Online Appendix

Merger Policy with Merger Choice

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Lemma 5. *Consider the function* $H(s_1,...,s_N) = \sum_n (s_n)^2$ *and two vectors* $\mathbf{s}' = (s'_1,...,s'_N)$ *and* $\mathbf{s}'' = (s''_1,...,s''_N)$ *having* $\sum_{n=1}^N s'_n = \sum_{n=1}^N s''_n$. *If for some* r, *(i)* $s'_r \geq s'_j$ *for all* $j \neq r$, *(ii)* $s''_r > s'_r$, *and (iii)* $s''_j \leq s'_j$ *for all* $j \neq r$, *then* $H(\mathbf{s}'') > H(\mathbf{s}')$.

Proof. Without loss of generality, take $r = 1$ and define $\Delta_n \equiv s'_n - s''_n$ for $n > 1$. Observe that $\Delta_n \geq 0$ for all $n > 1$ and $\Delta_n > 0$ for some $n > 1$. Define as well the vectors $\mathbf{s^n} \equiv (s'_1 + \sum_{t=2}^n \Delta_t, s'_2 - \Delta_2, ..., s'_n - \Delta_n, s'_{n+1}, ..., s'_N)$ for $n > 1$ and $\mathbf{s^1} \equiv \mathbf{s}'$. Note that $\mathbf{s^N} = \mathbf{s}''$. Then

$$H(\mathbf{s}'') - H(\mathbf{s}') = \sum_{n=1}^{N-1} [H(\mathbf{s^{n+1}}) - H(\mathbf{s^n})].$$

Now letting $\overline{s}_1^1 \equiv s'_1$ and $\overline{s}_1^n \equiv s'_1 + \sum_{t=2}^n \Delta_t \geq s'_1$ for all $n > 1$, each term in this sum is nonnegative,

$$\begin{aligned} H(\mathbf{s^{n+1}}) - H(\mathbf{s^n}) &= (\overline{s}_1^n + \Delta_{n+1})^2 + (s'_{n+1} - \Delta_{n+1})^2 - (\overline{s}_1^n)^2 - (s'_{n+1})^2 \\ &= 2\Delta_{n+1}(\overline{s}_1^n - s'_{n+1}) + 2(\Delta_{n+1})^2 \geq 0, \end{aligned}$$

and strictly positive if $\Delta_{n+1} > 0$. Since $\Delta_{n+1} > 0$ for some $n \geq 1$, the result follows. □

Proof of Proposition 2. Denote by $\mathcal{A}^*(\overline{\Delta\Pi}|J)$ a policy that is an element of $\arg\max_{\mathcal{A} \subseteq \Pi_{k \in J}[l,h_k]} ECS(\overline{\Delta\Pi}; \mathcal{A}, J)$ for a given J and $\overline{\Delta\Pi}$. Also, define $P(\overline{\Delta\Pi}|J, \mathcal{A}) \equiv \{k \in J : \Delta\Pi(k, \overline{a}_k) < \overline{\Delta\Pi}\}$ as the set of targets in J who may have an acceptable merger with profit below $\overline{\Delta\Pi}$ under policy $\mathcal{A} \subseteq \Pi_{k \in J}[l, h_k]$. Note that changes to $\mathcal{A}$ that alter acceptance sets only for $k \notin P(\overline{\Delta\Pi}|J, \mathcal{A})$ and leave $P(\overline{\Delta\Pi}|J, \mathcal{A})$ unchanged have no effect on the value of $ECS(\overline{\Delta\Pi}; \mathcal{A}, J)$. Finally, for any set $\mathcal{A}$, let $\mathcal{A}_J \equiv \Pi_{j \in J} \mathcal{A}_j$.

With these preliminaries, we now establish the result. Observe first that, for any J, a sufficient condition for M_k with $k \in J$ and $\Delta\Pi(M_k) < \overline{\Delta\Pi}$ to be approved in *any* solution to $\max_{\mathcal{A} \subseteq \Pi_{j \in J}[l,h_j]} ECS(\overline{\Delta\Pi}; \mathcal{A}, J)$ is that its CS-level, $\Delta CS(M_k)$, strictly exceeds $\max_{\mathcal{A} \subseteq \Pi_{j \in J \setminus k}[l,h_j]} ECS(\Delta\Pi(M_k); \mathcal{A}, J \setminus k)$. We will establish the result through an induction argument that shows that for all $\overline{\Delta\Pi}$ and any J such that $1 \in J$, if $\mathcal{A}^*(\overline{\Delta\Pi}|J) \in \arg\max_{\mathcal{A} \subseteq \Pi_{j \in J}[l,h_j]} ECS(\overline{\Delta\Pi}; \mathcal{A}, J)$ then

$$P(\overline{\Delta\Pi}|J, \mathcal{A}^*(\overline{\Delta\Pi}|J)) = P(\overline{\Delta\Pi}|J, \mathcal{A}^C(J)) \tag{25}$$

and

$$\mathcal{A}_k^*(\overline{\Delta\Pi}|J) = \mathcal{A}_k^C(J) \text{ for all } k \in P(\overline{\Delta\Pi}|J, \mathcal{A}^C(J)). \tag{26}$$

That is, any policy $\mathcal{A}$ that maximizes $ECS(\overline{\Delta\Pi}; \mathcal{A}, J)$ accepts with positive probability [conditional on the most profitable acceptable merger having $\Delta\Pi(M_j) \leq \overline{\Delta\Pi}$] mergers involving the same set of targets as does the cutoff policy $\mathcal{A}^C(J)$, and coincides with the cutoff policy $\mathcal{A}^C(J)$ for all such targets. In particular, this implies that the cutoff policy $\mathcal{A}^C(J) \in \arg\max_{\mathcal{A}\subseteq \Pi_{k\in J}[l,h_k]} ECS(\overline{\Delta\Pi}, \mathcal{A}, J)$ for all $\overline{\Delta\Pi}$ and any J such that $1 \in J$. Taking $\overline{\Delta\Pi} = \infty$ and $J = \mathcal{K}$ will then yield the result.

Consider first the set $J = \{1\}$. Then, we have $\overline{a}_1^C(J) = \widehat{c}_1(Q^\circ)$. Moreover, it is immediate – given our earlier discussion – that (25) and (26) hold for all $\overline{\Delta\Pi}$.

Now consider any set $J = J_n$ with $\#J_n = n$ and $1 \in J_n$, and assume:

Induction Hypothesis 1: Properties (25) and (26) hold for any set $J = J_{n'}$ with $1 \in J_{n'}$ and $n' < n$.

Number the targets in set J_n in increasing order of their pre-merger market share as $(1, ..., n)$. If $P(\overline{\Delta\Pi}|J, \mathcal{A}^C(J)) = \varnothing$, then $\overline{\Delta\Pi} \leq \Delta\Pi(1, \widehat{c}_1(Q^\circ))$. From Proposition 1, it follows immediately that $P(\overline{\Delta\Pi}|J, \mathcal{A}^*(\overline{\Delta\Pi}|J)) = \varnothing$. Hence, properties (25) and (26) hold for set J_n.

So suppose now instead that $P(\overline{\Delta\Pi}|J, \mathcal{A}^C(J)) \neq \varnothing$. Note that, since $\Delta\Pi(k, \overline{a}_k^C(J))$ is increasing in k, the set $P(\overline{\Delta\Pi}|J, \mathcal{A}^C(J))$ is of the form $P(\overline{\Delta\Pi}|J, \mathcal{A}^C(J)) = \{1, ..., \widehat{j}(\overline{\Delta\Pi})\}$ for some $\widehat{j}(\overline{\Delta\Pi})$.

Consider first the treatment of mergers with target 1. We have $\overline{a}_1^C(J) = \widehat{c}_1(Q^\circ)$. Moreover, the following two properties hold for all $\overline{\Delta\Pi}$: for any $\mathcal{A}^*(\overline{\Delta\Pi}|J_n) \in \arg\max_{\mathcal{A}\subseteq \Pi_{j\in J_n}[l,h_j]} ECS(\overline{\Delta\Pi}; \mathcal{A}, J_n)$,

$$1 \in P(\overline{\Delta\Pi}|J_n, \mathcal{A}^*(\overline{\Delta\Pi}|J_n)) \Leftrightarrow 1 \in P(\overline{\Delta\Pi}|J_n, \mathcal{A}^C(J_n)) \tag{27}$$

and

$$\mathcal{A}_1^*(\overline{\Delta\Pi}|J_n) = \mathcal{A}_1^C(J_n) \text{ if } 1 \in P(\overline{\Delta\Pi}|J_n, \mathcal{A}^C(J_n)). \tag{28}$$

These follow from the following facts: (i) No CS-decreasing merger M_1 can be accepted in $\mathcal{A}^*(\overline{\Delta\Pi}|J_n)$; (ii) for any $\mathcal{A} \subseteq \Pi_{j\in J_n}[l, h_j]$, $ECS(\Delta\Pi(1, \overline{c}_1); \mathcal{A}, J_n) < \Delta CS(1, \overline{c}_1)$ for all $\overline{c}_1 < \widehat{c}_1(Q^\circ)$, so all mergers $M_1 = (1, \overline{c}_1)$ such that $\overline{c}_1 < \widehat{c}_1(Q^\circ)$ and $\Delta\Pi(1, \overline{c}_1) \leq \overline{\Delta\Pi}$ must be in $\mathcal{A}^*(\overline{\Delta\Pi}|J_n)$, and (iii) accepting all mergers M_1 such that $\Delta\Pi(1, \overline{c}_1) > \overline{\Delta\Pi}$ maximizes $\Pr(\Delta\Pi(M_1) > \overline{\Delta\Pi}$ and $M_1 \in \mathcal{A}_1)$ and, since accepting the mergers described in (ii) is optimal, therefore maximizes $ECS(\overline{\Delta\Pi}; \mathcal{A}, J_n)$.

Now, consider a merger with target $k > 1$ and assume:

Induction Hypothesis 2: For all $k' < k$, the following two properties hold for all $\overline{\Delta\Pi}$: for any $\mathcal{A}^*(\overline{\Delta\Pi}|J_n) \in \arg\max_{\mathcal{A}\subseteq \Pi_{j\in J_n}[l,h_j]} ECS(\overline{\Delta\Pi}; \mathcal{A}, J_n)$,

$$k' \in P(\overline{\Delta\Pi}|J_n, \mathcal{A}^*(\Delta\Pi|J_n)) \Leftrightarrow k' \in P(\overline{\Delta\Pi}|J_n, \mathcal{A}^C(J_n)) \tag{29}$$

and

$$\mathcal{A}_{k'}^*(\overline{\Delta\Pi}|J_n) = \mathcal{A}_{k'}^C(J_n) \text{ if } k' \in P(\overline{\Delta\Pi}|J_n, \mathcal{A}^C(J_n)). \tag{30}$$

We will show that properties (29) and (30) hold as well for k so that Induction Hypothesis 2 holds for $k + 1$. Suppose, first, that $k \notin P(\overline{\Delta\Pi}|J_n, \mathcal{A}^C(J_n))$. Then every M_k with $\Delta\Pi(M_k) \leq \overline{\Delta\Pi}$ has $ECS(\Delta\Pi(M_k); \mathcal{A}^C_{J_n\backslash k}(J_n), J_n\backslash k) > \Delta CS(M_k)$. But by Induction Hypothesis 2 and Proposition 1 [which implies that in $\mathcal{A}^*(\overline{\Delta\Pi}|J_n)$ we must have $\underline{\Delta\Pi}_k < \underline{\Delta\Pi}_j$ for any $j > k$ such that $j \in P(\overline{\Delta\Pi}|J_n, \mathcal{A}^*(\overline{\Delta\Pi}|J_n))$], which implies that $ECS(\Delta\Pi(M_k); \mathcal{A}^*_{J_n\backslash k}(\overline{\Delta\Pi}|J_n), J_n\backslash k) = ECS(\Delta\Pi(M_k); \mathcal{A}^C_{J_n\backslash k}(J_n), J_n\backslash k)$. Hence, merger M_k cannot be in $\mathcal{A}^*(\overline{\Delta\Pi}|J_n)$; i.e., $k \notin P(\overline{\Delta\Pi}|J_n, \mathcal{A}^*(\overline{\Delta\Pi}|J_n))$.

Suppose now instead that $k \in P(\overline{\Delta\Pi}|J, A^C(J_n))$. Observe, first, that every $M_k = (k, \overline{c}_k)$ with $\overline{c}_k > \overline{a}_k^C(J_n)$ has $ECS(\Delta\Pi(M_k); \mathcal{A}^C_{J_n \backslash k}(J_n), J_n \backslash k) > \Delta CS(M_k)$, and since by Induction Hypothesis 2 and Proposition 1, $ECS(\Delta\Pi(M_k); \mathcal{A}^*_{J_n \backslash k}(\overline{\Delta\Pi}|J_n), J_n \backslash k) = ECS(\Delta\Pi(M_k); \mathcal{A}^C_{J_n \backslash k}(J_n), J_n \backslash k)$, the merger cannot be in $\mathcal{A}^*(\overline{\Delta\Pi}|J_n)$; i.e., $\mathcal{A}^*_k(\overline{\Delta\Pi}|J_n) \subseteq \mathcal{A}^C_k(J_n)$. Next, consider mergers $M_k = (k, \overline{c}_k)$ with $\overline{c}_k < \overline{a}_k^C(J_n)$. Condition (12) combined with Induction Hypotheses 1 and 2 imply that each of these mergers satisfies $\Delta CS(M_k) > ECS(\Delta\Pi(M_k); \mathcal{A}^C(J_n \backslash k), J_n \backslash k) = \max_{\mathcal{A} \subseteq \Pi_{j \in J_n \backslash k}[l, h_j]} ECS(\Delta\Pi(M_k), \mathcal{A}, J_n \backslash k)$, and hence must be included in $\mathcal{A}^*(\overline{\Delta\Pi}|J_n)$; i.e., $\mathcal{A}^C_k(J_n) \subseteq \mathcal{A}^*_k(\overline{\Delta\Pi}|J_n)$. We thus have $\mathcal{A}^C_k(J_n) = \mathcal{A}^*_k(\overline{\Delta\Pi}|J_n)$. Hence, properties (29) and (30) hold as well for k. Applying induction (twice) then yields the result. $\square$

Proof of Proposition 3. Let $\mathcal{A}$ denote the optimal approval policy with cutoffs $(\overline{a}_1, ..., \overline{a}_K)$, and $\mathcal{F}$ the set of feasible mergers, when $\Pr(\phi_k = 1) = \theta_k$. Similarly, let $\mathcal{A}'$ denote the optimal approval policy with cutoffs $(\overline{a}'_1, ..., \overline{a}'_K)$, and $\mathcal{F}'$ the set of feasible mergers, when $\Pr(\phi_k = 1) = \theta'_k$. From the recursive definition of the cutoffs, it follows immediately that a change in θ_k does not affect the cutoffs for any smaller merger M_j, $j < k$, nor the cutoff of merger M_k itself. Hence, $\underline{\Delta CS}'_j = \underline{\Delta CS}_j$ for all $j \leq k$.

Consider now the cutoff for merger M_{k+1}, $k+1 \leq \hat{K}$. We can write the cutoff condition as

$$\begin{aligned}
\underline{\Delta CS}_{k+1} &= E_{\mathcal{F}_{\{1,\dots,k\}}} \Delta CS\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \mid \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1})] \\
&= \Pr(\phi_k = 1 | \ \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1})) \\
&\quad \times E_{\mathcal{F}_{\{1,\dots,k\}}} \left[\Delta CS\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \mid \right. \\
&\quad \left. \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1}) \text{ and } \phi_k = 1\right] \\
&\quad + \left[1 - \Pr(\phi_k = 1 | \ \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1})\right] \\
&\quad \times E_{\mathcal{F}_{\{1,\dots,k\}}} \left[\Delta CS\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \mid \right. \\
&\quad \left. \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1}) \text{ and } \phi_k = 0\right],
\end{aligned}$$

where $\mathcal{A}_J \equiv \Pi_{j \in J} \mathcal{A}_j$.

Note first that the optimal policy must be such that

$$\begin{aligned}
& E_{\mathcal{F}_{\{1,\dots,k\}}} \left[\Delta CS\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \mid M_{k+1} = (k+1, \overline{a}_{k+1}), \right. \\
& \left. \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(M_{k+1}), \text{ and } \phi_k = 1\right] \\
> \ & E_{\mathcal{F}_{\{1,\dots,k\}}} \left[\Delta CS\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \mid M_{k+1} = (k+1, \overline{a}_{k+1}), \right. \\
& \left. \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(M_{k+1}), \text{ and } \phi_k = 0\right].
\end{aligned}$$

To see this, consider the case where $\phi_k = 1$ and $\Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1})$. Two cases can arise: (i) $M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right) \neq M_k$ and (ii) $M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right) = M_k$. In case (i), the outcome is the same as if M_k were not feasible ($\phi_k = 0$). In case (ii), merger M_k will be implemented. If merger M_k were not feasible, we would instead obtain the expected consumer surplus of the next-most profitable allowable merger. By the optimality of the approval policy, $\Delta CS(M_k)$ must weakly exceed (and, generically, strictly) the expected consumer surplus of the next-most profitable allowable merger.

Next, note that we can rewrite the conditional probability as

$$\begin{aligned}
& \Pr(\phi_k = 1 | \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1})) \\
= \; & \Pr(\; \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1}) | \phi_k = 1)\theta_k \\
& \times \left\{ \Pr(\Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1}) | \phi_k = 1)\theta_k \right. \\
& \left. + \Pr(\Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1}) | \phi_k = 0)(1-\theta_k) \right\}^{-1} \\
= \; & \left\{ 1 + \frac{\Pr(\Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1}) | \phi_k = 0)}{\Pr(\Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1}) | \phi_k = 1)} \left(\frac{1-\theta_k}{\theta_k}\right) \right\}^{-1}.
\end{aligned}$$

Hence, an increase in θ_k induces an increase in the conditional probability $\Pr(\phi_k = 1 | \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,k\}}, \mathcal{A}_{\{1,\dots,k\}}\right)\right) \leq \Delta\Pi(k+1, \overline{a}_{k+1}))$. But this implies that an increase in θ_k induces an increase in the RHS of the cutoff condition for merger M_{k+1}. Hence, $\underline{\Delta CS}'_{k+1} > \underline{\Delta CS}_{k+1}$.

Consider now the induction hypothesis that $\underline{\Delta CS}'_{k'} > \underline{\Delta CS}_{k'}$ for all $k < k' < j \leq \hat{K}$. In particular, $\underline{\Delta CS}'_{j-1} > \underline{\Delta CS}_{j-1}$. We claim that this implies that $\underline{\Delta CS}'_{j} > \underline{\Delta CS}_{j}$. To see this, note that we can decompose the effect of the increase in θ_k on the conditional expectation of the next-most profitable merger into two steps:

1. Increase the feasibility probability from θ_k to $\theta'_k > \theta_k$, holding fixed the approval policy $\mathcal{A}$.

2. Change the approval policy from $\mathcal{A}$ to $\mathcal{A}'$.

Consider first step (1). For the same reason as before, the increase in the feasibility probability must raise the expected increase in consumer surplus, conditional on the most profitable merger being less profitable than the marginal merger M_j; that is,

$$\begin{aligned}
& E_{\mathcal{F}_{\{1,\dots,j-1\}}}\left[\Delta CS(M^*(\mathcal{F}_{\{1,\dots,j-1\}}, \mathcal{A}_{\{1,\dots,j-1\}})) \mid \Delta\Pi\left(M^*\left(\mathcal{F}_{\{1,\dots,j-1\}}, \mathcal{A}_{\{1,\dots,j-1\}}\right)\right) \leq \Delta\Pi(j, \overline{a}_j)\right] \\
< \; & E_{\mathcal{F}'_{\{1,\dots,j-1\}}}\left[\Delta CS(M^*(\mathcal{F}'_{\{1,\dots,j-1\}}, \mathcal{A}_{\{1,\dots,j-1\}})) \mid \Delta\Pi\left(M^*\left(\mathcal{F}'_{\{1,\dots,j-1\}}, \mathcal{A}_{\{1,\dots,j-1\}}\right)\right) \leq \Delta\Pi(j, \overline{a}_j)\right].
\end{aligned}$$

Consider now step (2). The outcome under the two policies differs only in the event where $M^*(\mathcal{F}'_{\{1,\dots,j-1\}}, \mathcal{A}_{\{1,\dots,j-1\}}) \notin \mathcal{A}'$. Let $M_i = M^*(\mathcal{F}'_{\{1,\dots,j-1\}}, \mathcal{A}_{\{1,\dots,j-1\}})$. Under policy $\mathcal{A}$, the change in consumer surplus in this event is $\Delta CS(M_i)$. As $M_i \in \mathcal{A}$ and $M_i \notin \mathcal{A}'$ by assumption, we have $\Delta\Pi(i, \overline{c}_i) < \Delta\Pi(k, \overline{a}'_h)$ for all $h \geq i$. Under policy $\mathcal{A}'$, the expected change in consumer surplus in this event is therefore given by

$$\begin{aligned}
& E_{\mathcal{F}'_{\{1,\dots,i-1\}}}\left[\Delta CS(M^*(\mathcal{F}'_{\{1,\dots,i-1\}}, \mathcal{A}'_{\{1,\dots,i-1\}})) \mid \Delta\Pi(M^*(\mathcal{F}'_{\{1,\dots,i-1\}}, \mathcal{A}'_{\{1,\dots,i-1\}})) \leq \Delta\Pi(i, \overline{c}_i)\right] \\
> \; & \Delta CS(M_i),
\end{aligned}$$

where the inequality follows by the optimality of approval policy $\mathcal{A}'$. Thus,

$$\begin{aligned} & E_{\mathcal{F}'_{\{1,\dots,j-1\}}}\left[\Delta CS(M^*(\mathcal{F}'_{\{1,\dots,j-1\}},\mathcal{A}_{\{1,\dots,j-1\}})) \mid \Delta\Pi\left(M^*\left(\mathcal{F}'_{\{1,\dots,j-1\}},\mathcal{A}_{\{1,\dots,j-1\}}\right)\right) \le \Delta\Pi(j,\overline{a}_j)\right] \\ < \; & E_{\mathcal{F}'_{\{1,\dots,i-1\}}}\left[\Delta CS(M^*(\mathcal{F}'_{\{1,\dots,i-1\}},\mathcal{A}'_{\{1,\dots,i-1\}})) \mid \Delta\Pi(M^*(\mathcal{F}'_{\{1,\dots,i-1\}},\mathcal{A}'_{\{1,\dots,i-1\}})) \le \Delta\Pi(i,\overline{c}_i)\right]. \end{aligned}$$

As the expected consumer surplus increases at each step, we must have $\underline{\Delta CS}'_j > \underline{\Delta CS}_j$. □

Efficient Bargaining Among a Subset of Firms

Suppose instead that the outcome of the bargaining process maximizes the joint profit of only a subset of firms, $\mathcal{L}$, that includes firm 0 and all of the targets, i.e., $(\{0\} \cup \mathcal{K}) \subseteq \mathcal{L} \subset \mathcal{N}$. That is, the proposal rule is

$$M^*(\mathcal{F},\mathcal{A}) = \begin{cases} \arg\max_{M_k \in (\mathcal{F}\cap\mathcal{A})} \Delta\Pi_{\mathcal{L}}(M_k) & \text{if } \max_{M_k \in (\mathcal{F}\cap\mathcal{A})} \Delta\Pi_{\mathcal{L}}(M_k) > 0, \\ M^\circ & \text{otherwise.} \end{cases}$$

where $\Delta\Pi_{\mathcal{L}}(M_k)$ now denotes the induced change in the joint profit of the firms in set $\mathcal{L}$, $\Delta\Pi_{\mathcal{L}}(M_k) \equiv \sum_{i \in \mathcal{L}\setminus\{0\}} \pi_i(M_k) - \sum_{i\in\mathcal{L}} \pi_i^0$.

Under the same conditions as in the case of efficient bargaining, Proposition 1 carries over to this bargaining process. The key point is the following: If any CS-nondecreasing merger or any reduction in a merged firm's marginal cost induces an increase in aggregate profit, then it also induces an increase in the joint profit of the firms in set $\mathcal{L}$. This follows because both a CS-nondecreasing merger and a reduction in a firm's post-merger marginal cost weakly reduce the profit of any nonmerging firm, including the firm(s) not in set $\mathcal{L}$. This observation has several implications. First, it means that part (iv) of Lemma 4 continues to hold if we replace aggregate profit by the joint profit of the firms in set $\mathcal{L}$. Second, it also means that a reduction in the post-merger marginal cost $\overline{c}_k$ raises the joint profit of the firms in set $\mathcal{L}$ for any CS-nondecreasing merger. Third, Lemma 4 continues to hold if we replace the induced change in aggregate profit by the induced change in the joint profit of the firms in $\mathcal{L}$. This follows because the two mergers in the statement of the lemma, M_j and M_k, induce (by assumption) the same change in consumer surplus, so the profit of any firm $i \neq j,k$ is the same under both mergers. As a result, we can again draw a figure like Figure 1, and all of the steps in the proof of Proposition 1 carry over to this case.

Differentiated Products

In our analysis we have assumed that firms produce a homogeneous good and compete in a Cournot fashion. Focusing on the case of efficient bargaining between firms, we now show that our main insights carry over to the case where firms compete in prices and produce symmetrically differentiated goods with consumers having CES or multinomial logit demand. These models share with the Cournot model an important property: they are "aggregative games." Using this common structure, we show below that if merger M_k is CS-neutral, then it raises the joint profit of the merging firms as well as aggregate profit (Willingness to Propose). Moreover, a reduction in post-merger marginal cost increases the merged firm's profit and, provided pre-merger differences between firms are not too large, aggregate profit (Monotonicity). In addition, if any two mergers M_j and M_k, $k > j$, induce the same nonnegative change in consumer surplus, then the larger merger M_k induces a greater increase in aggregate profit than the smaller merger M_j (Ordered Bias). In sum, in the two differentiated goods models, the merger curves have the same features in $(\Delta\Pi_I, \Delta CS)$-space as in the Cournot model. Our main result, Proposition 1, therefore carries over as well.

Assumptions. Suppose an unmerged firm i's profit can be written as

$$\pi(\psi_i, c_i; \Psi),$$

where $\psi_i \geq 0$ is firm i's strategic variable, c_i the firm's constant marginal cost, and $\Psi \equiv \sum_j \psi_j$ an aggregator summarizing the "aggregate outcome." The firm's cumulative best reply, $r(\Psi; c_i) \equiv \arg\max_{\psi_i} \pi(\psi_i, c_i; \psi_i + \sum_{j \neq i} \psi_j)$, is assumed to be single-valued and decreasing in marginal cost c_i. Similarly, a merged firm k's profit is given by $2\pi(\psi_k, \overline{c}_k; \Psi)$, and its cumulative best reply, $\overline{r}(\Psi; \overline{c}_k) \equiv \arg\max_{\psi_k} 2\pi(\psi_k, \overline{c}_k; 2\psi_k + \sum_{j \neq 0,k} \psi_j)$, is single-valued and decreasing in $\overline{c}_k$. Consumer surplus, denoted $V(\Psi)$, is an increasing function of the aggregator and does not depend on the composition of the aggregator.

Suppose that there exists a unique stable equilibrium. Let $\psi_i(M_k)$ denote firm i's equilibrium action under market structure M_k, and $\Psi(M_k) \equiv \sum_j \psi_j(M_k)$. Further, suppose that firm i's equilibrium profit can be written as

$$\begin{aligned} g(\psi_i(M_k); \Psi(M_k)) &\equiv \max_{\psi_i} \pi(\psi_i, c_i; \Psi(M_k)) \text{ if firm } i \text{ is unmerged;} \\ g(2\psi_i(M_k); \Psi(M_k)) &\equiv \max_{\psi_i} 2\pi(\psi_i, \overline{c}_i; \Psi(M_k)) \text{ if firm } i = k \text{ is merged.} \end{aligned}$$

The equilibrium profit function g has the following properties: (i) $g(0; \Psi) = 0$; (ii) for $0 \leq \psi_i \leq \Psi$, $g(\psi_i; \Psi)$ is strictly increasing and strictly convex in ψ_i. We assume that a reduction in post-merger marginal cost $\overline{c}_k$ leads to (a) an increase in $\psi_k(M_k)$ and in the aggregate outcome $\Psi(M_k)$; (b) an increase in $\psi_k(M_k)/\Psi(M_k)$ and a decrease in $\psi_j(M_k)/\Psi(M_k)$, $j \neq 0, k$; and (c) an increase in the merged firm's equilibrium profit $g(2\psi_k(M_k), \Psi(M_k))$ and a reduction in any other firm i's equilibrium profit $g(\psi_i(M_k); \Psi(M_k))$.

Our assumptions hold for several textbook models of competition.

Example 3 (Cournot)**.** Consider the homogeneous goods Cournot model with constant marginal costs. All unmerged firms can be thought of as single-plant firms, whereas a merged firm can be thought of as running two plants at the same marginal cost (producing the same output at both plants). We impose the same assumptions on demand as in the main text. Let ψ_i denote the output of plant i and $\Psi \equiv \sum_i \psi_\iota$ aggregate output. Consumer surplus is $V(\Psi) = \int_0^\Psi [P(s) - P(\Psi)]\, ds$, which is strictly increasing in Ψ. The profit maximization problem of a single-plant firm i with marginal cost c_i can be written as

$$\max_{\psi_i} \left[P(\psi_i + \sum_{j \neq i} \psi_j) - c_i \right] \psi_i.$$

From the first-order condition of profit maximization, $P(\Psi) - c_i + \psi_i P'(\Psi) = 0$, we can write the equilibrium profit under merger M_k as

$$g(\psi_i(M_k); \Psi(M_k)) = -\left[\psi_i(M_k)\right]^2 P'(\Psi(M_k)).$$

The profit maximization problem of a merged firm k with marginal cost $\overline{c}_k$ (and two plants) can be written as

$$\max_{\psi_k} \left[P(2\psi_k + \sum_{j \neq 0,k} \psi_j) - \overline{c}_k \right] 2\psi_k.$$

From the first-order condition of profit maximization, $P(\Psi) - \overline{c}_k + 2\psi_k P'(\Psi) = 0$, so that we can write the merged firm's equilibrium profit under merger M_k as

$$g(2\psi_k(M_k); \Psi(M_k)) = -\left[2\psi_k(M_k)\right]^2 P'(\Psi(M_k)).$$

It can easily be verified that g has all of the required properties (it takes the value of zero if its first argument is zero and is increasing and convex in its first argument) and that a reduction in post-merger marginal cost $\overline{c}_k$ has the posited effects. (The other assumptions were shown to hold in the main text.)

Example 4 (CES)**.** In the CES model, the utility function of the representative consumer is given by

$$U = \left(\sum_{i=0}^{N} X_i^{\rho} \right)^{1/\rho} Z^{\alpha},$$

where $\rho \in (0,1)$ and $\alpha > 0$ are parameters, X_i is consumption of differentiated good i, and Z is consumption of the numeraire. Utility maximization implies that the representative consumer spends a constant fraction $1/(1+\alpha)$ of his income Y on the $N+1$ differentiated goods (and the remainder on the numeraire). Using the normalization $Y/(1+\alpha) \equiv 1$, the resulting demand for differentiated good i is

$$X_i = \frac{p_i^{-\lambda-1}}{\sum_{j=0}^{N} p_j^{-\lambda}},$$

where p_i is the price of good i, and $\lambda \equiv \rho/(1-\rho)$. The consumer's indirect utility can be written as

$$V = (1+\alpha)\ln Y + \frac{1}{\lambda} \ln \left(\sum_{j=0}^{N} p_j^{-\lambda} \right). \qquad (31)$$

Suppose that firms compete in prices, and that an unmerged firm produces a single good, while a merged firm produces two goods at the same marginal cost (thus optimally charging the same price for each). Let $\psi_i \equiv p_i^{-\lambda}$ and $\Psi \equiv \sum_i p_i^{-\lambda}$. By (31), $V(\Psi)$ is strictly increasing in Ψ. Consider first a single-product firm i. The profit maximization problem of a single-product firm i with marginal cost c_i can be written as

$$\max_{\psi_i}[\psi_i^{-1/\lambda} - c_i]\frac{\psi_i^{(\lambda+1)/\lambda}}{\psi_i + \sum_{j\neq i}\psi_j}.$$

From the first-order condition of profit maximization,

$$-\Psi + \left[\psi_i^{-1/\lambda} - c_i\right]\psi_i^{(\lambda+1)/\lambda}\left\{\frac{(\lambda+1)\Psi}{\psi_i} - \lambda\right\} = 0,$$

it can be seen that there is a unique cumulative best reply $r(\Psi; c_i)$ and that it is decreasing in the firm's marginal cost c_i. We can write the firm's equilibrium profit under merger M_k as

$$g\left(\psi_i(M_k); \Psi(M_k)\right) \equiv \left\{\frac{(\lambda+1)\Psi(M_k)}{\psi_i(M_k)} - \lambda\right\}^{-1}.$$

Consider now the merged firm k and suppose the firm produces two products at marginal cost $\overline{c}_k$. The profit maximization problem can be written as

$$\max_{\psi_k} 2[\psi_k^{-1/\lambda} - \overline{c}_k]\frac{\psi_k^{(\lambda+1)/\lambda}}{2\psi_k + \sum_{j\neq 0,k}\psi_j}.$$

(It can easily be verified that the firm optimally chooses the same value of ψ_k for each one of its two products.) From the first-order condition,

$$-\Psi + \left[\psi_k^{-1/\lambda} - \overline{c}_k\right]\psi_k^{(\lambda+1)/\lambda}\left\{\frac{(\lambda+1)\Psi}{\psi_k} - 2\lambda\right\} = 0,$$

it can be seen that there is a unique cumulative best reply $\overline{r}(\Psi; \overline{c}_k)$ and that it is decreasing in $\overline{c}_k$. We can write the merged firm's equilibrium profit under merger M_k as

$$g\left(2\psi_k(M_k); \Psi(M_k)\right) \equiv \left\{\frac{(\lambda+1)\Psi(M_k)}{2\psi_k(M_k)} - \lambda\right\}^{-1}.$$

It can easily be verified that our assumptions hold in the CES model. In particular, there exists a unique equilibrium and this equilibrium is stable.[21] Moreover, the equilibrium profit function g has all of the required properties (it takes the value of zero if its first argument is zero and is increasing and

[21] From the first-order condition for profit maximization, we obtain that $dr(\Psi; c_i)/d\Psi$ can be written as a decreasing and convex function of $\beta_i \equiv \Psi/r(\Psi; c_i)$:

$$\frac{dr(\Psi; c_i)}{d\Psi} = \frac{\lambda}{(\lambda+1)\beta_i(\beta_i - 1) + \lambda}.$$

This derivative attains its maximum of 1 if firm i is the only active firm (i.e., $r(\Psi; c_i) = \Psi$). [Similarly, for a merged firm M_k, we have

$$\frac{d2\overline{r}(\Psi; \overline{c}_k)}{d\Psi} = \frac{\lambda}{(\lambda+1)\overline{\beta}_k(\overline{\beta}_k - 1) + \lambda},$$

where $\overline{\beta}_k \equiv \Psi/\left[2\overline{r}(\Psi; \overline{c}_k)\right]$.] It follows that $\sum_i dr(\Psi; c_i)/d\Psi < 1$ [resp. $\sum_{i\neq 0,k} dr(\Psi; c_i)/d\Psi + 2\overline{r}(\Psi; \overline{c}_k)/d\Psi < 1$ after merger M_k] in any equilibrium with more than one active firm. Hence, any equilibrium must be stable. Moreover, as $r(0; c_i) \geq 0$ [resp. $\overline{r}(0; \overline{c}_k) \geq 0$] and $r(\Psi; c_i) = 0$ [resp. $\overline{r}(\Psi; \overline{c}_k) = 0$] for Ψ sufficiently large, this implies that there exists a unique Ψ that is consistent with equilibrium in the sense that $\Psi - \sum_i r(\Psi; c_i) = 0$ [resp. $\Psi - \sum_{i\neq 0,k} r(\Psi; c_i) - 2\overline{r}(\Psi; \overline{c}_k) = 0$ after merger M_k].

convex in its first argument). Consider a reduction in post-merger marginal cost $\overline{c}_k$. Since $\overline{r}(\Psi;\overline{c}_k)$ is decreasing in $\overline{c}_k$ and since $r(\Psi;c_i)$ and $\overline{r}(\Psi;\overline{c}_k)$ are increasing in Ψ, and since equilibrium is stable, the reduction in $\overline{c}_k$ induces a higher value of $\Psi = 2\overline{r}(\Psi;\overline{c}_k) + \sum_{i\neq 0,k} r(\Psi;c_i)$. Rewrite the first-order condition of an unmerged firm i:

$$-1 + \left[1 - c_i\left[r(\Psi;c_i)\right]^{1/\lambda}\right]\left\{(\lambda+1) - \lambda\frac{r(\Psi;c_i)}{\Psi}\right\} = 0.$$

As the induced increase in Ψ induces an increase in $r(\Psi;c_i)$ (i.e., prices are strategic complements), the ratio $r(\Psi;c_i)/\Psi$ must fall as otherwise the l.h.s. of the first-order condition would decrease. But as

$$\frac{2\overline{r}(\Psi;\overline{c}_k)}{\Psi} + \frac{\sum_{i\neq 0,k} r(\Psi;c_i)}{\Psi} = 1,$$

it follows that the same ratio for the merged firm, $\overline{r}(\Psi;\overline{c}_k)/\Psi$, must increase. From the expression for the equilibrium profits, we thus obtain that the profit of the merged firm, $g\left(2\overline{r}(\Psi(M_k);\overline{c}_k);\Psi(M_k)\right)$, increases and that of any unmerged firm i, $g\left(r(\Psi(M_k);c_i);\Psi(M_k)\right)$, decreases.

Example 5 (Multinomial Logit)**.** In the multinomial logit model, expected demand for product i is given by

$$X_i = \frac{\exp\left(\frac{a-p_i}{\mu}\right)}{\sum_{j=0}^{N}\exp\left(\frac{a-p_j}{\mu}\right)},$$

where $a > 0$ and $\mu > 0$ are parameters, and p_j the price of product j. Letting Y denote income, the indirect utility of the representative consumer can be written as

$$V = Y + \mu\ln\left[\sum_{j=0}^{N}\exp\left(\frac{a-p_j}{\mu}\right)\right]. \qquad (32)$$

Suppose that firms compete in prices, and that an unmerged firm produces a single good, while a merged firm produces two goods at the same marginal cost (and optimally charging the same price for each). Let $\psi_i \equiv \exp\left((a-p_i)/\mu\right)$ and $\Psi \equiv \sum_i \exp\left((a-p_i)/\mu\right)$. By (32), $V(\Psi)$ is strictly increasing in Ψ. Consider first a single-product firm i. The profit maximization problem of a single-product firm i with marginal cost c_i can be written as

$$\max_{\psi_i}\left[a - \mu\ln\psi_i - c_i\right]\frac{\psi_i}{\psi_i + \sum_{j\neq i}\psi_j}$$

From the first-order condition of profit maximization,

$$\left\{-\mu + a - \mu\ln\psi_i - c_i\right\}\Psi - \left[a - \mu\ln\psi_i - c_i\right]\psi_i = 0,$$

it can be seen that there is a unique cumulative best reply $r(\Psi;c_i)$ and that it is decreasing in the firm's marginal cost c_i. Firm i's equilibrium profit under merger M_k can be written as

$$g(\psi_i(M_k);\Psi(M_k)) = \mu\left\{\frac{\Psi(M_k)}{\psi_i(M_k)} - 1\right\}^{-1}.$$

Consider now the merged firm k and suppose the firm produces two products at marginal cost $\overline{c}_k$. The profit maximization problem can be written as

$$\max_{\psi_k} 2\left[a - \mu \ln \psi_k - \overline{c}_k\right] \frac{\psi_k}{2\psi_k + \sum_{j \neq 0,k} \psi_j}.$$

(It can easily be verified that the firm optimally chooses the same value of ψ_k for each one of its two products.) From the merged firm's first-order condition of profit maximization,

$$\left\{-\mu + a - \mu \ln \psi_k - \overline{c}_k\right\} \Psi - 2\left[a - \mu \ln \psi_k - \overline{c}_k\right] \psi_k = 0,$$

it can be seen that there is a unique cumulative best reply $\overline{r}(\Psi; \overline{c}_k)$ and that it is decreasing in $\overline{c}_k$. Firm k's equilibrium profit under merger M_k can be written as

$$g(2\psi_k(M_k); \Psi(M_k)) = \mu \left\{ \frac{\Psi(M_k)}{2\psi_k} - 1 \right\}^{-1}.$$

It can easily be verified that our assumptions hold in the multinomial logit model. In particular, there exists a unique equilibrium and this equilibrium is stable.[22] Moreover, the equilibrium profit function g has all of the required properties (it takes the value of zero if its first argument is zero and is increasing and convex in its first argument). Consider a reduction in post-merger marginal cost $\overline{c}_k$. Since $\overline{r}(\Psi; \overline{c}_k)$ is decreasing in $\overline{c}_k$ and since $r(\Psi; c_i)$ and $\overline{r}(\Psi; \overline{c}_k)$ are increasing in Ψ, and since equilibrium is stable, the reduction in $\overline{c}_k$ induces a higher value of $\Psi = 2\overline{r}(\Psi; \overline{c}_k) + \sum_{i \neq 0,k} r(\Psi; c_i)$. Rewrite the first-order condition of an unmerged firm i:

$$-1 + \left[1 - c_i \left[r(\Psi; c_i)\right]^{1/\lambda}\right] \left\{(\lambda + 1) - \lambda \frac{r(\Psi; c_i)}{\Psi}\right\} = 0.$$

As the induced increase in Ψ induces an increase in $r(\Psi; c_i)$ (i.e., prices are strategic complements), the ratio $r(\Psi; c_i)/\Psi$ must fall as otherwise the l.h.s. of the first-order condition would decrease. But as

$$\frac{-\mu + a - \mu \ln r(\Psi; c_i) - c_i}{a - \mu \ln r(\Psi; c_i) - c_i} = \frac{r(\Psi; c_i)}{\Psi}.$$

it follows that the same ratio for the merged firm, $\overline{r}(\Psi; \overline{c}_k)/\Psi$, must increase. From the expression for the equilibrium profits, we thus obtain that the profit of the merged firm, $g\left(2\overline{r}(\Psi(M_k); \overline{c}_k); \Psi(M_k)\right)$, increases and that of any unmerged firm i, $g\left(r(\Psi(M_k); c_i); \Psi(M_k)\right)$, decreases.

[22] From the first-order condition for profit maximization, we obtain that $dr(\Psi; c_i)/d\Psi$ can be written as a decreasing and convex function of $\beta_i \equiv \Psi / r(\Psi; c_i)$:

$$\frac{dr(\Psi; c_i)}{d\Psi} = \frac{1}{\beta_i(\beta_i - 1) + 1}.$$

This derivative attains its maximum of 1 if firm i is the only active firm (i.e., $r(\Psi; c_i) = \Psi$). [Similarly, for a merged firm M_k, we have

$$\frac{d2\overline{r}(\Psi; \overline{c}_k)}{d\Psi} = \frac{1}{\overline{\beta}_k(\overline{\beta}_k - 1) + 1},$$

where $\overline{\beta}_k \equiv \Psi / \left[2\overline{r}(\Psi; \overline{c}_k)\right]$.] It follows that $\sum_i dr(\Psi; c_i)/d\Psi < 1$ [resp. $\sum_{i \neq 0,k} dr(\Psi; c_i)/d\Psi + 2\overline{r}(\Psi; \overline{c}_k)/d\Psi < 1$ after merger M_k] in any equilibrium with more than one active firm. Hence, any equilibrium must be stable. Moreover, as $r(0; c_i) \geq 0$ [resp. $\overline{r}(0; \overline{c}_k) \geq 0$] and $r(\Psi; c_i) = 0$ [resp. $\overline{r}(\Psi; \overline{c}_k) = 0$] for Ψ sufficiently large, this implies that there exists a unique Ψ that is consistent with equilibrium in the sense that $\Psi - \sum_i r(\Psi; c_i) = 0$ [resp. $\Psi - \sum_{i \neq 0,k} r(\Psi; c_i) - 2\overline{r}(\Psi; \overline{c}_k) = 0$ after merger M_k].

Results. Let $\psi_i^\circ \equiv \psi_i(M_0)$ and $\Psi^\circ \equiv \Psi(M^\circ)$, and note that, as consumer surplus $V(\Psi)$ is strictly increasing in Ψ, merger M_k is CS-neutral if $\Psi(M_k) = \Psi^\circ$; it is CS-increasing if $\Psi(M_k) > \Psi^\circ$, and CS-decreasing if $\Psi(M_k) < \Psi^\circ$.

Lemma 6. *Merger M_k is CS-neutral if $2\psi_k(M_k) = \psi_0^\circ + \psi_k^\circ$, CS-increasing if $2\psi_k(M_k) > \psi_0^\circ + \psi_k^\circ$, and CS-decreasing if $2\psi_k(M_k) < \psi_0^\circ + \psi_k^\circ$.*

Proof. Suppose merger M_k is CS-neutral. Then, $\Psi(M_k) = \Psi^\circ$. From the profit maximization problem of any firm i not involved in the merger, it follows that $\psi_i(M_k) = r(\Psi(M_k); c_i) = r(\Psi^\circ; c_i) = \psi_i^\circ$. Hence, we must have $2\psi_k(M_k) = \psi_0^\circ + \psi_k^\circ$. The claim then follows from the observation that consumer surplus is increasing in Ψ and that the equilibrium is stable. □

Lemma 7. *If merger M_k is CS-neutral, it raises the joint profit of the merging firms as well as aggregate profit.*

Proof. It is immediate to see that the profit of any firm not involved in the merger remains unchanged as Ψ remains unchanged. It thus remains to show that

$$g(2\psi_k(M_k); \Psi(M_k)) > g(\psi_0^\circ; \Psi^\circ) + g(\psi_k^\circ; \Psi\circ).$$

But as M_k is CS-neutral, we have $\Psi(M_k) = \Psi^\circ$ and $2\psi_k(M_k) = \psi_0^\circ + \psi_k^\circ$. The above inequality can thus be rewritten as

$$g(\psi^\circ + \psi_k^\circ; \Psi^\circ) > g(\psi_0^\circ; \Psi^\circ) + g(\psi_k^\circ; \Psi^\circ).$$

But this follows from the assumed properties of the function g. □

As a reduction in post-merger marginal cost increases the merged firm's profit, any CS-nondecreasing merger is profitable. As in the Cournot model with efficient bargaining, we impose the following assumption:

Assumption 3. *If merger M_k, $k \geq 1$, is CS-nondecreasing, then reducing its post-merger marginal cost $\overline{c}_k$ increases the aggregate profit $\left[g(2\psi_k(M_k); \Psi(M_k)) + \sum_{i \in \mathcal{N}\setminus\{0,k\}} g(\psi_i(M_k); \Psi(M_k))\right]$.*

In the CES and multinomial logit models (and, as we have seen before, in the Cournot model), a sufficient condition for this assumption to hold is that pre-merger cost differences are not too large so that for every merger M_k, $(\psi_0^\circ + \psi_k^\circ)/\Psi^\circ > \max_{i \neq 0,k} \psi_i^\circ/\Psi^\circ$, i.e., the sum of the pre-merger shares of the merger partners exceeds the pre-merger share of the largest nonmerging firm.

Example 6 (CES)**.** In the CES model, if pre-merger marginal cost differences are not too large so that $(\psi_0^\circ + \psi_k^\circ)/\Psi^\circ > \max_{i \neq 0,k} \psi_i^\circ/\Psi^\circ$, then the reduction in post-merger marginal cost $\overline{c}_k$ following a CS-nondecreasing merger M_k increases aggregate profit. To see this, note that from the argument given in our exposition of the CES model above, the reduction in $\overline{c}_k$ induces a change from ψ_i/Ψ to

$(\psi_i/\Psi - \Delta_i)$, $i \neq 0, k$, $\Delta_i > 0$, and from $2\psi_k/\Psi$ to $(2\psi_k/\Psi + \sum_{i\neq 0,k} \Delta_i)$. It thus suffices to show that the joint profit of the merged firm k and any other firm i,

$$h_i(\Delta) \equiv \left\{ \frac{\sigma_k + \Delta}{(\lambda+1) - \lambda(\sigma_k + \Delta)} \right\} + \left\{ \frac{\sigma_i - \Delta}{(\lambda+1) - \lambda(\sigma_i - \Delta)} \right\},$$

where $\Delta \in [0, \Delta_i]$, $\sigma_i = \psi_i/\Psi$ and $2\psi_k/\Psi \leq \sigma_k \leq 2\psi_k/\Psi + \sum_{j\neq 0,k} \Delta_j$, is increasing in Δ. But this holds as we have

$$h_i'(\Delta) \equiv \frac{\lambda+1}{[(\lambda+1) - \lambda(\sigma_k + \Delta)]^2} - \frac{\lambda+1}{[(\lambda+1) - \lambda(\sigma_i - \Delta)]^2} > 0,$$

where the inequality follows since $\psi_0^0 + \psi_k^0 > \psi_i^0$ implies that $\sigma_k > \sigma_i$ for any CS-nondecreasing merger M_k.

Example 7 (Multinomial Logit)**.** In the multinomial logit model, if pre-merger marginal cost differences are not too large so that $(\psi_0^\circ + \psi_k^\circ)/\Psi^\circ > \max_{i\neq 0,k} \psi_i^\circ/\Psi^\circ$, then the reduction in post-merger marginal cost $\overline{c}_k$ following a CS-nondecreasing merger M_k increases aggregate profit. To see this, note that from the argument given in our exposition of the multinomial logit model above, the reduction in $\overline{c}_k$ induces a change from ψ_i/Ψ to $(\psi_i/\Psi - \Delta_i)$, $i \neq 0, k$, $\Delta_i > 0$, and from $2\psi_k/\Psi$ to $(2\psi_k/\Psi + \sum_{i\neq 0,k} \Delta_i)$. It thus suffices to show that the joint profit of the merged firm k and any other firm i,

$$h_i(\Delta) \equiv \mu \left\{ \frac{\sigma_k + \Delta}{1 - (\sigma_k + \Delta)} \right\} + \mu \left\{ \frac{\sigma_i - \Delta}{1 - (\sigma_i - \Delta)} \right\},$$

where $\Delta \in [0, \Delta_i]$, $\sigma_i = \psi_i/\Psi$ and $2\psi_k/\Psi \leq \sigma_k \leq 2\psi_k/\Psi + \sum_{j\neq 0,k} \Delta_j$, is increasing in Δ. But this holds as we have

$$h_i'(\Delta) \equiv \frac{\mu}{[1 - (\sigma_k + \Delta)]^2} - \frac{\mu}{[1 - (\sigma_i - \Delta)]^2} > 0,$$

where the inequality follows since $\psi_0^\circ + \psi^\circ > \psi_i^\circ$ implies that $\sigma_k > \sigma_i$ for any CS-nondecreasing merger M_k.

We are now in the position to extend Lemma 4 to this larger class of models:

Lemma 8. *Suppose mergers M_j and M_k, $k > j$, induce the same nonnegative change in consumer surplus so that $\Psi(M_j) = \Psi(M_k) \geq \Psi^\circ$. Then, the larger merger M_k induces a greater increase in aggregate profit than the smaller merger M_j.*

Proof. As the aggregate outcome Ψ is the same under both mergers, the profit of each firm not participating in either merger is also the same under both mergers. We thus only need to show that

$$g(2\psi_k(M_k); \overline{\Psi}) + g(\psi_j(M_k); \overline{\Psi}) > g(2\psi_j(M_j); \overline{\Psi}) + g(\psi_k(M_j); \overline{\Psi}),$$

where $\overline{\Psi} \equiv \Psi(M_j) = \Psi(M_k)$ is the common aggregate outcome after each of the two alternative mergers. As $\Psi(M_j) = \Psi(M_k)$, we must have

$$2\psi_k(M_k) + \psi_j(M_k) = 2\psi_j(M_j) + \psi_k(M_j).$$

Now, as $c_j > c_k$ and as $\Psi(M_j) = \Psi(M_k)$, we obtain (from the assumption that a firm's cumulative best reply is decreasing in its marginal cost) that

$$\psi_j(M_k) < \psi_k(M_j),$$

implying that

$$2\psi_k(M_k) > 2\psi_j(M_j).$$

Next, note that as a CS-nondecreasing merger increases the profit of the merging firms and reduces everyone else's profit, we have

$$\begin{aligned} g(2\psi_k(M_k), \Psi(M_k)) &> g(\psi_k^\circ, \Psi^\circ) \\ &\geq g(\psi_k(M_j), \Psi(M_j)). \end{aligned}$$

As $\Psi(M_k) = \Psi(M_j)$ and as g is strictly increasing in its first argument, this implies that

$$2\psi_k(M_k) > \psi_k(M_j).$$

Using the same type of argument, we also have

$$2\psi_j(M_j) > \psi_j(M_k).$$

We have thus shown that

$$2\psi_k(M_k) > \max\left\{2\psi_j(M_j), \psi_k(M_j)\right\} \geq \min\left\{2\psi_j(M_j), \psi_k(M_j)\right\} > \psi_j(M_k).$$

But since $2\psi_k(M_k) + \psi_j(M_k) = 2\psi_j(M_j) + \psi_k(M_j)$ and since g is strictly convex in its first argument, this implies that

$$g(2\psi_k(M_k); \overline{\Psi}) + g(\psi_j(M_k); \overline{\Psi}) > g(2\psi_j(M_j); \overline{\Psi}) + g(\psi_k(M_j); \overline{\Psi}).$$

□

Finally, note that if $\left|\overline{\Psi} - \Psi^\circ\right|$ is sufficiently small, where $\overline{\Psi} \equiv \Psi(M_j) = \Psi(M_k) \geq \Psi^\circ$, then the lemma also implies that the larger merger M_k induces a larger increase in the bilateral profit change than the smaller merger M_j. (This follows from the fact that if both mergers are CS-neutral, then the induced bilateral profit change is equal to the induced aggregate profit change.)

General Sets of Mergers

So far, we have assumed that there is a single firm, firm 0, that is part of every potential merger. Moreover, we have assumed that every merger involves only two firms, firm 0 and one target. In this section, we relax both of these assumptions by allowing for general sets of mergers. As the offer game no longer seems an appropriate bargaining process once there is no single firm that is party to every potential merger, we focus on efficient bargaining. We continue to assume that at most one merger can be proposed to the antitrust authority. We provide sufficient conditions under which the main result of the paper carries over to this more general setting. In particular, we show that the key criterion according to which the antitrust authority should optimally discriminate between alternative mergers is the naively-computed post-merger Herfindahl index. This naively-computed post-merger index is frequently used by antitrust authorities in merger analysis as it is entirely based on readily available information on pre-merger market structure.

To proceed, let $m_k \geq 2$ denote the number of merger partners in merger M_k and let $\overline{c}_{M_k}$ denote the realized post-merger marginal cost of merger M_k. It is straightforward to see that the characterization of CS-neutral mergers in Lemma 1 extends to any $m_k \geq 2$. In particular, any CS-neutral merger raises aggregate profit. In the main text, we have shown that aggregate profit following merger M_k is proportional to the post-merger Herfindahl index $H(M_k)$, where the proportionality factor depends only on the post-merger aggregate output $Q(M_k)$ [see (10)]. Observe that for a CS-*neutral* merger M_k, Lemma 1 implies that the actual post-merger Herfindahl index equals the naively-computed index:

$$\begin{aligned} H(M_k) &\equiv [s_k(M_k)]^2 + \sum_{i \notin M_k} [s_i(M_k)]^2 \\ &= \left[\sum_{i \in M_k} s_i^0\right]^2 + \sum_{i \notin M_k} \left[s_i^0\right]^2 \equiv H^{naive}(M_k). \end{aligned}$$

Thus, for any two CS-neutral mergers M_j and M_k, regardless of the number of merger partners, the merger that induces a greater naively-computed post-merger Herfindahl index also induces a greater increase in aggregate profit:

$$H^{naive}(M_k) > H^{naive}(M_j) \Leftrightarrow \Delta\Pi(M_k) > \Delta\Pi(M_j).$$

Hence, provided that merger curves slope upward in the positive orthant of $(\Delta\Pi, \Delta CS)$-space and do not intersect, Proposition 1 carries over to this more general setting, where a "larger" merger now refers to a merger that induces a greater increase in the naively-computed post-merger Herfindahl index.

Under what conditions do the curves for CS-nondecreasing mergers slope upward and not intersect? To identify such conditions, we first state the following result:

Lemma 9. *The slope of the curve for merger M_k in $(\Delta\Pi_I, \Delta CS)$-space is given by*

$$\frac{d\Delta\Pi_I(M_k)}{d\Delta CS(M_k)} = -2 - \left[\frac{P''(Q(M_k))Q(M_k)}{P'(Q(M_k))}\right] H(M_k) + \left[\frac{2}{P'(Q(M_k))Q(M_k)}\right]\left[\frac{r(Q(M_k);\overline{c}_{M_k})}{dQ(M_k)/d\overline{c}_{M_k}}\right], \quad (33)$$

where $r(Q;c) \equiv \{q|P(Q) - c + qP'(Q) = 0\}$ is the "cumulative best reply" of a firm with marginal cost c to aggregate output Q.

Proof. The change in ΔCS induced by a small increase in post-merger marginal cost is

$$\frac{d\Delta CS(M_k)}{d\overline{c}_{M_k}} = -P'(\overline{Q})\overline{Q}\frac{d\overline{Q}}{d\overline{c}_{M_k}},$$

where $\overline{Q} \equiv Q(M_k)$ is aggregate output following merger M_k. Recall that aggregate profit can be written as $\eta(\overline{Q})\overline{H}$, where $\overline{H} \equiv H(M_k)$ is the post-merger Herfindahl index and $\eta(\overline{Q}) \equiv -P'(\overline{Q})\overline{Q}^2$. The effect of a small increase in post-merger marginal cost on the change in aggregate profit induced by merger M_k is thus given by

$$\frac{d\Delta\Pi_I(M_k)}{d\overline{c}_{Mk}} = \eta'(\overline{Q})\frac{d\overline{Q}}{d\overline{c}_{M_k}}\overline{H} + \eta(\overline{Q})\frac{d\overline{H}}{d\overline{c}_{M_k}},$$

where

$$\eta'(\overline{Q}) = -[P''(\overline{Q})\overline{Q}^2 + 2P'(\overline{Q})\overline{Q}].$$

Putting this together, we obtain

$$\begin{aligned} \frac{d\Delta\Pi_I(M_k)}{d\Delta CS(M_k)} &= -\left[\frac{\eta'(\overline{Q})}{P'(\overline{Q})\overline{Q}}\right]\overline{H} - \left[\frac{\eta(\overline{Q})}{P'(\overline{Q})\overline{Q}}\right]\frac{(d\overline{H}/d\overline{c}_{M_k})}{(d\overline{Q}/d\overline{c}_{M_k})} \\ &= \left[2 + \frac{P''(\overline{Q})\overline{Q}}{P'(\overline{Q})}\right]\overline{H} + \overline{Q}\frac{(d\overline{H}/d\overline{c}_{M_k})}{(d\overline{Q}/d\overline{c}_{M_k})} \end{aligned} \tag{34}$$

Now, we have

$$\begin{aligned} \frac{d\overline{H}}{d\overline{c}_{M_k}} &= \frac{d}{d\overline{c}_{M_k}}\left[\frac{\sum_i r(\overline{Q};c_i)^2}{\overline{Q}^2}\right] \\ &= -\left(\frac{2}{\overline{Q}^3}\right)\frac{d\overline{Q}}{d\overline{c}_{M_k}}\left[\sum_i r(\overline{Q};c_i)^2\right] + \left(\frac{1}{\overline{Q}^2}\right)\left\{2r(\overline{Q};\overline{c}_{M_k})\frac{\partial r(\overline{Q};\overline{c}_{M_k})}{\partial \overline{c}_{M_k}} + 2\sum_i r(\overline{Q};c_i)\frac{dr(\overline{Q};c_i)}{d\overline{Q}}\frac{d\overline{Q}}{d\overline{c}_{M_k}}\right\} \\ &= -\left(\frac{2}{\overline{Q}}\right)\overline{H}\frac{d\overline{Q}}{d\overline{c}_{M_k}} + \left(\frac{2}{P'(\overline{Q})\overline{Q}^2}\right)r(\overline{Q};\overline{c}_{M_k}) - \left(\frac{2}{\overline{Q}^2}\right)\frac{d\overline{Q}}{d\overline{c}_{M_k}}\sum_i\left[r(\overline{Q};c_i) + \frac{P''(\overline{Q})}{P'(\overline{Q})}r(\overline{Q};c_i)^2\right] \\ &= -\left(\frac{2}{\overline{Q}}\right)\overline{H}\frac{d\overline{Q}}{d\overline{c}_{M_k}} + \left(\frac{2}{P'(\overline{Q})\overline{Q}^2}\right)r(\overline{Q};\overline{c}_{M_k}) - \left(\frac{2}{\overline{Q}}\right)\frac{d\overline{Q}}{d\overline{c}_{M_k}} - 2\frac{P''(\overline{Q})}{P'(\overline{Q})}\overline{H}\frac{d\overline{Q}}{d\overline{c}_{M_k}}, \end{aligned} \tag{35}$$

where the third equality follows using the facts that $\partial r(\overline{Q};\overline{c}_{M_k})/\partial\overline{c}_{M_k} = 1/P'(\overline{Q})$ and $dr(\overline{Q};c_i)/d\overline{Q} = -\left(1 + r(\overline{Q};c_i)P''(\overline{Q})/P'(\overline{Q})\right)$. Thus, we have:

$$\overline{Q}\frac{(d\overline{H}/d\overline{c}_{M_k})}{(d\overline{Q}/d\overline{c}_{M_k})} = -2\overline{H} - 2 - 2\left(\frac{P''(\overline{Q})\overline{Q}}{P'(\overline{Q})}\right)\overline{H} + \left(\frac{2}{P'(\overline{Q})\overline{Q}}\right)\frac{r(\overline{Q};\overline{c}_{M_k})}{(d\overline{Q}/d\overline{c}_{M_k})}. \tag{36}$$

Substituting (36) into (34), we obtain equation (33). □

We now use expression (33) to identify condtions under which the merger curves are upward-sloping and non-intersecting.[23] As earlier, merger curves are upward-sloping in the positive orthant whenever

[23] Condition (33) also offers an alternative method to establish Lemma 4. To see this, observe that, in our baseline model, if two mergers induce the same change in consumer surplus, ΔCS, and the same change in aggregate profit, $\Delta\Pi_I$, then the two mergers also induce the same aggregate output Q and the same post-merger Herfindahl index H. Moreover, in our baseline model, the firm resulting from a larger merger has a larger output $r(Q;\overline{c}_M)$ (as it faces a larger $\sum_{i\neq M} c_i$, and so must have a lower $\overline{c}_M$ if it induces an equal CS-level). Hence, (33) implies in that model that if there were a point of intersection, the curve of the larger merger would have a larger value of $d\Delta\Pi_I/d\Delta CS$, hence a flatter curve, which yields a contradiction since the larger merger's curve must cross from below at the first crossing since the larger merger's curve lies further to the right where $\Delta CS = 0$.

the pre-merger joint market share of any merging firms exceed the pre-merger share of the largest nonmerging firm. However, expression (33) allows us to derive a weaker condition than this:[24]

Lemma 10. *The merger curve of merger M_k slopes upward in the positive orthant of* $(\Delta\Pi_I, \Delta CS)$*-space if the merged firm's naively-computed post-merger market share $s_{M_k}^{naive} \equiv \sum_{i\in M_k} s_i^\circ$ and the naively-computed post-merger Herfindahl index $H^{naive}(M_k)$ satisfy*

$$s_{M_k}^{naive} \geq \frac{H^{naive}(M_k)}{2} \geq 1-(N-m_k+2)s_{M_k}^{naive}, \tag{37}$$

where $N+1$ is the pre-merger number of firms (and thus $N-m_k+2$ is the number of firms following merger M_k).

Proof. Let $\overline{Q} \equiv Q(M_k)$ denote post-merger aggregate output. Inserting

$$\frac{d\overline{Q}}{d\overline{c}_{M_k}} = \frac{1}{(N-m_k+3)P'(\overline{Q})+\overline{Q}P''(\overline{Q})}$$

into equation (33), we obtain

$$\begin{aligned}\frac{d\Delta\Pi_I(M_k)}{d\Delta CS(M_k)} &= -2-\frac{\overline{Q}P''(\overline{Q})}{P'(\overline{Q})}\overline{H}+\frac{2\overline{s}_{M_k}}{P'(\overline{Q})}\left[(N-m_k+3)P'(\overline{Q})+\overline{Q}P''(\overline{Q})\right] \\ &= 2\left[(N-m_k+3)\overline{s}_{M_k}-1\right]+\frac{\overline{Q}P''(\overline{Q})}{P'(\overline{Q})}\left[2\overline{s}_{M_k}-\overline{H}\right],\end{aligned}$$

where $\overline{s}_{M_k}$ is the actual market share of the merged firm and $\overline{H} \equiv H(M_k)$ the actual post-merger Herfindahl index.

Now, we claim that $s_{M_k}^{naive} \geq H^{naive}(M_k)/2$ implies that $2\overline{s}_{M_k} \geq \overline{H}$. To see this, note that the naively-computed inequality $s_{M_k}^{naive} \geq H^{naive}(M_k)/2$ corresponds to the case of a CS-neutral merger. As merger M_k is CS-nondecreasing by assumption, it involves a (weakly) lower level of $\overline{c}_{M_k}$ (and a weakly greater level of aggregate output) than a CS-neutral merger. It therefore suffices to show that a small reduction in $\overline{c}_{M_k}$ leads to a larger value of $\left[2\overline{s}_{M_k}-\overline{H}\right]$, i.e., $d\left[2\overline{s}_{M_k}-\overline{H}\right]>0$. But we have

$$\begin{aligned}d\left[2\overline{s}_{M_k}-\overline{H}\right] &= 2d\overline{s}_{M_k}-2\left(\overline{s}_{M_k}d\overline{s}_{M_k}+\sum_{i\notin M_k}\overline{s}_i d\overline{s}_i\right) \\ &= 2(1-\overline{s}_{M_k})\left(1-\sum_{i\notin M_k}d\overline{s}_i\right)-2\sum_{i\notin M_k}\overline{s}_i d\overline{s}_i \\ &> 0,\end{aligned}$$

where the inequality follows from the observation that the induced increase in aggregate output reduces the market share of each nonmerging firm i, i.e., $d\overline{s}_i<0$.

[24] That condition (37) below holds when $s_{M_k}^{naive} \geq \max_{i\notin M_k} s_i^\circ$ follows from the facts that in this case $H^{naive}(M_k) \leq s_{M_k}^{naive}$ and $(N-m_k+2)s_{M_k}^{naive} \geq 1$. (Note that, in general, the Herfindahl index is bounded above by the share of the largest firm.)

Since Assumption 1 implies that $\overline{Q}P''(\overline{Q})/P'(\overline{Q}) > -1$, we obtain

$$\begin{aligned}\frac{d\Delta\Pi_I(M_k)}{d\Delta CS(M_k)} &\geq 2\left[(N-m_k+3)\overline{s}_{M_k}-1\right]-\left[2\overline{s}_{M_k}-\overline{H}\right]\\ &= 2\left[(N-m_k+2)\overline{s}_{M_k}-1\right]+\overline{H}.\end{aligned}$$

The r.h.s. of the last equation is positive if and only if

$$\frac{\overline{H}}{2} \geq 1-(N-m_k+2)\overline{s}_{M_k}.$$

We claim that this inequality is implied by the naively-computed analog,

$$\frac{H^{naive}(M_k)}{2} \geq 1-(N-m_k+2)s^{naive}_{M_k}.$$

To see this, consider the effect of decreasing the post-merger marginal cost $\overline{c}_{M_k}$ on $\left[\overline{H}-2\left(1-(N-m_k+2)\overline{s}_{M_k}\right)\right]$:

$$\begin{aligned}d\left[\overline{H}-2\left(1-(N-m_k+2)\overline{s}_{M_k}\right)\right] &= 2\left(\overline{s}_{M_k}d\overline{s}_{M_k}+\sum_{i\notin M_k}\overline{s}_i d\overline{s}_i\right)+2(N-m_k+2)d\overline{s}_{M_k}\\ &= 2(N-m_k+2+\overline{s}_{M_k})\left(1-\sum_{i\notin M_k}d\overline{s}_i\right)+2\sum_{i\notin M_k}\overline{s}_i d\overline{s}_i\\ &= 2\left\{(N-m_k+2+\overline{s}_{M_k})-\sum_{i\notin M_k}(N-m_k+2+\overline{s}_{M_k}-\overline{s}_i)d\overline{s}_i\right\}\\ &> 0,\end{aligned}$$

where the inequality follows from the observation that $d\overline{s}_i < 0$ for all $i \notin M_k$ and $N-m_k+2 \geq 1$. □

Now consider when the merger curves are non-intersecting. We will use expression (33) to provide conditions under which two merger curves cannot cross; that is, their ranking must be the same as their ranking where $\Delta CS = 0$. We will show this by contradiction, showing that the curve further to the right at $\Delta CS = 0$ must have a smaller slope wherever the two curves cross. Since aggregate profit and consumer surplus are the same wherever the curves cross, so must be the industry Herfindahl index and aggregate quantity. By (33), this means the slopes at that point are ordered by the values of $r(Q(M_k);\overline{c}_{M_k})/(dQ(M_k)/d\overline{c}_{M_k})$. The following lemma provides a condition under which those quantities are ordered in the correct way to give us an analog of Lemma 4:

Lemma 11. *Consider two mergers M_j and M_k, with $m_j \geq m_k$. If the firms in M_k jointly produce more pre-merger than the firms in M_j (i.e., $\sum_{i\in M_k} s_i^\circ > \sum_{i\in M_j} s_i^\circ$) and if the naively-computed post-merger Herfindahl index is larger when M_k is implemented than when M_j is implemented [i.e., $H^{naive}(M_k) > H^{naive}(M_j)$], then the curve relating to merger M_k lies to the right of that relating to merger M_j in the positive orthant of $(\Delta\Pi_I, \Delta CS)$-space.*

Proof. Let $q^\circ \equiv \sum_{i\in M_l} q_i^\circ$, $l = j, k$. The pre-merger first-order conditions imply that

$$[m_l P(Q^\circ) - \sum_{i\in M_l} c_i] + P'(Q^\circ)q^\circ_{M_l} = 0 \text{ for } l = j, k,$$

so

$$(m_k - m_j)P(Q^0) - \sum_{i \in M_k} c_i + \sum_{i \in M_j} c_i = P'(Q^\circ)(q^\circ_{M_j} - q^\circ_{M_k}) > 0. \tag{38}$$

Next, summing up the post-merger first-order conditions, we have

$$(N - m_l + 2)P(\overline{Q}) - \sum_i c_i + \sum_{i \in M_l} c_i - \overline{c}_{M_l} + P'(\overline{Q})\overline{Q} = 0 \text{ for } l = j, k, \tag{39}$$

where $N+1$ is the number of firms prior to any merger. So,

$$-\left[(m_k - m_j)P(\overline{Q}) - \sum_{i \in M_k} c_i + \sum_{i \in M_j} c_i\right] - [\overline{c}_{M_k} - \overline{c}_{M_j}] = 0.$$

Since $-(m_k - m_j)P(\overline{Q}) \le -(m_k - m_j)P(Q)$ as the mergers are CS-nondecreasing by assumption, we have

$$-\left[(m_k - m_j)P(Q) - \sum_{i \in M_k} c_i + \sum_{i \in M_j} c_i\right] - [\overline{c}_{M_k} - \overline{c}_{M_j}] \ge 0,$$

so (38) implies that $\overline{c}_{M_k} - \overline{c}_{M_j} < 0$, which in turn implies that $r(\overline{Q}; \overline{c}_{M_k}) > r(\overline{Q}; \overline{c}_{M_j})$.

Applying the implicit function theorem to (39), yields

$$\frac{d\overline{Q}}{d\overline{c}_{M_l}} = \frac{1}{(N - m_l + 3)P'(\overline{Q}) + \overline{Q}P''(\overline{Q})}.$$

As $m_k \le m_j$, $P'(\overline{Q}) < 0$, $P'(\overline{Q}) + \overline{Q}P''(\overline{Q}) < 0$, and (from above) $r(\overline{Q}; \overline{c}_{M_k}) > r(\overline{Q}; \overline{c}_{M_j})$, we obtain

$$-\frac{r(\overline{Q}; \overline{c}_{M_k})}{d\overline{Q}/d\overline{c}_{M_k}} > -\frac{r(\overline{Q}; \overline{c}_{M_j})}{d\overline{Q}/d\overline{c}_{M_j}}.$$

Equation (33) implies that $d\Delta\Pi_I / d\Delta CS$ is larger for merger M_k than for M_j at any point where the curves cross, from which the assertion follows. □

Finally, if all mergers have the same minimum of the support of post-merger marginal costs, denoted l, the maximum CS-increase that the smaller merger can achieve is larger than that of the larger merger.[25] Hence, under the assumptions of Lemmas 10 and 11, the merger curves have all of the properties required to obtain our main result, the analog of Proposition 1.

For example, one special case in which this result can be applied arises where there are three potential mergers, one involving firms 1 and 2, a second involving firms 1 and 3, and a third involving firms 2 and 3. As before, the three mergers are mutually exclusive but, in contrast to the baseline model, there is no longer a single firm that is party to every potential merger. In this case, the two

[25] To see this, consider a larger merger M_k and a smaller merger M_j, $j < k$. The maximum ΔCS induced by the larger merger M_k is $\Delta CS(k, l)$. The assumption in Lemma 11 that the firms in M_k produce more pre-merger implies that $\Sigma_{i \in M_k} c_i < \Sigma_{i \in M_j} c_i$. Since agrgegate quantity depends only on the sum of firms' costs, this in turn implies that if the two mergers induce the same change in consumer surplus, then $\overline{c}_{M_k} < \overline{c}_{M_j}$. Thus, $\Delta CS(k, l) = \Delta CS(j, \overline{c}_{M_j})$ implies that $\overline{c}_{M_j} > l$. Since $\Delta CS(j, \overline{c}_{M_j})$ is decreasing in $\overline{c}_{M_j}$, the maximum CS-increase for the smaller merger, $\Delta CS(j, l)$, must be larger than that of the larger merger: i.e., $\Delta CS(j, l) > \Delta CS(k, l)$.

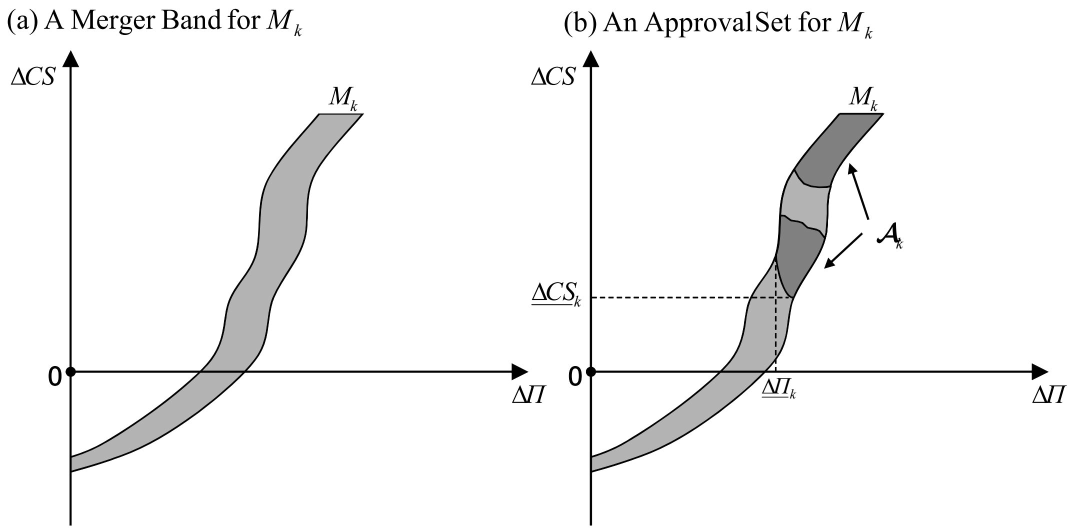

Figure 8: The figure depicts merger bands when mergers create both marginal and fixed cost savings in panel (a) and a possible approval set in panel (b).

conditions of Lemma 11 are satisfied if the mergers have the same ranking by both the product and the sum of the merging firms' pre-merger market shares.

Synergies in Fixed Costs

So far, we have assumed that firms have constant returns, implying that all merger-specific efficiencies involve marginal cost savings. We now consider the case where firms have to incur fixed costs, part of which may be saved by merging, and identify conditions under which our main result carries over to this setting. The discussion that follows applies to our baseline (offer-game) model as well to other scoring-rule bargaining models discussed in the article, with $\Delta\Pi$ appropriately reinterpreted.

Let f_i denote the fixed cost of firm i and asume that it is small enough that firm i remains active following any merger by other firms. A feasible merger M_k is now described by $M_k = (k, \overline{c}_k, \overline{f}_k)$, where $\overline{f}_k \in [\overline{f}^l_k, \overline{f}^h_k] \subset \mathbb{R}_+$ is the realization of its post-merger fixed cost. The merger induces a fixed cost saving if $f_0 + f_k - \overline{f}_k \equiv \alpha_k > 0$. Graphically, a fixed cost saving shifts the merger curve in a parallel fashion (by the amount of the saving) to the right in $(\Delta\Pi, \Delta CS)$-space. Thus, the possibility of fixed cost savings implies that the merger curves in $(\Delta\Pi, \Delta CS)$-space are now "broad bands," with each point in the band of merger M_k corresponding to a different realization of $(\overline{c}_k, \overline{f}_k)$, and with the horizontal width of the band given by $\left|\overline{f}^h_k - \overline{f}^l_k\right|$ at any $\Delta CS(M_k)$. Figure 8 depicts the merger band for merger M_k.

When a feasible merger is proposed, the antitrust authority can observe all aspects of that merger, including the induced fixed cost saving. The antitrust authority's approval set is now described by $\mathcal{A} \equiv \{M_k : (\overline{c}_k, \overline{f}_k) \in \mathcal{A}_k\} \cup M^\circ$, where $\mathcal{A}_k \subseteq [l, h_k] \times [\overline{f}_k^l, \overline{f}_k^h]$. Without loss of generality, we restrict attention to approval sets that are *regular* in the sense that every $\mathcal{A}_k$ is the closure of its interior, i.e., $\mathcal{A}_k = \text{cl}(\text{int}(\mathcal{A}_k))$. Let $\overline{a}_k(\overline{f}_k) \equiv \max\{\overline{c}_k : (\overline{c}_k, \overline{f}_k) \in \mathcal{A}_k\}$ denote the largest allowable post-merger marginal cost level for a merger between firms 0 and k, conditional on the realized post-merger fixed cost $\overline{f}_k$. Let $\underline{\Delta CS}_k(\overline{f}_k) \equiv \Delta CS(k, \overline{a}_k(\overline{f}_k), \overline{f}_k)$ and $\underline{\Delta\Pi}_k(\overline{f}_k) \equiv \Delta\Pi(k, \overline{a}_k(\overline{f}_k), \overline{f}_k)$ denote the changes in consumer surplus and bilateral profits, respectively, induced by the "marginal merger" between firms 0 and k given $\overline{f}_k$, and let $\underline{\Delta CS}_k \equiv \min_{\overline{f}_k \in [\overline{f}_k^l, \overline{f}_k^h]} \underline{\Delta CS}_k(\overline{f}_k)$ and $\underline{\Delta\Pi}_k \equiv \min_{\overline{f}_k \in [\overline{f}_k^l, \overline{f}_k^h]} \underline{\Delta\Pi}_k(\overline{f}_k)$ denote the lowest levels of ΔCS and $\Delta\Pi$, respectively, in any acceptable merger M_k. Figure 8(b) depicts an approval set for merger M_k and shows $\underline{\Delta CS}_k$ and $\underline{\Delta\Pi}_k$.

An immediate observation is the following. Suppose that fixed cost savings are nonnegative and perfectly correlated across mergers, so that $\alpha_k = \alpha \geq 0$ for every feasible merger $M_k \in \mathcal{F}$. Then the optimal approval set is constant in α in the sense that $(\overline{c}_k, f_0 + f_k - \alpha) \in \mathcal{A}_k$ if and only if $(\overline{c}_k, f_0 + f_k - \alpha') \in \mathcal{A}_k$, from which it follows that $\underline{\Delta CS}_k(\overline{f}_k) = \underline{\Delta CS}_k$ for all $\overline{f}_k$ and k. Moreover, as before, the optimal policy for any α is characterized by Proposition 1. To see this, note that the expected CS-maximizing antitrust authority cares about fixed cost savings only insofar as they affect firms' merger proposals. But if fixed cost savings are perfectly correlated and nonnegative, the profit ranking of mergers (and teh profitability of CS-nondecreasing mergers) is unaffected by the fixed cost realization and all CS-nondecreasing mergers remain profitable.

Suppose now that the realized fixed cost saving of merger M_k can be decomposed as follows:

$$\alpha_k = \alpha + \eta_k,$$

where $\alpha \in [\alpha^l, \alpha^h]$ is the (random or deterministic) component that is common across all feasible mergers (as above) and $\eta_k \in [\eta_k^l, \eta_k^h]$ is the component idiosyncratic to merger M_k. We assume that both the idiosyncratic shocks and post-merger marginal cost realizations are independent across mergers conditional on α, have full support, and no mass points. We assume as well that when merger M_k is proposed, the antitrust authority can observe α and η_k separately (and condition the approval set on both components separately).[26] Using the same arguments as above, it is straightforward to show that the optimal approval set is constant in α. Therefore, for notational simplicity, we will from now on assume that there is no common component (i.e., $\alpha \equiv 0$), so that $\overline{f}_k = f_0 + f_k - \eta_k$.

In the remainder of this section, we assume that $\left|\overline{f}_k^h - \overline{f}_k^l\right|$ is sufficiently small so that the bands of the different mergers are non-overlapping in the positive orthant, as depicted in Figure 9. Thus, if any two mergers M_j and M_k, $j < k$, induce the same nonnegative change in consumer surplus, then the larger merger is more profitable, regardless of the realized fixed cost savings. As fixed cost savings

[26] That is, a feasible merger M_k is described by $M_k = (k, \overline{c}_k, \alpha, \overline{f}_k)$, and the approval set by $\mathcal{A} \equiv \{M_k : (\overline{c}_k, \alpha, \overline{f}_k) \in \mathcal{A}_k\} \cup M_0$, where $\mathcal{A}_k \subseteq [l, h_k] \times [\alpha^l, \alpha^h] \times [\overline{f}_k^l, \overline{f}_k^h]$.

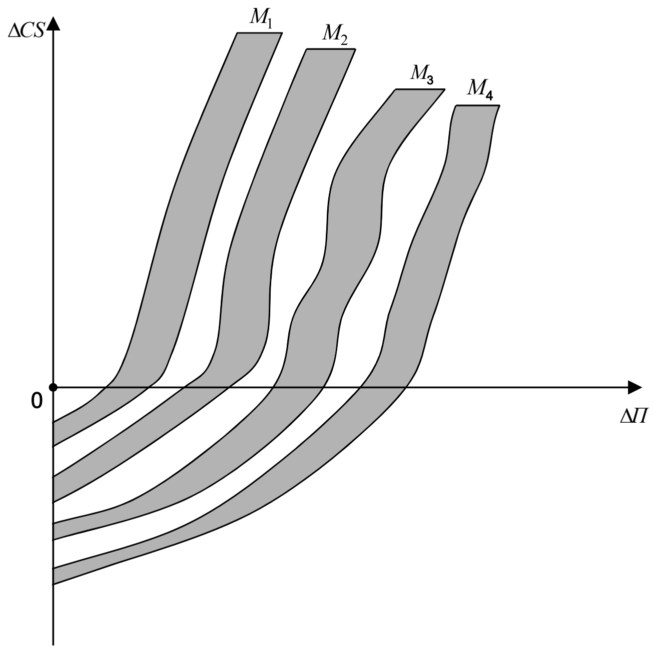

Figure 9: The figure shows merger bands for various possible mergers when mergers can create both fixed and marginal cost synergies.

are nonnegative by assumption, the conclusion of Lemma 1 – that a CS-neutral merger is profitable – continues to hold.

Our main result, Proposition 1, carries over to this setting:

Proposition 4. *In the model with fixed cost savings, any optimal approval policy $\mathcal{A}$ approves the smallest merger if and only if it is CS-nondecreasing, approves only mergers $k \in \mathcal{K}^+ \equiv \{1, ..., \widehat{K}\}$ with positive probability ($\widehat{K}$ may equal K), and satisfies $0 = \underline{\Delta CS}_1 < \underline{\Delta CS}_2 < ... < \underline{\Delta CS}_{\widehat{K}}$ for all $k \leq \widehat{K}$.*

Proof. Steps 1-2 proceed along the same lines as those in the proof of Proposition 1.

Step 3. As in the absence of fixed cost savings, any optimal policy has the property that, for all $k \in \mathcal{K}^+$ and any $\overline{f}_k$, $\underline{\Delta CS}_k(\overline{f}_k)$ is equal to the expected change in consumer surplus from the next-most profitable merger $M^*(\mathcal{F}\backslash(k, \overline{a}_k(\overline{f}_k), \overline{f}_k), \mathcal{A})$, conditional on the marginal merger $M_k = (k, \overline{a}_k(\overline{f}_k), \overline{f}_k)$ maximizing the change in the merging firms' bilateral profit in $\mathcal{F} \cap \mathcal{A}$. That is,

$$\begin{aligned} \underline{\Delta CS}_k(\overline{f}_k) &= E_k^{\mathcal{A}}(\overline{a}_k(\overline{f}_k), \overline{f}_k) \\ &\equiv E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}\backslash M_k, \mathcal{A})) | M_k = (k, \overline{a}_k(\overline{f}_k), \overline{f}_k) \text{ and } \Delta\Pi(M^*(\mathcal{F}\backslash M_k, \mathcal{A})) \leq \Delta\Pi(M_k)]. \end{aligned}$$

To see that this equation must hold for all $k \in \mathcal{K}^+$, suppose first that $\underline{\Delta CS}_{k'}(\overline{f}'_{k'}) > E_{k'}^{\mathcal{A}}(\overline{a}_{k'}(\overline{f}'_{k'}), \overline{f}'_{k'})$ for some firm $k' \in \mathcal{K}^+$ and fixed cost realization $\overline{f}'_{k'}$, and consider the alternative approval set $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$, where

$$\mathcal{A}_{k'}^{\varepsilon} \equiv \left\{ M_k : M_k = (k', \overline{c}_{k'}, \overline{f}_{k'}) \text{ with } \overline{c}_{k'} \in \left(\overline{a}_{k'}(\overline{f}'_{k'}), \overline{a}_{k'}(\overline{f}'_{k'}) + \varepsilon \right) \text{ and } \overline{f}_{k'} \in \left(\overline{f}'_{k'} - \varepsilon, \overline{f}'_{k'} + \varepsilon \right) \right\}.$$

Using the same type of argument as in the proof of Proposition 1, it is straightforward to show that, for $\varepsilon > 0$ small enough, the change in expected consumer surplus from changing the approval set from $\mathcal{A}$ to $\mathcal{A} \cup \mathcal{A}_{k'}^{\varepsilon}$ is strictly positive. A similar logic can be used to show that we cannot have $\underline{\Delta CS}_{k'}(\overline{f}'_{k'}) < E_{k'}^{\mathcal{A}}(\overline{a}_{k'}(\overline{f}'_{k'}), \overline{f}'_{k'})$.

Step 4. Let $\mathcal{M}_j^{CS} \equiv \{M_j : \Delta CS(M_j) = \underline{\Delta CS}_j \text{ and } M_j \in \mathcal{A}_j\}$ denote the set of marginal mergers M_j that induce a change in consumer surplus of $\underline{\Delta CS}_j$, and let $M_j^{CS} \in \mathcal{M}_j^{CS}$ denote the most profitable among these mergers, i.e., $\Delta\Pi(M_j^{CS}) \geq \Delta\Pi(M'_j)$ for all $M'_j \in \mathcal{M}_j^{CS}$. This merger is depicted in Figure 10 for $j = 2$. An optimal approval set must have the property that, for all $j < k$ such that $j, k \in \mathcal{K}^+$, we have $\Delta\Pi\left(M_j^{CS}\right) \leq \underline{\Delta\Pi}_k$. The argument is similar to (but slightly more involved than) Step 4 in the proof of Proposition 1: For $j \in \mathcal{K}^+$, let $k' \equiv \arg\min_{k \in \mathcal{K}^+, k > j} \underline{\Delta\Pi}_k$ and suppose that, contrary to our claim, $\underline{\Delta\Pi}_{k'} < \Delta\Pi(M_j^{CS})$. In Figure 10 we suppose that $k' = 3$. Let $M_{k'}^{\Pi} = (k', \overline{a}_{k'}(\overline{f}_{k'}^{\Pi}), \overline{f}_{k'}^{\Pi})$ denote the marginal merger $M_{k'}$ that induces the bilateral profit change $\underline{\Delta\Pi}_{k'}$, i.e., $\Delta\Pi\left(M_{k'}^{\Pi}\right) = \underline{\Delta\Pi}_{k'}$. By Step 3, $M_{k'}^{\Pi}$ is uniquely defined, and $\underline{\Delta CS}_{k'}(M_{k'}^{\Pi}) = E_{k'}^{\mathcal{A}}\left(\overline{a}_{k'}(\overline{f}_{k'}^{\Pi}), \overline{f}_{k'}^{\Pi}\right)$. Note that $E_{k'}^{\mathcal{A}}\left(\overline{a}_{k'}(\overline{f}_{k'}^{\Pi}), \overline{f}_{k'}^{\Pi}\right)$ can be written as a weighted average of

$$\tau_1 \equiv E_{\mathcal{F}}[\Delta CS(M_j, \mathcal{A}))|M_{k'} = M_{k'}^{\Pi},\ M_j \in M^*(\mathcal{F} \backslash M_{k'}, \mathcal{A}), \text{ and } \Delta\Pi(M^*(\mathcal{F} \backslash M_{k'}, \mathcal{A})) \leq \Delta\Pi(M_{k'})]$$

and

$$\tau_2 \equiv E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F} \backslash M_{k'}, \mathcal{A}))|M_{k'} = M_{k'}^{\Pi},\ M_j \notin M^*(\mathcal{F} \backslash M_{k'}, \mathcal{A}), \text{ and } \Delta\Pi(M^*(\mathcal{F} \backslash M_{k'}, \mathcal{A})) \leq \Delta\Pi(M_{k'})],$$

where the probability weight on τ_1 is positive if and only if $\underline{\Delta\Pi}_j < \underline{\Delta\Pi}_{k'}$. Note also that $\tau_1 \geq \underline{\Delta CS}_j > \Delta CS\left(M_{k'}^{\Pi}\right) = E_{k'}^{\mathcal{A}}\left(\overline{a}_{k'}(\overline{f}_{k'}^{\Pi}), \overline{f}_{k'}^{\Pi}\right)$. Hence, by Step 3,

$$\Delta CS\left(M_{k'}^{\Pi}\right) = E_{k'}^{\mathcal{A}}\left(\overline{a}_{k'}(\overline{f}_{k'}^{\Pi}), \overline{f}_{k'}^{\Pi}\right) \geq \tau_2. \tag{40}$$

Consider a change in the approval set from $\mathcal{A}$ to $\mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}$, where

$$\overline{\mathcal{A}}_j^{\varepsilon} \equiv \{M_j : \Delta\Pi(M_j) \in [\underline{\Delta\Pi}_{k'} - \varepsilon, \underline{\Delta\Pi}_{k'}]\},$$

and $\varepsilon > 0$. Note that, as shown in Figure 10, $\overline{\mathcal{A}}_j^{\varepsilon} \not\subseteq \mathcal{A}$. The change in expected consumer surplus from this change in the approval set equals $\Pr(M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \in (\mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \backslash \mathcal{A})$ [which is strictly positive as $\overline{\mathcal{A}}_j^{\varepsilon} \not\subseteq \mathcal{A}$] times

$$E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon})) - E_j^{\mathcal{A}}(\overline{c}_j, \overline{f}_j)|M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \in (\mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \backslash \mathcal{A}], \tag{41}$$

where $(\overline{c}_j, \overline{f}_j)$ is the pair of realized cost levels in the most profitable merger $M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon})$, which is a merger of firms 0 and j when the conditioning statement is satisfied. Now there exists a $\delta > 0$ such that for all $\varepsilon > 0$ the quantity in (41) is at least as large as

$$E_{\mathcal{F}}[\Delta CS(M_{k'}^{\Pi}) + \delta - E_j^{\mathcal{A}}(\overline{c}_j, \overline{f}_j)|M^*(\mathcal{F}, \mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \in (\mathcal{A} \cup \overline{\mathcal{A}}_j^{\varepsilon}) \backslash \mathcal{A}]. \tag{42}$$

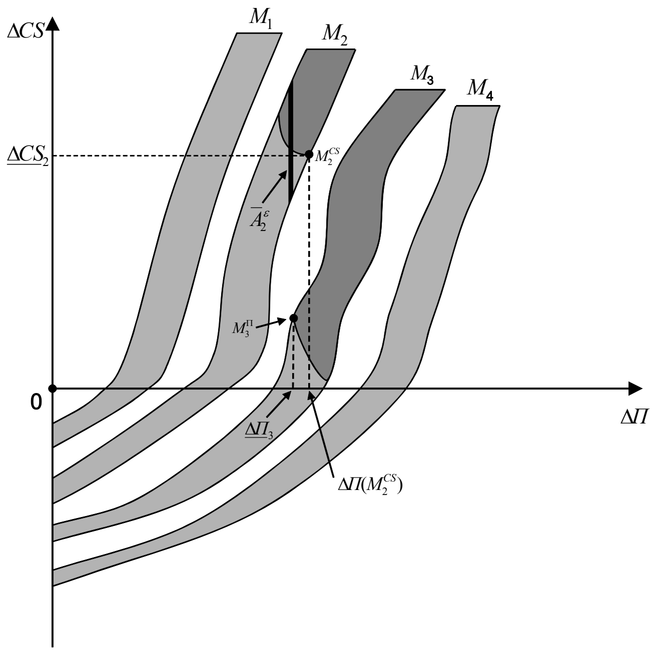

Figure 10: The figure illustrates the change considered in Step 4 of the proof of Proposition 4

As $\varepsilon \to 0$, the quantity in (42) converges to $[\Delta CS(M_{k'}^{\Pi}) - \tau_2] + \delta > 0$, so for small enough $\varepsilon > 0$ (40) implies that this change in the acceptance set is strictly beneficial.

Step 5. For all $j, k \in \mathcal{K}^+$, $j < k$, we must have $\underline{\Delta CS}_j < \underline{\Delta CS}_k$. Suppose otherwise so that for some $j, h \in \mathcal{K}^+$, $h > j$, we have $\underline{\Delta CS}_j \geq \underline{\Delta CS}_h$. Let $k \equiv \arg\min\{h \in \mathcal{K}^+ : h > j \text{ and } \underline{\Delta CS}_j \geq \underline{\Delta CS}_h\}$. Figure 11 shows such a case where $j = 2$ and $k = 3$. Let merger M_k^{CS}'s marginal and fixed costs be $\overline{c}_k^{CS} = \overline{a}_k(\overline{f}_k^{CS})$ and $\overline{f}_k^{CS}$, respectively. Given Step 4, $E_k^{\mathcal{A}}(\overline{a}_k(\overline{f}_k^{CS}), \overline{f}_k^{CS})$ can be written as a weighted average of two conditional expectations:

$$E_{\mathcal{F}}[\Delta CS(M^*(\mathcal{F}\backslash M_k,\mathcal{A}))|M_k = (k, \overline{c}_k^{CS}, \overline{f}_k^{CS}),\ M_k = M^*(\mathcal{F},\mathcal{A}), \text{ and } \Delta\Pi(M^*(\mathcal{F}\backslash M_k,\mathcal{A})) < \underline{\Delta\Pi}_j] \quad (43)$$

and

$$E_{\mathcal{F}}\Big[\Delta CS(M^*(\mathcal{F}\backslash M_k,\mathcal{A}))|M_k = (k, \overline{c}_k^{CS}, \overline{f}_k^{CS}),\ M_k = M^*(\mathcal{F},\mathcal{A}),$$
$$\text{and } \Delta\Pi(M^*(\mathcal{F}\backslash M_k,\mathcal{A})) \in [\underline{\Delta\Pi}_j, \Delta\Pi(M_k)]\Big]. \quad (44)$$

Now the term in (43) equals $E_k^{\mathcal{A}}(\overline{a}_k(\overline{f}_k^{\Pi}), \overline{f}_k^{\Pi})$, which by Step 3 equals $\Delta CS(M_j^{\Pi})$, which in turn is at least $\underline{\Delta CS}_j$ by definition. On the other hand, the term in (44) strictly exceeds $\underline{\Delta CS}_j$. Together, this implies that $E_k^{\mathcal{A}}(\overline{a}_k(\overline{f}_k^{CS}), \overline{f}_k^{CS}) > \underline{\Delta CS}_j$. Since, by Step 3, we must have $\underline{\Delta CS}_k = E_k^{\mathcal{A}}(\overline{a}_k(\overline{f}_k^{CS}), \overline{f}_k^{CS})$, this contradicts $\underline{\Delta CS}_j \geq \underline{\Delta CS}_k$.

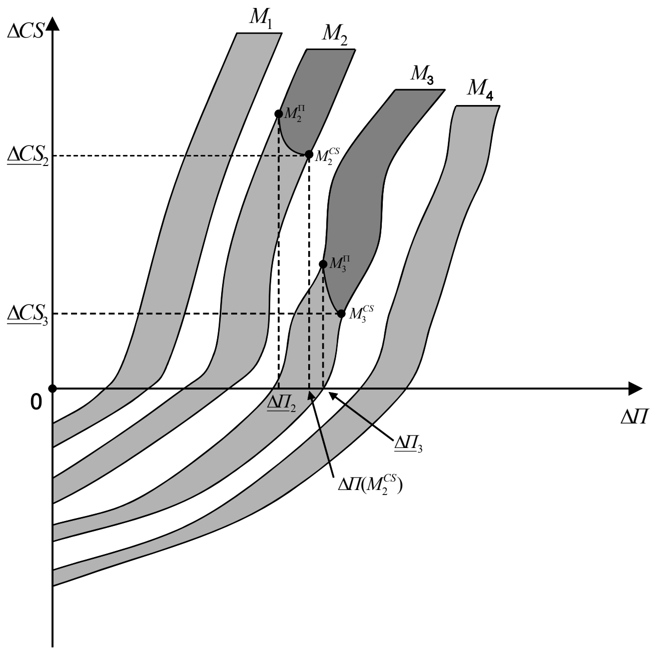

Figure 11: The figure shows the change considered in Step 5 of the proof of Proposition 4

Step 6. The argument proceeds proceeds along the same lines as that in the proof of Proposition 1. □

Thus, provided that idiosyncratic fixed cost synergies are small enough that merger bands do not overlap, it remains optimal to adopt a more stringent consumer surplus test for larger mergers. The restriction on the size of fixed cost synergies contrasts with the model of Armstrong and Vickers (2010). Their model, applied to the merger problem, assumes that the distribution of possible $(\Delta\Pi, \Delta CS)$ pairs are the same for each merger and has a rectangular support. An interesting open question is how projects that are ex ante asymmetric in terms of their distribution of $(\Delta\Pi, \Delta CS)$ pairs should be differentially treated when their supports overlap or even coincide.