Real Rigidity, Nominal Rigidity, and the Social Value of Information

_by_ George-Marios Angeletos, Luigi Iovino and Jennifer La'O[1]

Online Appendices

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This document contains the two online appendices for our article.

Appendix A: Proofs for the Baseline Model

This appendix contains the proofs of all the formal results that appear in Sections II-III of our paper. (The numbering of equations, lemmas, and propositions throughout this document is consistent with the one used in the paper.)

Derivation of equation (1). Let p_{it} be the price index for the consumption basket of the goods produced in island i. The optimal consumption decision satisfies

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\rho} C_t \quad \text{and} \quad c_{ijt} = \left(\frac{p_{ijt}}{p_{it}}\right)^{-\eta_{it}} c_{it},$$

for, respectively, the aforementioned basket and the particular good produced by firm j in island i. In equilibrium, consumption coincides with production. It follows that the inverse demand function faced by firm j in island i is given by

$$p_{ijt} = D_{it}\, y_{ijt}^{-\frac{1}{\eta_{it}}}, \tag{14}$$

where $D_{it} \equiv p_{it} y_{it}^{\frac{1}{\eta_{it}}} = P_t Y_t^{\frac{1}{\rho}} y_{it}^{\frac{1}{\eta_{it}} - \frac{1}{\rho}}$ is taken as given by the individual firm but is determined endogenously within the island.

[1] Angeletos: Department of Economics, MIT, and NBER, angelet@mit.edu; Iovino: Department of Economics, Bocconi University, and IGIER, luigi.iovino@unibocconi.it; La'O: Department of Economics, Columbia University, and NBER, jenlao@columbia.edu.

Consider now the optimal behavior of the individual firm. Given that the marginal value of (nominal) income for the representative household is $U'(Y_t)/P_t$, the firm's objective is simply the local expectation of its profit times $U'(Y_t)/P_t$. Using (14), this can be expressed as follows:

$$\mathbb{E}_{it}\left[\frac{U'(Y_t)}{P_t}\left(D_{it}y_{ijt}^{1-\frac{1}{\eta_{it}}} - w_{it}n_{ijt}\right)\right]$$

Using $y_{ijt} = A_i n_{ijt}$ and taking the FOC with respect to n_{ijt} gives

$$\mathbb{E}_{it}\left[\left(1-\frac{1}{\eta_{it}}\right)A_{it}U'(Y_t)\frac{D_{it}y_{ijt}^{-\frac{1}{\eta_{it}}}}{P_t} - U'(Y_t)\frac{w_{it}}{P_t}\right] = 0.$$

By the fact that all firms within a given island are symmetric, we have that, in equilibrium, $n_{ijt} = n_{it}$, $y_{ijt} = y_{it}$, and $p_{ijt} = p_{it}$. It follows that $D_{it}y_{ijt}^{-\frac{1}{\eta_{it}}} = P_t Y_t^{\frac{1}{\rho}} y_{it}^{-\frac{1}{\rho}}$ and the above condition reduces to

$$\mathbb{E}_{it}\left[U'(Y_t)\frac{w_{it}}{P_t}\right] = \mathbb{E}_{it}\left[\left(1-\frac{1}{\eta_{it}}\right)U'(Y_t)Y_t^{\frac{1}{\rho}}y_{it}^{-\frac{1}{\rho}}A_{it}\right]$$

Finally, consider the optimal labor supply in island i. The relevant FOC for the household is

$$\chi_{it}V'(n_{it}) = (1-\tau_{it})\mathbb{E}_{it}\left[U'(Y_t)\frac{w_{it}}{P_t}\right]$$

Combining the above two conditions and letting $\mathcal{M}_{it} \equiv \frac{1}{1-\tau_{it}}\frac{\eta_{it}}{\eta_{it}-1}$ gives condition (1). ■

Proof of Lemma 1. Taking logs of both sides of (1) and rearranging gives us

$$\left(\tfrac{1}{\rho}+\epsilon\right)\log y_{it} = -\mu_{it} + \log\mathbb{E}_{it}\left[Y_t^{\frac{1}{\rho}-\gamma}\right] + (1+\epsilon)\,a_{it}.$$

Assuming Y_t is log-normal (which we verify below), the latter can be rewritten as

$$\log y_{it} = \phi_0 + \phi_a a_{it} + \phi_\mu \mu_{it} + \alpha\mathbb{E}_{it}\left[\log Y_t\right],$$

where

$$\phi_0 \equiv \frac{1}{2}\tfrac{\rho}{1+\rho\epsilon}\left(\tfrac{1}{\rho}-\gamma\right)^2 Var\left(\log Y_t\right), \quad \phi_a \equiv \tfrac{\rho(1+\epsilon)}{1+\rho\epsilon}, \quad \phi_\mu \equiv -\tfrac{\rho}{1+\rho\epsilon}, \quad \alpha \equiv \tfrac{1-\rho\gamma}{1+\rho\epsilon}.$$

Note that $\phi_a > 0$ and $\phi_\mu < 0$, reflecting the fact that local output increases with local productivity and decreases with the local level of monopoly power. Finally, α could be either positive or negative, but it is necessarily less than 1. ■

Proof of Lemma 2. Welfare is given by

$$\mathcal{W} = \sum \beta^t W_t$$

where

$$W_t \equiv \mathbb{E}\left[\frac{Y_t^{1-\gamma}}{1-\gamma} - \tfrac{1}{1+\epsilon}\int \chi_{it}\left(\frac{y_{it}}{A_{it}}\right)^{1+\epsilon} di\right]$$

measures the unconditional expectation of the welfare flow in period t. Because the aggregate shocks are i.i.d. across time and all second moments are time-invariant,[2] the unconditional expectations of all the objects that enter into W_t are time-invariant, and hence W_t is itself a time-invariant function of the underlying preference, technology, and information parameters. To simplify the notation, we thus drop the time index t in the rest of this proof and proceed to develop a certain decomposition of the welfare flow W for an arbitrary period.

Before doing this, we highlight a property of log-normal distributions that is utilized repeatedly in this appendix. When a variable X is log-normal with $\ln X \sim \mathcal{N}\left(\bar{x}, \sigma^2\right)$, then, for any $\delta \in \mathbb{R}$, we have that

$$\mathbb{E}[X^\delta] = \exp\left(\delta\bar{x} + \tfrac{1}{2}\delta^2\sigma^2\right) = \left(\exp\left(\bar{x} + \tfrac{1}{2}\sigma^2\right)\right)^\delta \exp\left(\tfrac{1}{2}(\delta-1)\delta\sigma^2\right)$$

and therefore

$$\mathbb{E}[X^\delta] = (\mathbb{E}[X])^\delta \exp\left(\tfrac{1}{2}(\delta-1)\delta\sigma^2\right). \tag{15}$$

We use this property again and again in the derivations that follow, for various X and δ.

Consider the first component of W, which corresponds to the utility of consumption and which is given by $\frac{1}{1-\gamma}\mathbb{E}\left(Y^{1-\gamma}\right)$. Noting that equilibrium Y is log-normal and using the log-normal property, we have that

$$\mathbb{E}\left(Y^{1-\gamma}\right) = \left[\mathbb{E}\left(Y\right)\right]^{1-\gamma} \exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right) Var\left(\log Y\right)\right\} \tag{16}$$

Consider now the second component of W, which corresponds to the disutility of labor. Defining $b_i \equiv A_i^{1+\epsilon}/\chi_i$, letting B denote the cross-sectional mean of b_i, noting that $Y = \left(\int y_i^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}}$, and using once again the log-normal property, we can express the *realized* disutility of labor, in any given state, as follows:

$$\int \chi_i \left(\frac{y_i}{A_i}\right)^{1+\epsilon} di = \mathbb{E}\left[\int \frac{y_i^{1+\epsilon}}{b_i}\right] = \frac{Y^{1+\epsilon}}{B}\exp(H)$$

where

$$H \equiv \tfrac{1}{2}\left(\epsilon + \tfrac{1}{\rho}\right)(1+\epsilon)Var\left(\log y_i|\Theta\right) + \tfrac{1}{2}Var\left(\log b_i|\Theta\right) - (1+\epsilon)Cov\left(\log y_i, \log b_i|\Theta\right)$$

[2]These assumptions are for expositional simplicity; otherwise, the welfare results we document would have to be restated simply by distinguishing the information structure period by period.

and where $\Theta \equiv (Y, B)$ encapsulates the aggregate state of the economy. It follows that the *expected* disutility of labor is given by

$$\mathbb{E}\left[\int \chi_i \left(\frac{y_i}{A_i}\right)^{1+\epsilon} di\right] = \mathbb{E}\left[\frac{Y^{1+\epsilon}}{B}\right] \exp(H) = \frac{\mathbb{E}[Y]^{1+\epsilon}}{\mathbb{E}[B]} \exp(G) \tag{17}$$

where we have used once again the property from (15) to obtain

$$G \equiv H + \tfrac{1}{2}\epsilon(1+\epsilon)Var\left(\log Y\right) + Var\left(\log B\right) - (1+\epsilon)Cov\left(\log Y, \log B\right).$$

Because of our Gaussian specification, the variance and covariance terms that enter H and G above are constants (non-random and time-invariant), and hence H and G are themselves constants.

Combining (16) and (17), we infer that the per-period welfare is given by

$$W = \tfrac{1}{1-\gamma}\left[\mathbb{E}\left(Y\right)\right]^{1-\gamma} \exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y\right)\right\} - \tfrac{1}{1+\epsilon}\tfrac{\left[\mathbb{E}(Y)\right]^{1+\epsilon}}{\mathbb{E}(B)} \exp(G). \tag{18}$$

Next, let us define $\hat{Y}$ as the value of $\mathbb{E}\left(Y\right)$ that maximizes expression (18) for W, taking as given B, G, and $Var\left(\log Y\right)$. Clearly, this is given by taking the FOC of (18) with respect to $\mathbb{E}(Y)$ and equating this with 0, or equivalently by the solution to the following condition:

$$\hat{Y}^{1-\gamma} \exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y\right)\right\} = \tfrac{\hat{Y}^{1+\epsilon}}{\mathbb{E}(B)} \exp(G) \tag{19}$$

We can then restate W as follows:

$$W = \left\{\tfrac{1}{1-\gamma}\left[\frac{\mathbb{E}\left(Y\right)}{\hat{Y}}\right]^{1-\gamma} - \tfrac{1}{1+\epsilon}\left[\frac{\mathbb{E}\left(Y\right)}{\hat{Y}}\right]^{1+\epsilon}\right\} \tfrac{\hat{Y}^{1+\epsilon}}{\mathbb{E}(B)} \exp(G)$$

If $\mathbb{E}(Y)$ happens to equal $\hat{Y}$, then $W = \hat{W}$, where

$$\hat{W} \equiv \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)} \tfrac{\hat{Y}^{1+\epsilon}}{\mathbb{E}(B)} \exp(G). \tag{20}$$

Letting

$$\Delta \equiv \frac{\mathbb{E}(Y)}{\hat{Y}} \qquad \text{and} \qquad v(\Delta) \equiv \frac{U(\Delta) - V(\Delta)}{U(1) - V(1)} = \frac{\frac{1}{1-\gamma}\Delta^{1-\gamma} - \frac{1}{1+\epsilon}\Delta^{1+\epsilon}}{\frac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}},$$

we conclude that

$$W = v(\Delta)\hat{W}. \tag{21}$$

The term $v(\Delta)$ therefore identifies the wedge between actual welfare, W, and the reference level $\hat{W}$ that a planner could have afforded if he had a non-contingent subsidy that permitted him to scale up and down the mean level of output and could use it to maximize welfare. To see this more clearly, note that $v(\Delta)$ is strictly concave in Δ and reaches its maximum at $\Delta = 1$ when $\gamma < 1$, whereas it is strictly convex and reaches its minimum at $\Delta = 1$ when $\gamma > 1$. Along with the fact that $\hat{W} > 0$ when $\gamma < 1$ but $\hat{W} < 0$ when $\gamma > 1$ (this fact will be come clear momentarily), this means that $\hat{W}v(\Delta)$ is always strictly concave in Δ, with the maximum attained at $\Delta = 1$.

So far, we have decomposed the per-period welfare flow as $W = \hat{W} v(\Delta)$. In what follows, we proceed to decompose the reference level $\hat{W}$ itself into the product of two terms: the first-best level W^*; and a function of Λ, which encapsulates the welfare losses of volatility and dispersion.

From (19), we have that

$$\hat{Y} = [\mathbb{E}(B)]^{\frac{1}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{\epsilon+\gamma}\left[G + \tfrac{1}{2}\gamma(1-\gamma)Var(\log Y)\right]\right\},$$

which together with (20) gives

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}(B)]^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{G - \tfrac{1+\epsilon}{\epsilon+\gamma}\left[G + \tfrac{1}{2}\gamma(1-\gamma)Var(\log Y)\right]\right\}$$

Equivalently,

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}(B)]^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\hat{\Omega})\right\} \tag{22}$$

where

$$\begin{aligned}
\hat{\Omega} &\equiv \tfrac{2}{1+\epsilon}G + \gamma Var(\log Y) \\
&= (\epsilon+\gamma)Var(\log Y) + \tfrac{2}{1+\epsilon}Var(\log B) - 2Cov(\log Y, \log B) \\
&\quad + \left(\epsilon + \tfrac{1}{\rho}\right)Var(\log y_i|\Theta) + \tfrac{1}{1+\epsilon}Var(\log b_i|\Theta) - 2Cov(\log y_i, \log b_i|\Theta)
\end{aligned}$$

Now, note that the first-best levels of output are given by the fixed point to the following equation:

$$\log y_i^* = (1-\alpha)\tfrac{1}{\epsilon+\gamma}\log b_i + \alpha \log Y^*.$$

It follows that, up to some constants that we omit for notational simplicity,

$$\log Y^* = \tfrac{1}{\epsilon+\gamma}\log B \qquad \text{and} \qquad \log y_i^* - \log Y^* = (1-\alpha)\tfrac{1}{\epsilon+\gamma}(\log b_i - \log B)$$

Using this result towards replacing the terms in $\hat{\Omega}$ that involve b_i and B, we get

$$\begin{aligned}
\hat{\Omega} &= (\epsilon+\gamma)Var(\log Y) + 2\tfrac{(\epsilon+\gamma)^2}{(1+\epsilon)}Var(\log Y^*) - 2(\epsilon+\gamma)Cov(\log Y, \log Y^*) \\
&\quad + \left(\epsilon + \tfrac{1}{\rho}\right)Var(\log y_i|\Theta) + \tfrac{(\epsilon+\gamma)^2}{(1+\epsilon)(1-\alpha)^2}Var(\log y_i^*|\Theta) - 2\tfrac{\epsilon+\gamma}{1-\alpha}Cov(\log y_i, \log y_i^*|\Theta)
\end{aligned}$$

Furthermore, the first-best level of welfare is given by

$$W^* = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}(B)]^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\Omega^*)\right\} \tag{23}$$

where Ω^* obtains from $\hat{\Omega}$ once we replace y_i and Y with, respectively, y_i^* and Y^* (which have themselves been obtained above as functions of the exogenous objects b_i and B). We conclude that

$$\hat{W} = W^* \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\left(\hat{\Omega} - \Omega^*\right)\right\} \tag{24}$$

Finally, using the definitions of $\hat{\Omega}$ and Ω^* together with $1-\alpha = \frac{\epsilon+\gamma}{\epsilon+1/\rho}$, we have

$$\begin{aligned}\frac{\hat{\Omega}-\Omega^*}{\epsilon+\gamma} &= \{Var(\log Y) + Var(\log Y^*) - 2Cov(\log Y, \log Y^*)\} \\ &\quad + \frac{1}{1-\alpha}\{Var(\log y_i|\Theta) + Var(\log y_i^*|\Theta) - 2Cov(\log y_i, \log y_i^*|\Theta)\} \\ &= Var(\log Y - \log Y^*) + \frac{1}{1-\alpha} Var(\log y_i - \log y_i^*|\Theta)\end{aligned}$$

Note that conditioning on $\Theta \equiv (\log Y, \log B)$ is equivalent to conditioning on $(\log Y, \log Y^*)$. Furthermore, because $\log Y$ and $\log Y^*$ are the cross-sectional means (expectations) of, respectively, $\log y_i$ and $\log y_i^*$, we have that

$$\begin{aligned}&Var(\log y_i - \log y_i^* — \log Y, \log Y^*) = \\ &\quad = Var((\log y_i - \log Y) - (\log y_i^* - \log Y^*) — \log Y, \log Y^*) \\ &\quad = Var\left(\log \widetilde{y}_i - \log \widetilde{Y}\right)\end{aligned}$$

Combining the above results with the definitions of Σ, σ and Λ, yields

$$\frac{\hat{\Omega}-\Omega^*}{\epsilon+\gamma} = \Sigma + \frac{1}{1-\alpha}\sigma = \Lambda,$$

and therefore (24) can be restated as

$$\hat{W} = W^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}, \tag{25}$$

which gives the sought-after decomposition of $\hat{W}$.

Note from (23) that the sign of W^* is the same as the sign of $(1-\gamma)$. It follows that the sign of $\hat{W}$ is also the same as that of $(1-\gamma)$, which in turn verifies the claim made earlier on that the product $\hat{W}v(\Delta)$ is strictly convex in Δ with a maximum value of 1 attained at $\Delta = 1$.

Finally, combining (25) with (21), we conclude that

$$\mathcal{W} = v(\Delta)w(\Lambda)$$

where $w(x) \equiv \mathcal{W}^* \exp\left\{-\frac{1}{2}(1+\epsilon)(1-\gamma)x\right\}$ for every x and where $\mathcal{W}^* \equiv \frac{1}{1-\beta}W^*$ is the first-best level of (life-time) welfare. The proof is then completed by noting once again that W^* has the same sign as $1-\gamma$ and therefore that w is a strictly decreasing function of Λ, regardless of whether γ is greater or smaller than 1. The fact that $\mathcal{W}$ is strictly concave in Δ, with a maximum attained at $\Delta = 1$, follows directly from our earlier observation that $W = v(\Delta)\hat{W}$ has these exact properties. ■

Equilibrium with productivity shocks. Suppose the equilibrium production strategy takes a log-linear form:

$$\log y_{it} = \varphi_0 + \varphi_a a_{it} + \varphi_x x_{it} + \varphi_z z_t, \tag{26}$$

for some coefficients $(\varphi_a, \varphi_x, \varphi_z)$. Aggregate output is then given by

$$\log Y_t = \varphi_0 + X + (\varphi_a + \varphi_x)\,\bar{a}_t + \varphi_z z_t$$

where

$$X \equiv \frac{1}{2}\left(\frac{\rho-1}{\rho}\right) Var(\log y_{it}|\Theta) = \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\left[\frac{\varphi_a^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_a\varphi_x}{\kappa_x}\right]$$

adjusts for the curvature in the CES aggregator. It follows that Y_t is log-normal, with

$$\mathbb{E}_{it}[\log Y_t] = \varphi_0 + X + (\varphi_a + \varphi_x)\,\mathbb{E}_{it}[\bar{a}_t] + \varphi_z z_t \tag{27}$$

$$Var_{it}[\log Y_t] = (\varphi_a + \varphi_x)^2\, Var_{it}[\bar{a}_t] \tag{28}$$

where, by standard Gaussian updating,

$$\mathbb{E}_{it}[\bar{a}_t] = \tfrac{\kappa_x}{\kappa_a+\kappa_x+\kappa_z} x_{it} + \tfrac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z} z_t \tag{29}$$

$$Var_{it}[\bar{a}_t] = \tfrac{1}{\kappa_a+\kappa_x+\kappa_z} \tag{30}$$

Because of the log-normality of Y_t, the fixed-point condition (1) reduces to following:

$$\log y_{it} = (1-\alpha)(\Psi a_{it} - \Psi' \log \bar{\mathcal{M}}) + \alpha \mathbb{E}_{it}[\log Y_t] + \Gamma \tag{31}$$

where $\Psi \equiv \frac{1+\epsilon}{\epsilon+\gamma} > 0$, $\Psi' \equiv \frac{1}{\epsilon+\gamma} > 0$, $\log \bar{\mathcal{M}} \equiv -\log\left[\left(\frac{\bar{\eta}-1}{\bar{\eta}}\right)(1-\bar{\tau})\right] \approx \bar{\mu} + \bar{\tau} > 0$ is the overall distortion caused by the monopoly markup and the labor wedge (which are both constant because we are herein focusing on the case with only productivity shocks), and

$$\Gamma = \tfrac{1}{2}\alpha\left(\tfrac{1}{\rho} - \gamma\right) Var_{it}[\log Y_t] = \tfrac{1}{2}\alpha^2\left(\tfrac{1}{\rho} + \epsilon\right) Var_{it}[\log Y_t] > 0$$

Next, combining (31) with (27) and (29), we obtain

$$\begin{aligned} \log y_{it} &= \Gamma - (1-\alpha)\Psi'\lambda_s + (1-\alpha)\,\Psi a_{it} + \alpha\,(\varphi_0 + X + \varphi_z z_t) \\ &\quad + \alpha\,(\varphi_a + \varphi_x)\left(\tfrac{\kappa_x}{\kappa_a+\kappa_x+\kappa_z} x_{it} + \tfrac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z} z_t\right) \end{aligned}$$

For this to coincide with our initial guess in (26) for every realization of shocks and signals, it is necessary and sufficient that the coefficients $(\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ solve the following system:

$$\begin{aligned} \varphi_0 &= \Gamma - (1-\alpha)\Psi' \log \bar{\mathcal{M}} + \alpha(\varphi_0 + X) \\ \varphi_a &= (1-\alpha)\,\Psi \\ \varphi_x &= \alpha\,(\varphi_a + \varphi_x)\,\tfrac{\kappa_x}{\kappa_a+\kappa_x+\kappa_z} \\ \varphi_z &= \alpha\varphi_z + \alpha\,(\varphi_a + \varphi_x)\,\tfrac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z} \end{aligned}$$

The unique solution to this system is given by the following:

$$\varphi_a = (1-\alpha)\,\Psi > 0, \quad \varphi_x = \frac{(1-\alpha)\,\kappa_x}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\alpha\Psi,$$

$$\varphi_z = \tfrac{\kappa_z}{\kappa_a+(1-\alpha)\kappa_x+\kappa_z}\alpha\Psi, \qquad \text{and} \qquad \varphi_0 = -\Psi'\lambda_s + \tfrac{1}{1-\alpha}\left(\alpha X + \Gamma\right)$$

Note then that the coefficients φ_x and φ_z, which capture the individual response to expectations of the aggregate state, are positive if and only if $\alpha > 0$. ■

Proof of Proposition 1. Using the characterization of the equilibrium allocation in the preceding proof along with that of the first best in the proof of Lemma 2, we can calculate the equilibrium value of the aggregate and local output gaps as follows:

$$\begin{aligned}
\log Y_t - \log Y_t^* &= (\varphi_a + \varphi_x + \varphi_z)\,\bar{a}_t + \varphi_z \varepsilon_t - \Psi \bar{a}_t \\
\log y_{it} - \log y_{it}^* &= \varphi_x u_{it}
\end{aligned}$$

It follows that the volatility of the aggregate output gap is

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{(\varphi_a + \varphi_x + \varphi_z - \Psi)^2}{\kappa_a} = \frac{\alpha^2\,(\kappa_a + \kappa_z)}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^2}\Psi^2$$

and the cross-sectional dispersion of the local output gaps is

$$\sigma = \frac{\varphi_x^2}{\kappa_x} = \frac{\alpha^2\,(1-\alpha)^2\,\kappa_x}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^2}\Psi^2.$$

Taking the derivative of Σ with respect to the precision of public information gives

$$\frac{\partial \Sigma}{\partial \kappa_z} = \frac{(1-\alpha)\kappa_x - (\kappa_a + \kappa_z)}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3}\alpha^2\Psi^2$$

which is negative if and only if $\kappa_z > (1-\alpha)\kappa_x - \kappa_a$, while taking the derivative of σ gives

$$\frac{\partial \sigma}{\partial \kappa_z} = -2\frac{\alpha^2\,(1-\alpha)^2\,\kappa_x}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3}\Psi^2$$

which is necessarily negative.

Similarly, taking the derivatives of Σ and σ with respect to the precision of private information, we obtain

$$\frac{\partial \Sigma}{\partial \kappa_x} = -\frac{2\,(1-\alpha)\,(\kappa_a + \kappa_z)}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3}\alpha^2\left(\Psi'\right)^2$$

which is necessarily negative and

$$\frac{\partial \sigma}{\partial \kappa_x} = \tfrac{\kappa_z + \kappa_a - (1-\alpha)\kappa_x}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3}\left(1-\alpha\right)^2\alpha^2\left(\Psi'\right)^2$$

which is negative if and only if $(1-\alpha)\kappa_x > \kappa_z + \kappa_a$. ■

Proof of Theorem 1. From the proof of Proposition 1, we can rewrite Λ as

$$\Lambda = \Sigma + \frac{1}{1-\alpha}\sigma = \frac{\alpha^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)}\Psi^2$$

from which it is immediate that Λ is decreasing in the precision of either public or private information, regardless of the sign of α. Furthermore,

$$\frac{\partial^2 \Lambda}{\partial \kappa_z \partial \alpha} = -\frac{2\alpha\left(\kappa_x + \kappa_z + \kappa_a\right)}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_a\right)^3}\Psi^2$$

which is itself negative if and only if $\alpha > 0$. Finally, note that the distortion in the mean level of output is given by

$$\Delta = \bar{\mathcal{M}}^{\frac{1}{\epsilon+\gamma}} \equiv \left[\left(\tfrac{\bar{\eta}-1}{\eta}\right)(1-\bar{\tau})\right]^{\frac{1}{\epsilon+\gamma}} < 1$$

where $\frac{\bar{\eta}-1}{\eta}$ is the monopoly wedge (the reciprocal of the markup) and $1-\bar{\tau}$ is the labor wedge. Since Δ is invariant to the information structure, the welfare effects of either type of information are captured by the comparative statics of Λ alone, which have been established above. ■

Equilibrium with markup shocks. This follows very similar steps as the characterization of equilibrium in the case with productivity shocks. Suppose equilibrium output takes a log-linear form:

$$\log y_{it} = \varphi_0 + \varphi_\mu \mu_{it} + \varphi_x x_{it} + \varphi_z z_t,$$

for some coefficients $(\varphi_\mu, \varphi_x, \varphi_z)$. This guarantees that aggregate output is log-normal, which in turn implies that the fixed-point condition (1) now reduces to

$$\log y_{it} = (1-\alpha)(\Psi\bar{a} - \Psi'\mu_{it}) + \alpha\mathbb{E}_{it}[\log Y_t] + \Gamma$$

where Ψ, Ψ', and Γ are defined as in the case with productivity shocks. Following similar steps as in that case, we can then show that the unique equilibrium coefficients are given by the following:

$$\varphi_\mu = -\left(1-\alpha\right)\Psi' < 0, \quad \varphi_x = -\tfrac{(1-\alpha)\kappa_x}{\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z}\alpha\Psi',$$

$$\varphi_z = -\tfrac{\kappa_z}{\kappa_\mu + (1-\alpha)\kappa_x + \kappa_z}\alpha\Psi', \qquad \text{and} \qquad \varphi_0 = \Psi\bar{a} + \tfrac{1}{1-\alpha}\left(\alpha X + \Gamma\right)$$

Note that the sign of the coefficients φ_x and φ_z is once again pinned down by the sign of α. ■

Proof of Proposition 2. With only markup shocks, the first-best levels of output are constant. The volatility of aggregate output gaps and the dispersion of local output gaps are thus given by the following:

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{\left(\varphi_\mu + \varphi_x + \varphi_z\right)^2}{\kappa_\mu} = \frac{\alpha^2\kappa_\mu\kappa_z + \left(\left(1-\alpha\right)\kappa_\mu + \left(1-\alpha\right)\kappa_x + \kappa_z\right)^2}{\kappa_\mu\left(\kappa_\mu + \left(1-\alpha\right)\kappa_x + \kappa_z\right)^2}\left(\Psi'\right)^2 \tag{32}$$

$$\sigma = \frac{\varphi_\mu^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_\mu \varphi_x}{\kappa_x} = \frac{(1-\alpha)^2}{\kappa_\xi}\left(\Psi'\right)^2 + \frac{\alpha(1-\alpha)^2(2\kappa_\mu + (2-\alpha)\kappa_x + 2\kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2}\left(\Psi'\right)^2 \tag{33}$$

Next, taking the derivatives with respect to the precision of public information, we obtain

$$\frac{\partial \Sigma}{\partial \kappa_z} = \frac{(2+\alpha)(1-\alpha)\kappa_x + (\kappa_z + \kappa_\mu)(2-\alpha)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}\alpha\left(\Psi'\right)^2$$

which is positive if $\alpha > 0$ and

$$\frac{\partial \sigma}{\partial \kappa_z} = -\frac{2(1-\alpha)^2(\kappa_\mu + \kappa_x + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}\alpha\left(\Psi'\right)^2$$

which is negative if (and only if) $\alpha > 0$.

Similarly, taking the derivatives of Σ and σ with respect to the precision of private information, we obtain

$$\frac{\partial \Sigma}{\partial \kappa_x} = \frac{2(1-\alpha)^2(\kappa_x + \kappa_z + \kappa_\mu)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}\alpha\left(\Psi'\right)^2$$

which is positive if (and only if) $\alpha > 0$ and

$$\frac{\partial \sigma}{\partial \kappa_x} = -\frac{(1-\alpha)^2\left[(2-\alpha)(1-\alpha)\kappa_x + (2-3\alpha)(\kappa_\mu + \kappa_z)\right]}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^3}\alpha\left(\Psi'\right)^2,$$

which is in general ambiguous. ■

Proof of Proposition 3. Using (32) and (33), we can obtain the equilibrium value of Λ as

$$\Lambda = \frac{1-\alpha}{\kappa_\xi}\left(\Psi'\right)^2 + \frac{(1-\alpha)\kappa_x + \kappa_z + (1-\alpha^2)\kappa_\mu}{\kappa_\mu((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)}\left(\Psi'\right)^2$$

(Note that the first term captures the distortion caused by cross-sectional dispersion in actual markups, whereas the second terms captures the distortion caused by the firms' response to their information about the aggregate markup.) It follows that, regardless of the sign of α,

$$\frac{\partial \Lambda}{\partial \kappa_z} = \frac{\alpha^2\left(\Psi'\right)^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2} > 0 \tag{34}$$

$$\frac{\partial \Lambda}{\partial \kappa_x} = \frac{(1-\alpha)\alpha^2\left(\Psi'\right)^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu)^2} > 0, \tag{35}$$

which proves that Λ increases with the precision of either public or private information, regardless of the sign of α. ■

Proof of Proposition 4. Recall that Δ is given by the ratio of the equilibrium value of expected output, $\mathbb{E}[Y]$, to the corresponding optimal value, $\hat{Y}$. The former can be computed from the preceding equilibrium characterization and the latter from condition (19). After some tedious algebra (which is available upon request), we can thus show that

$$\Delta \equiv \frac{\mathbb{E}[Y]}{\hat{Y}} = \exp\left[-\Psi'\left(\bar{\mu} + \tfrac{1}{2}D\right)\right],$$

where

$$\begin{aligned} D &\equiv Var\left(\mu_i\right) + 2\left(1+\epsilon\right)Cov\left(y_i,\mu_i\right) \\ &= \tfrac{1}{\kappa_\xi} + \tfrac{1}{\kappa_\mu} + 2\left(1+\epsilon\right)\left(\frac{\varphi_\mu}{\kappa_\xi} + \frac{\varphi_x}{\kappa_x} + \frac{\varphi_\mu+\varphi_x+\varphi_z}{\kappa_\mu}\right) \\ &= \tfrac{1}{\kappa_\xi} + \tfrac{1}{\kappa_\mu} - 2\left(1+\epsilon\right)\left(\frac{1-\alpha}{\kappa_\xi} + \tfrac{1}{\kappa_\mu} - \frac{\alpha^2}{\kappa_\mu + \left(1-\alpha\right)\kappa_x + \kappa_z}\right)\Psi' \end{aligned}$$

It follows that

$$\frac{\partial \Delta}{\partial \kappa_z} = -\tfrac{1}{2}\Delta\Psi'\frac{\partial D}{\partial \kappa_z} = \frac{\alpha^2\left(\Psi'\right)^2}{\left(\left(1-\alpha\right)\kappa_x + \kappa_z + \kappa_\mu\right)^2}\left(1+\epsilon\right)\Delta > 0 \tag{36}$$

$$\frac{\partial \Delta}{\partial \kappa_x} = -\tfrac{1}{2}\Delta\Psi'\frac{\partial D}{\partial \kappa_x} = \frac{(1-\alpha)\alpha^2\left(\Psi'\right)^2}{\left(\left(1-\alpha\right)\kappa_x + \kappa_z + \kappa_\mu\right)^2}\left(1+\epsilon\right)\Delta > 0. \tag{37}$$

That is, Δ is increasing in the precision of either public or private information, irrespective of whether α is positive or negative. ■

Proof of Theorem 2. To obtain the overall welfare effect, recall that welfare is given by

$$\mathcal{W} = v(\Delta)w(\Lambda) = \mathcal{W}^* v(\Delta)\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}$$

Consider first the case of public information. From the above, we have that

$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = \mathcal{W}^*\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}\left(v'(\Delta)\frac{\partial \Delta}{\partial \kappa_z} - \tfrac{1}{2}v(\Delta)(1+\epsilon)(1-\gamma)\frac{\partial \Lambda}{\partial \kappa_z}\right)$$

From (34) and (36), we have that

$$\frac{\partial \Delta}{\partial \kappa_z} = \frac{\partial \Lambda}{\partial \kappa_z}(1+\epsilon)\Delta$$

It follows that

$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = \mathcal{W}^*\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}\frac{\partial \Lambda}{\partial \kappa_z}H$$

where

$$H \equiv v'(\Delta)(1+\epsilon)\Delta - \tfrac{1}{2}(1-\gamma)(1+\epsilon)v(\Delta) = \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\left[(1+\epsilon)\Delta^{1-\gamma} - (1+2\epsilon+\gamma)\Delta^{1+\epsilon}\right],$$

and, therefore,

$$\frac{\partial \mathcal{W}}{\partial \kappa_z} = \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\mathcal{W}^*\exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}\frac{\partial \Lambda}{\partial \kappa_z}\Delta^{1-\gamma}\left[(1+\epsilon) - (1+2\epsilon+\gamma)\Delta^{\epsilon+\gamma}\right]$$

Note then that the sign of $\mathcal{W}^*$ is the same as that of $(1-\gamma)$ which, together with the facts that $\frac{\partial\Lambda}{\partial\kappa_z} > 0$ and $\Delta > 0$, implies that the sign of $\frac{\partial\mathcal{W}}{\partial\kappa_z}$ is the same as the sign of $(1+\epsilon)-(1+2\epsilon+\gamma)\Delta^{\epsilon+\gamma}$. We conclude that

$$\frac{\partial\mathcal{W}}{\partial\kappa_z} < 0 \quad \text{iff} \quad \Delta > \hat{\Delta},$$

where

$$\hat{\Delta} \equiv \left(\frac{1+\epsilon}{1+2\epsilon+\gamma}\right)^{\frac{1}{\epsilon+\gamma}} \in (0,1).$$

Consider next the case of private information. From (35) and (37), we have that

$$\frac{\partial\Delta}{\partial\kappa_x} = \frac{\partial\Lambda}{\partial\kappa_x}(1+\epsilon)\Delta,$$

as in the case of public information. It follows that

$$\begin{aligned}\frac{\partial\mathcal{W}}{\partial\kappa_x} &= \mathcal{W}^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}\left(v'(\Delta)\frac{\partial\Delta}{\partial\kappa_x} - \tfrac{1}{2}v(\Delta)(1+\epsilon)(1-\gamma)\frac{\partial\Lambda}{\partial\kappa_x}\right) \\ &= \mathcal{W}^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}\frac{\partial\Lambda}{\partial\kappa_x}H\end{aligned}$$

where H is defined as before. By direct implication,

$$\frac{\partial\mathcal{W}}{\partial\kappa_x} < 0 \quad \text{iff} \quad \Delta > \hat{\Delta},$$

where $\hat{\Delta}$ is the same threshold as the one in the case of public information.

Clearly, when a non-contingent subsidy is set optimally, $\mathbb{E}[Y] = \hat{Y}$ or $\Delta = 1 > \hat{\Delta}$. ■

Appendix B: Auxiliary Results and Proofs for Extended Model

This appendix contains the proofs for Section 4, along with a number of auxiliary results. In Section B.1, we provide a characterization of the set of implementable allocations, that is, the set of all allocations that can be part of an equilibrium for *some* monetary policy; we also show that this set remains the same whether monetary policy responds to the state within the same period or with a lag. In Section B.2, we develop a preliminary welfare decomposition, which forms the basis of the particular decompositions that appear in the main text. In Section B.4, we use a numerical example to illustrate an argument made in the main text. In Section B.4, we collect the proofs for all results that appear either in the main text or in Sections B.1 and B.2 of this appendix.

B.1 Equilibrium and Implementability

The equilibrium is defined in a similar manner as in the baseline model, modulo the fact that prices are now set on the basis of incomplete information. Consider the FOCs of firm i, who chooses n_{it} and p_{it} so as to maximize the expected valuation of its profit. Combining these conditions with the household's FOC for labor supply and for the demand of the different commodities, we obtain the following conditions:

$$0 = n_{it}^{1+\epsilon} - \mathbb{E}_{it}\left[\mathcal{M}_{it}U'\left(Y_t\right)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\theta c_{it}\right] \tag{38}$$

$$0 = \mathbb{E}_{it}\left[\left(l_{it}^{1+\epsilon} - \mathcal{M}_{it}U'\left(Y_t\right)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\eta y_{it}\right)\right] \tag{39}$$

where

$$Y_t = \left[\int \left(q_{it}l_{it}^{\eta}\right)^{\frac{\rho-1}{\rho}} di\right]^{\frac{\rho}{\rho-1}}; \tag{40}$$

These conditions are the analogue of condition (1) from the baseline model and identify two of the four key implementability conditions of the general model. The third condition follows form the household's optimal demand for the different commodities and ties relative prices to relative quantities:

$$\frac{p_{it}}{P_t} = \left(\frac{q_{it}l_{it}^{\eta}}{Y_t}\right)^{-\frac{1}{\rho}}; \tag{41}$$

The last condition follows from the Euler condition of the household and ties the nominal interest rate to output growth and inflation:

$$\log(1+R_t) = const + \gamma\left(\mathbb{E}_t[\log Y_{t+1}] - \log Y_t]\right) + \left(\mathbb{E}_t[\log P_{t+1}] - \log P_t\right) \tag{42}$$

To recap, a combination of quantities and prices are part of an equilibrium if and only if (i) the quantities and prices satisfy conditions (38) through (41) and (ii) monetary policy satisfies (42).

We now proceed to restate these conditions in a manner that facilitates our subsequent analysis. For expositional purposes, this is done in three steps. First, in Lemma 6, we restrict attention to the subset of equilibria in which the interest rate is measurable in the current fundamental and the current public signal. Next, in Lemma 7, we show that exactly the same real outcomes as those in Lemma 6 obtain if we instead consider the subset of equilibria in which the interest rate is measurable in the value of the shock at some past period (i.e., if policy reacts with the lag). It is then immediate that the set of implementable allocations remain the same if we also consider the more general case in which the interest rate is arbitrary function of the entire history of the shock. Clearly, the same applies for the public signal. We thus conclude the characterization in Lemma 8 by considering the residual case in which the interest rate depends also on a shock that is orthogonal to the entire history of the fundamental and the public signal. Throughout, we let s_{it} and $\bar{s}_t$ denote the idiosyncratic and aggregate shocks to fundamentals (technology or markups).

Lemma 6 *Suppose that the nominal interest rate satisfies*

$$\log(1+R_t) = \rho_s \bar{s}_t + \rho_z z_t,$$

for some coefficients $r_{\bar{s}}$ and r_z, and consider the following pair of strategies:[3]

$$\log q_{it} = \varphi_0 + \varphi_s s_{it} + \varphi_x x_{it} + \varphi_z z_t \qquad \text{and} \qquad \log l_{it} = l_0 + l_{\bar{s}} \bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t. \tag{43}$$

(i) When prices are set on the basis of incomplete information, a pair of strategies as in (43) can be implemented as part of an equilibrium if and only if the following conditions are satisfied:[4]

$$\varphi_s = \hat{\phi}_s \tag{44}$$

$$\varphi_x = \Gamma_x + \Gamma_x' l_{\bar{s}} \tag{45}$$

$$\varphi_z = \Gamma_z + \Gamma_z' l_{\bar{s}} \tag{46}$$

$$l_s = \frac{1}{\theta}(\varphi_s - \mathbf{1}_{s=a}) \tag{47}$$

$$l_x = \frac{1}{\theta}\varphi_x - \frac{\kappa_x}{\kappa_s+\kappa_x+\kappa_z} l_{\bar{s}} \tag{48}$$

$$l_z = \frac{1}{\theta}\varphi_z - \frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z} l_{\bar{s}}, \tag{49}$$

where $\hat{\phi}_s$ is given in (52), $\mathbf{1}_{s=a}$ is an indicator that takes the value 1 in the case of technology shocks ($s=a$) and 0 in the case of markup shocks ($s=\mu$), Γ_x, Γ_x', Γ_z, Γ_z' are scalars given in the proof, and $l_{\bar{s}}$ is an arbitrary coefficient.

(ii) When instead prices are flexible (i.e., free to adjust to the true sate), there exists a unique pair of strategies as in (43) that can obtain in equilibrium, and this pair is pinned down by the combination of conditions (44)-(49) along with the following condition:

$$l_{\bar{s}} = l_{\bar{s}}^* \equiv \frac{(1-\rho\gamma)\left(1+\frac{\eta}{\theta}\right)\left(\hat{\phi}_s + \Gamma_x\right) - \eta(1-\rho\gamma)\mathbf{1}_{s=a}}{\rho(1+\epsilon-\eta)+\eta-(1-\rho\gamma)\left(\left(1+\frac{\eta}{\theta}\right)\Gamma_x' + \eta\frac{\kappa_s+\kappa_z}{\kappa_s+\kappa_x+\kappa_z}\right)}. \tag{50}$$

This lemma identifies the precise way in which monetary policy can control real allocations. When prices are set on the basis of incomplete information, by appropriately designing the response of the interest rate to the realized shock, the policy maker can choose at will the coefficient $l_{\bar{s}}$, that is, the response of the second-stage labor input (the margin of adjustment in quantities) to the realized technology or markup shock. Conditional on choosing this coefficient, however, the monetary authority has no further control over the real allocation. In this sense, the coefficient

[3]By "strategies" we refer to functions that map the information set of a firm to its employment and production.

[4]There is also a pair of restrictions on φ_0 and l_0, which we omit because these are of no interest: φ_0 and l_0 are irrelevant for the stochastic properties of the equilibrium.

coefficient $l_{\bar{s}}$ is the only "free variable" at the disposal of the policy maker. Finally, when prices are flexible (free to adjust to the realized shock), this variable ceases to be free, and the policy maker has, of course, no control over real allocations (although he can still control the nominal price level).

We now proceed to show that the set of implementable allocations remains the same whether monetary policy responds to the realized state within the same period or with an arbitrary lag.

Lemma 7 *Suppose that the nominal interest rate satisfies*

$$\log(1+R_t) = \rho_{-k}\bar{s}_{t-k} + \rho_z z_t \tag{51}$$

for some $k \geq 1$ and some scalars ρ_{-k} and ρ_z. Parts (i) and (ii) of Lemma 6 continue to hold. That is, the set of implementable allocations remains the same.

By a similar argument as the one found in the proof of this lemma, the set of implementable allocations remains the same if we consider the more general class of policies in which the interest rate is an arbitrary function of the entire history of the fundamental and the public signal. We thus conclude this section by extending Lemma 6 to the only case that has not been allowed so far, namely allowing for the interest rate to contain a pure monetary shock, by which we mean a shock orthogonal to both the fundamental and the public signal (and the histories thereof). This makes no essential difference to the logic underlying the implementability constraints we derived in Lemma 6. It only introduces a mechanical response of output to the monetary shock.[5]

Lemma 8 *Suppose that the nominal interest rate satisfies*

$$\log(1+R_t) = \rho_s\bar{s}_t + \rho_z z_t + r_t,$$

where r_t is a Normally distributed random variable that is orthogonal to both $\bar{s}_t$ and z_t and that is unpredictable by the firms. Then, the second-period labor choice satisfies:

$$\log l_{it} = l_0 + l_{\bar{s}}\bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t - \frac{1}{\eta\gamma} r_t.$$

However, the strategy for q_{it} remains the same and the implementability conditions (44)-(49) are also not affected.

B.2 Welfare

In this subsection, we obtain a preliminary welfare decomposition, which extends Lemma 2 from the baseline model to the more general model under consideration.

[5] This response would itself be more complicated if firms had information about the monetary shock at the moment they make their pricing and production decisions, a possibility which we only briefly discuss in the end of Section 5.

To this goal, we first introduce certain notation:

$$
\begin{aligned}
\hat{\epsilon} &\equiv \tfrac{1+\epsilon}{\theta}-1, \quad \hat{\gamma}\equiv 1-\tfrac{(1-\gamma)(1+\epsilon)}{1+\epsilon-\eta(1-\gamma)}, \quad \hat{\rho}\equiv\tfrac{\rho(1+\epsilon-\eta)+\eta}{1+\epsilon+\eta(1-\rho)}=\tfrac{1}{1-\rho\nu+\nu}, \\
\hat{\alpha} &\equiv \tfrac{1-\hat{\rho}\hat{\gamma}}{1+\hat{\rho}\hat{\epsilon}}, \quad \chi\equiv\tfrac{1+\epsilon}{1+\epsilon-\eta+\gamma\eta}>0, \quad \nu\equiv\tfrac{1+\epsilon}{\rho(1+\epsilon-\eta)+\eta}, \\
\hat{\phi}_a &\equiv \tfrac{\hat{\rho}(1+\hat{\epsilon})}{1+\hat{\rho}\hat{\epsilon}}, \quad \hat{\phi}_\mu\equiv-\tfrac{\hat{\rho}+(\hat{\rho}-1)\frac{\eta}{1+\epsilon}}{1+\hat{\rho}\hat{\epsilon}}, \quad \bar{\alpha}\equiv\tfrac{(1-\rho\gamma)\eta}{\rho(1+\epsilon-\eta)+\eta}.
\end{aligned}
\tag{52}
$$

As in the main text, we also let $q_{it}\equiv A_{it}n_{it}^{\theta}$ denote the component of output that is fixed on the basis of the firm's incomplete information of the state of the economy, and define the corresponding aggregate as

$$
Q_t \equiv \left[\int_I (q_{it})^{\frac{\hat{\rho}-1}{\hat{\rho}}}\,di\right]^{\frac{\hat{\rho}}{\hat{\rho}-1}}.
$$

Next, we denote with $\log\bar{y}_{it}$ and $\log\bar{Y}_t$ the socially optimal levels of, respectively, local and aggregate output, *conditional* on an arbitrary allocation of the q's; and with $\log q_{it}^{*}$ and $\log Q_t^{*}$ the first-best levels of, respectively, $\log q_{it}$ and $\log Q_t$. Finally, we let Σ_Q and σ_q denote, respectively, the volatility of $\log Q_t - \log Q_t^{*}$ and the cross-sectional dispersion of $\log q_{it}-\log q_{it}^{*}$, and similarly we let Σ_Y and σ_y denote, respectively, the volatility of $\log Y_t - \log\bar{Y}_t$ and the cross-sectional dispersion of $\log y_{it}-\log\bar{y}_{it}$.

The first of the following two lemmas characterizes the aforementioned reference points, the first best and the allocation that is optimal conditional on q's. The second lemma then develops the desired welfare decomposition in terms of gaps relative to these reference points.

Lemma 9 *For any given distribution of q in the cross-section, the optimal output levels solve the following fixed-point relation:*

$$
\log\bar{y}_{it}=\rho\nu\log q_{it}+\bar{\alpha}\log\bar{Y}_t. \tag{53}
$$

The first-best allocation satisfies the following fixed-point relation:

$$
\log q_{it}^{*}=\hat{\phi}_a a_{it}+\hat{\alpha}\log Q_t^{*}. \tag{54}
$$

Lemma 10 *There exists a decreasing function w, which is invariant to the information structure, such that welfare satisfies*

$$
\mathcal{W}=w(\Lambda'), \tag{55}
$$

with

$$
\Lambda'=\left(\Sigma_Q+\frac{1}{1-\hat{\alpha}}\sigma_q\right)+\xi\left(\Sigma_Y+\frac{1}{1-\bar{\alpha}}\sigma_y\right), \tag{56}
$$

and where $\hat{\gamma}$ and $\hat{\epsilon}$ are given in (52) and ξ is a positive scalar pinned down by $(\gamma,\epsilon,\theta,\eta)$.

Like Lemma 2 in the baseline model, Lemma 10 is not particularly surprising. It simply decomposes the welfare losses that obtain relative to the first-best in two components. The first component, namely the sum $\Sigma_Q + \frac{1}{1-\tilde{\alpha}}\sigma_q$, capture the distortions (if any) that obtain in the first-stage production decisions, that is, those that must be set on the basis of incomplete information. The second component, namely the sum $\Sigma_Y + \frac{1}{1-\tilde{\alpha}}\sigma_y$, captures the distortions (if any) that obtain in the second-stage production decisions, that is, those that are free to adjust to the realized state. Each of these components contains a volatility and a dispersion subcomponent, reflecting the fact that some distortions are aggregate whereas others are idiosyncratic.

What is interesting, however, is how these components are affected by the information frictions and by the associated types of rigidity. To develop intuition, let us abstract from markup shocks. When information is complete, all distortions vanish, $\Sigma_Q = \Sigma_q = \Sigma_Y = \Sigma_y = 0$, and hence $\Lambda' = 0$. When, instead, information is incomplete, the nature of the distortions depends on whether the incompleteness of information is only the source of real rigidity or also the source of nominal rigidity. In the former case, Σ_Q and Σ_q are positive, reflecting the measurability constraint on quantities, but $\Sigma_Y = \Sigma_y = 0$, reflecting the margin of adjustment in second-stage production to the realized state. In the latter case, by contrast, whether Σ_Y and Σ_y coincide or diverge from zero depends on whether monetary policy coincides or diverges from the benchmark of replicating flexible prices. This discussion therefore underscores how the two types of rigidity map into different kinds of potential distortions, an issue that is further explored in the main text.

B.3 An Example With Different Policy Targets

In the main text, we noted that, if monetary policy tries to stabilize either the price level or the output gap, more precise information may help increase welfare not only by attenuating the real rigidity but also by alleviating the policy suboptimality. We now illustrate the logic behind this argument in Figure B1, with the help of a numerical example. This example assumes the following parameterization: $\theta = \eta = 0.5$, $\epsilon = 1$, $\gamma = 0.25$, $\rho = 0.5$, $\sigma_a = \sigma_x = \sigma_z = .02$, and $\sigma_m = \sigma_\epsilon = 0$.

The figure reports the welfare effects of public information (namely the value of Λ against the value of σ_z) under three alternative specifications of the monetary policy: the optimal policy (solid blue line), a policy that stabilizes the aggregate output gap (dotted red line), and a policy that stabilizes the price level (dashed green line). The last two policies are suboptimal in our model due to the presence of the informational friction, but are useful reference points because they are optimal in the prototypical New-Keynesian model.

For any level of noise $\sigma_z > 0$, the welfare losses associated with targeting either the price level or the output gap are higher than those associated with the optimal policy. Nevertheless, the welfare gap between these policies and the optimal one decreases with the precision of the available information, and vanishes in the limit as $\sigma_z \to 0$. It follows that, under either of these two policies, more information improves welfare via two effects: by bringing the equilibrium allocation closer to the optimal one; and by raising the welfare attained at the optimal allocation.

We hope that these findings give some guidance about the potential role of policies which are

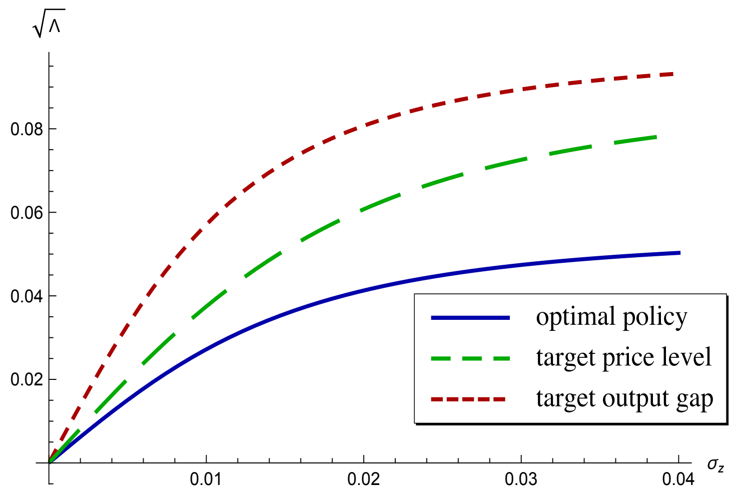

Figure 1: An illustration of the welfare losses due to incomplete information, under different levels of noise in the public signal and under different policy rules.

not exactly optimal in our setting but are perhaps closer to real-world practice, such as a policies that follow a Taylor rule. We nevertheless have to leave any serious quantitative exploration of the issue for future work.

B.4 Proofs

In this subsection, we provide first the proofs of the auxiliary results we stated in parts B.1 and B.2 of this appendix and next the proofs of the results that appear in Section 4.

Proof of Lemma 6. *Part (i).* The proof combines the first-order conditions of the firm's problem with the resource constraint and the monetary policy rule. These are used to derive the conditions that the coefficients in (43) have to satisfy in order to be part of an equilibrium.

We now seek to translate these properties in terms of the relevant coefficients that parameterize the allocations, prices, and policy under a log-normal specification. First note that, since all shocks are independent over time, $\mathbb{E}_t[\log Y_{t+1}]$ and $\mathbb{E}_t[\log P_{t+1}]$ in (42) will be constant. Thus, let (omitting unimportant constants)

$$\begin{aligned}
\log q_{it} &= \varphi_s s_{it} + \varphi_x x_{it} + \varphi_z z_t \\
\log l_{it} &= l_{\bar{s}} \bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t \\
\log Y_t &= c_{\bar{s}} \bar{s}_t + c_z z_t \\
\log p_{it} &= \psi_s s_{it} + \psi_x x_{it} + \psi_z z_t
\end{aligned}$$

for some coefficients $(\varphi_s, \varphi_x, ..., \psi_z)$, and with the understanding that s stands either for a or μ, depending on whether we are considering the case of technology or markup shocks. Note that the resource constraint (40) is satisfied if and only if

$$c_{\bar{s}} = (\varphi_s + \varphi_x) + \eta(l_s + l_x + l_{\bar{s}}) \tag{57}$$

$$c_z = \varphi_z + \eta l_z \tag{58}$$

while (42) is satisfied if and only if

$$\rho_s = -\gamma c_{\bar{s}} - (\psi_s + \psi_x) \tag{59}$$

$$\rho_z = -\gamma c_z - \psi_z. \tag{60}$$

Consider the consumer's demand function (41), which can be expressed as follows:

$$-\rho\left(\log p_{it} - \log P_t\right) = \left(\log y_{it} - \log Y_t\right)$$

Using market clearing, the production function, and the proposed strategies, we can express the output of good i as follows:

$$\begin{aligned}\log y_{it} &= \log q_{it} + \eta \log l_{it} \\ &= (\varphi_s + \eta l_s)s_{it} + (\varphi_x + \eta l_x)x_{it} + (\varphi_z + \eta l_z)z_t + \eta l_{\bar{s}}\bar{s}_t,\end{aligned}$$

By implication, aggregate output satisfies

$$\log Y_{it} = (\varphi_s + \eta l_s + \varphi_x + \eta l_x)\bar{s}_t + (\varphi_z + \eta l_z)z_t.$$

Substituting the above two results in the consumer's demand function, and doing a similar substitution for p_{it} and P_t, we infer that the following must hold for all realizations of shocks and signals:

$$-\rho(\psi_s s_{it} + \psi_x x_{it} - (\psi_s + \psi_x)\bar{s}_t) = (\varphi_s + \eta l_s)s_{it} + (\varphi_x + \eta l_x)x_{it} - (\varphi_s + \eta l_s + \varphi_x + \eta l_x)\bar{s}_t.$$

This is true if and only if

$$\psi_s = -\tfrac{1}{\rho}(\varphi_s + \eta l_s) \qquad \text{and} \qquad \psi_x = -\tfrac{1}{\rho}(\varphi_x + \eta l_x). \tag{61}$$

Finally, taking the logs of conditions (38) and (39) and using the properties of log-normal distributions, we can rewrite these conditions as follows:

$$\mathbb{E}_{it}[\log l_{it}] = \tfrac{1}{\theta}\left(\log q_{it} - a_{it}\right) \tag{62}$$

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \tfrac{\hat{\alpha}}{\chi}\mathbb{E}_{it}\left[\log Y_t\right]. \tag{63}$$

Clearly, condition (62) holds for all i if and only if

$$l_s = \tfrac{1}{\theta}\left(\varphi_s - \mathbf{1}_{s=a}\right) \tag{64}$$

$$l_x = \tfrac{1}{\theta}\varphi_x - l_{\bar{s}}\tfrac{\kappa_x}{\kappa_s + \kappa_x + \kappa_z} \tag{65}$$

$$l_z = \tfrac{1}{\theta}\varphi_z - l_{\bar{s}}\tfrac{\kappa_z}{\kappa_s + \kappa_x + \kappa_z}, \tag{66}$$

while condition (63) holds for all i if and only if

$$\varphi_s = \hat{\phi}_s \tag{67}$$

$$\varphi_x = \frac{\hat{\alpha}}{\chi} c_{\bar{s}} \frac{\kappa_x}{\kappa_s+\kappa_x+\kappa_z} \tag{68}$$

$$\varphi_z = \frac{\hat{\alpha}}{\chi} \left(c_{\bar{s}} \frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z} + c_z \right). \tag{69}$$

We can now use (57)-(58) to replace $c_{\bar{s}}$ and c_z in (68) and (69) to get

$$\varphi_x = \frac{\hat{\alpha}}{\chi} \left(\varphi_s + \varphi_x + \eta \left(l_s + l_x + l_{\bar{s}} \right) \right) \frac{\kappa_x}{\kappa_s+\kappa_x+\kappa_z}$$

and

$$\varphi_z = \frac{\hat{\alpha}}{\chi} \left(\varphi_s + \varphi_x + \eta \left(l_s + l_x + l_{\bar{s}} \right) \right) \frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z} + \frac{\hat{\alpha}}{\chi} \left(\varphi_z + \eta l_z \right),$$

with the understanding that s stands for either a or μ, depending on the case under consideration. Using (64), (65), (66), and (67) and rearranging gives

$$\left(\frac{\kappa_s+\kappa_x+\kappa_z}{\kappa_x} \frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta} \right) \varphi_x = \hat{\phi}_s + \frac{\eta}{\theta} \left(\hat{\phi}_s - \mathbf{1}_{s=a} \right) + \eta \frac{\kappa_s+\kappa_z}{\kappa_s+\kappa_x+\kappa_z} l_{\bar{s}}$$

and

$$\left(\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta} \right) \varphi_z = \frac{\chi}{\hat{\alpha}} \frac{\kappa_z}{\kappa_x} \varphi_x - \eta \frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z} l_{\bar{s}}.$$

or

$$\varphi_x = \Gamma_x + \Gamma'_x l_{\bar{s}},$$

and

$$\varphi_z = \Gamma_z + \Gamma'_z l_{\bar{s}},$$

where

$$\Gamma_x \equiv \frac{\hat{\phi}_s + \frac{\eta}{\theta}\left(\hat{\phi}_s - \mathbf{1}_{s=a}\right)}{\frac{\kappa_s+\kappa_x+\kappa_z}{\kappa_x} \frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}}, \quad \Gamma'_x \equiv \frac{\eta \kappa_x}{\kappa_s+\kappa_x+\kappa_z} \frac{\kappa_s+\kappa_z}{(\kappa_s+\kappa_x+\kappa_z)\frac{\chi}{\hat{\alpha}} - \kappa_x \left(1+\frac{\eta}{\theta}\right)},$$

$$\Gamma_z \equiv \frac{\frac{\chi}{\hat{\alpha}} \frac{\kappa_z}{\kappa_x}}{\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}} \Gamma_x, \quad \Gamma'_z \equiv \frac{\frac{\chi}{\hat{\alpha}} \frac{\kappa_z}{\kappa_x} \Gamma'_x - \eta \frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z}}{\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}}.$$

This completes the proof of the necessity of conditions (44)-(49) for an allocation to be part of an equilibrium.

We now prove sufficiency. Pick arbitrary $l_{\bar{s}}$ and let $(\varphi_s, \varphi_x, \varphi_z, l_s, l_x, l_z)$ be the unique vector that satisfies conditions (44) through (49) for the given $l_{\bar{s}}$. Next, let $(c_{\bar{s}}, c_z, \rho_{\bar{s}}, \rho_z, \psi_s, \psi_x)$ be determined as in (57)-(61) and let $\psi_z = -\gamma c_z$. By construction, the allocations, prices and policies defined in this way constitute an equilibrium, which completes the sufficiency argument.

Part (ii). This proof is the same as that of part (i), except for one key difference: now the marginal costs and returns of second-state employment must be equated state-by-state, not just in expectation. It is this additional restriction that pins down $l_{\bar{s}}$ at the value stated in equation (50) in the lemma. A detailed derivation is available upon request. ■

Proof of Lemma 7. Before proceeding, it is useful to recall two key facts about Lemma 6. First, Lemma 6 established that, when the interest rate is determined according to (42), an allocation as in (43) can be implemented as part of an equilibrium if and only if conditions (44)-(49) are satisfied. Second, the nominal prices that supported such an allocation (i.e., that were consistent with the firm's optimal price-setting behavior) were left outside the statement of that lemma, but were constructed as part of its proof. With this in mind, the strategy underlying the present proof is to show that the class of policies in (51) spans exactly the same set of allocations as the class of policies in (42), but now the nominal prices that support any such allocation are different.

Thus let us start by collecting, once again, the key equilibrium conditions:

$$0 = n_{it}^{1+\epsilon} - \mathbb{E}_{it}\left[\mathcal{M}_{it}U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\theta c_{it}\right] \tag{70}$$

$$0 = \mathbb{E}_{it}\left[\left(l_{it}^{1+\epsilon} - \mathcal{M}_{it}U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\eta y_{it}\right)\right] \tag{71}$$

$$Y_t = \left[\int (q_{it}l_{it}^{\eta})^{\frac{\rho-1}{\rho}} di\right]^{\frac{\rho}{\rho-1}}; \tag{72}$$

$$\frac{p_{it}}{P_t} = \left(\frac{q_{it}l_{it}^{\eta}}{Y_t}\right)^{-\frac{1}{\rho}}; \tag{73}$$

$$\log(1+R_t) = \gamma\left(\mathbb{E}_t[\log Y_{t+1}] - \log Y_t\right) + \left(\mathbb{E}_t[\log P_{t+1}] - \log P_t\right) \tag{74}$$

Clearly, these conditions are necessary and sufficient for equilibrium *regardless* of how R_t is determined.

Now, let us restrict R_t to be determined according to (51) and let us consider the following strategies (omitting unimportant constants):

$$\begin{aligned}
\log q_{it} &= \varphi_s s_{it} + \varphi_x x_{it} + \varphi_z z_t \\
\log l_{it} &= l_{\bar{s}}\bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t \\
\log p_{it} &= \psi_s s_{it} + \psi_x x_{it} + \psi_z z_t + \psi_{-k}\bar{s}_{t-k}
\end{aligned}$$

for some coefficients $(\varphi_s, \varphi_x, ..., \psi_{-k})$. The proof proceeds by obtaining the restrictions that equilibrium imposes on these coefficients and by showing that the restrictions imposed on the φ's and the l's (the quantity-related coefficients) are the same as those found in Lemma 6, whereas those that are imposed on the ψ's (the price-related coefficients) are different. The proof then concludes by obtaining the scalars ρ_{-k} and ρ_z that implement these coefficients.

First, consider conditions (70) and (71), which encapsulated the optimal behavior of the firms and the workers. These conditions can be restated as:

$$\begin{aligned}
\mathbb{E}_{it}[\log l_{it}] &= \tfrac{1}{\theta}\left(\log q_{it} - a_{it}\right) \\
\log q_{it} &= \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \tfrac{\hat{\alpha}}{\chi}\mathbb{E}_{it}\left[\log Y_t\right].
\end{aligned}$$

Because the proposed strategies for q_{it} and l_{it} are the same as those in Lemma 6, and because neither the prices nor the interest rate enter into the above conditions, these conditions reduce to exactly the same restrictions on the ϕ and the l coefficients as those in Lemma 6.

Next, consider condition (72). Under the proposed strategies, this reduces to

$$\log Y_t = c_{\bar{s}}\bar{s}_t + c_z z_t$$

with

$$c_{\bar{s}} = (\varphi_s + \varphi_x) + \eta(l_s + l_x + l_{\bar{s}}) \qquad \text{and} \qquad c_z = \varphi_z + \eta l_z \tag{75}$$

Similarly to the case of the ϕ and the l coefficients, the restrictions that pertain to the coefficients $c_{\bar{s}}$ and c_s are therefore the same as the corresponding ones in the proof of Lemma 6.

Next, consider condition (73), which can be expressed as follows:

$$-\rho\left(\log p_{it} - \log P_t\right) = \left(\log y_{it} - \log Y_t\right)$$

Under the proposed strategies, this reduces to the following:

$$-\rho(\psi_s s_{it} + \psi_x x_{it} - (\psi_s + \psi_x)\bar{s}_t) = (\varphi_s + \eta l_s)s_{it} + (\varphi_x + \eta l_x)x_{it} - (\varphi_s + \eta l_s + \varphi_x + \eta l_x)\bar{s}_t.$$

This in turn is true if and only if

$$\psi_s = -\tfrac{1}{\rho}(\varphi_s + \eta l_s) \qquad \text{and} \qquad \psi_x = -\tfrac{1}{\rho}(\varphi_x + \eta l_x). \tag{76}$$

The restrictions on the coefficients ψ_s and ψ_x are therefore also the same as the corresponding ones in the proof of Lemma 6. (Also note that, up to this point, the coefficients ψ_{-k} and ψ_z are "free".)

Finally, consider the Euler condition (74). Let $X_t \equiv \gamma \log Y_t + \log P_t$ and rewrite (74) as follows:

$$X_t = -\log(1 + R_t) + \mathbb{E}_t[X_{t+1}].$$

Iterating this condition forward k times yields

$$X_t = -\sum_{j=0}^{k} \mathbb{E}_t\left[\log(1 + R_{t+j})\right] + \mathbb{E}_t\left[X_{t+k+1}\right] \tag{77}$$

Under the proposed strategies, the output and price level in period $t + k + 1$ satisfy

$$\begin{aligned}
\mathbb{E}_t\left[\log Y_{t+k+1}\right]\right] &= \mathbb{E}_t\left[c_{\bar{s}}\bar{s}_{t+k+1} + c_z z_{t+k+1}\right] = \mathit{constant} \\
\mathbb{E}_t\left[\log P_{t+k+1}\right]\right] &= \mathbb{E}_t\left[\left(\psi_s + \psi_x\right)\bar{s}_{t+k+1} + \psi_z z_{t+k+1} + \psi_{-k}\bar{s}_{t+1}\right]\right] = \mathit{constant},
\end{aligned}$$

From (51), the interest rate satisfies

$$\begin{aligned}
\mathbb{E}_t\left[\log(1 + R_{t+j})\right] &= \mathbb{E}_t\left[\rho_{-k}\bar{s}_{t-k+j} + \rho_z z_{t+j}\right] \\
&= \begin{cases} \rho_{-k}\bar{s}_{t-k} + \rho_z z_t, & \text{for } j = 0 \\ \mathit{constant}, & \text{for } j \in \{1, ..., k-1\} \\ \mathit{constant} + \rho_{-k}\bar{s}_t, & \text{for } j = k \end{cases}
\end{aligned}$$

It follows that (77) reduces to the following (omitting constants):

$$X_t = -\rho_{-k}\bar{s}_{t-k} - \rho_z z_t - \rho_{-k}\bar{s}_t$$

At the same time, the proposed strategies imply

$$\begin{aligned} X_t \equiv \gamma \log Y_t + \log P_t &= \gamma\left(c_{\bar{s}}\bar{s}_t + c_z z_t\right) + \left(\left(\psi_s + \psi_x\right)\bar{s}_t + \psi_z z_t + \psi_{-k}\bar{s}_{t-k}\right) \\ &= \left(\gamma c_{\bar{s}} + \psi_s + \psi_x\right)\bar{s}_t + \left(\gamma c_z + \psi_z\right) z_t + \psi_{-k}\bar{s}_{t-k} \end{aligned}$$

The two are consistent if and only if the following hold true:

$$\psi_{-k} = \gamma c_{\bar{s}} + \psi_s + \psi_x \tag{78}$$

$$\rho_{-k} = -\psi_{-k} \tag{79}$$

$$\rho_z = -\gamma c_z - \psi_z \tag{80}$$

Now, pick any allocation that is implementable under (42). This is equivalent to choosing an arbitrary value for $l_{\bar{s}}$ and letting the ϕ and l coefficients be determined by conditions (44)-(49). Any such choice also pins down the values of $c_{\bar{s}}$, c_z, ψ_s, and ψ_x according to the restrictions (75)-(76). But then this pins down ψ_{-k} from (78), which in turn pins down ρ_{-k} from (79). We have thus arrived to the value of ρ_{-k} which is necessary and sufficient for the policy in (51) to implement the allocation under consideration. Finally, what we are left with then is the familiar indeterminacy of the response of the price level to the public signal: for any allocation that has been implemented, we can still pick an arbitrary ψ_z and let ρ_z be determined from (80). ■

Proof of Lemma 8. Since prices are set without any information about the monetary shock (a maintained assumption throughout), employment and aggregate output will respond to $\tilde{r}_t$ one-to-one. Thus, if we conjecture

$$\begin{aligned} \log l_{it} &= l_{\bar{s}}\bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t + l_r r_t \\ \log Y_t &= c_{\bar{s}}\bar{s}_t + c_z z_t + c_r r_t \end{aligned}$$

and follow steps analogous to those in the proof of part (i) of Lemma 6, we obtain $c_r = \eta l_m = -1/\gamma$. Modulo these changes, it is immediate to see that conditions (44)-(49) remain true. ■

Proof of Lemma 9. As in the baseline model, once we use the equilibrium conditions for the wages and the prices, the first-best level of output *conditional* on $\log q_{it}$ satisfies the following FOC of the firm's profit with respect to the second input:

$$l_{it}^{\epsilon} = U'\left(Y_t\right)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\eta\frac{y_{it}}{l_{it}}\right). \tag{81}$$

Furthermore, the first-best level of output satisfies (81) and, in addition, the following FOC of the firm's profit with respect to the first input:

$$n_{it}^{\epsilon} = U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\theta\frac{y_{it}}{n_{it}}\right) \tag{82}$$

Note that, by definition of the first-best level of output, the markup and the expectation operator are absent from both conditions.

Rearranging (81) to solve for $\log l_{it}$ (and omitting unimportant constants)

$$(1+\epsilon)\log l_{it} = \left(\tfrac{1}{\rho}-\gamma\right)\log Y_t + \left(1-\tfrac{1}{\rho}\right)\log y_{it} \tag{83}$$

and using $\log y_{it} = \log q_{it} + \eta \log l_{it}$ to replace l_{it}, we can restate the above as

$$\log y_{it} = \rho\nu \log q_{it} + \bar{\alpha}\log Y_t,$$

which proves (53). Using (53) to replace y_{it} in (82), taking logs, and noting that $\log q_{it} = a_{it} + \theta \log n_{it}$, we arrive at

$$\log q_{it}^{*} = \hat{\phi}_a a_{it} + \tfrac{\hat{\alpha}}{\chi}\log Y_t, \tag{84}$$

where $\hat{\phi}_a$, $\hat{\alpha}$, χ, and ν are defined in (52). Finally, integrating (83) across different islands

$$\int \log l_{it} di = \frac{1-\gamma}{1+\epsilon}\log Y_t$$

and combining the latter with (40) gives

$$\log Y_t = \chi \log Q_t$$

and therefore (84) gives (54). ■

Proof of Lemma 10. As with the proof of Lemma 2, we drop the time index t and focus on the characterization of the per-period welfare flow. The latter is now defined as

$$W = \mathbb{E}\left[\frac{Y^{1-\gamma}}{1-\gamma} - \frac{1}{1+\epsilon}\int\left(\frac{q_i}{A_i}\right)^{\frac{1+\epsilon}{\theta}} di - \frac{1}{1+\epsilon}\int\left(\frac{y_i}{q_i}\right)^{\frac{1+\epsilon}{\eta}} di\right]$$

We will proceed to establish that the above can be expressed as follows:

$$W = W^{*}\exp\left(-\frac{1}{2}(1-\hat{\gamma})(1+\hat{\epsilon})\Lambda'\right).$$

where W^{*} is the first-best level of W.

We first focus on the first and third component of W:

$$\mathbb{E}\left[\frac{Y^{1-\gamma}}{1-\gamma} - \frac{1}{1+\epsilon}\int\left(\frac{y_i}{q_i}\right)^{\frac{1+\epsilon}{\eta}} di\right].$$

This expression resembles the expression for welfare in the baseline model with q_i replacing local productivity a_i. We can thus follow the same steps as in the proof of Lemma 2 and rewrite the latter expression as (omitting unimportant constants)

$$\mathbb{E}\left[Q\right]^{\frac{1-\gamma}{\frac{1+\epsilon}{\eta}-1+\gamma}\frac{1+\epsilon}{1-\eta}} \exp\left(-\frac{1}{2}\frac{\frac{1+\epsilon}{\eta}(1-\gamma)}{\frac{1+\epsilon}{\eta}-1+\gamma}\bar{\Omega}\right) \exp\left(-\frac{1}{2}\frac{1+\epsilon}{\eta}\left(1-\gamma\right)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right),$$

with

$$\bar{\Omega} \equiv \left(\frac{1+\epsilon}{\eta}\gamma + \gamma - 1\right)\frac{\frac{1+\epsilon}{\eta}+\gamma-1}{\left(\frac{1+\epsilon}{\eta}\right)^2} Var\left(\log \bar{Y}\right) = \frac{\frac{1+\epsilon}{\eta}\gamma+\gamma-1}{\frac{1+\epsilon}{\eta}+\gamma-1} Var\left(\log Q\right),$$

where the second line uses the fact that (53) implies $\log \bar{Y} = \frac{\rho\nu}{1-\bar{\alpha}}\log Q$.

We can use these results to rewrite W as follows:

$$W = \mathbb{E}\left[Q\right]^{1-\hat{\gamma}} \exp\left(-\frac{1}{2}\frac{\frac{1+\epsilon}{\eta}(1-\gamma)}{\frac{1+\epsilon}{\eta}-1+\gamma}\bar{\Omega} - \frac{1}{2}\frac{1+\epsilon}{\eta}\left(1-\gamma\right)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right) - \frac{1}{1+\epsilon}\mathbb{E}\left[\int\left(\frac{q_i}{A_i}\right)^{\frac{1+\epsilon}{\theta}} di\right]$$

Using property (15),

$$E\left[Q\right]^{1-\hat{\gamma}} = E\left[Q^{1-\hat{\gamma}}\right] e^{\frac{1}{2}\hat{\gamma}(1-\hat{\gamma})Var(\log Q)}$$

we have that

$$W = \frac{1}{1-\hat{\gamma}}\mathbb{E}\left[Q^{1-\hat{\gamma}}\right]\Xi - \frac{1}{1+\hat{\epsilon}}\int\left(\frac{q_i}{A_i}\right)^{1+\hat{\epsilon}} di$$

where

$$\Xi \equiv \left(1-\hat{\gamma}\right)\theta \exp\left(\frac{1}{2}\hat{\gamma}\left(1-\hat{\gamma}\right)Var\left(\log Q\right) - \frac{1}{2}\frac{\frac{1+\epsilon}{\eta}(1-\gamma)}{\frac{1+\epsilon}{\eta}-1+\gamma}\bar{\Omega} - \frac{1}{2}\frac{1+\epsilon}{\eta}\left(1-\gamma\right)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right),$$

or, using the definition of $\bar{\Omega}$,

$$\Xi = \left(1-\hat{\gamma}\right)\theta \exp\left(-\frac{1}{2}\frac{1+\epsilon}{\eta}\left(1-\gamma\right)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right).$$

Note then that the functional that maps the strategy q into the welfare level $\mathcal{W}$ is the same as the one in the proof of Lemma 2, provided that we make two changes: we replace ρ, γ, and ϵ with, respectively, $\hat{\rho}$, $\hat{\gamma}$, and $\hat{\epsilon}$; and we accommodate the constant Ξ. Thus, following the same steps as in that proof, we can obtain the following characterization of the per-period welfare flow:

$$W = W^* \exp\left\{-\frac{1}{2}\left(1+\hat{\epsilon}\right)\left(1-\hat{\gamma}\right)\left(\Sigma_Q - \frac{1}{1-\hat{\alpha}}\sigma_q\right)\right\}\Xi^{\frac{1+\hat{\epsilon}}{\hat{\epsilon}+\hat{\gamma}}}.$$

Finally, using the definition of Ξ and rearranging gives us

$$\begin{aligned} W &= W^* \exp\left\{-\tfrac{1}{2}(1-\hat{\gamma})\left(1+\hat{\epsilon}\right)\left(\Sigma_Q - \tfrac{1}{1-\hat{\alpha}}\sigma_q\right)\right\} \exp\left(-\tfrac{1}{2}\tfrac{1+\hat{\epsilon}}{\hat{\epsilon}+\hat{\gamma}}\left(1-\gamma\right)\tfrac{1+\epsilon}{\eta}\left(\Sigma_Y + \tfrac{1}{1-\bar{\alpha}}\sigma_y\right)\right) \\ &= W^* \exp\left\{-\tfrac{1}{2}(1-\hat{\gamma})\left(1+\hat{\epsilon}\right)\left[\left(\Sigma_Q - \tfrac{1}{1-\hat{\alpha}}\sigma_q\right) + \xi\left(\Sigma_Y + \tfrac{1}{1-\bar{\alpha}}\sigma_y\right)\right]\right\}, \end{aligned}$$

where

$$\xi \equiv \frac{\frac{1+\epsilon}{\eta} + \gamma - 1}{\hat{\epsilon} + \hat{\gamma}}.$$

The result then follows from translating the above in terms of life-time welfare and defining the function $w\left(\cdot\right)$ as $w(x) \equiv \mathcal{W}^* \exp\left\{-\frac{1}{2}(1-\hat{\gamma})\left(1+\hat{\epsilon}\right)x\right)$. ■

Proof of Lemma 3. *Part (i).* This part follows trivially from projecting (regressing) the equilibrium nominal GDP on the fundamental and the public signal, and letting (λ_s, λ_z) be the projection coefficients and m_t the residual.

Part (ii). From Lemma 8, the sum $c_{\bar{s}} + (\psi_s + \psi_x)$, which gives the response of $\log M_t$ to the fundamental, is a linear function of ρ_s. Furthermore, c_z is invariant to ρ_z, whereas ψ_z is inversely related to it. It follows that, for any pair (λ_s, λ_z), we can find a value of the pair (ρ_s, ρ_z) so that

$$c_{\bar{s}} + (\psi_s + \psi_x) = \lambda_s \qquad \text{and} \qquad c_z + \psi_z = \lambda_z$$

The result then follows from considering the policy that is identified by the combination of this particular value for the pair (ρ_s, ρ_z) with $r_t = -\frac{1}{\gamma} m_t$.

Part (iii). This follows directly from the analysis in Section B.1. ■

Proof of Lemma 4. From part (ii) of Lemma 6 and the proof of Lemma 3 we conclude that a monetary policy as in (7) replicates flexible-price allocations if and only if $\lambda_s = \lambda_s^*$, where $\lambda_s^* \equiv \bar{\Pi} + \Pi l_{\bar{s}}^*$ and $l_{\bar{s}}^*$ is given by (50). We can then use Lemma 3 to translate these results in terms of the interest rate rule. ■

Proof of Proposition 5. *Part (i).* As in the proof of Lemma 6, once we use the equilibrium conditions for the wages and the prices, the FOCs of the firm's profit with respect to the two inputs reduce to the following:

$$n_{it}^{\epsilon} = \mathbb{E}_{it}\left[\mathcal{M}_{it} U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\theta \frac{y_{it}}{n_{it}}\right)\right] \tag{85}$$

$$l_{it}^{\epsilon} = \mathcal{M}_{it} U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\eta \frac{y_{it}}{l_{it}}\right) \tag{86}$$

Note that the first condition is the same as (38). The second condition, by contrast, does not feature an expectation operator, because the absence of nominal rigidity means that the stage-2 input adjusts so as to equate marginal cost and marginal revenue state-by-state. Rearranging (86) to solve for $\log l_{it}$ (and omitting unimportant constants)

$$(1+\epsilon)\log l_{it} = -\mu_{it} + \left(\tfrac{1}{\rho} - \gamma\right)\log Y_t + \left(1 - \tfrac{1}{\rho}\right)\log y_{it} \tag{87}$$

and using $\log y_{it} = \log q_{it} + \eta \log l_{it}$ to replace l_{it}, we can restate the above as

$$\log y_{it} = b_q \log q_{it} - b_\mu \mu_{it} + \bar{\alpha}\log Y_t$$

where

$$b_q \equiv \rho\nu \quad \text{and} \quad b_\mu \equiv \tfrac{\eta}{1+\epsilon}\rho\nu.$$

Using the above result to replace y_{it} in (85), taking logs, and noting that $\log q_{it} = a_{it} + \theta \log n_{it}$, we arrive at

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \tfrac{\hat{\alpha}}{\chi}\mathbb{E}_{it}[\log Y_t],$$

where $\hat{\phi}_a$, $\hat{\phi}_\mu$, $\hat{\alpha}$, χ, and ν are defined in (52). Finally, integrating (87) across different islands

$$\int \log l_{it} di = \frac{1-\gamma}{1+\epsilon}\log Y_t - \frac{1}{1+\epsilon}\bar{\mu}_t$$

and combining the latter with (40) gives us

$$\log Y_t = \chi\left(\log Q_t - \tfrac{\eta}{1+\epsilon}\bar{\mu}_t\right).$$

Thus,

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \hat{\phi}_{\bar{\mu}}\mathbb{E}_{it}\left[\bar{\mu}_t\right] + \hat{\alpha}\mathbb{E}_{it}\left[\log Q_t\right],$$

where $\hat{\phi}_{\bar{\mu}} \equiv -\hat{\alpha}\frac{\eta}{1+\epsilon}$.

Part (ii). From Lemma 10 we have that, irrespective of whether the nominal rigidity is present or not, welfare is a decreasing function of

$$\Lambda' = \left(\Sigma_Q + \frac{1}{1-\hat{\alpha}}\sigma_q\right) + \xi\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right).$$

When the policy of Lemma 4 is in place, the equilibrium quantities satisfy the FOC (86) which, as shown in part (i), can be rearranged as

$$\log y_{it} = b_q \log q_{it} - b_\mu \mu_{it} + \bar{\alpha}\log Y_t.$$

On the other hand, Lemma 9 shows that the first-best level of output conditional on the first-period equilibrium production decision satisfies

$$\log \bar{y}_{it} = b_q \log q_{it} + \bar{\alpha}\log \bar{Y}_t.$$

Therefore, the aggregate and local gaps are given by, respectively,

$$\log Y_t - \log \bar{Y}_t = -b_{\bar{\mu}}\bar{\mu}_t$$

where $b_{\bar{\mu}} \equiv \frac{b_\mu}{1-\bar{\alpha}}$ and

$$\log y_{it} - \log \bar{y}_{it} = -b_\mu \mu_{it} - \bar{\alpha} b_{\bar{\mu}}\bar{\mu}_t.$$

Importantly, note that b_μ and $b_{\bar{\mu}}$ are independent of the information structure. It follows that the second term in Λ' reduces to

$$\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y = b_{\bar{\mu}}^2\sigma_{\bar{\mu}}^2 + \frac{1}{1-\bar{\alpha}}b_\mu^2\sigma_\xi^2$$

which is independent of the information structure. Therefore, (56) reduces to

$$\Lambda' = \Sigma_Q + \frac{1}{1-\hat{\alpha}}\sigma_q + \omega$$

where $\omega \equiv \xi\frac{b_\mu^2}{1-\bar{\alpha}}\left(\frac{1}{1-\bar{\alpha}}\sigma_{\bar{\mu}}^2 + \sigma_\xi^2\right)$, corresponding to equation (11) in the main text. ■

Proof of Theorem 3. From Proposition 5, when the policy that replicates flexible prices is in place, welfare is a decreasing function of

$$\Lambda' = \Sigma + \frac{1}{1-\hat{\alpha}}\sigma + \omega.$$

Since ω is independent of the information structure, the effects of the precision of private and public information are determined only by the changes in the volatility and dispersion of the first-period gaps.

Consider first the case with technology shocks. In that case $\hat{\phi}_\mu = \hat{\phi}_{\bar{\mu}} = 0$ and $\omega = 0$ and the welfare effects coincide with those found in Theorem 1.

In the case of markup shocks, $\hat{\phi}_\mu$ is given in (52), $\hat{\phi}_{\bar{\mu}} \equiv -\hat{\alpha}\frac{\eta}{1+\epsilon}$, ω is given in Proposition 5, and

$$\Lambda' = \frac{\left(\hat{\alpha}\eta-(1+\epsilon)\hat{\phi}_\mu\right)^2((1-\hat{\alpha})\kappa_x+\kappa_z)-(1-\hat{\alpha})(1+\epsilon)\left(2\alpha\eta-(1+\epsilon)(1+\hat{\alpha})\hat{\phi}_\mu\right)\kappa_\mu\hat{\phi}_\mu}{(1-\hat{\alpha})^2(1+\epsilon)^2(\kappa_\mu+(1-\hat{\alpha})\kappa_x+\kappa_z)\kappa_\mu} + \frac{\hat{\phi}_\mu^2}{(1-\hat{\alpha})\kappa_\xi} + \omega.$$

Differentiating Λ' with respect to the precision of, respectively, public and private information gives

$$\frac{\partial \Lambda'}{\partial \kappa_z} = \frac{\hat{\alpha}^2\left(\eta-(1+\epsilon)\hat{\phi}_\mu\right)^2}{(1-\hat{\alpha})^2(1+\epsilon)^2(\kappa_\mu+(1-\hat{\alpha})\kappa_x+\kappa_z)^2}$$

and

$$\frac{\partial \Lambda'}{\partial \kappa_x} = \frac{\hat{\alpha}^2\left(\eta-(1+\epsilon)\hat{\phi}_\mu\right)^2}{(1-\hat{\alpha})(1+\epsilon)^2(\kappa_\mu+(1-\hat{\alpha})\kappa_x+\kappa_z)^2},$$

which are both always positive. ■

Proof of Proposition 6. While derivations are lengthy, the idea of the proof is simple. From Lemma 10 we know that welfare losses Λ' depend on the volatility and dispersion of the first-period and second-period gaps between equilibrium and first-best production. We thus take each gap and decompose it into two new gaps: the first captures the deviation of equilibrium production from production with flexible prices; and the second captures the deviation of flexible-price production from first-best production. Finally, we rewrite the volatility and dispersion of the original gaps in terms of the volatility and dispersion of the new gaps.

We introduce some notation which will simplify the expressions in the proof. First, we let $\mathring{q}_{it}$ denote first-period output when monetary policy replicates flexible prices. Next, we decompose the first-period output gap as follows

$$\log \tilde{q}_{it} \equiv \log q_{it} - \log q^*_{it} = \log \hat{q}_{it} + \log \check{q}_{it},$$

where $\log \hat{q}_{it} \equiv \log q_{it} - \log \mathring{q}_{it}$ denotes the deviation of first-period equilibrium production from the flexible-price benchmark, and where $\log \check{q}_{it} \equiv \log \mathring{q}_{it} - \log q^*_{it}$ denotes the usual output gap with flexible prices. With the same notation we obtain a similar decomposition for the second-period output gap:

$$\log \tilde{y}_{it} \equiv \log y_{it} - \log \bar{y}_{it} = \log \hat{y}_{it} + \log \check{y}_{it}.$$

Finally, in the proof we use analogous decompositions for the first and second-period aggregate gaps, $\log \tilde{Q}_t$ and $\log \tilde{Y}_t$.

The volatility of the first-period aggregate gap can thus be expressed as follows

$$\Sigma_Q \equiv Var\left(\log \tilde{Q}_t\right) = Var\left(\log \hat{Q}_t\right) + 2Cov\left(\log \hat{Q}_t, \log \check{Q}_t\right) + \check{\Sigma}_Q,$$

where $\check{\Sigma}_Q$ is the aggregate volatility of the first-period aggregate gap with flexible prices. Similarly, the dispersion of the first-period local gaps can be rewritten as

$$\begin{aligned}\sigma_q &\equiv Var\left(\log \tilde{q}_{it} - \log \tilde{Q}_t\right) \\ &= Var\left(\log \hat{q}_{it} - \log \hat{Q}_t\right) + 2Cov\left(\log \hat{q}_{it} - \log \hat{Q}_t, \log \check{q}_{it} - \log \check{Q}_t\right) + \check{\sigma}_q,\end{aligned}$$

where $\check{\sigma}_q$ is the dispersion of the first-period local gaps with flexible prices.

From part (i) of Lemma 6, first-period production satisfies (43) with φ_x and φ_z given by (45) and (46), respectively. We denote by $\mathring{\varphi}_x$ and $\mathring{\varphi}_z$ the coefficients φ_x and φ_z when the policy replicating flexible prices is in place. Combining parts (i) and (iii) of Lemma 6, $\mathring{\varphi}_x$ and $\mathring{\varphi}_z$ satisfy

$$\begin{aligned}\mathring{\varphi}_x &= \Gamma_x + \Gamma_x' l_{\bar{a}}^* = \tfrac{\hat{\phi}_a}{\kappa_a + (1-\hat{\alpha})\kappa_x + \kappa_z} \hat{\alpha}\kappa_x, \\ \mathring{\varphi}_z &= \Gamma_z + \Gamma_z' l_{\bar{a}}^* = \tfrac{\hat{\phi}_a}{\kappa_a + (1-\hat{\alpha})\kappa_x + \kappa_z} \tfrac{\hat{\alpha}\kappa_z}{1-\hat{\alpha}}.\end{aligned}$$

Finally, first-best production is given by (54). We can thus compute all the gaps defined above as follows

$$\begin{aligned}\log \hat{Q}_t &= \hat{Q}_{\bar{a}}^a \bar{a}_t + \hat{Q}_z^a \varepsilon_t \\ \log \check{Q}_t &= \check{Q}_{\bar{a}}^a \bar{a}_t + \check{Q}_z^a \varepsilon_t,\end{aligned}$$

where $\hat{Q}_{\bar{a}}^a \equiv \Gamma_x' + \Gamma_z'$, $\hat{Q}_z^a \equiv \Gamma_z'$, $\check{Q}_{\bar{a}}^a \equiv -\frac{\hat{\alpha}}{1-\hat{\alpha}}\hat{\phi}_a + \Gamma_x + \Gamma_x' l_{\bar{a}}^* + \Gamma_z + \Gamma_z' l_{\bar{a}}^*$, and $\check{Q}_z^a \equiv \Gamma_z + \Gamma_z' l_{\bar{a}}^*$. Therefore,

$$\begin{aligned}Var\left(\log \hat{Q}_t\right) &= \left(\hat{Q}_{\bar{a}}^a\right)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \tfrac{1}{\kappa_a} + \left(\hat{Q}_z^a\right)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \tfrac{1}{\kappa_z} \\ Cov\left(\log \hat{Q}_t, \log \check{Q}_t\right) &= \hat{Q}_{\bar{a}}^a \check{Q}_{\bar{a}}^a (l_{\bar{a}} - l_{\bar{a}}^*) \frac{1}{\kappa_a} + \hat{Q}_z^a \check{Q}_z^a (l_{\bar{a}} - l_{\bar{a}}^*) \tfrac{1}{\kappa_z}.\end{aligned}$$

Similarly, the terms capturing the dispersion of the first-period local gaps can be expressed as

$$\begin{aligned}\log \hat{q}_{it} - \log \hat{Q}_t &= \hat{q}_x^a (l_{\bar{a}} - l_{\bar{a}}^*) u_{it} \\ \log \check{q}_{it} - \log \check{Q}_t &= \check{q}_x^a u_{it}.\end{aligned}$$

where $\hat{q}_x^a \equiv \Gamma_x'$ and $\check{q}_x^a \equiv \Gamma_x + \Gamma_x' l_{\bar{a}}^*$. Therefore,

$$\begin{aligned}Var\left(\log \hat{q}_{it} - \log \hat{Q}_t\right) &= (\hat{q}_x^a)^2 (l_{\bar{a}} - l_{\bar{a}}^*)^2 \frac{1}{\kappa_x} \\ Cov\left(\log \hat{q}_{it} - \log \hat{Q}_t, \log \check{q}_{it} - \log \check{Q}_t\right) &= \hat{q}_x^a \check{q}_x^a (l_{\bar{a}} - l_{\bar{a}}^*) \tfrac{1}{\kappa_x}.\end{aligned}$$

Furthermore, it turns out that, in the case of technology shocks, the covariance terms do not contribute to the welfare losses, that is,

$$\begin{aligned} & Cov\left(\log\hat{Q}_t, \log\check{Q}_t\right) + \frac{1}{1-\hat{\alpha}} Cov\left(\log\hat{q}_{it} - \log\hat{Q}_t, \log\check{q}_{it} - \log\check{Q}_t\right) \\ = & \left(\hat{Q}^a_{\bar{a}}\check{Q}^a_{\bar{a}}\tfrac{1}{\kappa_a} + \hat{Q}^a_z\check{Q}^a_z\tfrac{1}{\kappa_z} + \frac{1}{1-\hat{\alpha}}\hat{q}^a_x\check{q}^a_x\tfrac{1}{\kappa_x}\right)(l_{\bar{a}} - l^*_{\bar{a}}) = 0. \end{aligned}$$

Collecting all the remaining terms together, the second-order welfare losses associated with the first-period gaps can be rewritten as

$$\Sigma_Q + \frac{1}{1-\hat{\alpha}}\sigma_q = \vartheta_a (l_{\bar{a}} - l^*_{\bar{a}})^2 + \check{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\check{\sigma}_q, \tag{88}$$

where

$$\vartheta_a \equiv \left(\hat{Q}^a_{\bar{a}}\right)^2 \tfrac{1}{\kappa_a} + \left(\hat{Q}^a_z\right)^2 \tfrac{1}{\kappa_z} + \tfrac{1}{1-\hat{\alpha}}(\hat{q}^a_x)^2 \tfrac{1}{\kappa_x}.$$

Let's now turn to the second-period output gaps. With technology shocks flexible-price allocations satisfy (86) with a constant markup. If we rearrange (86) we can then obtain an expression similar to (53), except for a constant capturing the markup. Thus, with technology shocks flexible-price allocations and first-best allocations conditional on equilibrium first-period production differ only by a constant and, thus, there is no use in decomposing the second-period gaps as we did for the first-period gaps. Using the conditions in Lemma 6, the second-period aggregate and local gaps are, respectively,

$$\begin{aligned} \log\tilde{Y}_t &= \tilde{Y}^a_{\bar{a}}(l_{\bar{a}} - l^*_{\bar{a}})\bar{a}_t + \tilde{Y}^a_z(l_{\bar{a}} - l^*_{\bar{a}})\varepsilon_t \\ \log\tilde{y}_{it} &= \tilde{Y}^a_x(l_{\bar{a}} - l^*_{\bar{a}})u_{it} \end{aligned}$$

where $\tilde{Y}^a_{\bar{a}} \equiv \left(1+\frac{\eta}{\theta}\right)\Gamma'_x + \left(1+\frac{\eta}{\theta}\right)\Gamma'_z + \frac{\eta\kappa_a}{\kappa_a+\kappa_x+\kappa_z}$, $\tilde{Y}^a_z \equiv \left(1+\frac{\eta}{\theta}\right)\Gamma'_z - \frac{\eta\kappa_z}{\kappa_a+\kappa_x+\kappa_z}$, and $\tilde{Y}^a_x \equiv \left(1+\frac{\eta}{\theta}\right)\Gamma'_x - \frac{\eta\kappa_x}{\kappa_a+\kappa_x+\kappa_z}$. Thus, volatility and dispersion of second-period gaps are, respectively,

$$\begin{aligned} \Sigma_Y &= \left(\tilde{Y}^a_{\bar{a}}\right)^2 (l_{\bar{a}} - l^*_{\bar{a}})^2 \tfrac{1}{\kappa_a} + \left(\tilde{Y}^a_z\right)^2 (l_{\bar{a}} - l^*_{\bar{a}})^2 \tfrac{1}{\kappa_z} + \sigma^2_m \\ \sigma_y &= \left(\tilde{Y}^a_x\right)^2 (l_{\bar{a}} - l^*_{\bar{a}})^2 \tfrac{1}{\kappa_x}. \end{aligned}$$

Collecting all terms, the second-order welfare losses associated with the second-period gaps can be rewritten as

$$\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y = \vartheta'_a (l_{\bar{a}} - l^*_{\bar{a}})^2 + \sigma^2_m,$$

where

$$\vartheta'_a \equiv \left(\tilde{Y}^a_{\bar{a}}\right)^2 \tfrac{1}{\kappa_a} + \left(\tilde{Y}^a_z\right)^2 \tfrac{1}{\kappa_z} + \tfrac{1}{1-\bar{\alpha}}\left(\tilde{Y}^a_x\right)^2 \tfrac{1}{\kappa_x}.$$

Finally, in the proof of Lemma 8 we show that we can rewrite $l_{\bar{a}} - l^*_{\bar{a}}$ as $(\lambda_s - \lambda^*_s)/\Pi$. Therefore, if we let $\Lambda = \check{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\check{\sigma}_q$, $\mathcal{T} = \sigma^2_m$, and

$$\Theta \equiv \frac{\vartheta_a + \xi\vartheta'_a}{\Pi^2\sigma^2_a},$$

the result, for the case with technology shocks, follows directly from the welfare decomposition in Lemma 10.

The proof for the case with markup shocks closely resembles the proof with technology shocks; here we simply report the terms required to derive $\mathcal{K}$ and $\mathcal{T}$. Since markups are absent from first-best allocations, the latter are constant when the business cycle is driven by markup shocks. Using parts (i) and (iii) of Lemma 6, the first-period aggregate gap $\log \tilde{Q}_t$ is given by the sum of the following gaps

$$\begin{aligned} \log \check{Q}_t &= \hat{Q}^{\mu}_{\bar{\mu}} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) \bar{\mu}_t + \hat{Q}^{\mu}_{z} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) \varepsilon_t \\ \log \hat{Q}_t &= \check{Q}^{\mu}_{\bar{\mu}} \bar{\mu}_t + \check{Q}^{\mu}_{z} \varepsilon_t. \end{aligned}$$

where $\hat{Q}^{\mu}_{\bar{\mu}} \equiv \Gamma'_x + \Gamma'_z$, $\hat{Q}^{\mu}_{z} \equiv \Gamma'_z$, $\check{Q}^{\mu}_{\bar{\mu}} \equiv \hat{\phi}_{\mu} + \Gamma_x + \Gamma'_x l^*_{\bar{\mu}} + \Gamma_z + \Gamma'_z l^*_{\bar{\mu}}$, and $\check{Q}^{\mu}_{z} \equiv \Gamma_z + \Gamma'_z l^*_{\bar{\mu}}$. Therefore,

$$\begin{aligned} Var\left(\log \check{Q}_t\right) &= \left(\hat{Q}^{\mu}_{\bar{\mu}}\right)^2 \tfrac{1}{\kappa_\mu} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 + \left(\hat{Q}^{\mu}_{z}\right)^2 \tfrac{1}{\kappa_z} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 \\ Cov\left(\log \check{Q}_t, \log \hat{Q}_t\right) &= \hat{Q}^{\mu}_{\bar{\mu}} \check{Q}^{\mu}_{\bar{\mu}} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) \tfrac{1}{\kappa_\mu} + \hat{Q}^{\mu}_{z} \check{Q}^{\mu}_{z} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) \tfrac{1}{\kappa_z}. \end{aligned}$$

Similarly, the terms capturing the dispersion of the first-period local gaps can be expressed as

$$\begin{aligned} \log \hat{q}_{it} - \log \hat{Q}_t &= \hat{q}^{\mu}_{x} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) u_{it} \\ \log \check{q}_{it} - \log \check{Q}_t &= \check{q}^{\mu}_{\xi} \xi_{it} + \check{q}^{\mu}_{x} u_{it}, \end{aligned}$$

where $\hat{q}^{\mu}_{x} \equiv \Gamma'_x$, $\check{q}^{\mu}_{\xi} \equiv \hat{\phi}_{\mu}$, and $\check{q}^{\mu}_{x} \equiv \Gamma_x + \Gamma'_x l^*_{\bar{\mu}}$. Therefore,

$$\begin{aligned} Var\left(\log \hat{q}_{it} - \log \hat{Q}_t\right) &= \left(\hat{q}^{\mu}_{x}\right)^2 \tfrac{1}{\kappa_x} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 \\ Cov\left(\log \hat{q}_{it} - \log \hat{Q}_t, \log \check{q}_{it} - \log \check{Q}_t\right) &= \hat{q}^{\mu}_{x} \left(\check{q}^{\mu}_{\xi} + \check{q}^{\mu}_{x}\right) \tfrac{1}{\kappa_x} \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right). \end{aligned}$$

Collecting all terms, the second-order welfare losses associated with the first-period gaps can be rewritten as

$$\Sigma_Q + \frac{1}{1-\hat{\alpha}} \sigma_q = \vartheta_\mu \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 + 2\tilde{\vartheta}_\mu \left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) + \check{\Sigma}_Q + \frac{1}{1-\hat{\alpha}} \check{\sigma}_q,$$

where

$$\begin{aligned} \vartheta_\mu &\equiv \left(\hat{Q}^{\mu}_{\bar{\mu}}\right)^2 \tfrac{1}{\kappa_\mu} + \left(\hat{Q}^{\mu}_{z}\right)^2 \frac{1}{\kappa_z} + \frac{1}{1-\hat{\alpha}} \left(\hat{q}^{\mu}_{x}\right)^2 \tfrac{1}{\kappa_x} \\ \tilde{\vartheta}_\mu &\equiv \hat{Q}^{\mu}_{\bar{\mu}} \check{Q}^{\mu}_{\bar{\mu}} \tfrac{1}{\kappa_\mu} + \hat{Q}^{\mu}_{z} \check{Q}^{\mu}_{z} \tfrac{1}{\kappa_z} + \frac{1}{1-\hat{\alpha}} \hat{q}^{\mu}_{x} \left(\check{q}^{\mu}_{\xi} + \check{q}^{\mu}_{x}\right) \tfrac{1}{\kappa_x}. \end{aligned}$$

Let us now turn to the second-period output gaps. With markup shocks, flexible-price output and first-best output conditional on first-period equilibrium production are no longer related only by a constant. In particular, the latter satisfies (53) and the corresponding aggregate level is obtained by aggregating (53) across islands:

$$\log \bar{Y}_t = \frac{\rho\nu}{1-\bar{\alpha}} \log Q_t.$$

In contrast, equilibrium and flexible-price allocations can be obtained using the conditions in parts (i), (iii), and (iv) of Lemma 6.

Combining terms, the second-period aggregate gaps are given by the following:

$$\begin{aligned}\log \hat{Y}_t &= \hat{Y}^{\mu}_{\bar{\mu}}\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)\bar{\mu}_t + \hat{Y}^{\mu}_{z}\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)\varepsilon_t + m_t \\ \log \check{Y}_t &= \hat{Y}^{\mu}_{x}\bar{\mu}_t.\end{aligned}$$

where $\hat{Y}^{\mu}_{\bar{\mu}} \equiv \left(1+\frac{\eta}{\theta}\right)\left(\Gamma'_x + \Gamma'_z\right) + \frac{\eta\kappa_\mu}{\kappa_\mu+\kappa_x+\kappa_z}$, $\hat{Y}^{\mu}_{z} \equiv \left(1+\frac{\eta}{\theta}\right)\Gamma'_z - \frac{\eta\kappa_z}{\kappa_\mu+\kappa_x+\kappa_z}$, and $\hat{Y}^{\mu}_{x} \equiv -\frac{\eta}{1+\varepsilon}\frac{\rho\nu}{1-\bar{\alpha}}$. Therefore,

$$\begin{aligned}Var\left(\log \hat{Y}_t\right) &= \left(\hat{Y}^{\mu}_{\bar{\mu}}\right)^2\frac{1}{\kappa_\mu}\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 + \left(\hat{Y}^{\mu}_{z}\right)^2\frac{1}{\kappa_z}\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 + \sigma^2_m \\ Cov\left(\log \hat{Y}_t, \log \check{Y}_t\right) &= \hat{Y}^{\mu}_{x}\hat{Y}^{\mu}_{\bar{\mu}}\frac{1}{\kappa_\mu}\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right).\end{aligned}$$

Similarly, the terms capturing the dispersion of the second-period local gaps can be expressed as

$$\begin{aligned}\log \hat{y}_{it} - \log \hat{Y}_t &= \hat{y}^{\mu}_{x}\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)u_{it} \\ \log \check{y}_{it} - \log \check{Y}_t &= \check{y}^{\mu}_{\xi}\xi_{it},\end{aligned}$$

where $\hat{y}^{\mu}_{x} \equiv \left(1+\frac{\eta}{\theta}\right)\Gamma'_x - \frac{\eta\kappa_x}{\kappa_\mu+\kappa_x+\kappa_z}$ and $\check{y}^{\mu}_{x} \equiv -\frac{\eta}{1+\varepsilon}\rho\nu$. Therefore,

$$\begin{aligned}Var\left(\log \hat{y}_{it} - \log \hat{Y}_t\right) &= \left(\hat{y}^{\mu}_{x}\right)^2\frac{1}{\kappa_x}\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 \\ Cov\left(\log \hat{y}_{it} - \log \hat{Y}_t, \log \check{y}_{it} - \log \check{Y}_t\right) &= \check{y}^{\mu}_{\xi}\hat{y}^{\mu}_{x}\frac{1}{\kappa_x}\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right).\end{aligned}$$

Collecting all terms together, the second-order welfare losses associated with the second-period gaps can be rewritten as

$$\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y = \vartheta'_\mu\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right)^2 + 2\tilde{\vartheta}'_\mu\left(l_{\bar{\mu}} - l^*_{\bar{\mu}}\right) + \check{\Sigma}_Y + \frac{1}{1-\bar{\alpha}}\check{\sigma}_y + \sigma^2_m,$$

where

$$\begin{aligned}\vartheta'_\mu &\equiv \left(\hat{Y}^{\mu}_{\bar{\mu}}\right)^2\frac{1}{\kappa_\mu} + \left(\hat{Y}^{\mu}_{z}\right)^2\frac{1}{\kappa_z} + \frac{1}{1-\bar{\alpha}}\left(\hat{y}^{\mu}_{x}\right)^2\frac{1}{\kappa_x} \\ \tilde{\vartheta}'_\mu &\equiv \hat{Y}^{\mu}_{x}\hat{Y}^{\mu}_{\bar{\mu}}\frac{1}{\kappa_\mu} + \frac{1}{1-\bar{\alpha}}\check{y}^{\mu}_{\xi}\hat{y}^{\mu}_{x}\frac{1}{\kappa_x}.\end{aligned}$$

Finally, in the proof of Lemma 8 we show that we can rewrite $l_{\bar{\mu}} - l^*_{\bar{\mu}}$ as $\left(\lambda_s - \lambda^*_s\right)/\Pi$, thus, if we let

$$\begin{aligned}\Lambda &= \check{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\check{\sigma}_q + \xi\left(\check{\Sigma}_Y + \frac{1}{1-\bar{\alpha}}\check{\sigma}_y\right), \quad \mathcal{T} = \sigma^2_m, \\ \Theta_1 &= \frac{\tilde{\vartheta}_\mu + \xi\tilde{\vartheta}'_\mu}{\Pi^2\sigma^2_\mu}, \quad \text{and} \quad \Theta_2 = -\frac{\vartheta_\mu + \xi\vartheta'_\mu}{\Pi^2\sigma^2_\mu},\end{aligned}$$

the statement of the proposition follows directly from the welfare decomposition in Lemma 10. ■

Proof of Lemma 5. From Proposition 6, the optimal policy can be found by minimizing the term $\Lambda+\mathcal{K}+\mathcal{T}$. The only terms that depend on the monetary policy are $\mathcal{K}$ and $\mathcal{T}$. The minimum value for $\mathcal{T}$ is clearly achieved when $\sigma_m^2=0$. From (12), $\lambda_s^{**}=\arg\min_{\lambda_s}\mathcal{K}(\lambda_s)=\Theta_1/\Theta_2$ and the minimum value is $K(\kappa_x,\kappa_z)=-\Theta_1^2/\Theta_2$. Finally, note that, since $\mathcal{K}(0)=0$, it has to be the case that $K(\kappa_x,\kappa_z)\leq 0$.

Part (i). When real rigidities are absent ($\theta=0$),

$$K(\kappa_x,\kappa_z)=-\frac{(1-\gamma\rho)^2}{(\gamma+\epsilon)^2(1+\epsilon\rho)((1+\epsilon\rho)(\kappa_\mu+\kappa_z)+\rho(\gamma+\epsilon)\kappa_x)},$$

which is clearly increasing in both κ_x and κ_z.

Part (ii). This is proved by example. When

$$(\gamma,\epsilon,\eta,\theta,\rho,\kappa_s,\kappa_\xi,\kappa_x,\kappa_z)=(1,1,.5,.5,2,0,.1,.1,.9),$$

$K(\kappa_x,\kappa_z)$ is strictly increasing in both κ_x and κ_z. On the contrary, when

$$(\gamma,\epsilon,\eta,\theta,\rho,\kappa_s,\kappa_\xi,\kappa_x,\kappa_z)=(2,1,.5,.5,2,0,.5,.5,.5),$$

$K(\kappa_x,\kappa_z)$ is striclty decreasing in κ_x; and when

$$(\gamma,\epsilon,\eta,\theta,\rho,\kappa_s,\kappa_\xi,\kappa_x,\kappa_z)=(.1,.45,.03,.36,.09,0,.1,.1,.9),$$

$K(\kappa_x,\kappa_z)$ is strictly decreasing in κ_z. (By continuity, the aforementioned monotonicities continue to hold in the neighborhood of the considered parameter values.)

Part (iii). First note that both Θ_1 and Θ_2 are linear in $\kappa=\kappa_x+\kappa_z$. Thus, $\bar{K}(\kappa,\varrho)$ is linear in κ and the statement follows from the fact that $-\Theta_1^2/\Theta_2\leq 0$. ■

Proof of Theorem 4. *Part (i).* This follows from Proposition 5 along with the fact that, in the case of technology shocks, the optimal policy replicates flexible prices.

Part (ii). This follows from Proposition 5, Lemma 5, and the discussion in the main text. ■