0.44	0.05 0.44	0.08 0.44	0.12 0.44	0.17 0.44	0.20 0.44
Mode of type distribution		0.0	6(.02/.08)	6(.02/.06)	5(.02/.03)			
		0.1	4 (.01/.06)	4(.01/.02)	2(.00/.01)			
		0.2						
		0.3						
		0.4						
		0.5						
		0.6						
		0.7						
		0.8						
		0.9						

Table B.2: Summary of search for turning points for 3600 different parameter values (C = 0.2)

We systematically searched for turning points over 36 runs (3600 data points). As the results show, the only examples of turning points occur when the probability of bankruptcy is very low and at an interest rate very close to the mean $\mu = 2.0$. Thus all these examples are in the family of "case (ii)" extreme cases discussed in the paper. Results are summarized in Tables B.3-B.7 below.

Number of cases in which there is a turning point	**C=0.4**	σ_0	0.1	0.2	0.25	0.3	0.35	0.4
		σ_1	1	1	1	1	1	1
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1}	0.00003 0.34	0.02 0.34	0.05 0.34	0.09 0.34	0.13 0.34	0.16 0.34
Mode of type distribution		0.0	10 (.03/.17)	10 (.03/.08)	5 (.01/.03)			
		0.1	10 (.02/.07)	10 (.02/.05)				
		0.2	10 (.02/.12)	10 (.02/.04)				
		0.3	10 (.02/.07)	8 (.01/.03)				
		0.4	7 (.01/.06)	5 (.01/.02)				
		0.5	2 (.01/.06)	2 (.01/.02)				
		0.6	2 (.01/.05)	2 (.01/.02)				
		0.7	4 (.01/.05)	3 (.01/.02)				
		0.8	2 (.01/.04)	3 (.01/.02)				
		0.9	4 (.01/0.06)	3 (.01/.02)				

Table B.3: Summary of search for turning points for 3600 different parameter values (C = 0.4)

Number of cases in which there is a turning point	C=0.6	σ_0	0.2	0.3	0.4	0.5	0.6	0.7
		σ_1	2	2	2	2	2	2
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1}	0.001 0.33	0.02 0.33	0.07 0.33	0.12 0.33	0.16 0.33	0.19 0.33
Mode of type distribution	0.0		10 (.02/.08)	10 (.02/.04)				
	0.1		10 (.02/.06)	10 (.01/.02)				
	0.2		10 (.01/.05)	10 (.00/.01)				
	0.3		9 (.01/.03)	6 (.00/.01)				
	0.4		4 (.01/.03)	1 (.00/.01)				
	0.5		1 (.01/.03)					
	0.6		1 (.01/.03)					
	0.7		1 (.01/0.3)					
	0.8		1 (.01/.03)					
	0.9		1 (.01/.03)					

Table B.4: Summary of search for turning points for 3600 different parameter values (C = 0.6)

Number of cases in which there is a turning point	C=0.8	σ_0	0.3	0.4	0.5	0.6	0.7	0.8
		σ_1	2	2	2	2	2	5
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1}	0.004 0.27	0.023 0.27	0.05 0.27	0.09 0.27	0.12 0.27	0.15 0.3
Mode of type distribution	0.0		10(.01/.05)	7 (.01/.02)				
	0.1		10 (.01/.04)	6 (.01/.01)				
	0.2		10 (.01/.03)					
	0.3		7 (.00/.02)					
	0.4		1 (.00/.02)					
	0.5							
	0.6							
	0.7							
	0.8							
	0.9							

Table B.5: Summary of search for turning points for 3600 different parameter values (C = 0.8)

Number of cases in which there is a turning point	C=1.0	σ_0	0.2	0.4	0.5	0.6	0.7	0.8
		σ_1	2	2	2	2	2	2
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1}	0.00000 0.22	0.006 0.22	0.02 0.22	0.05 0.22	0.07 0.22	0.10 0.22
Mode of type distribution	0.0		10 (.01/.05)	10 (.01/.02)				
	0.1		10 (.01/.04)	10 (.01/.02)				
	0.2		10 (.01/.03)	10 (.00/0.1)				
	0.3		9 (.00/.03)					
	0.4							
	0.5							
	0.6							
	0.7							
	0.8							
	0.9							

Table B.6: Summary of search for turning points for 3600 different parameter values (C =1.0)

Number of cases in which there is a turning point	C=1.2	σ_0	0.3	0.4	0.5	0.6	0.7	0.8
		σ_1	2	2	2	2	2	2
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1}	0.00003 0.17	0.001 0.17	0.01 0.17	0.02 0.17	0.04 0.17	0.06 0.17
Mode of type distribution	0.0							
	0.1							
	0.2							
	0.3							
	0.4							
	0.5							
	0.6							
	0.7							
	0.8							
	0.9							

Table B.7: Summary of search for turning points for 3600 different parameter values (C = 1.2)

As the tables show, the turning points are concentrated at low values of the mode parameter and when the probability of bankruptcy is low for a mass of low risks.

We then extended the analysis to examine type distributions with the mode equal to the lower support, that is, the mode parameter is negative.

MODE	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
PR{tail}	**0.005**	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**	**0.09**

Table B.8: The mode of the type distribution and tail probabilities

Tables B.9-B.11 below show that when the slope of the density function is sufficiently negative at t=0, the results noted above are magnified, but only slightly. The following figures (analogous to B.1-B.4) are representative.

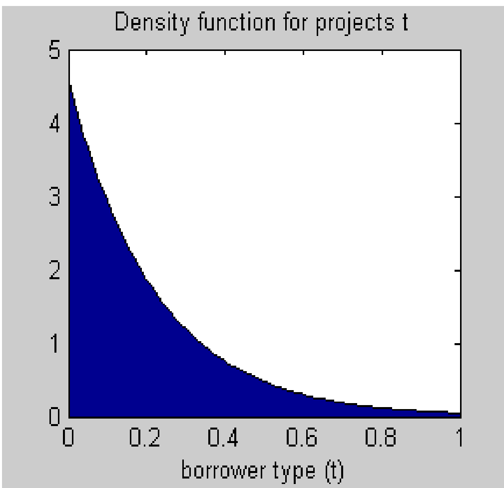

Fig. B.5: Density function of borrower types

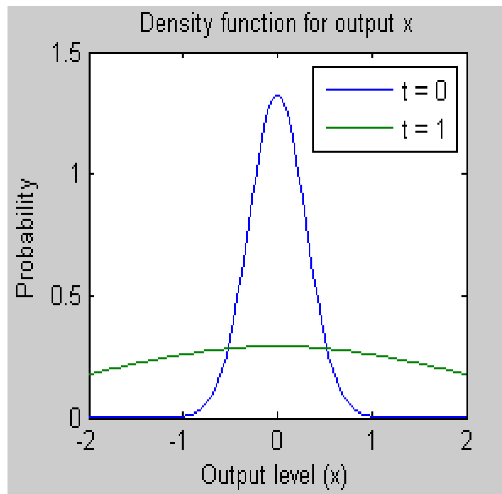

Fig. B.6: Extreme density functions for output

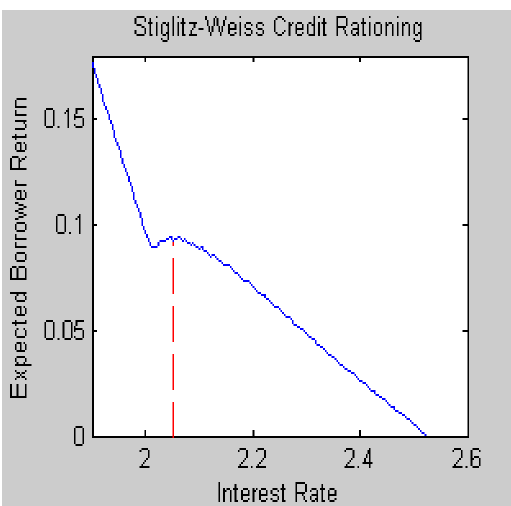

Fig.B.7: Mean borrower return $\bar{U}(R)$

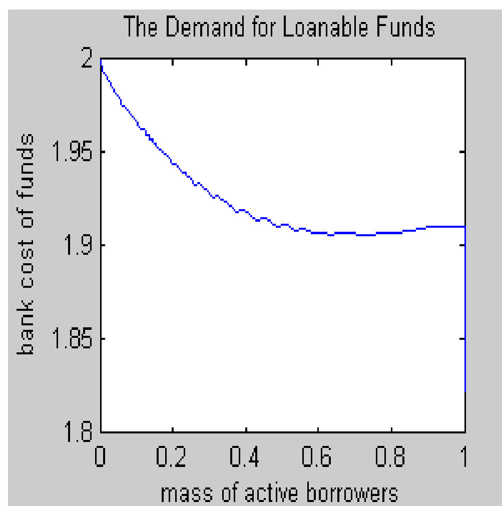

Fig. B.8: Implied demand for loanable funds

Note that the non-monotonicity is “small” and, as a result, the interest rates are close when there is rationing. Moreover the derived demand curve for loanable funds is almost horizontal. Thus even in this extreme example, with a high the mode at t=0 the scope for rationing is small.

Number of cases in which there is a turning point	**C=0.4**	σ_0	0.2	0.25	0.3	0.4	0.5	0.5
		σ_1	2	2	2	2	2	2
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1}	0.02 0.38	0.06 0.38	0.09 0.38	0.16 0.38	0.21 0.38	0.26 0.38
Mode of type distribution	-10		10(.04/.10)	10(.03/.05)				
	-9		10(.04/.10)	10(.03/.05)				
	-8		10(.04/.10)	10(.03/.05)				
	-7		10(.04/.10)	10(.03/.05)				
	-6		10(.04/.10)	10(.03/.05)				
	-5		10(.03/.10)	10(.03/.05)				
	-4		10(.03/.09)	10(.03/.05)				
	-3		10(.03/.09)	10(.03/.05)				
	-2		10(.03/.09)	10(.03/.05)				
	-1		10(.03/.08)	10(.03/.04)				

Table B.9: Summary of search for turning points for 3600 different parameter values (C = 0.4)

Number of cases in which there is a turning point	**C=0.6**	σ_0	0.2	0.3	0.4	0.5	0.6	0.7
		σ_1	2	2	2	2	2	2
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1}	0.001 0.33	0.02 0.33	0.07 0.33	0.12 0.33	0.16 0.33	0.19 0.33
Mode of type distribution	-10		10(.02/.10)	10(.02/.06)				
	-9		10(.02/.10)	10(.02/.06)				
	-8		10(.02/.10)	10(.02/.06)				
	-7		10(.02/.10)	10(.02/.06)				
	-6		10(.02/.10)	10(.02/.06)				
	-5		10(.02/.10)	10(.02/.06)				
	-4		10(.02/.10)	10(.02/.06)				
	-3		10(.02/.10)	10(.02/.06)				
	-2		10(.02/.09)	10(.02/.05)				
	-1		10(.02/.09)	10(.02/.05)				

Table B.10: Summary of search for turning points for 3600 different parameter values (C = 0.6)

Number of cases in which there is a turning point	**C=0.8**	σ_0	0.2	0.3	0.4	0.5	0.6	0.7
		σ_1	2	2	2	2	2	2
		PR{bk|R= μ ,t=0} PR{bk|R= μ ,t=1}	0.00003	0.004	0.02	0.05	0.09	0.13
Mode of type distribution	-10		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-9		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-8		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-7		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-6		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-5		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-4		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-3		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-2		10(.02/.08)	10(.02/.06)	10(.02/.02)			
	-1		10(.02/.07)	10(.01/.05)	10(.01/.02)			

Table B.11: Summary of search for turning points for 3600 different parameter values (C = 0.8)