Exchange Rates, Interest Rates and the Risk Premium

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Online Appendix

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Table A1
Coefficient estimates from VECMs
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Canada

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-0.4584	0.0733	-0.0144
$s_{t-1} - p_{t-1}^R$	-0.0187	0.0022	-0.0004
i_{t-1}^R	-1.3804	0.3659	-0.1102
$s_{t-1} - s_{t-2}$	-0.0101	0.0324	0.0040
$p_{t-1}^R - p_{t-2}^R$	-0.0281	0.0566	-0.0170
$i_{t-1}^R - i_{t-2}^R$	-4.6764	0.3031	-0.0848
$s_{t-2} - s_{t-3}$	0.0636	0.0053	0.0043
$p_{t-2}^R - p_{t-3}^R$	-0.1542	0.0368	0.0195
$i_{t-2}^R - i_{t-3}^R$	0.3412	-0.2015	0.0166
$s_{t-3} - s_{t-4}$	0.0390	-0.0063	-0.0010
$p_{t-3}^R - p_{t-4}^R$	0.3320	0.0543	-0.0212
$i_{t-3}^R - i_{t-4}^R$	-0.5013	0.3728	0.0190
R^2	0.05	0.08	0.13

France

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-4.8745	0.3984	0.1067
$s_{t-1} - p_{t-1}^R$	-0.0283	0.0022	0.0007
i_{t-1}^R	-0.5996	0.2497	-0.1243
$s_{t-1} - s_{t-2}$	0.0521	0.0180	0.0016
$p_{t-1}^R - p_{t-2}^R$	0.1457	0.2221	0.0148
$i_{t-1}^R - i_{t-2}^R$	-0.1733	0.0804	-0.3283
$s_{t-2} - s_{t-3}$	0.0483	0.0093	0.0046
$p_{t-2}^R - p_{t-3}^R$	0.7696	-0.0421	0.0206
$i_{t-2}^R - i_{t-3}^R$	-0.5356	0.1822	-0.0430
$s_{t-3} - s_{t-4}$	0.0820	-0.0016	0.0004
$p_{t-3}^R - p_{t-4}^R$	0.6618	0.1393	-0.0739
$i_{t-3}^R - i_{t-4}^R$	1.5395	-0.0271	0.0032
R^2	0.05	0.22	0.22

Germany

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-1.4739	0.2004	0.0067
$s_{t-1} - p_{t-1}^R$	-0.0335	0.0029	0.0001
i_{t-1}^R	-2.2007	0.4995	-0.0381
$s_{t-1} - s_{t-2}$	0.0486	0.0199	0.0016
$p_{t-1}^R - p_{t-2}^R$	0.6154	0.0814	0.0055
$i_{t-1}^R - i_{t-2}^R$	-2.0641	0.2419	0.1724
$s_{t-2} - s_{t-3}$	0.0403	0.0060	-0.0004
$p_{t-2}^R - p_{t-3}^R$	-0.2929	0.0291	0.0056
$i_{t-2}^R - i_{t-3}^R$	0.7971	0.1484	-0.1350
$s_{t-3} - s_{t-4}$	0.0639	-0.0031	0.0001
$p_{t-3}^R - p_{t-4}^R$	0.5742	-0.1026	-0.0048
$i_{t-3}^R - i_{t-4}^R$	536820	-0.1789	-0.0276
R^2	0.06	0.12	0.07

Italy

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-16.2931	2.6931	-0.0715
$s_{t-1} - p_{t-1}^R$	-0.0221	0.0036	-0.0001
i_{t-1}^R	0.1056	0.4070	-0.1151
$s_{t-1} - s_{t-2}$	0.0773	0.0113	0.0003
$p_{t-1}^R - p_{t-2}^R$	-0.3443	0.3778	-0.0068
$i_{t-1}^R - i_{t-2}^R$	-1.7433	0.1120	-0.1153
$s_{t-2} - s_{t-3}$	0.0714	0.0028	0.0024
$p_{t-2}^R - p_{t-3}^R$	0.7718	-0.0683	0.0043
$i_{t-2}^R - i_{t-3}^R$	0.1222	-0.0132	0.1360
$s_{t-3} - s_{t-4}$	0.0774	-0.0008	0.0039
$p_{t-3}^R - p_{t-4}^R$	0.6177	0.0512	0.0186
$i_{t-3}^R - i_{t-4}^R$	1.7392	0.2441	-0.1464
R^2	0.06	0.40	0.15

Japan

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-10.0751	0.5716	0.1129
$s_{t-1} - p_{t-1}^R$	-0.0242	0.0008	0.0002
i_{t-1}^R	-3.1859	0.3014	-0.0525
$s_{t-1} - s_{t-2}$	0.0143	-0.0140	0.0004
$p_{t-1}^R - p_{t-2}^R$	-0.5921	0.0647	-0.0018
$i_{t-1}^R - i_{t-2}^R$	0.3740	-0.0520	0.1390
$s_{t-2} - s_{t-3}$	0.0207	0.0111	0.0001
$p_{t-2}^R - p_{t-3}^R$	-0.0698	-0.1941	0.0122
$i_{t-2}^R - i_{t-3}^R$	3.0030	0.6690	0.0940
$s_{t-3} - s_{t-4}$	0.0136	0.0020	0.0005
$p_{t-3}^R - p_{t-4}^R$	-0.0416	-0.1583	-0.0237
$i_{t-3}^R - i_{t-4}^R$	3.9180	0.0092	0.0418
R^2	0.05	0.11	0.10

United Kingdom

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	1.8366	-0.0188	-0.0323
$s_{t-1} - p_{t-1}^R$	-0.0400	0.0007	0.0004
i_{t-1}^R	-1.9722	0.3071	-0.0674
$s_{t-1} - s_{t-2}$	0.0941	0.0255	0.0008
$p_{t-1}^R - p_{t-2}^R$	-0.1056	0.0290	0.0032
$i_{t-1}^R - i_{t-2}^R$	-0.3261	-0.3684	0.0684
$s_{t-2} - s_{t-3}$	0.0309	0.0106	0.0004
$p_{t-2}^R - p_{t-3}^R$	0.3732	-0.0583	-0.0023
$i_{t-2}^R - i_{t-3}^R$	-0.8468	0.5442	0.0627
$s_{t-3} - s_{t-4}$	0.0238	0.0094	0.0002
$p_{t-3}^R - p_{t-4}^R$	0.0332	-0.1061	0.0037
$i_{t-3}^R - i_{t-4}^R$	-1.6905	1.0814	-0.0531
R^2	0.06	0.08	0.06

G6

	$s_t - s_{t-1}$	$p_t^R - p_{t-1}^R$	$i_t^R - i_{t-1}^R$
(Intercept)	-6.3496	0.7123	0.0534
$s_{t-1} - p_{t-1}^R$	-0.0297	0.0031	0.0002
i_{t-1}^R	-2.2496	0.3228	-0.0641
$s_{t-1} - s_{t-2}$	0.0589	0.0196	0.0011
$p_{t-1}^R - p_{t-2}^R$	0.1636	0.1552	-0.0094
$i_{t-1}^R - i_{t-2}^R$	-0.2604	0.2365	-0.0349
$s_{t-2} - s_{t-3}$	0.0368	0.0098	0.0022
$p_{t-2}^R - p_{t-3}^R$	0.3971	-0.0207	0.0058
$i_{t-2}^R - i_{t-3}^R$	0.1497	0.2370	0.0393
$s_{t-3} - s_{t-4}$	0.0489	-0.0019	0.0003
$p_{t-3}^R - p_{t-4}^R$	0.8704	-0.0538	-0.0203
$i_{t-3}^R - i_{t-4}^R$	3.3658	0.1735	-0.0417
R^2	0.05	0.12	0.07

A.2 Bootstraps

This appendix describes five bootstrap methods. The first, to construct the standard errors reported in Table 2, are constructed under the null hypothesis of no cointegration between relative prices and the nominal exchange rate. Tables 3, 4 and 5 construct bootstraps from pseudo-data generated from the estimated VECMs, using the percentile and the percentile-t methods. We use a fourth method of bootstrapping to test the joint null hypotheses in Tables 2, 3, 4, and 5. These bootstraps draw from the joint empirical distribution for the G6 currencies. The fifth method, which is not reported in the text, but reported in this appendix, uses Kilian's (1998) "bootstrap after bootstrap" method to correct for possible small sample biases in the coefficient estimates of the parameters of the VECM that are used to construct the percentile and percentile-t confidence intervals for Tables 3, 4 and 5.

For Table 2, we first estimate a VAR with four lags in the variables $s_t - s_{t-1}$, $p_t - p_t^* - \left(p_{t-1} - p_{t-1}^*\right)$, and $i_t - i_t^*$. In other words, this VAR is estimated under the assumption that s_t and $p_t - p_t^*$ each have a unit root but $i_t - i_t^*$ is stationary. We use the estimates from this VAR to construct pseudo-data. Initial values are set at the sample means. We draw the error terms with equal probability from the vector of error terms estimated by the VAR at each date. We generate samples of 500+*T* observations, then use the last *T* observations, where *T* corresponds to the length of the time series we use in estimation. For each pseudo-sample, we then estimate the VECM of equation (6). Table 2 reports the 1%, 5% and 10% left tails of the empirical distribution of g_{11}, g_{21} and $g_{11} - g_{21}$.

For both bootstraps in the results reported in Tables 3, 4, and 5, we construct pseudo-samples using the VECM estimates. Initial values are set as described in the previous paragraph. We draw the error terms with equal probability from the vector of error terms estimated by the VECM at each date. For each pseudo-sample, we estimate the VECM. We estimate all of the regression coefficients reported in Tables 3, 4 and 5, and calculate the Newey-West standard errors for each of those regressions. We repeat this exercise 1000 times.

The first confidence interval based on the bootstraps uses the coefficient estimates reported in the tables. Let $\hat{\beta}$ refer to any of the coefficient estimates reported in Tables 3, 4 and 5. From the regressions on the pseudo-samples, we order the coefficient estimates from these 1000 replications from smallest to largest - $\hat{\beta}_1$ is the smallest and $\hat{\beta}_{1000}$ be the largest. The confidence interval reported in the tables is based on

$([\hat{\beta}-(\hat{\beta}_{950}-\hat{\beta}),\hat{\beta}+(\hat{\beta}-\hat{\beta}_{50})]$. That is, the reported confidence interval corrects for the asymmetry in the distribution of $\hat{\beta}_i$ from the regressions on the pseudo-samples.

Hansen (2010) argues that the first bootstrap method performs poorly when the $\hat{\beta}_i$ do not have a symmetric distribution. Instead, he recommends the following procedure. As above, let $\hat{\beta}$ refer to the estimated coefficient in the data, and $\hat{\sigma}$ to be the Newey-West standard error in the data. For each pseudo-sample *i*, we will record analogous estimates: $\hat{\beta}_i$ and $\hat{\sigma}_i$. θ_i is defined by: $\theta_i = \frac{\hat{\beta}_i - \hat{\beta}}{\hat{\sigma}_i}$. We arrange these θ_i from smallest to largest, so that θ_1 is the smallest and θ_{1000} is the largest. The third confidence interval reported for each coefficient estimate is given by $[\hat{\beta}-\hat{\sigma}\theta_{950},\hat{\beta}-\hat{\sigma}\theta_{50}]$. It turns out that our two bootstraps generally produce very similar confidence intervals.

In each of Tables 3, 4, and 5, the slope coefficient estimates for each of the G6 currencies are all of the same sign. This allows us to construct a very simple test of the joint null hypotheses reported in the text. We take the errors generated from the estimated VECMs, as we did in constructing the confidence intervals in tables 3, 4 and 5, described above. However, now we take the vector of errors for each date from all six of the individual currency VECMs jointly as a point in the empirical distribution. That is, in constructing pseudo-data, we draw from the 18x1 vector of errors for each date – 3x1 for each of six countries. With this data, we then estimate VECMs as in equation (6) for each country (country by country). We estimate the coefficients of Tables 3, 4 and 5 using each pseudo-sample, repeating this process 2000 times. We record for example for Table 3 (but analogously for Tables 4 and 5), the proportion of times all six coefficient estimates are negative, which gives us the probability reported in the text.

Kilian (1998) suggests that bootstrap distributions based on VAR (or VECM) estimates may be biased in small samples. We follow Kilian's procedure for producing unbiased distributions. Note that this is only a problem for bootstraps such as in Tables 3 and 5 that are not constructed under the null hypothesis, as opposed to those reported in Table 2 in which the pseudo-data is constructed under the null. Here is a concise account of how Kilian's method works: Estimate the VECM. Construct pseudo samples based on data generated using the estimated coefficients from the VECM. For each pseudo sample, re-estimate the VECM. Construct the mean parameter estimate from the pseudo samples for each parameter. Then adjust the original parameter estimates. For example, if the original estimate of parameter a_{ij} is given by $\hat{a}^0_{ij}$, and the mean of the parameter estimate from the pseudo sample is $\overline{a}^0_{ij}$, then construct the adjusted parameter as: $\hat{a}^1_{ij} = 2\hat{a}^0_{ij} - \overline{a}^0_{ij}$. Then construct new pseudo-samples using the adjusted parameters, and proceed as before.

We do not report the results using this method in the text because it turns out that the bootstrap distributions for the coefficient estimates reported in Tables 3 and 5 are not much different under the original bootstrap and the Kilian bootstrap. Here we report the median and mean parameter bias in the estimates from

Tables 3 and 5 for the "bias corrected" bootstraps and compared the bootstraps used in Tables 3 and 5. (The bias is reported as the mean or median coefficient estimate from the bias corrected bootstrap less the corresponding estimate from the original bootstrap.) In all cases, the bias correction in those coefficient estimates is small and does not affect our conclusions:

	Canada	France	Germany	Italy	Japan	U.K.	G6
Table 3							
Coefficient estimate	0.722	1.482	1.733	0.431	2.360	1.850	1.983
Mean bias	0.137	0.076	0.198	0.190	0.148	0.043	-0.037
Median bias	0.153	0.079	0.210	0.190	0.138	0.070	-0.056
Table 5							
Coefficient estimate	-24.762	-13.983	-33.895	-26.556	-15.225	-10.717	-30.890
Mean bias	2.34	-0.933	-1.291	-2.979	-3.949	0.890	-1.8822
Median bias	2.856	-2.039	-3.491	-2.368	-3.519	0.761	-2.961

Kilian, Lutz. 1998. "Small-Sample Confidence Intervals for Impulse Response Functions." Review of Economics and Statistics 80, 218-230.

A.3 Derivation of log-linearization for model with Epstein-Zin preferences

This section derives the expressions for the risk premium and interest differential in the model with Epstein-Zin preferences. The derivation follows that in Backus et. al. (2010) closely.

The utility function is given by:

$$U_t = \left\{(1-\beta)C_t^{\rho} + \beta\left[E_t\left(U_{t+1}^{\alpha}\right)^{\rho/\alpha}\right]\right\}^{1/\rho} .$$

The stochastic discount factor is given by (see Backus et. al. (2010) or Bansal and Shaliastovich (2013)):

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1}\left(\frac{U_{t+1}}{\left(E_t\left(U_{t+1}^{\alpha}\right)\right)^{1/\alpha}}\right)^{\alpha-\rho} .$$

Taking logs, we can write:

$$m_{t+1} = \ln(\beta) + (\rho-1)x_{t+1} + (\alpha-\rho)(z_{t+1}+x_{t+1}) - \left(\frac{\alpha-\rho}{\alpha}\right)\ln\left[E_t\left(e^{\alpha(z_{t+1}+x_{t+1})}\right)\right],$$

where $x_{t+1} \equiv c_{t+1} - c_t$ and $z_{t+1} \equiv \ln(U_{t+1}) - c_{t+1}$. Assuming log-normality, we can simplify this to:

$$m_{t+1} = \ln(\beta) + (\rho-1)x_{t+1} + (\alpha-\rho)\left[(z_{t+1}+x_{t+1}) - E_t(z_{t+1}+x_{t+1})\right] - \frac{\alpha(\alpha-\rho)}{2}Var_t(z_{t+1}+x_{t+1})$$

We can write the utility function as:

$$z_t = \frac{1}{\rho}\ln\left[1-\beta+\beta e^{\rho E_t(z_{t+1}+x_{t+1}) + \frac{\rho\alpha}{2}\mathrm{var}_t(z_{t+1}+x_{t+1})}\right] .$$

We take a log-linear approximation around the point where $E_t(z_{t+1}+x_{t+1}) + \frac{\alpha}{2}Var_t(z_{t+1}+x_{t+1})$ equals $\bar{m}$:

$$z_t = \bar{\kappa} + \kappa\left(E_t(z_{t+1}+x_{t+1}) + \frac{\alpha}{2}Var_t(z_{t+1}+x_{t+1}) - \bar{m}\right),$$

where $\bar{\kappa} \equiv \frac{1}{\rho}\ln\left(1-\beta+\beta e^{\rho\bar{m}}\right)$ and $\kappa = \beta e^{\rho\bar{m}} / \left(1-\beta+\beta e^{\rho\bar{m}}\right)$. If we choose $\bar{m}$ to equal zero, then $\kappa = \beta$.

Recall that the dynamics of consumption is given in the equations:

$$x_{t+1} = \mu + l_t + \sqrt{u_t}\varepsilon_{t+1}^{x}, \text{ where } u_t \equiv u_t^h + u_t^c$$
$$l_{t+1} = \varphi_l l_t + \sqrt{w_t}\varepsilon_{t+1}^{l}, \text{ where } w_t \equiv w_t^h + w_t^c$$
$$u_{t+1} = (1-\varphi_u)\theta_u + \varphi_u u_t + \sigma_u\varepsilon_{t+1}^{u}$$

$$w_{t+1} = (1-\varphi_w)\theta_w + \varphi_w w_t + \sigma_w \varepsilon_{t+1}^w$$

Then conjecture a solution for z_t of the form:

$$z_t = \omega_l l_t + \omega_u u_t + \omega_w w_t ,$$

where we have suppressed the intercept term. We will suppress constant terms from here on.

Then

$$z_{t+1} + x_{t+1} = \omega_l \left(\varphi_l l_t + \sqrt{w_t}\varepsilon_{t+1}^l\right) + \omega_u \left(\varphi_u u_t + \sigma_u \varepsilon_{t+1}^u\right) + \omega_w \left(\varphi_w w_t + \sigma_w \varepsilon_{t+1}^w\right) + l_t + \sqrt{u_t}\varepsilon_{t+1}^x .$$

So,

$$E_t\left(z_{t+1} + x_{t+1}\right) = \left(1 + \omega_l \varphi_l\right) l_t + \omega_u \varphi_u u_t + \omega_w \varphi_w w_t$$

$$Var_t\left(z_{t+1} + x_{t+1}\right) = u_t + \omega_l^2 w_t$$

We have then that

$$z_t = \kappa\left(\left(1 + \omega_l \varphi_l\right) l_t + \omega_u \varphi_u u_t + \omega_w \varphi_w w_t + \frac{\alpha}{2}\left(u_t + \omega_l^2 w_t\right)\right).$$

Solving for the undetermined coefficients, we find:

$$\omega_l = \frac{\kappa}{1-\kappa\varphi_l} , \ \omega_u = \frac{\alpha}{2}\frac{\kappa}{1-\kappa\varphi_u}, \ \omega_w = \omega_l^2 \frac{\alpha}{2}\frac{\kappa}{1-\kappa\varphi_w} .$$

Substituting this solution for z_t back into the expression for m_{t+1}, we find:

$$\begin{aligned} m_{t+1} &= \left(\rho - 1\right) l_t - \frac{\alpha}{2}\left(\alpha - \rho\right) u_t - \frac{\alpha}{2}\left(\alpha - \rho\right)\omega_l^2 w_t + \left(\alpha - 1\right)\sqrt{u_t}\varepsilon_{t+1}^x \\ &\quad + \left(\alpha - \rho\right)\left[\omega_l \sqrt{w_t}\varepsilon_{t+1}^l + \omega_u \sigma_u \varepsilon_{t+1}^u + \omega_w \sigma_w \varepsilon_{t+1}^w\right] \end{aligned}$$

From this expression, we can derive immediately the equations for $E_t m_{t+1}$ and $Var_t\left(m_{t+1}\right)$ that are used in the text.

A.4 Verdelhan (2010) Model

In Verdelhan (2010) there are two symmetric countries. The objective of Home household i is to maximize

(1) $$E_t \sum_{j=0}^{\infty} \beta^j (C_{i,t+j} - H_{t+j})^{1-\gamma} / (1-\gamma),$$

where γ is the coefficient of relative risk aversion, and H_t represents an external habit. H_t is defined implicitly by defining the "surplus", $s_t \equiv \ln\left((C_t - H_t)/C_t\right)$, where C_t is aggregate consumption, and s_t is assumed to follow the stochastic process:

(2) $$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \mu(s_t)(c_{t+1} - c_t - g), \quad 0 < \phi < 1.$$

Here, ϕ and $\bar{s}$ are parameters, and $c_t \equiv \ln(C_t)$ is assumed to follow a simple random walk:

(3) $$c_{t+1} = g + c_t + u_{t+1}, \text{ where } u_{t+1} \sim i.i.d.\ N(0,\sigma^2).$$

$\mu(s_t)$ represents the sensitivity of the surplus to consumption growth, and is given by:

(4) $$\mu(s_t) \equiv \frac{1}{\bar{S}}\sqrt{1-2(s_t - \bar{s})} - 1, \text{ when } s_t \le s_{\max}, \text{ 0 elsewhere.}$$

The log of the stochastic discount factor is given by:

(5) $$m_{t+1} = \ln(\beta) - \gamma\left[g + (\phi - 1)(s_t - \bar{s}) + (1+\mu(s_t))(c_{t+1} - c_t - g)\right]$$

When the parameters $\bar{S}$ and $s_{\max}$ are suitably normalized, Verdelhan shows we can write the expected rate of depreciation as:

(6) $$r_t^* - r_t = \left[\gamma(1-\phi) - \left(\gamma^2\sigma^2 / \bar{S}^2\right)\right](s_t - s_t^*),$$

where s_t^* is the Foreign surplus. The excess return is given by:

(7) $$E_t \rho_{t+1} = -(\gamma^2\sigma^2 / \bar{S}^2)(s_t - s_t^*).$$

Under the assumption of Verdelhan (2010) that $\gamma\left(1-\phi\right) < \gamma^2\sigma^2 / \bar{S}^2$, this model can account for the empirical finding of $\text{cov}(E_t\rho_{t+1}, r_t^* - r_t) > 0$. However, because $s_t - s_t^*$ follows a first-order autoregressive process, the model also implies $\text{cov}\left(E_t \sum_0^{\infty} \rho_{t+j+1}, r_t^* - r_t\right) > 0$, contrary to our empirical findings.

A.5 Derivation of model based on Nagel (2014)

Here we derive the extension of Nagel (2014) to the two-country setting. In each country, households can hold deposits that are perfectly liquid, local-currency denominated deposits or bonds that provide a liquidity service but are less liquid than deposits, and foreign-currency denominated bonds that provide no liquidity service.

Following Nagel, we assume in the Home country that households maximize

$$E_t \sum_{j=0}^{\infty} \beta^j u\left(C_{t+j}, Q_{t+j}\right), \qquad 0 < \beta < 1$$

where C_t denotes consumption and Q_t denotes liquidity services. As in Nagel, assume *u* is concave, increasing in its arguments, and that utility is additively separable in the utility from consumption and utility from liquidity services. We assume:

$$Q_t = \frac{D_t}{P_t} + \kappa\left(\varepsilon_t\right)\frac{B_t}{P_t} \ .$$

Here, D_t are liquid nominal deposits, and B_t are less-liquid domestic currency interest-bearing assets. We assume $0 < \kappa\left(\varepsilon_t\right) < 1$, so that B_t is less liquid than D_t. ε_t is a shock to the liquidity of B_t, with $\kappa' > 0$.

Monetary policy changes affect the amount of liquid assets. A monetary expansion would increase Q_t. This would clearly be true if the increase occurred through a transfer to households that increased D_t, perhaps financed with lump-sum taxes. But even an open-market operation in which the central bank purchased B_t from the public in exchange for an equal amount of D_t, would increase liquidity because $\kappa < 1$

The household receives a nominal endowment of Y_t each period. The budget constraint is:

$$P_t C_t + D_t + B_t + S_t B_t^* = Y_t + D_{t-1} + \left(1 + i_{t-1}\right) B_{t-1} + S_t \left(1 + i_{t-1}^*\right) B_{t-1}^* \ .$$

We can derive the following first-order conditions:

$$\beta\left(1 + i_t^*\right) E_t \left[\frac{u_C\left(C_{t+1}, Q_{t+1}\right)}{u_C\left(C_t, Q_t\right)} \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} \right] = 1$$

$$\beta\left(1 + i_t\right) E_t \left[\frac{u_C\left(C_{t+1}, Q_{t+1}\right)}{u_C\left(C_t, Q_t\right)} \frac{P_t}{P_{t+1}} \right] = 1 - \kappa\left(\varepsilon_t\right) \frac{u_Q\left(C_t, Q_t\right)}{u_C\left(C_t, Q_t\right)}$$

$$\beta E_t\left[\frac{u_C\left(C_{t+1},Q_{t+1}\right)}{u_C\left(C_t,Q_t\right)}\frac{P_t}{P_{t+1}}\right]=1-\frac{u_Q\left(C_t,Q_t\right)}{u_C\left(C_t,Q_t\right)}$$

We can log-linearize around a steady state in which C_t and Q_t are constant, where the Home and Foreign real interest rates are constant. For simplicity, we assume no inflation in either country in steady state, and no change in the exchange rate.

In the steady state, we have:

$$\beta\left(1+\bar{i}^*\right)=1$$

$$\beta\left(1+\bar{i}\right)=1-\bar{\kappa}\frac{\bar{u}_Q}{\bar{u}_C}$$

$$\beta=1-\frac{\bar{u}_Q}{\bar{u}_C}$$

Note that these equations imply $0<\frac{\bar{u}_Q}{\bar{u}_C}<1$, $\bar{i}>0$ (since $\bar{\kappa}<1$), and $1-\beta\left(1+\bar{i}\right)>0$.

Define Then define $X_t\equiv\frac{u_Q\left(C_t,Q_t\right)}{u_C\left(C_t,Q_t\right)}$. Linearizing the first-order conditions, ignoring intercept terms, and using lower case letters to denote the logs of upper case letters, we find:

$$i_t^*-\gamma E_t\left(c_{t+1}-c_t\right)+E_t\left(s_{t+1}-s_t\right)-E_t\left(p_{t+1}-p_t\right)=0\ ,$$

$$\gamma E_t\left(c_{t+1}-c_t\right)+E_t\left(p_{t+1}-p_t\right)=\frac{1}{\beta}X_t.$$

$$i_t-\gamma E_t\left(c_{t+1}-c_t\right)-E_t\left(p_{t+1}-p_t\right)=-\frac{\left(1-\beta\right)\bar{\kappa}'}{\left(1+\bar{i}\right)\beta}\varepsilon_t-\frac{\bar{\kappa}}{\left(1+\bar{i}\right)\beta}X_t\ ,$$

where we define the steady state coefficient of relative risk aversion as $\gamma=-\frac{\bar{U}''\bar{C}}{\bar{U}'}$.

We can interpret X_t as the marginal rate of substitution between liquidity services and consumption. X_t increases as the amount of liquidity, Q_t, falls.

We then derive an expression for the ex ante excess return on the foreign interest-bearing claim:

$$E_t\rho_{t+1}\equiv i_t^*+E_t\left(s_{t+1}-s_t\right)-i_t=\frac{\left(1-\beta\right)\bar{\kappa}'}{\left(1+\bar{i}\right)\beta}\varepsilon_t+\frac{\bar{\kappa}}{\left(1+\bar{i}\right)\beta}X_t\ .$$

But using the second and third linearized first-order conditions, we can write:

$$i_t = -\frac{(1-\beta)\bar{\kappa}'}{(1+\bar{i})\beta}\varepsilon_t + \frac{1}{\beta}\left(\frac{1+\bar{i}-\bar{\kappa}}{1+\bar{i}}\right)X_t .$$

When ε_t rises, the near-money becomes more valued for its liquidity services, so *ceteris paribus*, it pays a lower interest rate. When the marginal value of liquidity X_t increases, because the amount of liquidity, Q_t, falls, the interest rate rises *ceteris paribus*. This occurs because the near-money is less valued for liquidity than liquid deposits, so its return rises.

We can write the last equation as:

$$X_t = \beta\left(\frac{1+\bar{i}}{1+\bar{i}-\bar{\kappa}}\right)i_t + \frac{(1-\beta)\bar{\kappa}'}{(1+\bar{i}-\bar{\kappa})}\varepsilon_t .$$

Substitute this into the expression above for $E_t\rho_{t+1}$:

$$E_t\rho_{t+1} = \left(\frac{\bar{\kappa}}{1+\bar{i}-\bar{\kappa}}\right)i_t + \frac{1-\beta}{\beta}\left(\frac{\bar{\kappa}'}{1+\bar{i}-\bar{\kappa}}\right)\varepsilon_t .$$

When the liquidity of the near-money asset rises, its return falls relative to the return on Foreign assets, so $E_t\rho_{t+1}$ rises when ε_t increases. When there is a decrease in liquidity that causes X_t and therefore i_t to rise, the excess return on the Foreign asset nonetheless rises. The domestic near-money is more liquid than the Foreign asset, so the ex ante relative pecuniary return on the Foreign asset rises. So $E_t\rho_{t+1}$ rises when i_t goes up.

Symmetrically, the Foreign household receives liquidity services from Foreign liquid nominal deposits, and the Foreign less-liquid interest-bearing assets. We have:

$$E_t\rho_{t+1} = -\left(\frac{\bar{\kappa}}{1+\bar{i}-\bar{\kappa}}\right)i_t^* - \frac{1-\beta}{\beta}\left(\frac{\bar{\kappa}'}{1+\bar{i}-\bar{\kappa}}\right)\varepsilon_t^* .$$

Average the last two equations to get:

$$E_t\rho_{t+1} = -\frac{1}{2}\left(\frac{\bar{\kappa}}{1+\bar{i}-\bar{\kappa}}\right)(i_t^* - i_t) + \frac{1-\beta}{\beta}\left(\frac{\bar{\kappa}'}{1+\bar{i}-\bar{\kappa}}\right)(\varepsilon_t - \varepsilon_t^*) .$$

We can write this as in the text of the paper as:

$$E_t\rho_{t+1} = -\alpha(i_t^* - i_t) + \eta_t ,$$

with $\alpha \equiv \frac{1}{2}\left(\frac{\bar{\kappa}}{1+\bar{i}-\bar{\kappa}}\right)$ and $\eta_t \equiv \frac{1-\beta}{\beta}\left(\frac{\bar{\kappa}'}{1+\bar{i}-\bar{\kappa}}\right)(\varepsilon_t - \varepsilon_t^*)$.

A.6 Model Simulations

As stated in the text, the baseline parameters are given by $\beta = 0.998$, $\delta = 0.014$, $\sigma = 0.1275$, $\phi = 0.915$, $\alpha = 0.15$, and $\xi = 0.99$, where the latter is the serial correlation of $\bar{q}_t$ assuming that variable follows a first-order autoregressive process. In addition, the variance of η_t is set equal to 0.04 times the variance of $\bar{q}_t$: $\text{var}(\eta_t)/\text{var}(\bar{q}_t) = 0.04$.

The price stickiness parameter is set as a compromise. According to Monacelli (2004), a standard parameterization assumes that the expected price duration is one year. However, Bils and Klenow (2004) find that the half-life of prices in the data for the U.S. consumer price index is around 5.5 months, excluding sale items, implying an expected duration of around 8 months. The parameter δ is calculated from the formula $\delta \equiv (1-\theta)(1-\theta\beta)/\theta$, where θ is the probability of not adjusting the price in any period in the Calvo model.

We do not observe the equilibrium real exchange rate. This calibration assumes a half-life of five years for the equilibrium real exchange rate. This is based, first, on Rogoff's (1996) well-cited claim that the consensus is that among high-income countries, the half-life of real exchange rates is 3 to 5 years. The upper end of that range is appropriate because Rogoff's consensus applies to the actual real exchange rate, rather than the equilibrium rate, so some of the adjustment involves convergence of the disequilibrium component that might be due to price stickiness. Rogoff's "purchasing power parity puzzle", however, refers in part to the fact that the actual real exchange rate is too persistent to be accounted for by adjustment of the disequilibrium component. Second, Engel (2000) decomposes the real exchange rate for the U.S. relative to the U.K. into a component identified as an equilibrium component – based on the relative price of nontraded to traded goods – and a disequilibrium component. The equilibrium component has a serial correlation of 0.97 in quarterly data, implying a serial correlation of around 0.99 in monthly data.

The value of α is based on Nagel's (2014) estimates in a regression of his measure of the liquidity premium on the U.S. fed funds rate. That parameter ranges from a low of around 0.05 to a high of around 0.11, depending on the regression specification and the measure of the liquidity premium. The liquidity premium refers to the spread between the repo rate on repurchase agreements with Treasury collateral and the T-bill rate, which Nagel argues captures mostly liquidity return because there is very little risk in the repos. We choose a value of α slightly larger than Nagel's estimates in order to capture the idea that the foreign deposits may be even less liquid than these repos.

In the table below, we alter each parameter individually, leaving the others unchanged. (In the case of the Taylor rule, in the first three rows, we consider changes in the smoothing parameter, ϕ, leaving the long-run response of policy interest rate to inflation unchanged ($\phi/(1-\sigma) = 1.5$.) We report the slope coefficient for the regression of short-run excess returns on the real interest differential as in Table 3, the regression of the cumulated expected excess returns on the real interest differential, as in Table 5, and the correlation of real and nominal interest rates (which is 0.79 in our data for the real and nominal interest rates of the U.S. relative to the G6 average.)

Parameter	value	Slope Coefficient for Table 3	Slope Coefficient for Table 5	Correlation of real and nominal interest rates
baseline		1.81	-20.66	0.77
ϕ	0.95	1.09	-10.86	0.61
ϕ	0.90	2.09	-23.52	0.81
ϕ	0.85	2.93	-29.21	0.89
ϕ	0.95	1.93	-16.98	0.91
ϕ	0.90	1.63	-18.23	0.62
ϕ	0.88	1.03	-1.25	0.23
σ	0.10	1.04	-8.06	0.41
σ	0.20	2.53	-19.70	0.95
σ	0.30	3.18	-16.11	0.98
δ	0.01	1.21	-22.10	0.78
δ	0.015	1.94	-20.37	0.77
δ	0.02	2.66	-18.83	0.76
ξ	0.995	7.62	-49.81	0.85
ξ	0.98	0.39	-6.74	0.66
ξ	0.90	0.05	-0.11	0.45
α	0.10	1.55	-13.51	0.77
α	0.20	2.10	-27.70	0.77
α	0.30	2.78	-41.44	0.77
$\mathrm{var}(\eta_t)/\mathrm{var}(\bar{q}_t)$	0.02	0.78	-21.89	0.77
$\mathrm{var}(\eta_t)/\mathrm{var}(\bar{q}_t)$	0.05	2.32	-20.06	0.77
$\mathrm{var}(\eta_t)/\mathrm{var}(\bar{q}_t)$	0.10	4.79	-17.11	0.77

Bils, Mark and Peter J. Klenow. 2004. “Some Evidence on the Importance of Sticky Prices.” Journal of Political Economy 112, 947-985.

Engel, Charles. 2000. “Long-Run PPP May Not Hold After All.” Journal of International Economics 57, 243-273.

Monacelli, Tommaso. 2004. “Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy.” Journal of International Economics 62, 191-217.

A.7 Relation to Froot and Ramadorai (2005)

On the surface, there appears to be a strong relationship between some of the empirical work in Froot and Ramadorai (2005, hereinafter FR), and the empirical findings of this paper. FR write out an expression for the real exchange rate similar to (3), and estimate a VAR that is similar to the VECM estimated in this paper.

The main focus of FR is on the contribution of institutional-investor currency flows to the determination of exchange rates. They ask whether those flows are contributing to movements of the exchange rate because they contribute to the component determined by the sum of expected real interest differentials or to the sum of expected excess returns. They also ask whether it is news about future trades that affects the exchange rate, or the actual current trades.

There is a small section of the paper (Tables 5 and 6, pp. 1558-1560) that focuses on comovements of real interest rates and excess returns. However, their focus is on the comovement of innovations of real interest rates and excess returns at different horizons, while this paper focuses on the unconditional covariances of ex ante returns and real interest rates, $\text{cov}(E_t\rho_{t+1}, r_t^* - r_t)$ and $\text{cov}\left(E_t\sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right)$. That is, this study is concerned with unconditional covariances of returns that are known at time *t*, while FR are concerned with covariances of innovations of returns with innovations in real interest rates. While these covariances are not completely unrelated, as we will show here, they look at different aspects of the data. FR do not cite any of the literature on the interest parity puzzle, and do not relate their findings to that work. They do have a notion of overreaction of the exchange rate to interest rate changes, but that is different than our notion of excess comovement – again, it reflects the different between covariances of shocks versus unconditional covariances.

The comparison of the moments estimated in the two papers is made more difficult by a problem with FR’s estimation. Their system is overidentified, meaning that there are an infinite number of estimates of the moments they are concerned with that could be generated from their VAR. This presents a difficulty in comparing the moments they estimate because their Tables V and VI label moments using the notation for the sample moments they calculate, but the overidentification causes some murkiness in translating those to population moments.

This section of the appendix proceeds in three parts. In part A.7.a, some general observations are made about the different information that is embodied in unconditional correlations and correlations of shocks. Part A.7.b discusses some of the problems with FR’s estimation, and some general issues with estimating this type of model. Part A.7.c then compares the moments estimated in this paper to the moments estimated in FR.

A.7.a

A useful place to point to the distinction between unconditional covariances and covariances of innovations is in the context of the model of delayed overshooting of section 3.2 of this paper. In that section, nominal prices are held constant, and the interest differential follows an exogenously given process. The uncovered interest parity puzzle concerns $\operatorname{cov}(E_t\rho_{t+1}, i_t^* - i_t)$, or $\operatorname{cov}(E_t s_{t+1} - s_t, i_t^* - i_t)$. The delayed overshooting literature has focused on impulse response functions from identified monetary shocks. If the monetary shock were the only shock, we could express these innovations as $\operatorname{cov}\left(s_{t+j} - E_{t-1}s_{t+j}, i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right)$.

It is common in the literature to invoke a "hump shape" in the impulse response function as an explanation for the Fama regression finding of $\operatorname{cov}(E_t s_{t+1} - s_t, i_t^* - i_t) > 0$. (Note that the literature usually expresses both the Fama regression and the impulse responses in terms of $i_t - i_t^*$ rather than $i_t^* - i_t$ as in this paper.) The "delayed overshooting" literature finds that initially the domestic currency appreciates as the home interest rate increases, so $\operatorname{cov}\left(s_t - E_{t-1}s_t, i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right) > 0$. The impulse response of future changes in the exchange rate is in the direction of further appreciation: $\operatorname{cov}\left(s_{t+1} - s_t - E_{t-1}\left(s_{t+1} - s_t\right), i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right) > 0$, for example. The impulse responses for subsequent changes in the exchange rate have the same sign for a number of periods before reversing sign, so that the maximum impulse response does not occur in the first period. This is in contrast to the implications of the Dornbusch model, and has been called delayed overshooting.

Does the finding of $\operatorname{cov}\left(s_{t+1} - s_t - E_{t-1}\left(s_{t+1} - s_t\right), i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right) > 0$ offer an explanation for $\operatorname{cov}(E_t s_{t+1} - s_t, i_t^* - i_t) > 0$? Not necessarily. The model of section 3.2 offers a nice illustration of the point. It was noted in that section that $\operatorname{cov}(s_{t+1} - s_t, i_t - i_t^*) < 0$ holds if and only if $\alpha\left(1-\delta\right)\left(1-\theta^2\right) < \theta\delta - 1$. Recall that α determines the initial response of the exchange rate to the interest-rate shock, θ measures the persistence of the interest differential (modeled as a first-order autoregression), and δ determines the speed of adjustment of the exchange rate to the equilibrium exchange rate. We assumed $0 \le \delta < 1$, $0 \le \theta < 1$, and $\alpha < 0$. On the other hand, $\operatorname{cov}\left(s_{t+1} - s_t - E_{t-1}\left(s_{t+1} - s_t\right), i_t^* - i_t - E_{t-1}\left(i_t^* - i_t\right)\right) > 0$ holds if and only if $\alpha\left(1-\delta\right) < -1$. The two conditions happen to be equivalent if $\theta = \delta$, but there is no economic reason that condition should hold.

Notice that the condition for positive covariance of the impulse response does not depend at all on the persistence of the interest rate differential. Even if the impulse response function is hump-shaped, if the interest differential is persistent enough, it will not account for the uncovered interest parity puzzle in the form $\operatorname{cov}(E_t s_{t+1} - s_t, i_t^* - i_t) > 0$. That is not a trivial consideration given the persistence of interest rates among the

G7 countries. So a hump-shaped impulse response function is not sufficient to account for the interest-parity puzzle.

A.7.b

FR consider a VAR with the vector of variables $z_t' = [\rho_t \quad f_t \quad d_t \quad q_t]$. Here, ρ_t is defined as in this paper, and corresponds to FR's variable r_t. f_t is FR's measure of institutional investor flows, and does not play a role in this discussion. We adopt FR's notation for d_t, which is defined as:

$$d_t \equiv i_{t-1}^* - \pi_t^* - (i_{t-1} - \pi_t) \ .$$

q_t is the real exchange rate as defined in this paper, and corresponds to the variable δ_t in FR.

FR specify a VAR for z_t of the form:

$$z_t = \Gamma z_{t-1} + u_t \ .$$

FR do not impose any restrictions on the matrix Γ.

Recall $\rho_t \equiv i_{t-1}^* + s_t - s_{t-1} - i_{t-1} = d_t + q_t - q_{t-1}$.

Using FR's notation where ej is a column vector with a one in position *j* and 0 elsewhere, there should be a restriction of the form $e1'z_t = (e3' + e4')z_t - e4'z_{t-1}$. The excess return ρ_t is defined in terms of d_t, q_t and q_{t-1}, and so cannot have unrestricted dynamics that are defined independently. A key variable in FR's analysis is the innovation in the excess return, $\rho_t - E_{t-1}\rho_t$. Given the definition of ρ_t, we have $\rho_t - E_{t-1}\rho_t = d_t - E_{t-1}d_t + q_t - E_{t-1}q_t$, or $e1'u_t = (e3' + e4')u_t$. FR do not impose any restriction that insures this equality holds. So, one could use as measures of $\rho_t - E_{t-1}\rho_t$ either $e1'u_t$ or $(e3' + e4')u_t$ or any weighted average of the two. FR report only one of these measures, and do not test whether the overidentifying restriction is valid.

Actually, had FR estimated the system by OLS, equation-by-equation, the restriction would be imposed by the estimation. The covariance matrix for the system would be singular, but that is how it should be. To see this, write out the system in detail:

$$\begin{bmatrix} \rho_t \\ f_t \\ d_t \\ q_t \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} \end{bmatrix} \begin{bmatrix} \rho_{t-1} \\ f_{t-1} \\ d_{t-1} \\ q_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \\ u_{4t} \end{bmatrix} .$$

Let a ^ represent the OLS estimate of the parameter. Since $q_t - q_{t-1} = \rho_t - d_t$, one estimated relationship from this equation has:

$$q_t - q_{t-1} = (\hat{\gamma}_{11} - \hat{\gamma}_{31})\rho_{t-1} + (\hat{\gamma}_{12} - \hat{\gamma}_{32}) f_{t-1} + (\hat{\gamma}_{13} - \hat{\gamma}_{33}) d_{t-1} + (\hat{\gamma}_{14} - \hat{\gamma}_{34}) q_{t-1} + \hat{u}_{1t} - \hat{u}_{3t} \ .$$

Another estimated relationship is:

$$q_t - q_{t-1} = \hat{\gamma}_{41}\rho_{t-1} + \hat{\gamma}_{42} f_{t-1} + \hat{\gamma}_{43} d_{t-1} + (\hat{\gamma}_{44} - 1) q_{t-1} + \hat{u}_{4t}.$$

From the properties of OLS estimators, it is the case that $\hat{\gamma}_{41} = \hat{\gamma}_{11} - \hat{\gamma}_{31}$, $\hat{\gamma}_{42} = \hat{\gamma}_{12} - \hat{\gamma}_{32}$, $\hat{\gamma}_{43} = \hat{\gamma}_{13} - \hat{\gamma}_{33}$, and $\hat{\gamma}_{44} - 1 = \hat{\gamma}_{14} - \hat{\gamma}_{34}$, so $\hat{u}_{4t} = \hat{u}_{1t} - \hat{u}_{3t}$. However, it is apparent from the discussion of estimation in FR on pages 1544-1545 that the overidentifying restrictions are not imposed, either deliberately or by dint of the estimation technique.

In fact, FR report moments involving the residuals of the first three elements of u_t but not the fourth. It is tempting to believe that an easy fix to the problem is for FR to drop q_t from their VAR. However, that points to another issue. Suppose their VAR only contains ρ_t and d_t but not q_t (we ignore f_t for simplicity.) So let $x_t' = [\rho_t \quad d_t]$, and consider a VAR of the form $x_t = \mathrm{B} x_{t-1} + v_t$. In this case, there is only one measure of $\rho_t - E_{t-1}\rho_t$ given by $e1'u_t$. However, FR base their analysis on this version of equation (3) above:

$$q_t = E_t \sum_{j=0}^{\infty} \left(r_{t+j}^* - r_{t+j} \right) - E_t \sum_{j=0}^{\infty} \rho_{t+j+1},$$

where, to make the correspondence between notation clear, $E_t\left(r_{t+j}^* - r_{t+j} \right) = E_t d_{t+j+1}$ for $j \ge 0$. One can construct measures of $E_t \sum_{j=0}^{\infty} \left(r_{t+j}^* - r_{t+j} \right)$ and $E_t \sum_{j=0}^{\infty} \rho_{t+j+1}$ from the VAR, as FR do from their VAR. However, the value of q_t constructed this way does not satisfy the identity $q_t = q_{t+1} + d_{t+1} - \rho_{t+1}$. Crucially for FR it therefore does not satisfy the relationship $\rho_t - E_{t-1}\rho_t = d_t - E_{t-1}d_t + q_t - E_{t-1}q_t$. Why not? The easiest way to see this is that the VAR involving the variables ρ_t and d_t is equivalent to a VAR in the two variables d_t and $q_t - q_{t-1}$, since ρ_t is constructed as the sum of d_t and $q_t - q_{t-1}$. However, such a VAR allows $q_t - q_{t-1}$ to be nonstationary, contrary to FR's assumption that it is stationary. FR's forward looking equation for the real exchange rate is not correct if the real exchange rate is nonstationary. It requires the additional term $\lim_{k\to\infty}(E_t q_{t+k})$, the permanent component of the real exchange rate, that appears in equation (3) above. That is,

$$q_t = \lim_{k\to\infty}(E_t q_{t+k}) + E_t \sum_{j=0}^{\infty} \left(r_{t+j}^* - r_{t+j} \right) - E_t \sum_{j=0}^{\infty} \rho_{t+j+1}.$$

If the VAR were instead defined in terms of d_t and q_t, which is a simplified version of the specification in this paper, no problem arises. In this specification ρ_t is a residual, calculated to satisfy $\rho_t = d_t + q_t - q_{t-1}$, and it follows that the relationship $\rho_t - E_{t-1}\rho_t = d_t - E_{t-1}d_t + q_t - E_{t-1}q_t$ holds, and there are no overidentifying restrictions that are ignored.

A.7.c

In order to compare the moments calculated in this paper with those that FR intended to calculate, following the logic of the previous paragraph, consider an example of a stochastic system involving d_t and q_t. It is useful to consider an MA representation. Assume:

$$q_t = \sum_{i=0}^{\infty} \alpha_i \varepsilon_{t-i} + \sum_{i=0}^{\infty} \beta_i u_{t-i}$$

$$d_t = \sum_{i=0}^{\infty} \gamma_i \varepsilon_{t-i} + \sum_{i=0}^{\infty} \delta_i u_{t-i} \,.$$

Here ε_t and u_t are i.i.d., mean zero, mutually uncorrelated random variables. We specify d_t and q_t as the sum of two moving average processes to facilitate intuition for the empirical findings in this paper.

The two moments we focus on in this paper are $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t)$ and $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right)$, which equal $\text{cov}(E_t \rho_{t+1}, E_t d_{t+1})$ and $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, E_t d_{t+1}\right)$, respectively. With a bit of work, one finds:

$$\text{cov}(E_t \rho_{t+1}, E_t d_{t+1}) = \text{var}(\varepsilon) \cdot \sum_{i=1}^{\infty} \gamma_i (\alpha_i - \alpha_{i-1} - \gamma_i) + \text{var}(u) \cdot \sum_{i=1}^{\infty} \delta_i (\beta_i - \beta_{i-1} - \delta_i)$$

$$\text{cov}\left(E_t \sum\nolimits_0^\infty \rho_{t+j+1}, E_t d_{t+1}\right) = \text{var}(\varepsilon) \left[\sum_{i=1}^{\infty} \gamma_i \sum_{j=i}^{\infty} \gamma_i - \sum_{i=1}^{\infty} \gamma_i \alpha_{i-1} \right] + \text{var}(u) \left[\sum_{i=1}^{\infty} \delta_i \sum_{j=i}^{\infty} \delta_i - \sum_{i=1}^{\infty} \delta_i \beta_{i-1} \right].$$

The finding that $\text{cov}(E_t \rho_{t+1}, r_t^* - r_t)$ is positive and $\text{cov}\left(E_t \sum_0^\infty \rho_{t+j+1}, r_t^* - r_t\right)$ is negative is easy to reconcile in this general unstructured setting. With two economic forces driving real exchange rates and interest rates, one might be more important in the short run and one more important in the longer run. For example we might have $\text{var}(\varepsilon) > \text{var}(u)$, but the coefficients on ε in the moving average processes for d_t and q_t might die out more quickly than those on *u*.

FR's Tables V and VI calculate covariances of four innovations related to excess returns with four innovations related to interest rate differentials. In terms of our MA example, the innovations in excess returns (reading across the top row of FR's tables) are:

$$\rho_t - E_{t-1} \rho_t = (\alpha_0 + \gamma_0) \varepsilon_t + (\beta_0 + \delta_0) u_t$$

$$(E_t - E_{t-1})(\rho_{t+1} + \rho_{t+2} + \ldots + \rho_{t+k}) = \left(\alpha_k - \alpha_0 + \sum_{i=1}^{k} \gamma_i \right) \varepsilon_t + \left(\beta_k - \beta_0 + \sum_{i=1}^{k} \delta_i \right) u_t$$

$$(E_t - E_{t-1})(\rho_{t+k+1} + \rho_{t+k+2} + \ldots) = \left(-\alpha_k + \sum_{i=k+1}^{\infty} \gamma_i \right) \varepsilon_t + \left(-\beta_k + \sum_{i=k+1}^{\infty} \delta_i \right) u_t$$

$$(E_t - E_{t-1})(d_t + d_{t+1} + \ldots) = \sum_{i=0}^{\infty} \gamma_i \varepsilon_t + \sum_{i=0}^{\infty} \delta_i u_t$$

The innovations in interest rates (reading down the column in FR's tables) are:

$$d_t - E_{t-1}d_t = \gamma_0\varepsilon_t + \delta_0 u_t$$

$$\left(E_t - E_{t-1}\right)\left(d_{t+1} + d_{t+2} + \ldots + d_{t+k}\right) = \sum_{i=1}^{k}\gamma_i\varepsilon_t + \sum_{i=1}^{k}\delta_i u_t$$

$$\left(E_t - E_{t-1}\right)\left(\rho_{t+k+1} + \rho_{t+k+2} + \ldots\right) = \sum_{i=k+1}^{\infty}\gamma_i\varepsilon_t + \sum_{i=k+1}^{\infty}\delta_i u_t$$

$$\left(E_t - E_{t-1}\right)\left(d_t + d_{t+1} + \ldots\right) = \sum_{i=0}^{\infty}\gamma_i\varepsilon_t + \sum_{i=0}^{\infty}\delta_i u_t$$

(Note that the last innovation in each group is the same, but in FR is different because of their overidentification. Also, in deriving the expressions from FR, it is helpful to note a typo in that paper. In the lower part of page 1543, FR state $v_{er,t} = e1'\Phi u_t$ and $v_{iv,t} = e1'\Psi u_t$. But given how $v_{er,t}$ and $v_{iv,t}$ are defined on page 1541, these are expressions for $v_{er,t-1}$ and $v_{iv,t-1}$.)

Inspecting the covariances of the first set of innovations with the second set, we find no moments that correspond to $\text{cov}(E_t\rho_{t+1}, E_t d_{t+1})$ and $\text{cov}\left(E_t \sum_0^{\infty}\rho_{t+j+1}, E_t d_{t+1}\right)$ as calculated above. This is not surprising given the example in A.5.a, which showed how covariances of innovations may give quite different information than unconditional covariances.