Online Appendix
of

Heterogeneity in Returns to Wealth and the Measurement of Wealth Inequality

By ANDREAS FAGERENG, LUIGI GUISO, DAVIDE MALACRINO AND LUIGI PISTAFERRI

This appendix complements the evidence shown in the text.

1. Simulations

Figure A1 and Figure A2 show the results of Monte Carlo simulations based on 200 replications and 100,000 individuals per sample. In Figure A1 we assume that wealth and returns are independent. Figure A2 relaxes this assumption. In both cases, we start by generating two standard normal (with correlation coefficient $\tilde{\rho}$, which we set equal to 0 in Figure A1). From these, we then generate a wealth distribution that is Pareto with shape parameter α and a distribution for the gross returns $(1+r)$ that is lognormal. We assume $\alpha = 1.3$ (a value consistent with the data), and that the mean of net returns is 0.03. In Figure A1 we look at the bias induced by returns heterogeneity and hence run simulations for different values of the standard deviation of returns σ. In Figure A2 we set $\sigma = 0.04$ (a value consistent with the data) and run simulations for different values of the implied (median) correlation between returns and wealth, ρ.

The top left panel of the figures plots the ratio between the Gini coefficient using imputed wealth and the same statistics using actual wealth. The other three panels repeat the exercise for measures of wealth concentration (the top 5%, 1%, and 0.1% wealth shares). Imputed wealth is constructed replicating the procedure used by Saez and Zucman (2015). In particular, to avoid imputing a negative wealth value, whenever we draw a negative value of capital income we set it to zero before computing the capitalization factor.

The figures show the median, 5th and 95th percentile ratio of the 200 draws. Two interesting patterns emerge. First, the Gini ratio and the top 5%, 1% and 0.1% wealth share ratios are both greater than 1 and increasing with the level of heterogeneity in rates of return to wealth, even when rates of return and wealth are independent (Figure A1). Second, a positive correlation between returns and wealth widens the gap between the Gini measure on imputed and actual wealth, at least at non-negligible correlation levels. For instance, in the absence of correlation, the ratio $G(\hat{w})/G(w)$ is 1.26 for a standard deviation of returns σ=0.04 (Figure A1, top left panel). Holding the standard deviation constant, a positive correlation of returns and wealth of 0.05 results in a larger ratio (1.35) between imputed and actual Gini index (Figure A2, top left panel). The gap increases more if the correlation is just slightly higher at 0.08 (ratio 1.39). As the narrow confidence intervals show, the Gini coefficient is consistently overestimated by the simulated capitalization method either when returns are independent or when they are correlated with wealth.

As for the shares, simulations results depend both on the magnitude of the correlation and the standard deviation as well as on which top share we focus on. If the correlation is large enough, the capitalization method overstates inequality when measured by top shares. However, for low correlations, capitalization can *understate* inequality when measured by the very top shares such as the top 1% or 0.1%, even when capitalization overstates inequality measured by the Gini. This is clear from the fact that the confidence band widens considerably as we look at higher fractiles of the wealth distribution. For example, when the correlation between returns and wealth is 0.01, the Gini index on imputed wealth is overstated by 28% in median and the estimation interval is very contained. On the other hand, the top 0.1% wealth share of imputed wealth, while overstated at the median, can be short of the actual in more than 1 out of 20 cases. In other words, summarizing inequality with top shares when the capitalization method is used can generate an upward or downward bias compared to the actual top share, depending on the degree of correlation between rates of returns and wealth. This ambiguity is absent if inequality is summarized by the more comprehensive Gini coefficient. Figure A3 shows that this property is present in our data. The two panels plot the top 1% and 0.1% shares of wealth from capitalized returns and true wealth. While the Gini measure and the top 5% share of capitalized tax returns always overstate their true counterpart, the top 1% and top 0.1% sometimes overstate and sometimes understate the corresponding actual shares.

2. Regression Evidence

Both heterogeneity in returns and correlation between returns and wealth can overstate measured inequality from capitalized tax returns. The discussion in the main text shows that both features are present in the data (see Figure 1). Table A1 complements the evidence presented in Table 1 by showing also the results of OLS regressions of the difference between the 1% and 0.1% share of imputed and actual wealth on the standard deviation of individual returns and the correlation between wealth and returns.

In columns (1), (3), (5) and (7) we control only for the standard deviation of returns, and find that all three gaps increase with the extent of return heterogeneity. However, in columns (2), (4), (6) and (8) we find that the gap between imputed and actual Gini and the imputed and actual top wealth shares are mostly sensitive to variation in the correlation between individual returns and wealth, while the effect of the standard deviation of returns turns statistically insignificant. Hence, as discussed in the main text, we conclude that it is the extent of systematic heterogeneity of returns across the wealth distribution that explains the gap between measures of inequality based on imputed and actual wealth.

Figure A1. Simulating the effect of return heterogeneity on the bias in inequality measures from capitalizing tax returns: independent returns

The Figure shows the results of a Monte Carlo simulation of the ratio between the Gini coefficient and three top wealth shares using imputed and actual wealth. The imputation assumes that true wealth is Pareto with shape parameter $\alpha = 1.3$ and the individual gross rate of return of wealth is distributed log normally and independently of wealth in the cross section with mean $\mu = 1.03$ and standard deviation σ. The figure shows the bias as we vary the value of the standard deviation of returns. The mean return is the average return observed in the data over the 1994-2013 period. Imputed wealth is computed by capitalizing the individual returns (computed as the product between the individual rate of returns and individual wealth) using the mean rate of return. To comply with the Saez and Zucman (2015) method we set at zero negative realizations of returns.

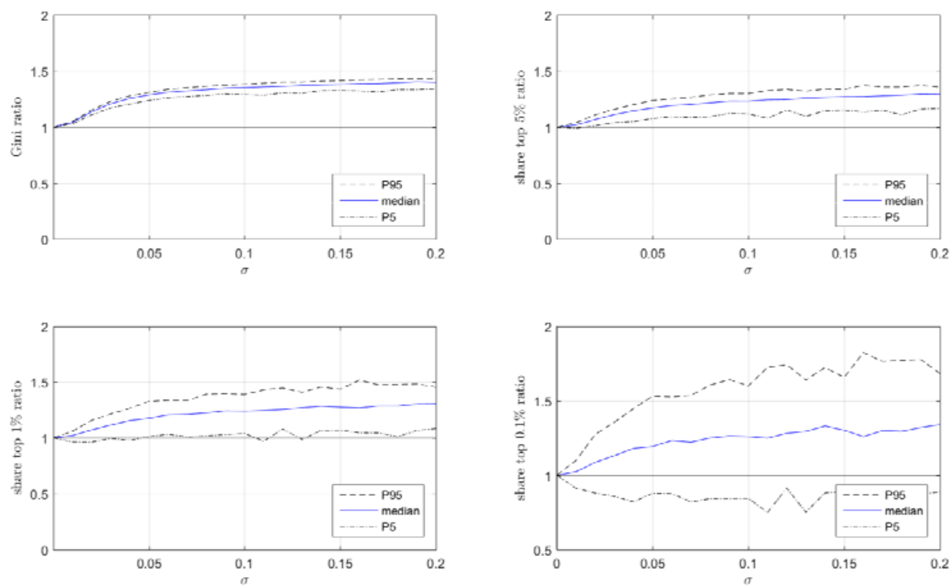

Figure A2. Simulating the effect of return heterogeneity on the bias in inequality measures from capitalizing tax returns: correlated returns

The Figure shows the results of a Monte Carlo simulation of the ratio between the Gini coefficient and three top wealth shares using imputed and actual wealth. The imputation assumes that true wealth is Pareto with shape parameter $\alpha = 1.3$ and the individual gross rate of return of wealth is distributed log normally in the cross section with mean $\mu = 1.03$ and standard deviation $\sigma = 0.04$, with median correlation with wealth equal to ρ. The figure shows the bias as we vary the value of the correlation parameter. The mean and standard deviation of returns are the average and the standard deviation of returns observed in the data over the 1994-2013 period. Imputed wealth is computed by capitalizing the individual returns (computed as the product between the individual rate of returns and individual wealth) using the mean rate of return. To comply with the Saez and Zucman (2015) method we set at zero negative realizations of returns.

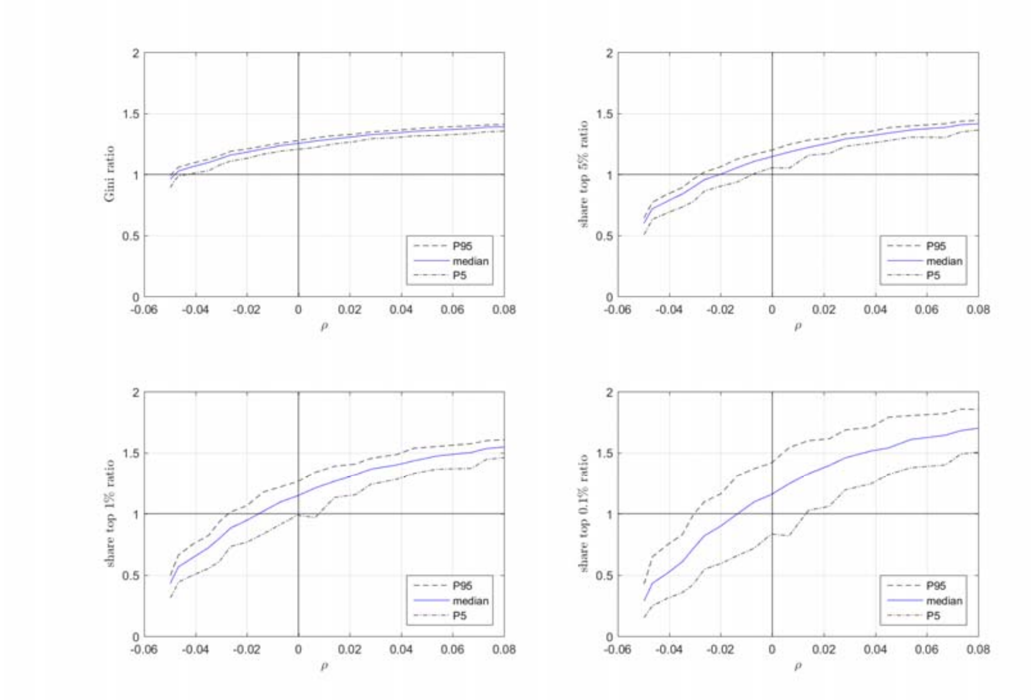

Figure A3. Shares of wealth to the top 1 and 0.1 percent of the population

The figure shows the pattern over time of the top 1% (top panel) and top 0.1% share (bottom panel) of the wealth estimated using the capitalization method and from the actual value of wealth.

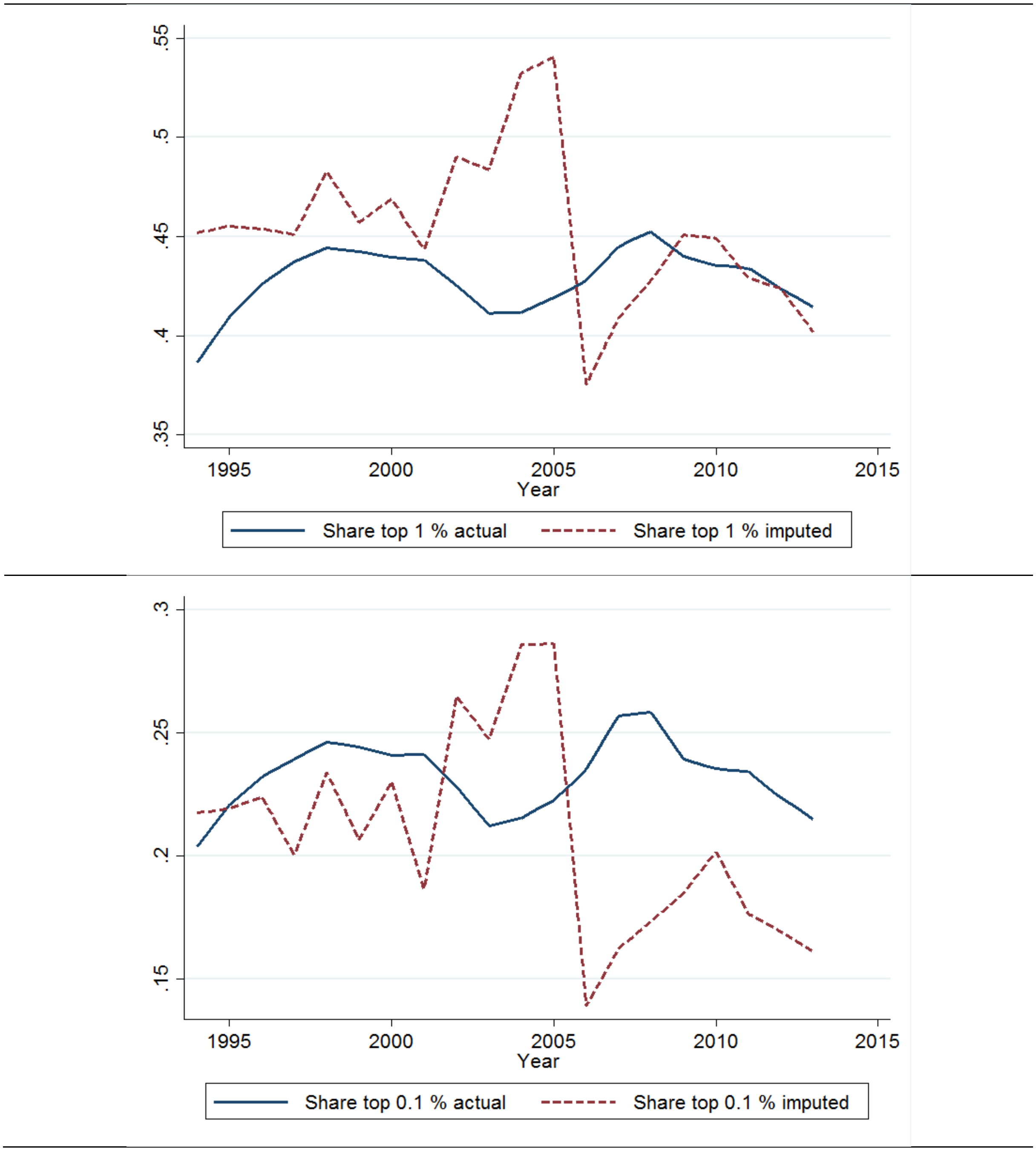

Table A1. Explaining the gap between imputed and actual inequality

	$G(\hat{w}) - G(w)$		$S_5(\hat{w}) - S_5(w)$		$S_1(\hat{w}) - S_1(w)$		$S_{0.1}(\hat{w}) - S_{0.1}(w)$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
St.dev. returns	0.81* (0.44)	-0.15 (0.24)	1.55* (0.83)	-0.24 (0.46)	2.24 (1.30)	-0.50 (0.78)	2.45* (1.37)	-0.39 (0.86)
Correl. Returns/wealth		0.69*** (0.09)		1.29*** (0.17)		1.98*** (0.28)		2.06*** (0.31)
Obs.	20	20	20	20	20	20	20	20
R^2	0.16	0.83	0.16	0.82	0.14	0.78	0.15	0.76