Online Appendix for "Using Lagged Outcomes to Evaluate Bias in Value-Added Model"

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In this appendix, we provide further detail on how we estimate teacher value-added. The Stata code used to generate the simulated data, estimate VA, and produce the results is contained in two files: (1) simulations_table1_final.do, which generates the results in Table 1 and (2) simulations_fig1_final.do, which generates the results in Figure 1.

We estimate the value-added model in four steps:

1. Calculate student test-score gains in year $t = 2$ as $\Delta A_{i,t=2} = A_{i,t=2} - A_{i,t=1}$ and classroom-specific average gains $\Delta \bar{A}_{c,t=2} = \sum_{i \in c} \Delta A_{i,t=2}$. To simplify notation, we drop the $t = 2$ subscript for the remainder of this description.

2. Decompose the variance of test score gains into its four constituent pieces according to

$$Var\left(\Delta A_{i,t=2}\right) = Var\left(\psi_{st} - \psi_{s,t-1}\right) + Var\left(\mu_j\right) + Var\left(\theta_t\right) + Var\left(\varepsilon_{it} - \widetilde{\varepsilon}_{i,t-1}\right)$$

   where $\widetilde{\varepsilon}_{i,t-1} = \varepsilon_{i,t-1} + \theta_{c(i,t-1),t-1} + \mu_{j(i,t-1)}$. We estimate these variance components as follows:

   (a) Track-Level Variance: We estimate the track-level variance component of score gains as the covariance between classroom average scores in classrooms in the same track taught by different teachers:

$$\widehat{Var\left(\psi_{st} - \psi_{s,t-1}\right)} = Cov\left(\Delta \bar{A}_c, \Delta \bar{A}_{c'}\right)\Big|_{s(c)=s(c'),j(c)\neq j(c')}$$

   (b) Teacher-Level Variance: We estimate the sum of track-level and teacher-level variance as the covariance between average scores in classrooms taught by the same teacher. We then subtract the estimate of track-level variance from (a) to estimate teacher-level variance:

$$\widehat{\sigma_\mu^2} = \widehat{Var\left(\mu_j\right)} = Cov\left(\Delta \bar{A}_c, \Delta \bar{A}_{c'}\right)\Big|_{j(c)=j(c')} - \widehat{Var\left(\psi_{st} - \psi_{s,t-1}\right)}$$

   (c) Individual-Level Variance: We estimate the individual level variance as the variance of test scores within classrooms, adjusted for the degrees of freedom. Note that the shock $\varepsilon_{i,t-1}$ has greater variance than ε_{it} since it includes both the lagged teacher shock and lagged classroom shock (neither of which aggregate to the classroom level because students are reshuffled across classrooms in practice):

$$\widehat{Var\left(\varepsilon_{it} - \widetilde{\varepsilon}_{i,t-1}\right)} = Var\left(\Delta A_i - \Delta \bar{A}_c\right) * \frac{I}{I-1}$$

(d) Class-Level Variance: We estimate the class-level variance as the residual variance present in the aggregate variance of test score gains after subtracting out the other three components:

$$\widehat{\sigma_\theta^2} = \widehat{Var\left(\theta_t\right)} = Var\left(\Delta A_i\right) - Var\left(\widehat{\psi_{st} - \psi_{s,t-1}}\right) - \widehat{\sigma_\mu^2} - Var\left(\widehat{\varepsilon_{it} - \widetilde{\varepsilon}_{i,t-1}}\right)$$

3. Calculate the shrinkage factor using the four component variances:

$$\lambda = \frac{\widehat{\sigma_\mu^2}}{\widehat{\sigma_\mu^2} + Var\left(\widehat{\psi_{st} - \psi_{s,t-1}}\right) + \frac{\widehat{\sigma_\theta^2}}{C} + \frac{Var(\widehat{\varepsilon_{it} - \widetilde{\varepsilon}_{i,t-1}})}{C*I}}$$

4. Estimate value-added for each teacher:

$$\hat{\mu}_j = \lambda \Delta \bar{A}_j$$

Appendix Table 1. Baseline Parameters for Monte Carlo Simulations

Parameter	Value
Number of Schools	2000
Number of Tracks per School	5
Number of Teachers per Track	4
Number of Classrooms per Teacher (C)	4
Number of Students per Classroom (I)	25
SD of Student Ability Levels (σ_δ)	0.88
SD Of Student Ability Trends (σ_α)	0.15
SD Of Teacher Value-Added (σ_μ)	0.10
SD of Classroom Shocks (σ_θ)	0.08
SD of Track Level Shocks (σ_ψ)	0.06
Correlation of Track Shocks Across Grades (ρ)	1.00
Degree of Sorting on Levels: Corr(δ,μ)	0.25
Degree of Sorting on Trends: Corr(α,μ)	0.00