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"Competition Through Commissions and Kickbacks"

Online Appendix

1. Beliefs

For the case without disclosure we have specified passive beliefs, as commonly assumed in the literature on vertical contracting. That literature has also analyzed the following alternative specification with wary beliefs. When observing a deviating contract offer from a common supplier, a downstream firm tries to infer how the supplier should have *optimally* (and secretly) adjusted contracts offered to other, competing downstream firms. In what follows, we show that when we apply to our model a restriction on beliefs that is in the same spirit, we can support the same unique equilibrium outcome as in Proposition 1.

Hence, a customer who observes a deviating price from some firm, say $\widehat{p}_A \neq p_A$, tries to infer how firm A, in anticipation of this deviation, should have optimally adjusted its commission. When firm A still expects the customer to follow the adviser's recommendation, this commission maximizes π_A and thus is given by (8), where profits are now evaluated at $\widehat{p}_A$. Profits would be zero regardless of firm A's choice of commission if, instead, the customer no longer followed the adviser's recommendation to buy good A.

Formally, for a given candidate equilibrium, we thus specify the following refinement on beliefs. When observing a deviating price $\widehat{p}_A \neq p_A$, beliefs assign all probability to commissions $\widehat{f}_A$ that maximize $[\widehat{p}_A - \widehat{f}_A - c_A][1 - G(\widehat{q}^*)]$, where $\widehat{q}^*$ is obtained from substituting $\widehat{f}_A$ and f_B into (5); when $\widehat{p}_B \neq p_B$, beliefs assign all probability to commissions $\widehat{f}_B$ that maximize $[\widehat{p}_B - \widehat{f}_B - c_A]G(\widehat{q}^*)$, where $\widehat{q}^*$ is now obtained from substituting $\widehat{f}_B$ and f_A into (5). When both $\widehat{p}_A \neq p_A$ and $\widehat{p}_B \neq p_B$, then this still applies for each $\widehat{f}_n$ individually. (Recall that without disclosure firms do not observe each others' choice of commissions.)

With this specification of beliefs, we now set up firms' optimization problems. For this note first that from (8) we can define for each p_A a unique value $\widehat{f}_A(p_A)$ and, together with the (equilibrium) $f_B = \widehat{f}_B$, a perceived cutoff $\widehat{q}^*(p_A)$, which is increasing in p_A. That is, beliefs assign probability one to a single commission, $\widehat{f}_A(p_A)$. Consequently, a customer who holds these beliefs will only follow the recommendation to buy A if, using (6), $p_A \leq P_A(\widehat{q}^*(p_A))$.[15] Note that, for given f_B, there is thus a unique price p_A for which this holds with equality. Given that this upper bound is independent of the actually

[15] Note that the customer can rightly believe that the adviser still applies a cutoff $\widehat{q}^*$ when facing commissions $\widehat{f}_A$ and f_B in case $p_A \leq P_A(\widehat{q}^*)$ and $p_B \leq P_B(\widehat{q}^*)$.

chosen commission, it is optimal for firm A to set p_A so that $p_A = P_A(\widehat{q}^*(p_A))$. This implies, however, that it is indeed optimal to choose f_A according to (8), and that consequently $q^* = \widehat{q}^*$. Given that the same logic applies to firm B, the equilibrium is pinned down by the same requirements as with passive beliefs (Proposition 1).

2. Agency

Our analysis, similar to standard contracting problems in the literature, focuses on a product sale to a single customer. We discuss now to what extent this matters. To do so, we consider the opposite extreme with a very large number of customers. Precisely, suppose that a unit mass of customers arrives simultaneously, after commissions and prices have been specified, as previously. The adviser's belief about each customer is an independent draw from $G(q)$. Generally, firms can now condition their commission payments on the mass (quantity) of sales, q_A and q_B, where $q_A + q_B = 1$. We denote such payments by $F_A(q_A)$ and $F_B(q_B)$. It is immediate to see that by optimality the agent once again chooses a cutoff rule q^* and realizes $q_A = 1 - q^*$ and $q_B = q^*$. With passive beliefs that do not depend on p_n, firms' profits as a function of q^* are given by

$$\begin{aligned} \pi_A &= (p_A - c_A)[1 - G(q^*)] - F_A(1 - q^*), \\ \pi_B &= (p_B - c_B)G(q^*) - F_B(q^*). \end{aligned}$$

For given q^*, the adviser's utility is

$$u = w_l + w\left[\int_0^{q^*} (1-q)dG(q) + \int_{q^*}^1 qdG(q)\right] + F_A(1-q^*) + F_B(q^*).$$

We now restrict consideration to continuously differentiable payments F_A and F_B, so that we obtain from optimality of the adviser

$$\frac{du}{dq^*} = wg(q^*)(1-2q^*) + F_B'(q^*) - F_A'(1-q^*) = 0.$$

From this, we immediately obtain firms' marginal cost of implementing a different cutoff q^*. Substituting this into the firms' respective first-order condition, as obtained from π_A and π_B, and using $p_A = P_A(q^*)$ and $p_B = P_B(q^*)$, we obtain for the unique equilibrium cutoff without disclosure, which we now denote by $\overline{q}^{ND}$,

$$\left[E[v_A(q) \mid q \geq \overline{q}^{ND}] - c_A\right] - \left[E[v_B(q) \mid q < \overline{q}^{ND}] - c_B\right] = w(1 - 2\overline{q}^{ND}). \qquad (28)$$

Even though with continuous differentiability of F_A and F_B we have pinned down a unique equilibrium cutoff and thus unique market shares, firms' profits are not uniquely pinned down. These depend, instead, on the level of F_A and F_B at the cutoff $\overline{q}^{ND}$. In equilibrium, $F_B(\overline{q}^{ND})$ and $F_A(1-\overline{q}^{ND})$ must be set so that the adviser has no incentive to deviate non-locally, by choosing any other cutoff $q^* \neq \overline{q}^{ND}$. As is well known in the literature on common agency (cf. Bernheim and Whinston 1986), there are multiple equilibria, given that the off-equilibrium values of F_n are not payoff relevant for the respective firm n, but determine the adviser's outside option and thus affect the value of $F_{n'}$ that is necessary at $\overline{q}^{ND}$ to keep the adviser from deviating.

We next compare (28) with (10), which we obtained in the text for the baseline case with a single customer. The only difference is that the term $2\frac{1-2G(q^{ND})}{g(q^{ND})}$ on the respective right-hand side is now absent. Recall that this term relates to the need for each firm to pay the incremental commission also inframarginally (i.e., for all realizations $q < q^{ND}$ for firm B and all realizations $q > q^{ND}$ for firm A) when steering advice to capture an additional customer. For $c_A < c_B$ we thus have that $\overline{q}^{ND} < q^{ND}$. However, the comparative statics result from Proposition 1 still holds.

Proceeding analogously for the case with disclosure, we obtain for the equilibrium cutoff $\overline{q}^D$ the requirement

$$\left[v_A(\overline{q}^D) - c_A\right] - \left[v_B(\overline{q}^D) - c_B\right] = w\left(1 - 2\overline{q}^D\right). \tag{29}$$

Comparing now (29) with (13), we have again $\overline{q}^D < q^D$ when $c_A < c_B$. The comparative statics result from Proposition 2 still holds, and the comparison of disclosure with non-disclosure in Proposition 3 also applies.

Finally, we discuss the robustness of our normative implications. Consider first the cases analyzed in Sections 5 and 6. There, from the definition of q_{FB} we still have that $\overline{q}^D > q_{FB}$ when $c_A < c_B$, and it is then immediate that all results of these two sections still apply, once the thresholds for w are appropriately modified. For Section 7's application, where the adviser's concern for suitability is part of the welfare criterion, we have instead that $\overline{q}^D = \widetilde{q}_{FB}$, regardless of the value of γ.

3. Deep Undercutting and Direct Sales

In this appendix, we first analyze how the results of Proposition 1 extend when assumption (3) is relaxed, so that there is scope for trade even without advice because the unconditional average valuation exceeds at least the cost of firm A. In this case the possibility arises for firms to *deviate* by undercutting the rival's price so as to sell to the customer against the adviser's recommendation. In a second step we then expand on the discussion reported at the end of Section 3 by analyzing the scope for direct sales without the adviser's information in equilibrium.

Deep Undercutting. Recall that assumption (3) ruled out the possibility that either firm can cut its price sufficiently so as to induce customers to purchase its product even against the recommendation of the adviser. As we noted in the main text, assumption (3) would no longer be needed in case such a strategy was ruled out as the customer was simply prevented from purchasing a product against the adviser's recommendation. The customer has then only the option of either purchasing the prescribed product or not purchasing at all—as is the case of a physician's prescription of a pharmaceutical product or medical device. For the case in which this restriction of the customer's choice is not feasible, however, we analyze next how condition (3) can be relaxed.

Note first that when (3) is relaxed, to ensure that still $0 < q^{ND} < 1$ holds for all w, so that both products are sold in equilibrium, the cost difference must not be too large:

$$c_B - c_A < \frac{v_h - v_l}{2}.$$

Given passive beliefs, the discussed deep undercutting does not strictly pay for either firm when

$$\begin{aligned} \pi_A^{ND} &\geq \int_0^{q^{ND}} v_A(q) \frac{g(q)}{G(q^{ND})} dq - c_A, \\ \pi_B^{ND} &\geq \int_{q^{ND}}^1 v_B(q) \frac{g(q)}{1 - G(q^{ND})} dq - c_B. \end{aligned} \qquad (30)$$

Note that both conditional valuations at the right-hand side of expressions (30) are strictly smaller than the unconditional valuation $(v_l + v_h)/2$. They capture the respective valuations of a customer who decides to buy a product against the adviser's recommendation. Conditions (30) relax the restriction imposed in (3). When these conditions do not hold and when the adviser cannot prescribe the purchase of a particular product, as discussed above, then no equilibrium in pure strategies satisfies our imposed restrictions (of passive beliefs and of an informative equilibrium).

Direct Sales. As discussed in the main text, when either firm chooses in $t = 0$ to only directly sell its product, then advice is always uninformative in the equilibrium of the resulting subgame. When (3) holds, profits for both firms are zero. And when (3) does not hold, they are still always zero for firm B and again zero for both firms when $c_A = c_B$.

Hence, to see whether direct sales can be an outcome of the expanded game, we only have to consider profits for firm A in case $c_A < c_B$ and when (3) does not hold.

Given that direct sales lead to (undifferentiated) Bertrand competition, we have profits of $c_B - c_A$ for firm A. Recall that when (3) does not hold, we have $c_B \leq \frac{v_h+v_l}{2}$. Hence, the considered deviation of firm A to direct sales is strictly profitable when

$$\pi_A^{ND} \geq c_B - c_A, \tag{31}$$

given that profits are given by π_n^{ND} when both firms choose intermediated sales with advice, assuming that conditions (30) hold. Generally, conditions (30) do not imply (31) and vice versa. In particular, note that when, holding all else constant, we change only the less cost-efficient firm's cost c_B, condition (31) becomes stricter while conditions (30) are relaxed. In particular, when c_B becomes low enough to be equal to c_A, we know that (31) is always satisfied, even though (30) may not hold.

4. Not Fully Covered Market

In the main text, we restricted consideration to the case in which, both on and off the equilibrium path, the adviser's recommendation was essentially restricted to that of either product A or product B. This followed from our specification that $w_l > w_0$, and it ensured that firms always were in direct competition through commissions. An increase in one firm's commission, even if only marginal, pushes up the firm's market share at the expense of the rival. In the terminology of Hotelling competition, the market is fully covered. In what follows, we consider the case in which the market is not necessarily fully covered. To be specific, we suppose that the adviser is subject to a penalty following an unsuitable sale but obtains no direct benefits from a sale: $w_h = w_0 = 0$ and $w = -w_l > 0$.

When, for given commissions, the adviser prefers that the customer sometimes does not purchase any product, two cutoffs arise, $0 \leq q_B^* \leq q_A^* \leq 1$. The adviser recommends a purchase of B when $q \leq q_B^*$, a purchase of A when $q > q_A^*$, and no purchase when $q_B^* < q < q_A^*$. Provided that the customer follows the respective recommendations and provided that cutoffs are interior, we have

$$\begin{aligned} q_A^* &= \frac{(w_0 - w_l) - f_A}{w} = 1 - \frac{f_A}{w}, \\ q_B^* &= \frac{(w_h - w_0) + f_B}{w} = \frac{f_B}{w}. \end{aligned}$$

Note that each cutoff depends only on the commission of the corresponding firm.

Consider first the case with disclosure, where we obtain a unique pair f_n^D together with a unique pair q_n^D that are determined from

$$\begin{aligned}
\frac{d\pi_A}{df_A} &= \left[v_A(q_A^D) - c_A - f_A^D\right] \frac{g(q_A^D)}{w} - \left[1 - G(q_A^D)\right] = 0, \\
\frac{d\pi_B}{df_B} &= \left[v_B(q_B^D) - c_B - f_B^D\right] \frac{g(q_B^D)}{w} - G(q_B^D) = 0,
\end{aligned}$$

so that we have that $f_A^D > 0$ and $f_B^D > 0$ in equilibrium. From (1) and (2) these values are uniquely determined. From substitution we have

$$\begin{aligned}
v_A(q_A^D) - c_A &= w\left[(1 - q_A^D) + \frac{1 - G(q_A^D)}{g(q_A^D)}\right], \\
v_B(q_B^D) - c_B &= w\left[q_B^D + \frac{G(q_B^D)}{g(q_B^D)}\right].
\end{aligned}$$

We can proceed similarly for the case without disclosure. There, provided that the market again is not fully covered in equilibrium, we have a unique equilibrium with $f_n^{ND} > 0$ and q_n^{ND} determined by

$$\begin{aligned}
E[v_A(q) \mid q \geq q_A^{ND}] - c_A = w\left[(1 - q_A^{ND}) + \frac{1 - G(q_A^{ND})}{g(q_A^{ND})}\right], \\
E[v_B(q) \mid q < q_B^{ND}] - c_B = w\left[q_B^D + \frac{G(q_B^D)}{g(q_B^D)}\right].
\end{aligned} \tag{32}$$

As long as the market is still not covered, it is immediate that sales of *both* products expand when w is reduced. The same holds also when commissions are not disclosed. These two observations highlight the key difference to the case of a fully covered market, in which a higher market share for one product comes at the expense of the share of the rival product. In the latter case, which firm's market share expands depended then on how a change (such as a shift in w or a change in the disclosure regime) affects the *relative* incentives of firms to pay commissions.

With respect to efficiency, it is also immediate that with disclosure sales of either product are too low in equilibrium, provided that the market is not fully covered. Instead, now the sales of *both* products can overshoot when w is low and commissions are not disclosed. To see this, suppose for simplicity that $c_A = c_B$. Observe that from (3) we obtain two efficient cutoffs $q_{A,FB}$ and $q_{B,FB}$ from the respective requirements that $v(q_{n,FB}) = c$, with $c_n = c$. These satisfy $0 < q_{B,FB} < 1/2 < q_{A,FB} < 1$. From (32) we have for low w that $q_B^{ND} > q_{B,FB}$ and $q_A^{ND} < q_{A,FB}$.

5. Information Example

Consider the two signal-generating distributions $F_A(s;a) = s^{a+1}$ and $F_B(s;a) = 1 - (1-s)^{a+1}$, where $a \geq 0$. By Bayes' rule the posterior belief is $q(s) = s^a / [s^a + (1-s)^a]$, with $q(0) = 0$ and $q(1) = 1$. Given that the unconditional distribution of the signal is $F(s) = 1/2 + [s^{a+1} - (1-s)^{a+1}]/2$, the distribution of the posterior in this power-signal example is

$$G(\widetilde{q};a) = \frac{1}{2} + \frac{[q^{-1}(\widetilde{q})]^{a+1} - [1-q^{-1}(\widetilde{q})]^{a+1}}{2}.$$

We now show that a induces a mean-preserving rotation in $G(\widetilde{q};a)$, according to (18). To see that this is the case, note first that for $\widetilde{s} := q^{-1}(\widetilde{q})$ we obtain, after some transformations, $d\widetilde{s}/da = -\widetilde{s}(1-\widetilde{s})[\ln(\widetilde{s}) - \ln(1-\widetilde{s})]/a$, so that

$$\begin{aligned} \frac{dG(\widetilde{q};a)}{da} &= \frac{1}{2}\{[\widetilde{s}^{a+1}\ln(\widetilde{s}) - (1-\widetilde{s})^{a+1}\ln(1-\widetilde{s})] \\ &\quad -\frac{a+1}{a}(\ln(\widetilde{s}) - \ln(1-\widetilde{s}))[\widetilde{s}^{a+1}(1-\widetilde{s}) + \widetilde{s}(1-\widetilde{s})^{a+1}]\}. \end{aligned} \tag{33}$$

Consider first $\widetilde{s} = 1/2$, for which it is straightforward that $dG(\widetilde{q};a)/da = 0$. Next, for $\widetilde{s} < 1/2$ we can use from (33) that

$$\begin{aligned} \frac{dG(q;a)}{da} &> \frac{1}{2}\{[\widetilde{s}^{a+1}\ln(\widetilde{s}) - (1-\widetilde{s})^{a+1}\ln(1-\widetilde{s})] \\ &\quad -(\ln(\widetilde{s}) - \ln(1-\widetilde{s}))[\widetilde{s}^{a+1}(1-\widetilde{s}) + \widetilde{s}(1-\widetilde{s})^{a+1}]\} \\ &= \frac{1}{2}\{[\underbrace{\widetilde{s}\ln(\widetilde{s}) + (1-\widetilde{s})\ln(1-\widetilde{s})}_{<0}][\underbrace{\widetilde{s}^{a+1} - (1-\widetilde{s})^{a+1}}_{<0}] > 0. \end{aligned}$$

Similarly, for $\widetilde{s} > 1/2$ we have

$$\frac{dG(q;a)}{da} < \frac{1}{2}\{[\underbrace{\widetilde{s}\ln(\widetilde{s}) + (1-\widetilde{s})\ln(1-\widetilde{s})}_{<0}][\underbrace{\widetilde{s}^{a+1} - (1-\widetilde{s})^{a+1}}_{>0}] < 0.$$

Finally, we show that $G(\widetilde{q};a)$ also satisfies the hazard-rate property (1) for all $\widetilde{q}$ when $a \leq 1$. For this note that

$$\begin{aligned} &\frac{d}{d\widetilde{q}}\frac{g(\widetilde{q};a)}{1-G(\widetilde{q};a)} \\ = \;& \frac{1+a}{2a}\frac{[\widetilde{s}^a + (1-\widetilde{s})^a]^2}{(1-\widetilde{s})^{a-1}\widetilde{s}^{a-1}}\left(\frac{a[1-(1-\widetilde{s}^{a-1})(1+(1-\widetilde{s})^{a-1})] + [\widetilde{s}^a + (1-\widetilde{s})^a]^2}{[1-\widetilde{s}^{a+1} + (1-\widetilde{s})^{a+1}]^2}\right). \end{aligned}$$

which is positive if $a[\widetilde{s}^{a-1} - (1-\widetilde{s}^{a-1})(1-\widetilde{s})^{a-1}] + [\widetilde{s}^a + (1-\widetilde{s})^a]^2 > 0$. For all $a < 1$, the first term on the left-hand side of this inequality is positive for all $\widetilde{s}$. When $a = 1$, the inequality holds strictly for all $\widetilde{s} > 0$ and weakly at $\widetilde{s} = 0$.

6. Covert Information Acquisition

Suppose that the quality of the adviser's information depends on a non-observable effort a exerted by the adviser at $t = 3$, i.e., after being matched with the customer. Consider the case with non-disclosure, in which the adviser's payoff is $u^{ND} - k(a)$, provided the recommendation is followed. The set of optimal values a depends only on the chosen cutoff $q^* = q^{ND}$, which is in turn independent of a. Importantly, the adviser's choice of effort thus does not depend directly on the level of commissions. In the symmetric case, where $c_A = c_B$ and $q^* = q^{ND} = q^D = 1/2$, this further implies that the effort chosen in $t = 3$ is independent of whether commissions are disclosed or not.

Continuing with the analysis, suppose for brevity that $k(a)$ is sufficiently convex so that the adviser's problem in $t = 3$ generates a unique solution. Depending on the choice of cutoff q^*, this level is uniquely determined from the first-order condition

$$w \left[\int_0^{q^*} \frac{dG(q;a)}{da} dq - \int_{q^*}^{1} \frac{dG(q;a)}{da} dq \right] = k'(a)$$

and denoted by $a^*(q^*)$. By implicit differentiation we have that $da^*/dq^* = 0$ at $q^* = 1/2$. This implies that the firms' first-order conditions for the optimal choice of commissions evaluated at the symmetric outcome $f_n = f$ and $q^* = 1/2$, both with and without disclosure, do *not* change. Provided that a unique equilibrium in pure strategies exists both with and without disclosure, we can then fully apply the analysis from the main text by using $G(q) = G(q; a^*(1/2))$.

However, given that for non-marginal deviations from $f_n = f$ and thus $q^* = 1/2$ commissions affect the quality of information, the monotone hazard rate condition is no longer sufficient to ensure that the firms' program is strictly quasiconcave or that best responses intersect only once. See also the working paper version for an analysis when $q^D < 1/2$ and $q^{ND} < 1/2$ hold for the case of a monopoly firm.