Online Appendix
The Growth of Low Skill Service Jobs and the Polarization of the U.S. Labor Market

By David H. Autor and David Dorn

I. Online Appendix Tables

Online Appendix Table 1. Routine Employment Share and Growth of Service Employment within Commuting Zones, 1980 - 2005: Stacked First Differences (2SLS Estimates) Robustness Checks.
Dependent Variable: 10 × Annual Change in Share of Non-College Employment in Service Occupations

	I. Baseline	II. Alternative Definitions of Routine Intensity			III. Alternative Measurements of Routine Share		
	ln(R) -ln(M) -ln(A)	ln(R) -ln(M)	$\ln(R^{sts})$ -ln(M) -ln(A)	$\ln(R^{fdex})$ -ln(M) -ln(A)	ln(R) -ln(M) -ln(A)	ln(R) -ln(M) -ln(A)	ln(R) -ln(M) -ln(A)
	Emp in 33% occs w/high RTI	Emp in 33% occs w/high RTI	Emp in 33% occs w/high RTI	Emp in 33% occs w/high RTI	Non-clg emp in 33% w/ high RTI	Emp in 25% occs w/high RTI	Emp in 40% occs w/high RTI
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	A. 2SLS Estimates						
Routine Occ Share_{-1}	0.192 **	0.165 **	0.203 **	0.174 **	0.151 **	0.233 **	0.205 **
	(0.035)	(0.027)	(0.048)	(0.026)	(0.028)	(0.044)	(0.033)
R^2	0.17	0.19	0.15	0.17	0.18	0.15	0.15
	B. Effect Size: CZ at 80th vs 20th Percentile of Routine Employment Share Measure						
P80 - P20	0.07	0.09	0.07	0.08	0.10	0.06	0.08
Effect size	1.37	1.45	1.33	1.36	1.55	1.34	1.54

N=2166 (3 time periods x 722 commuting zones). The variable for share of routine occupations is instrumented by interactions between the 1950 industry mix instrument and time dummies. In each model, both the routine share variable and the industry mix instrument use the definition of routine intensity and the measurement of routine share that is indicated at the top of the column. The three task variables, R, M and A, refer to Routine, Manual and Abstract measures from Autor, Levy and Murnane (2003) based on the Dictionary of Occupational Titles. Columns (3) and (4) exclude one-by-one the two task measures that comprise the Routine scale: column 3 omits routine-cognitive tasks (DOT variable "set limits, tolerances, or standards"); and column 4 omits routine-physical tasks (DOT variable "finger dexterity"). All models include an intercept, time dummies and state dummies. Robust standard errors in parentheses are clustered on state. Models are weighted by start of period commuting zone share of national population. ~ $p \leq 0.10$, * $p \leq 0.05$, ** $p \leq 0.01$.

Online Appendix Table 2. Changes in Educational Composition, 1980-2005 (2SLS Estimates). Dependent Variable: 10 × Annual Change in Education Shares; Difference in Education Shares between Migrant Workers (Out-of-State Five Years Ago) and Non-Migrant Workers.

	College Graduates		Some College		HS Graduates		HS Dropouts	
	(1)		(2)		(3)		(4)	
	A. Δ Education Shares among Workers, 1980-2005							
Share of Routine Occs.$_{-1}$	0.484	**	-0.257	**	-0.507	**	0.279	**
	(0.066)		(0.052)		(0.095)		(0.058)	
R^2	0.341		0.808		0.267		0.838	
	B. Difference Education Shares of Migrant Workers vs. Non-Migrant Workers, 1980-2000							
Share of Routine Occs.$_{-1}$	0.560	**	-0.790	**	0.002		0.228	
	(0.161)		(0.158)		(0.122)		(0.148)	
R^2	0.544		0.787		0.603		0.813	

N=2166 (3 time periods x 722 commuting zones) in Panel A, N=1444 (2 time periods x 722 commuting zones) in Panel B. Share of routine occupations is instrumented by interactions between the 1950 industry mix instrument and time dummies. All models include an intercept, state dummies, and time dummies. Robust standard errors in parentheses are clustered on state. Models are weighted by start of period commuting zone share of national population. ~ $p \leq 0.10$, * $p \leq 0.05$, ** $p \leq 0.01$.

II. Theory Appendix

A. The planner's problem

Given $p_k(t)$ at time t, the social planner's problem at time t is to solve:

$$
\begin{aligned}
&\max_{K,\eta}\left(C_s^{\frac{\sigma-1}{\sigma}}+C_g^{\frac{\sigma-1}{\sigma}}\right)^{\sigma/(\sigma-1)}\\
&\text{s.t. } C_g=Y_g-p_k(t)K\\
&\qquad C_s=Y_s=L_m=1-\exp(-\eta^*)\\
&\text{where } Y_g=L_a^{1-\beta}X^{\beta}\\
&\qquad X\equiv\left[(\alpha_r L_r)^{\mu}+(\alpha_k K)^{\mu}\right]^{1/\mu}\\
&\qquad L_r=(\eta^*+1)\exp(-\eta^*)\\
&\qquad L_a=1,
\end{aligned}
$$

where we write σ_c as σ to simplify notation. The above problem can further be simplified to:

$$
\begin{aligned}
(1)\qquad &\max_{K,L_m}\left(L_m^{\frac{\sigma-1}{\sigma}}+\left(Y_g-p_k(t)K\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\\
\text{where}\qquad & Y_g=X^{\beta}\text{ and } L_r=g(L_m)\equiv\left(1-\log(1-L_m)\right)(1-L_m),
\end{aligned}
$$

and $g(\cdot)$ is a function with the property that $g(0)=1$ and $g(1)=0$. Note that the social planner essentially chooses the level of capital, $K(t)$, and the allocation of labor $L_m(t)$ to manual tasks in the service sector (and thus, also the allocation $L_r(t)=g(L_m(t))$ to routine tasks in the goods sector).

We next characterize the solution to problem (1). The first order conditions with respect to capital K and labor L_m respectively give:

$$
\begin{aligned}
(2)\qquad \frac{\partial Y_g}{\partial K} &= p_k(t),\\
(3)\qquad L_m^{-1/\sigma} &= (Y_g-p_kK)^{-1/\sigma}\frac{\partial Y_g}{\partial X}\frac{\partial X}{\partial L_r}\left(-\log(1-L_m)\right),
\end{aligned}
$$

where we have used

$$g'(L_m)=\log(1-L_m)=-\eta^*.$$

The system in (2) - (3) contains two unknowns (L_m, X) in two equations and uniquely solves for the equilibrium at any time t.

We next characterize the behavior of the solution. We first consider the asymptotic equilibrium as $t\to\infty$ (or equivalently, as $p_k(t)\to 0$). We then characterize the dynamics of this equilibrium.

B. Asymptotic allocation of labor

Note that the intermediate good X is produced with a CES production function with elasticity $\frac{1}{1-\mu} > 1$ over the inputs L_r and K. Bearing in mind that L_r is bounded from above, it can be seen that equation (2) holds as $p_k \to 0$ only if $K \to \infty$. In other words, we have

$$\lim_{t\to\infty} K(t) = \infty. \tag{4}$$

Since L_r is bounded from above, and since L_r and K are gross substitutes in the production of X, the production of X in the limit will be essentially determined by the capital level. Formally, we have

$$\lim_{t\to\infty} X/\alpha_k K = 1.$$

Let $x \sim y$ be a shorthand for the notation that $\lim_{t\to\infty} x/y = 1$. Then, the previous limit expression can be written as

$$X \sim \alpha_k K. \tag{5}$$

Using Eq. (2) and Eq. (3) respectively, we further have

$$Y_g \sim (\alpha_k K)^\beta \text{ and } p_k K \sim \beta (\alpha_k K)^\beta. \tag{6}$$

From these expressions, net output (consumption) satisfies

$$C_g = Y_g - p_k K \sim \kappa_1 K^\beta, \tag{7}$$

where we define $\kappa_1 \equiv (1-\beta)\,\alpha_k^\beta$. Using the expressions in Eq. (3), it can be seen that the asymptotic manual labor choice $L_m^* \equiv \lim_{p_k\to 0} L_m(p_k)$ is the solution to:

$$(L_m^*)^{-1/\sigma} = \kappa_1^{-1/\sigma}\kappa_2 K^{\beta-\mu-\beta/\sigma} L_r^{\mu-1}\left(-\log\left(1-L_m^*\right)\right) \tag{8}$$

where $L_r = g(L_m^*)$, and we define $\kappa_2 \equiv \beta\alpha_k^{\beta-\mu}\alpha_r^\mu$.

Using this equation and Eq. (4), the asymptotic level of L_m^* is uniquely solved as follows:[1]

$$
L_m^* = \begin{cases} 1 & \text{if } \frac{1}{\sigma} > \frac{\beta-\mu}{\beta} \\ \bar{L}_m \in (0,1) & \text{if } \frac{1}{\sigma} = \frac{\beta-\mu}{\beta} \\ 0 & \text{if } \frac{1}{\sigma} < \frac{\beta-\mu}{\beta} \end{cases} . \tag{9}
$$

C. Dynamics of equilibrium in the aggregate economy case

Recall that L_r is bounded from above and K limits to ∞ (cf. Eq. (4)). Hence, $\frac{L_r(t)}{K(t)}$ will be decreasing for sufficiently large t. Suppose that $p_k(0)$ is sufficiently small so that $\frac{L_r(t)}{K(t)}$ is decreasing for all t (intuitively, use of machines relative to routine labor monotonically increases). Under this initial parameterization, the dynamics of the model are straightforward. Note that:

$$
\frac{X}{K} = \left[\alpha_r^\mu \left(\frac{L_r}{K}\right)^\mu + \alpha_k^\mu\right]^{1/\mu}
$$

will be strictly decreasing and it will limit to α_κ. Then, in summary, we have the following dynamics:

$$
X \sim \alpha_k K, \qquad Y_g \sim \alpha_k^\beta K^\beta, \qquad p_k K \sim \beta \alpha_k^\beta K^\beta, \text{ and } C_g \sim \kappa_1 K^\beta. \tag{10}
$$

The dynamics of $L_m(t)$ can be obtained by using these expressions in Eq. (3).

D. Asymptotic wages

We normalize the price of the good g to 1 at each time t. Factors are paid their marginal products. Hence,

$$
w_a = \frac{dY_g}{dL_a} = (1-\beta)\, Y_g \sim \kappa_1 K^\beta, \tag{11}
$$

where the last line uses the dynamics in Eq. (10). Similarly, note that

$$
w_r = \frac{\partial Y_g}{\partial X}\frac{\partial X}{\partial L_r} = \beta X^{\beta-\mu} \alpha_r^\mu g\,(L_m)^{\mu-1} \sim \kappa_2 K^{\beta-\mu} g\,(L_m)^{\mu-1} \tag{12}
$$

[1]Here, $\bar{L}_m$ is the solution to the equation:

$$
\left(\bar{L}_m\right)^{-1/\sigma} = \kappa_1^{-1/\sigma}\kappa_2 g\left(\bar{L}_m\right)^{\mu-1}\left(-\log\left(1-\bar{L}_m\right)\right).
$$

where we used $L_r = g(L_m)$.[2]

Finally,

$$w_m = p_s = \left(\frac{C_s}{C_g}\right)^{-1/\sigma} = (L_m)^{-1/\sigma} C_g^{1/\sigma} \sim (L_m)^{-1/\sigma} \kappa_1^{1/\sigma} K^{\beta/\sigma}. \tag{13}$$

E. Asymptotic wage ratios

From these expressions, relative wages and their dynamics can be determined. We are most interested in $\frac{w_m}{w_r}$, the relative wage of low skill workers in goods versus services production. To obtain the asymptotics of this ratio, note that the first order condition (3) can also be written as:

$$w_m = w_r \eta^* = w_r \left(-\log\left(1 - L_m^*\right)\right).$$

Then, using the characterization in (9), we have

$$\frac{w_m}{w_r} = \begin{cases} \infty \text{ if } \frac{1}{\sigma} > \frac{\beta-\mu}{\beta} \\ -\log\left(1 - L_m^*\right) \text{ if } \frac{1}{\sigma} = \frac{\beta-\mu}{\beta} \\ 0 \text{ if } \frac{1}{\sigma} < \frac{\beta-\mu}{\beta} \end{cases}. \tag{14}$$

We are also interested in the behavior of the ratio $\frac{w_a}{w_m}$. Using equations (11) and (13), we have

$$\frac{w_a}{w_m} \sim \frac{\kappa_1 K^{\beta}}{(L_m)^{-1/\sigma} \kappa_1^{1/\sigma} K^{\beta/\sigma}}. \tag{15}$$

If $\frac{1}{\sigma} > \frac{\beta-\mu}{\beta}$, then equation (15) shows that the asymptotic behavior of $\frac{w_a}{w_m}$ depends on σ. In particular,

$$\frac{w_a}{w_m} = \begin{cases} 0 \text{ if } \sigma < 1 \\ 1 \text{ if } \sigma = 1 \\ \infty \text{ if } \sigma > 1 \end{cases}, \text{ when } \frac{1}{\sigma} > \frac{\beta-\mu}{\beta}. \tag{16}$$

If instead $\frac{1}{\sigma} < \frac{\beta-\mu}{\beta}$ (which is greater than 1), then Eq. (9) shows that $L_m^* = 0$.

[2]Note that, unlike w_a, the dynamic behavior of w_r is not necessarily monotonic. In particular, Eq. (12) can also be written as

$$w_r = \frac{\partial Y_g}{\partial K} \frac{\partial X/\partial L_r}{\partial X/\partial K} = p_k \frac{\alpha_r^u}{\alpha_k^u} \left(\frac{L_r}{K}\right)^{\mu-1}.$$

The fact that p_k is decreasing drives down w_r because routine labor and machines are gross substitutes. On the other hand, $\frac{L_r}{K}$ is falling because of the increases in capital use. When $\mu < 1$ (so that the inputs are not perfect substitutes), the increase in the use of the complementary factors (capital) also tends to push up the wages of routine labor. Hence, the dynamic path of routine wages might be non-monotonic.

But then, Eq. (15) shows that the ratio $\frac{w_a}{w_m}$ is indeterminate. This indeterminacy follows from a rather superficial reason. Although employment in the service sector limits to zero, the wages of the few remaining workers in this sector near the limit may be high. This suggests that the right object to consider may be the *wage bill* of manual labor. When we consider this object, we indeed have:

$$\lim_{t\to\infty} \frac{L_a w_a}{L_m w_m} \sim \frac{\kappa_1 K^{\beta}}{(L_m)^{1-1/\sigma} \kappa_1^{1/\sigma} K^{\beta/\sigma}} = 0,$$

where the last equality follows because $\sigma > 1$ (so that $1-1/\sigma > 0$, and $(L_m)^{1-1/\sigma} = 0$).

Lastly, we derive the dynamics of the wage ratio between abstract and routine tasks. Eqs. (11) and (12) show that

$$\lim_{t\to\infty} \frac{w_a}{w_r} = \frac{\kappa_1 K^{\beta}}{\kappa_2 K^{\beta-\mu} g\left(L_m\right)^{\mu-1}} = \frac{\kappa_1 K^{\mu}}{\kappa_2 g\left(L_m\right)^{\mu-1}} = \infty \text{ when } \frac{1}{\sigma} \le \frac{\beta-\mu}{\beta},$$

where the last equality follows since $K \to \infty$, and since $L_r = g\left(L_m\right) > 0$ when $\frac{1}{\sigma} < \frac{\beta-\mu}{\beta}$ (so that $g\left(L_m\right)^{\mu-1}$ is bounded from above). But the empirically relevant case corresponds to the parametric condition $\frac{1}{\sigma} > \frac{\beta-\mu}{\beta}$. In this case, the ratio $\frac{w_a}{w_r}$ does not necessarily limit to ∞, because $L_r = g\left(L_m\right)$ decreases to zero, and $g\left(L_m\right)^{\mu-1}$ might also limit to ∞ (it does so when $\mu < 1$). Note, however, that in this case, the wages of routine labor are kept high for a reason analogous to above: routine tasks are not very important in production, and thus the economy allocates labor away from routine tasks; as there are very few workers remaining in routine tasks, each might be receiving a significant wage.

This intuition suggests that the routine sector overall should be receiving a lower wage payment, even though each routine worker might be receiving a high wage. In other words, the intuition suggests that we should instead attempt to prove the following:

$$\lim_{t\to\infty} \frac{L_a w_a}{L_r w_r} = \infty \text{ when } \frac{1}{\sigma} > \frac{\beta-\mu}{\beta}. \tag{17}$$

That is, the share of abstract labor relative to the share of routine labor limits to infinity. To prove this, consider Eqs. (11) and (12) (and use $L_r = g\left(L_m\right)$) to get:

$$\lim_{t\to\infty} \frac{L_a w_a}{L_r w_r} = \lim_{t\to\infty} \frac{\kappa_1 K^{\beta}}{\kappa_2 K^{\beta-\mu} g\left(L_m\right)^{\mu}} = \frac{\kappa_1}{\kappa_2}\left(\frac{K}{g\left(L_m\right)}\right)^{\mu} = \infty,$$

where the last equality follows since K increases but $g\left(L_m\right)$ is bounded from above. This proves the limit in (17), and completes our analysis for relative wages.

F. Derivation of spatial equilibrium

Let $\{L_{a,j}(t), L_{m,j}(t), K_j(t)\}$ and $\{w_{m,j}(t), w_{a,j}(t), w_{s,j}(t), p_s(t)\}$ denote the factor allocations and prices in region j in the asymptotic equilibrium. As in the above static economy, we normalize the good price in each region to 1, i.e., $p_{g,j}(t) = 1$ for each j. Define also the ideal price index for the consumption aggregator, $\left(C_{s,j}^{\frac{\sigma-1}{\sigma}} + C_{g,j}^{\frac{\sigma-1}{\sigma}}\right)^{\sigma/(\sigma-1)}$, as

$$P_j(t) = \left(p_{s,j}(t)^{1-\sigma} + 1\right)^{1/(1-\sigma)}. \tag{18}$$

Here, $P_j(t)$ is the cost of increasing the consumption aggregator by one unit. The spatial equilibrium condition in the high skill labor market can be written as

$$L_{a,j}(t) > 0 \text{ only if } w_{a,j}(t)/P_j(t) = w_{a,j^{\max}}(t)/P_{j^{\max}}(t). \tag{19}$$

A geographic equilibrium at time t is a collection of factor allocations $\{L_{a,j}(t), L_{m,j}(t), K_j(t)\}$, and prices $\{w_{m,j}(t), w_{a,j}(t), w_{s,j}(t), p_{s,j}(t)\}$, such that two conditions hold:

1) Local market equilibrium: The allocations $\{L_{a,j}(t), L_{m,j}(t), K_j(t)\}$ and prices $\{w_{m,j}(t), w_{a,j}(t), w_{s,j}(t), p_{s,j}(t)\}$ constitute a static equilibrium of the region j given high skill labor supply $L_{a,j}(t)$ and the price $p_k(t)$ (as described in the previous section).

2) Spatial equilibrium: The market for high skill labor is in spatial equilibrium when (19) holds for each region j, so that high skill workers have identical real earnings across all regions.

We conjecture that the asymptotic equilibrium allocations take the following form:

$$\begin{aligned} \frac{\dot{K}_j(t)}{K_j(t)} &= g_{K,j} \text{ for some } g_{K,j} > 0, \text{ for each } j, \\ L_{m,j}(t) &\to 1, \\ L_{a,\bar{j}}(t) &\to L_a \equiv \sum_j L_{a,j}.\text{for one region } \bar{j} \text{ (in particular, } g_{L,\bar{j}} = 0). \\ \frac{\dot{L}_{a,j}(t)}{L_{a,j}(t)} &= g_{L,j} \text{ for some } g_{L,j} < 0, \text{ for each region } j \neq \bar{j}. \end{aligned} \tag{20}$$

The rationale behind this equilibrium conjecture is as follows: The conjecture that $K_j(t)$ grows in every region intuitively follows from the fact that $p_K(t) \to 0$. The conjecture that $K_j(t)$ grows at a constant rate follows from the analysis in the previous section, which shows that the production function is asymptotically Cobb-Douglas. The conjecture that $L_{m,j}(t) \to 1$ follows from the parametric assumption $\sigma = 1$, which ensures that each region in isolation would allocate all low skill labor to manual tasks.

The last two conjectures rely on the observation that all other factor allocations asymptotically grow at the constant rate. This suggests that high skill labor also asymptotically grows at a constant rate. However, this is only possible if all high skill labor is eventually allocated to a single region, and the rest of the regions lose high skill labor at an asymptotically constant rate (it is also possible if high skill labor is asymptotically constant in all regions, but this case can be ruled out). The identity of the region, $\bar{j}$, along with the constant growth terms $\{g_{K,j}, g_{L,j}\}$, are yet to be determined. We also conjecture that

$$g_{Y,j} \equiv g_{L,j}(1-\beta_j) + \beta_j g_{K,j} > 0 \text{ for each } j \tag{21}$$

(which will be verified below), which ensures that $Y_{g,j}(t)$ and $C_{g,j}(t)$ asymptotically grow at a positive rate.

Under these conjectures, much of the discussion of the closed economy model also applies to each region in spatial equilibrium. In particular, it can be seen that

$$\begin{aligned} X_j(t) &\sim \alpha_k K_j(t), \\ p_k(t) K(t) &\sim \beta L_{a,j}(t)^{1-\beta} (\alpha_k K_j(t))^{\beta}, \\ C_{g,j}(t) &\sim \kappa_1 L_{a,j}(t)^{1-\beta} (\alpha_k K_j(t))^{\beta}. \end{aligned} \tag{22}$$

Moreover, $C_{g,j}(t)$ and $p_k(t) K(t)$ also grow at the constant rate $g_{Y,j} > 0$ (defined in (21)). This also implies that $g_{K,j} = g_{Y,j} + \delta$. Plugging in the definition of (21), we can also solve for $g_{K,j}$ in terms of $g_{L,j}$:

$$g_{K,j} = g_{L,j} + \frac{\delta}{1-\beta_j}. \tag{23}$$

This expression is intuitive. On the one hand, capital grows in response to the technological progress. On the other hand, capital growth is potentially slowed by the fact that high skill labor may be leaving a region (note that, under our conjecture, $g_{L,j}$ is negative for all regions but one).

Wages in each region may be calculated exactly as in the closed economy model. Thus we have:

$$\begin{aligned} w_{s,j}(t) &\sim \kappa_1 L_{a,j}(t)^{-\beta_j} K(t)^{\beta_j}, \\ w_{m,j}(t) &= \left(\frac{C_{g,j}(t)}{C_{s,j}(t)}\right)^{1/\sigma} \sim \kappa_1^{1/\sigma} L_{a,j}(t)^{(1-\beta_j)/\sigma} K_j(t)^{\beta_j/\sigma}, \\ w_{a,j}(t) &= \kappa_1 L_{a,j}(t)^{-\beta_j} K_j(t)^{\beta_j}. \end{aligned} \tag{24}$$

Note that $w_{m,j}(t)$ grows at rate $g_{Y,j}/\sigma$. In particular $w_{m,j}(t) \to \infty$. Using this observation and $p_{s,j}(t) = w_{m,j}(t)$, Eq. (18) can be simplified to

$$P_j(t) \sim w_{m,j}(t)^{1/2} \text{ for each } j. \tag{25}$$

Moreover, under the conjecture in (20), the labor market equilibrium condition (19) will be satisfied with equality for each region. Substituting Eq. (25), the labor market equilibrium condition can be written as follows:

$$\frac{w_{a,j}(t)}{w_{m,j}(t)^{1/2}} \sim \omega(t) \text{ for each } j \tag{26}$$

where $\omega(t)$ is a function that is independent of region j.

G. *Equilibrium mobility and wages of high skill labor*

The spatial equilibrium condition for the high skill labor market can be written as

$$L_{a,j}(t) > 0 \text{ only if } w_{a,j}(t)/P_j(t) = w_{a,j^{\max}}(t)/P_{j^{\max}}(t). \tag{27}$$

where

$$P_j(t) = \left(p_{s,j}(t)^{1-\sigma} + 1\right)^{1/(1-\sigma)} \tag{28}$$

is the cost of increasing the consumption aggregator in local labor market j, $\left(C_{s,j}^{\frac{\sigma-1}{\sigma}} + C_{g,j}^{\frac{\sigma-1}{\sigma}}\right)^{\sigma/(\sigma-1)}$, by one unit.[3] Eq. (27) says that region j will have non-zero abstract labor supply if its real wage for abstract tasks matches the real abstract wage in the region $j^{\max}$.

There are two forces operating in equation (28) that influence the decision of high skill labor to migrate. First, an increase in the skilled wage $w_{a,j}(t)$ creates an incentive for high skill labor to migrate to region j. Second, an increase in local prices $P_j(t)$ creates an incentive for high skill labor to migrate *away* from region j. The equilibrium allocation of skilled labor balances these two forces, so that high skill workers have identical real earnings across all regions.

Using the expressions for wages in (24), the labor market equilibrium condition in (26) can be written as:

$$\frac{\kappa_1 L_{a,j}(t)^{-\beta_j} K_j(t)^{\beta_j}}{\kappa_1^{1/2} L_{a,j}(t)^{(1-\beta_j)/2} K_j(t)^{\beta_j/2}} = \omega(t). \tag{29}$$

[3] As in the above static economy, we normalize the goods price in each region to 1, i.e., $p_{g,j}(t) = 1$ for each j.

where

$$\lim_{t\to\infty}\frac{w_{a,j}(t)}{w_{m,j}(t)^{1/2}}=\omega(t)\ \text{ for each } j$$

and $\omega(t)$ is a function that is independent of region j. The term $K_j(t)^{\beta_j}$ in the numerator of this expression captures the positive effect of capital growth on the share of high skill labor (since the two factors are complements). The term $L_{a,j}(t)^{-\beta_j}$ in the numerator captures the effect of the scarcity of high skill labor on wages. The denominator of this expression captures the effect of the capital growth on the price of service goods: as the economy grows faster, services (which are produced by scarce factors) become more expensive, which has a negative effect on the welfare of a high skill worker. In equilibrium, labor flows across regions until these forces are in balance and equation (29) is satisfied for each j.

Using the conjectures that $K_j(t)$ and $L_{a,j}(t)$ grow (or shrink) at asymptotically constant rates, equation (29) holds only if:

$$\beta_j\frac{g_{K,j}}{2}=\left(\frac{1-\beta_j}{2}+\beta_j\right)g_{L,j}+\eta\text{ for each } j,$$

where η is some constant. Recall that region $\bar{j}$ has asymptotically zero high skill labor growth (see the conjecture in (20)). Hence, considering the previous equation for region $\bar{j}$ gives $\eta=\beta_j\frac{\sigma-1}{\sigma}g_{K,\bar{j}}$, which implies

$$\beta_j\frac{g_{K,j}}{2}-\beta_{\bar{j}}\frac{g_{K,\bar{j}}}{2}=\left(\frac{1-\beta_j}{2}+\beta_j\right)g_{L,j}\text{ for each } j.$$

Plugging in the expression (23) and solving for $g_{L,j}$ gives:

$$g_{L,j}=\frac{\delta}{1-\beta_j}-\frac{\delta\beta_{\bar{j}}}{1-\beta_{\bar{j}}}\text{ for each } j. \tag{30}$$

Using Eq. (23), the growth rate of capital is also characterized as:

$$g_{K,j}=\delta\left(\frac{\beta_j}{1-\beta_j}-\frac{\beta_{\bar{j}}}{1-\beta_{\bar{j}}}\right)\text{ for each } j. \tag{31}$$

Note that the conjecture $g_{L,j}<0$ for each $j\neq\bar{j}$ holds only if $\beta_j<\beta_{\bar{j}}$ for each $j\neq\bar{j}$. This implies that $\bar{j}=j^{\max}$. In other words, high skill labor is attracted at a constant rate to the region with the greatest β_j, which is the region that benefits most from the declining price of computer capital.[4]

[4]To see the intuition for this equilibrium, consider a setting in which high skill labor is initially at a positive constant level in each region. Regions with greater β_j will have faster growth of capital and goods consumption. With a unit elasticity of substitution between goods and services, this leads to a proportional effect on the price of services. Also because $\sigma=1$, the good Y_g asymptotically has a positive

As skilled labor leaves other regions $j \neq \bar{j}$, the price of services decreases in these regions and the welfare of high skill workers increases. In equilibrium, the rate of skilled labor's departure from other regions ensures that in every period t, the remaining high skill workers are indifferent between their current geographic region and all alternatives. In the asymptotic equilibrium, all high skill labor is attracted to the region $\bar{j} = j^{\max}$ with the highest β_j.

Eqs. (30) and (31) completely characterize the constant growth rates in (20). It can be checked that the constructed allocation is an equilibrium.

share of the consumption aggregator, which implies that $P_j(t) = w_{m,j}(t)^{1/2}$ grows at a rate slower than $w_{m,j}(t)$. Hence, regions with greater β_j have faster growth of $w_{a,j}(t)$ and identical growth of $w_{m,j}(t)$, and therefore faster growth of welfare for high skill labor, $w_{a,j}(t)/w_{m,j}(t)^{1/2}$. Moreover, welfare rises equivalently for low skill workers in high β_j regions since productivity gains accrue proportionately to both skill groups given Cobb-Douglas preferences.