Online Appendixes to “The Long Slump”

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A Constructing the Index of Lending Standards

The model of bank i's change in standards is

$$\delta_i = \Delta x + \mu + \epsilon_i, \tag{44}$$

where ϵ_i has the standard normal distribution. Then

$$\begin{aligned} \text{Pr[Bank } i \text{ eased]} &= \Pr[\delta_i < -\nu] && (45)\\ &= \Pr[\Delta x + \mu + \epsilon_i < -\nu] && (46)\\ &= \Pr[\epsilon_i < -\Delta x - \mu - \nu] && (47)\\ &= \Phi(-\Delta x - \mu - \nu) && (48) \end{aligned}$$

and, by similar logic,

$$\text{Pr[Bank } i \text{ tightened]} = \Phi(\Delta x + \mu - \nu). \tag{49}$$

Thus

$$\begin{aligned} \text{Net change} &= \text{Pr[Bank } i \text{ tightened]} - \text{Pr[Bank } i \text{ eased]} && (50)\\ &= \Phi(\Delta x + \mu - \nu) - \Phi(-\Delta x - \mu - \nu) && (51) \end{aligned}$$

B Model Details

To facilitate understanding of the Matlab code for the model, I distinguish between core equations, which form the system of equations to be solved, and auxiliary equations, which yield the values of variables that can be calculated once the core variables are known. The solution process uses fsolve.m in Matlab. A Matlab function delivers the discrepancies associated with trial values of the core variables to fsolve.m, which then finds the values of the core variables that set the vector of discrepancies to zero.

B.1 Stationary model

Auxiliary:

$$r = \frac{1}{\beta} - 1 \tag{52}$$

$$n = \bar{n} \tag{53}$$

$$p_k = r + \delta_k \tag{54}$$

$$p_d = r + \delta_d \tag{55}$$

$$c_y = p_c c \phi \tag{56}$$

Core:

$$y = n^{\alpha} k^{1-\alpha} \tag{57}$$

$$k = (1-\alpha)\frac{y}{p_k} \tag{58}$$

$$p_d d = (1-\phi) p_c c \tag{59}$$

$$y = c_y + \delta_k k + \delta_d d + \gamma \nu \bar{n} \tag{60}$$

$$p_c = \phi^{-\phi}(1-\phi)^{-(1-\phi)} p d^{1-\phi} \tag{61}$$

B.2 Dynamic model during period of pinned real interest rate

Because Matlab starts vectors at location 1 rather than 0, the observation numbering convention here is one higher than in the body of the paper—initial values are dated 1 and the first calculated values are dated 2.

Timing: Output in t uses workers n_t and end-of-period capital k_{t-1}. Consumption occurs at the end of t and comes out of output at the end of t, y_t. The real interest rate from t to $t+1$ is r_t. The rental price for capital used during t is $p_{k,t}$. The net return to capital in use during period t is r_{t-1}. That capital was acquired in $t-1$ at price $q_{k,t-1}$. The discounter from t to $t+1$ is μ_t. All prices are restated here are restated in output units; they correspond to prices divided by $p_{y,t}$ in the body of the paper.

Core variables: $p_d, t \in [2,T](T-1)$ values, $\tilde{c}, t \in [2,T](T-1)$ values, $n, t \in [2,T](T-1)$ values, $k, t \in [2,T-1](T-2)$ values, and $d, t \in [2,T-1](T-2)$ values. Total number of values to solve for: $5T-6$

Auxiliary:

$$y: \ y_t = n_t^{\alpha}(x_t k_{t-1})^{1-\alpha}, t \in [2,T] \tag{62}$$

$$x: \ x_t = \frac{n_t}{\bar{n}}, t \in [2,T] \tag{63}$$

$$p_k: \ p_{k,t} = (1-\alpha)\left(\frac{x_t k_{t-1}}{n_t}\right)^{-\alpha}, t \in [2,T] \tag{64}$$

$$p_c: \ p_{c,t} = \phi^{-\phi}(1-\phi)^{-(1-\phi)} p_{d,t}^{1-\phi}, t \in [2,T] \tag{65}$$

$$\mu: \ \mu_t = \beta \frac{p_{c,t}}{p_{c,t+1}}\left(\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right)^{-1/\sigma}, t \in [2,T-1] \tag{66}$$

$$\tilde{c}_T = C(k_T, d_T, \omega, s_T, f_{k,T}, f_{d,T}) \tag{67}$$

$$\bar{c}: \ p_{c,t}\bar{c}_t = \omega y_t - s_t, t \in [2,T] \tag{68}$$

$$c_y: \ c_{y,t} = \phi p_{c,t}(\tilde{c}_t + \bar{c}_t), t \in [2,T] \tag{69}$$

$$q_k: \ \kappa_k \frac{k_t - k_{t-1}}{k_{t-1}} = q_{k,t} - 1, t \in [2,T] \tag{70}$$

$$q_d: \ \kappa_d \frac{d_t - d_{t-1}}{d_{t-1}} = q_{d,t} - 1, t \in [2,T] \tag{71}$$

$$r: \ r_t = -\pi, t \in [2,T-1] \tag{72}$$

Core discrepancies:

$$\text{Uses: } k_t + \frac{\kappa_k}{2}\frac{(k_t - k_{t-1})^2}{k_{t-1}} + d_t + \frac{\kappa_d}{2}\frac{(d_t - d_{t-1})^2}{d_{t-1}} + c_{y,t} + \nu\gamma\frac{n_t}{q(n_t)} \tag{73}$$

$$\text{Sources: } y_t + (1-\delta_k)k_{t-1} + (1-\delta_d)d_{t-1} \tag{74}$$

$$\text{Uses} - \text{Sources }, t \in [2, T](T-1) \tag{75}$$

$$(1 + r_t)\mu_t - 1, t \in [2, T-1](T-2) \tag{76}$$

$$p_{k,t} - [(1 + r_{t-1})(1 + f_{k,t})q_{k,t-1} - (1-\delta_k)q_{k,t}], t \in [3, T](T-2) \tag{77}$$

$$p_{d,t} - [(1 + r_{t-1})(1 + f_{d,t})q_{d,t-1} - (1-\delta_d)q_{d,t}], t \in [3, T](T-2) \tag{78}$$

$$p_{d,t}d_{t-1} - (1-\phi)p_{c,t}(\tilde{c}_t + \bar{c}_t),\ t \in [2, T](T-1) \tag{79}$$

Total core discrepancies: $T - 1 + T - 2 + T - 2 + T - 2 + T - 1 = 5T - 8$

Initial conditions: $k_1 = k^*$ and $d_1 = 1.14d^*$.

B.3 Dynamic model with free real interest rate

The model is the same as above except that the initial values of the state variables $(k_T, d_T, \omega, s_T, f_{k,t}, f_{d,t})$ are supplied as the arguments of the function C, equation (67) and equation (72) are omitted, and employment is determined by

$$n_t = \bar{n}. \tag{80}$$

B.4 The pasting function $C(k_T, d_T, \omega, s_T, f_{k,T}, f_{d,T})$

I solved the model for 16 values of the state variables, with all possible combinations of one of two values for each variable (taking $f_{k,T} = f_{d,T}$). I used separate equations for $\omega = 0$ and $\omega = 0.58$. I regressed the calculated value of c_2 on the initial state variables and all multiplicative interaction pairs, to get the approximation shown in Table 3. The R^2 for both equations is 1.000000.

	ω=0	ω=0.58
k	0.0154	0.0055
	(0.000)	(0.000)
d	0.0179	0.0053
	(0.000)	(0.000)
f	17.2026	2.4512
	(0.173)	(0.101)
s	2.3405	1.1249
	(0.017)	(0.010)
kd	0.0000	0.0000
	(0.000)	(0.000)
kf	0.2535	0.1091
	(0.002)	(0.001)
ks	0.0067	0.0049
	(0.000)	(0.000)
df	0.3428	0.1141
	(0.002)	(0.001)
ds	0.0053	0.0036
	(0.000)	(0.000)
fs	32.1424	30.3103
	(0.180)	(0.105)
Intercept	2.1441	0.1663
	(0.007)	(0.004)

Table 3: Coefficients of the Approximation to $C(k_T, d_T, \omega, s_T, f_{k,T}, f_{d,T})$

C Characterizing the Properties of a Dynamic Model

A dynamic model imposes a relationship on a vector of variables y_t of the form

$$y_t = F(y_{t-1}, \epsilon_t). \tag{81}$$

Here y_t is a vector of exogenous and endogenous variables and ϵ_t is a vector of random shocks. The relationship is a reduced form that derives from behavioral principles expressed in structural equations, including the life-cycle principle that calls for consumers to formulate consumption plans by looking into the indefinite future. In the limited class of models with a single representative agent and no wedges or externalities, which can be reformulated as dynamic programs, the function F comprises the policy functions and laws of motion of the dynamic program.

The sequence of derivatives

$$\frac{\partial F}{\partial \epsilon_t}, \ \frac{\partial F}{\partial y_t}\frac{\partial F}{\partial \epsilon_t}, \ \frac{\partial F}{\partial y_{t+1}}\frac{\partial F}{\partial y_t}\frac{\partial F}{\partial \epsilon_t}, \cdots \tag{82}$$

is the set of *impulse response functions* of the model. It shows how the variables in the model respond in period t and later to shocks in period t. Elements representing the response of exogenous variables to shocks in equations that determine endogenous variables are zero by definition.

The sequence of derivatives

$$\frac{\partial F}{\partial y_0}, \ \frac{\partial F}{\partial y_1}\frac{\partial F}{\partial y_0}, \ \frac{\partial F}{\partial y_2}\frac{\partial F}{\partial y_1}\frac{\partial F}{\partial y_0}, \cdots \tag{83}$$

describes how the variables respond to the initial conditions y_0. Most authors do not consider these responses and they do not have a widely used name. But the nature of the current investigation gives them an important role. I will call them *initial condition response functions.*

I noted earlier that the model does not treat uncertainty explicitly, even though it is obvious that households and producers lack anything like perfect foresight. I exploit the

Law of Dynamic Responses

The initial condition response functions of the perfect-foresight version of a model are surprisingly good estimates of the impulse response functions of a stochastic version of the *same model, where the perfect-foresight version is solved as if the realization of a random shock had just entered the initial conditions of the model.*

The law is on the same footing as Krusell and Smith's (1998) finding that a small number of well-chosen moments can stand in for an entire distribution in a dynamic general-equilibrium model. It is less than a universal truth.

D Sources Cited in Table 2

Hall (2004), Hall (2009), Silva and Toledo (2008)