Appendix B: Welfare Effects in a Benchmark Quality Ladder Model

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Suppose that in each period t firms engage in Bertrand competition to make sales. Thus, $\pi_E = \pi_m = \Delta$ and $\pi_I = 0$.[1] Specializing (IB*) to this case, the innovation prize is given by

$$W(\phi) = \left[\frac{\Delta(1-\delta) + \delta[\phi\Delta + (1-\phi)\Delta + c(\phi)]}{1-\delta+\delta(\phi+u(\phi))}\right] \tag{B1}$$

$$= \left[\frac{\Delta + \delta c(\phi)}{1-\delta+\delta(\phi+u(\phi))}\right]. \tag{B1}$$

For now we need not be specific about the nature of innovation supply; we assume only that it is described by a continuous nondecreasing innovation supply function.

Since we now have a fully-specified consumer side (unlike in Sections I and II), we can compare the equilibrium innovation rate to the symmetric innovation rate that maximizes aggregate welfare.[2] Let us begin by considering the social innovation benefit. To this end, observe that a technological advancement in period t raises gross consumer surplus in every subsequent period by Δ. The *social innovation prize* w_s is therefore equal to the present discounted value of this change, $w_s = \left(\frac{\Delta}{1-\delta}\right)$. From (VE*) we see that $V_E \geq 0$ implies $u(\phi)w - c(\phi) \geq 0$. Substituting from this inequality for $c(\phi)$ in (B1) implies that

$$w \leq \frac{\Delta}{1-\delta+\delta\phi}.$$

[1] We focus here on the undominated equilibrium in which the incumbent (who makes no sales) charges a price equal to cost and the entrant with technology $j_t + 1$ charges a price of Δ.

[2] In general, the socially optimal innovation plan may be asymmetric. We focus here on the best symmetric plan since our aim is to see how changes in the symmetric equilibrium innovation rate affect welfare.

Thus, $w \leq \frac{\Delta}{1-\delta} = w_s$, so that the (private) innovation benefit curve always lies weakly below the social innovation benefit curve, as in Figure 3. This difference is due to the "Schumpeterian effect" that arises because an innovator is eventually replaced even though its innovation raises surplus forever.

Figure 3 Here

Given the social benefit w_s (which is independent of ϕ), we can determine the socially optimal symmetric innovation rate by constructing a social innovation supply curve $\Phi_s(\cdot)$ giving the socially optimal symmetric innovation rate for a given level of the social innovation prize. Given the relation between the social and private innovation benefit shown in Figure 3, it is immediate that if the social innovation supply curve coincides with the private one then the equilibrium rate of innovation must be below the socially optimal rate (note that a nondecreasing innovation supply function and the fact that w_s is independent of ϕ implies a unique socially optimal innovation rate). This is true, for example, when there is a single potential entrant (and, hence, a single research lab) and when there is free entry with a randomly-arriving idea.

In contrast, innovation may be excessive when there is a fixed number $N > 1$ of potential entrants or with free entry. For example, consider the case with N potential entrants where the patent is awarded randomly among the firms who make a discovery and $c(\cdot)$ is differentiable. In this case, $r_N(\psi)$ takes the value in footnote 14. Given an innovation prize w, the socially efficient innovation rate would obtain by letting a successful innovator internalize its contribution to social surplus by giving it the innovation prize only when no other firm has innovated, which happens, conditional on innovation, with probability $(1-\psi)^{N-1}$, Comparing with the private incentive to innovate in equation (5) in the text, where a successful innovator receives prize w with probability $r_N(\psi) > (1-\psi)^{N-1}$, we see that the private innovation supply exceeds the social: $\Phi_s(w) \leq \Phi(w)$, with strict inequality for all $w > c'(0)$. (Formally, the comparison follows from applying Milgrom and Roberts' [1994] Corollary 2 to compare the fixed points of the firms' private and social best-response correspondences.) The same comparison obtains in the case of free entry, where socially efficient innovation would obtain by giving a successful innovator prize w with probability $1-\phi$, while in equilibrium he receives it with probability $\overline{r}(\phi) > 1-\phi$. The difference between private and social innovation supply is due to the "business stealing effect" that arises because a potential entrant is sure to get a patent when all other firms have failed (in which case the innovation is socially useful), but also gets the patent in some cases when another firm has succeeded (in which case it is not socially useful).[3]

As is evident in Figure 3, these two differences make the comparison between the equilibrium and socially optimal rates of innovation ambiguous when there are $N > 1$ potential entrants or with free entry. As $\delta \to 0$ the social and private innovation prizes both converge to Δ, and so only the latter innovation supply difference is present. As Figure 3 suggests (and Proposition 6 in Section V shows formally), in this case any equilibrium innovation rate will exceed the socially optimal rate. At the other extreme, when $\delta \to 1$, we have $w_s \to \infty$ while $w \leq \Delta/\phi$. Thus, as long as $\lim_{\phi \to 1} \gamma'(\phi) = \infty$, the equilibrium innovation rate will be bounded below 1, while the socially optimal innovation rate converges to 1.

[3] In Aghion and Howitt [1992], two additional distortions are present: an "appropriability effect" (an incumbent monopolist captures less than his full incremental contribution to social surplus in a period) and a "monopoly distortion" effect (an incumbent produces less than the socially optimal quantity in each period). These two distortions are absent here because of our assumption of homogeneous consumer valuations and Bertrand competition.