Web Appendix:
Relational Contracts and the Value of Relationships

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I. Equilibrium concepts

This section defines the equilibrium concepts used in the paper.

Given party i's offer b^i, let $d^j \in \{0,1\}$ denote party j's decision to accept or reject. Let $h_t = (b_t^i, d_t^j, y_t, W_t^i)$ denote the public outcome at time t, and $h^t = (h_0, ..., h_{t-1})$ the public history up to time t. A public strategy for a type-θ principal is a triple $\sigma_{\theta t} = (g_\theta(h^t, b), a_\theta(h^t, b), k_\theta(h^t, b^i))$, where g_θ is the probability with which she offers contract b (when she is the contract offerer), a_θ is the probability with which she accepts contract b (when the offeree), and k_θ is the probability with which she honors the contract (i.e., pays the agent when $y = \bar{y}$). A public strategy for the agent is analogously defined as $\sigma_{At} = (g_A(h^t, b), a_A(h^t, b), e(h^t, b^i), k_A(h^t, b^i))$, where e is the agent's effort choice.

Let $g_\theta(h^t, b) \equiv g_\theta$, $a_\theta(h^t, b) \equiv a_\theta$, $k_\theta(h^t, b^i) \equiv k_\theta^i$. A PPBE is a quintuple $(\sigma_\ell, \sigma_h, \sigma_A, \mu, \phi)$ such that Assumptions 1 and 2 are satisfied and

1. $\sigma_\ell, \sigma_h,$ and σ_A are mutual best responses for all t and h^t,
2. $\mu(p|b^P) = \dfrac{p\ g_\ell}{p\ g_\ell + (1-p)g_h}$ $\forall b^P$ s.t. $g_\theta > 0$ for some θ,
3. $\mu(p|b^A) = \dfrac{p\ a_\ell}{p\ a_\ell + (1-p)a_h}$ $\forall b^A$ s.t. $a_\theta > 0$ for some θ,
4. $\mu(p|\text{reject } b^A) = \dfrac{p(1-a_\ell)}{p(1-a_\ell) + (1-p)(1-a_h)}$ $\forall b^A$ s.t. $a_\theta < 1$ for some θ,
5. $\phi(\mu(p)|w^i + \bar{b}^i) = \dfrac{\mu(p|b^i)k_\ell^i}{\mu(p|b^i)k_\ell^i + (1-\mu(p|b^i))k_h^i}$ $\forall b^i$ s.t. $k_\theta^i > 0$ for some θ,
6. $\phi(\mu(p)|w^i) = \dfrac{\mu(p|b^i)(1-k_\ell^i)}{\mu(p|b^i)(1-k_\ell^i) + (1-\mu(p|b^i))(1-k_h^i)}$ $\forall b^i$ s.t. $k_\theta^i < 1$ for some θ.

A WMPBE is a PPBE where strategies are weak Markov and beliefs Markov as defined in the text. To define the weak Markov strategies formally, let the parties make a decision to continue or end the relationship at the beginning of each period t. Denote the probabilities with which a type-θ principal and the agent decide to continue by $\gamma_{\theta t}$ and γ_{At}. Let $\Gamma_t = 1$ if the principal and agent's observed decisions at time t are both to continue, and $\Gamma_t = 0$ otherwise. A weak Markov strategy for a type-θ principal is $\sigma_{\theta t}^{wm} = (\gamma_\theta(\mu_t, h_{t-1}), g_\theta(\Gamma_t, \mu_t, b), a_\theta(\Gamma_t, \mu_t, b), k_\theta(\Gamma_t, \mu_t, b^i))$, and for the agent it is $\sigma_{At}^{wm} = (\gamma_A(\mu_t, h_{t-1}), (g_A(\Gamma_t, \mu_t, b), a_A(\Gamma_t, \mu_t, b), e(\Gamma_t, \mu_t, b^i), k_A(\Gamma_t, \mu_t, b^i))$.

II. Consequences of default, rejection, and unexpected offers

This section states and proves the results discussed in Section IVB of the paper.

Proposition A1. *If a Pareto-optimal equilibrium exists, there exists a Pareto-optimal equilibrium where, following default, the relationship ends with positive probability and continues on the Pareto-optimal frontier otherwise, and where the parties' expected payoffs are the same.*

PROOF:

Suppose that no default occurs in equilibrium. Then the worst punishment for default is optimal and terminating the relationship with probability one is without loss.

Suppose next that a default occurs in equilibrium. Then after default, ℓ and h's continuation payoffs must be different; otherwise, both types would want to honor or both to renege, but then a default cannot occur in equilibrium. Thus, since ℓ and h only differ in their outside options, it must be that a default is followed by a contract that involves no trade with strictly positive probability. Then assuming that the relationship ends with positive probability after default is without loss.

Finally, suppose there exists a Pareto-optimal equilibrium where, after default, the relationship ends with probability $1 - \gamma > 0$ and continues on an inefficient path of play with probability γ. Consider a second equilibrium where, after default, the relationship ends with probability $1 - \gamma' > 0$ and continues on an efficient path of play with probability γ'. Let γ' be such that h's continuation payoff after default is the same as in the first equilibrium. (It is straightforward to show that such γ' exists.) Then ℓ's continuation payoff after default is lower than in the first equilibrium. Hence, the second equilibrium allows to implement the same or higher self-enforcing incentives as the first equilibrium while a default does not occur, and to lower the punishment for default for h conditional on ℓ's enforcement constraint holding. The result follows.

Proposition A2. *If an equilibrium is Pareto optimal under Assumptions 2 and 3, then it is Pareto optimal when Assumptions 2 and 3 are not imposed.*

PROOF:

First, note that the first part of Assumption 2 is without loss by Proposition A1. Next, note that any equilibrium under Assumptions 2-3 is also an equilibrium when these assumptions are not imposed. Finally, suppose by contradiction that there exists a Pareto-optimal equilibrium under Assumptions 2-3 that is not

Pareto optimal when these assumptions are not imposed. Let the expected payoffs generated by this equilibrium be u, π_ℓ, and π_h. Then it must be that, when Assumptions 2-3 are not imposed, there exists a Pareto-optimal equilibrium that (i) is not an equilibrium under Assumptions 2-3, and (ii) generates expected payoffs $u' \geqslant u$, $\pi'_\ell \geqslant \pi_\ell$, and $\pi'_h \geqslant \pi_h$, with at least one of these inequalities strict. Now (i) and (ii) imply that such an equilibrium must induce separation of types by either (a) prescribing inefficient play following a rejection by the principal or (b) prescribing inefficient play following an unexpected offer by the principal. But then, given that the continuation play following separation must be such that h is willing to reject in case (a) and make an unexpected offer in case (b), it must be that at least one of the inequalities in (ii) is not satisfied. Contradiction.

Proposition A3. *Suppose that the parties may end the relationship with positive probability after an unexpected offer. Then a contract-separating equilibrium exists for all $\lambda \in (0, 1]$.*

PROOF:

Let the agent's beliefs be $\mu(p_0|w_1, b_1) = 1$ for some contract (w_1, b_1) and $\mu(p_0|w, b) = 0$ for any contract $(w, b) \neq (w_1, b_1)$. Let $b_1 = b_\ell$ and w_1 be such that $\pi^P_\ell(1, w_1, b_1) = r_\ell$. Suppose that the agent ends the relationship with probability one if the principal offers $(w, b) \neq (w_1, b_1)$. Then it is immediate that ℓ is indifferent between (w_1, b_1) and $(w, b) \neq (w_1, b_1)$, while h strictly prefers $(w, b) \neq (w_1, b_1)$. The claim follows.

Proposition A4. *Regardless of the bargaining protocol and the restrictions on strategies, a separating equilibrium where trade occurs with probability one on the equilibrium path does not exist.*

PROOF:

Suppose that trade occurs with probability one on the equilibrium path. Then since the two types differ only in their outside options, it must be that ℓ and h take the same actions in equilibrium. But then no separation can occur. Contradiction.