

Online Appendix

“Feedback Effects, Asymmetric Trading, and the Limits to Arbitrage”

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A Proofs of Propositions and Corollaries in the Paper

Proof of Proposition 1

This proof only provides material supplementary to what is in the main text.

No Trade Equilibrium NT. The order flows of $X = -2$ and $X = 2$ are off the equilibrium path and the posteriors are given by 0 and 1, respectively, as these are the only posteriors that satisfy the Intuitive Criterion (as stated in the main text). The order flows of $X \in \{-1, 0, 1\}$ are on the equilibrium path and so the posteriors can be calculated by Bayes' rule:

$$\begin{aligned} q(X) &= \Pr(H|X) \\ &= \frac{\Pr(X|H)}{\Pr(X|H) + \Pr(X|L)}. \end{aligned}$$

We thus have:

$$\begin{aligned} q(-1) &= \frac{\lambda(1/3) + (1 - \lambda)(1/3)}{\lambda(1/3) + (1 - \lambda)(1/3) + \lambda(1/3) + (1 - \lambda)(1/3)} \\ &= \frac{1}{2}, \end{aligned}$$

and $q(0)$ and $q(1)$ are calculated in exactly the same way. Sequential rationality leads to the

decisions d and prices p as given by the Table in the proof in the main text.

We now turn to calculating the speculator's payoff (gross of the transaction cost κ) under different trading strategies, which comprises of the value of her final stake (of -1 , 0 , or 1 share), plus (minus) the price received (paid) for any share sold (bought). Under the positively-informed speculator's equilibrium strategy of not trading, we have $X \in \{-1, 0, 1\}$ and so her payoff is 0 . If she deviates to buying:

- With probability (w.p.) $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0 .
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

Thus, her expected gross gain from deviating to buying is given by:

$$\frac{1}{3}(R_H - R_L) \equiv \kappa_{NT}. \quad (1)$$

A similar calculation shows that, if the negatively-informed speculator sells, her expected gross gain is also given by (1). Thus, if and only if $\kappa \geq \kappa_{NT}$, the no-trade equilibrium is sustainable. The above calculations apply both in the case of feedback ($\frac{1}{2-\lambda} > \gamma_1$ and $\frac{1-\lambda}{2-\lambda} < \gamma_{-1}$) and no feedback ($\frac{1}{2-\lambda} \leq \gamma_1$ and $\frac{1-\lambda}{2-\lambda} \geq \gamma_{-1}$).

Partial Trade Equilibrium BNS. The order flow of $X = -2$ is off the equilibrium path and the posterior is given by 0 . The posteriors of the other order flows are given as follows:

$$\begin{aligned} q(-1) &= \frac{(1-\lambda)(1/3)}{(1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1-\lambda}{2-\lambda}, \\ q(0) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2}, \\ q(1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2}, \\ q(2) &= \frac{\lambda(1/3)}{\lambda(1/3)} = 1. \end{aligned}$$

Under the positively-informed speculator's equilibrium strategy of buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0 .
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

If she deviates to not trading, her payoff is 0. Thus, her expected gross gain from deviating to not trading is $-\kappa_{NT}$ (as given by (1)) in the cases of both feedback and no feedback.

Under the positively-informed speculator's equilibrium strategy of not trading, her payoff is 0. If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{1}{3}$, $X = -1$. In the case of feedback, she receives $\frac{1-\lambda}{2-\lambda}(R_H - x - c) + \frac{1}{2-\lambda}(R_L + x - c)$ per share, and so her payoff is $-(R_L + x - c) + (\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c) = \frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$. In the case of no feedback, she receives $\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L$ per share, and so her payoff is $-R_L + (\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L) = \frac{1-\lambda}{2-\lambda}(R_H - R_L)$.
- W.p. $\frac{1}{3}$, $X = 0$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_L + (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

Thus, her expected gross gain from deviating to selling is given by:

$$\frac{1}{3} \left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L) \right] \equiv \kappa_T \quad (2)$$

in the case of feedback, and

$$\frac{1}{3} \left[\frac{1-\lambda}{2-\lambda} + \frac{1}{2} \right] (R_H - R_L) \equiv \kappa_{NF} \quad (3)$$

in the case of no feedback.

Thus, the *BNS* equilibrium is sustainable if and only if $\kappa_T \leq \kappa < \kappa_{NT}$ in the case of feedback, and $\kappa_{NF} \leq \kappa < \kappa_{NT}$ in the case of no feedback.

Partial Trade Equilibrium SNB. The order flow of $X = 2$ is off the equilibrium path and the posterior is given by 1. The posteriors of the other order flows are given as follows:

$$\begin{aligned} q(-2) &= \frac{0}{\lambda(1/3)} = 0, \\ q(-1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2}, \\ q(0) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2}, \\ q(1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + (1-\lambda)(1/3)} = \frac{1}{2-\lambda}. \end{aligned}$$

Under the negatively-informed speculator's equilibrium strategy of selling:

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_L + (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

If she deviates to not trading, her payoff is 0. Thus, her expected gross gain from deviating to not trading is $-\kappa_{NT}$ (as given by (1)) in the cases of both feedback and no feedback.

Under the positively-informed speculator's equilibrium strategy of not trading, her payoff is 0. If she deviates to buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{1}{3}$, $X = 1$. In the case of feedback, she pays $\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c$ per share, and so her payoff is $(R_H + x - c) - (\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c) = \frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x)$. In the case of no feedback, she pays $\frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L$ per share, and so her payoff is $R_H - (\frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L) = \frac{1-\lambda}{2-\lambda}(R_H - R_L)$.
- W.p. $\frac{1}{3}$, $X = 0$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

Thus, her expected gross gain from deviating to buying is given by:

$$\frac{1}{3} \left[\frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x) + \frac{1}{2}(R_H - R_L) \right] \equiv \kappa_{SNB}$$

in the case of feedback, and κ_{NF} (as given by (3)) in the case of no feedback.

Thus, the *SNB* equilibrium is sustainable if and only if $\kappa_{SNB} \leq \kappa < \kappa_{NT}$ in the case of feedback, and $\kappa_{NF} \leq \kappa < \kappa_{NT}$ in the case of no feedback.

Trade Equilibrium T. All order flows are on the equilibrium path and so the posteriors are

given as follows:

$$\begin{aligned}
q(-2) &= \frac{0}{\lambda(1/3)} = 0, \\
q(-1) &= \frac{(1-\lambda)(1/3)}{(1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1-\lambda}{2-\lambda}, \\
q(0) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + \lambda(1/3) + (1-\lambda)(1/3)} = \frac{1}{2}, \\
q(1) &= \frac{\lambda(1/3) + (1-\lambda)(1/3)}{\lambda(1/3) + (1-\lambda)(1/3) + (1-\lambda)(1/3)} = \frac{1}{2-\lambda}, \\
q(2) &= \frac{\lambda(1/3)}{\lambda(1/3)} = 1.
\end{aligned}$$

Under the negatively-informed speculator's equilibrium strategy of selling:

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{1}{3}$, $X = -1$. In the case of feedback, she receives $\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c$ per share, and so her payoff is $-(R_L + x - c) + (\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c) = \frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$. In the case of no feedback, she receives $\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L$ per share, and so her payoff is $-R_L + (\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L) = \frac{1-\lambda}{2-\lambda}(R_H - R_L)$.
- W.p. $\frac{1}{3}$, $X = 0$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_L + (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

If she deviates to not trading, her payoff is 0. Thus, her expected gross gain from deviating to not trading is $-\kappa_T$ (as given by (2)) in the case of feedback, and $-\kappa_{NF}$ (as given by (3)) in the case of no feedback.

A similar calculation shows that, if the positively-informed speculator deviates to not trading, her gross gain is $-\kappa_{SNB}$ ($\kappa_{SNB} > \kappa_T$) in the case of feedback and $-\kappa_{NF}$ in the case of no feedback. Thus, the trade equilibrium is sustainable if and only if $\kappa < \kappa_T$ in the case of feedback, and $\kappa < \kappa_{NF}$ in the case of no feedback.

We now turn to the range of parameter values in which *BNS* is the only pure-strategy equilibrium in the case of feedback. If $\kappa_T \leq \kappa < \kappa_{NT}$, then the conditions for both the *NT* and *T* equilibrium to exist are violated. In addition, this is also the range where *BNS* equilibrium exists. We thus must derive conditions under which the *SNB* equilibrium does not hold, so that *BNS* is the unique equilibrium. There are two cases to consider. (i) If $\kappa_{SNB} \geq \kappa_{NT}$, the

SNB equilibrium never exists, and so $\kappa_T \leq \kappa < \kappa_{NT}$ is sufficient for *BNS* to be the unique equilibrium. (ii) If $\kappa_T < \kappa_{SNB} \leq \kappa_{NT}$, the *SNB* equilibrium exists unless $\kappa < \kappa_{SNB}$. Thus, *BNS* is the unique equilibrium if $\kappa_T \leq \kappa < \kappa_{SNB}$. Combining the two cases gives the range $\kappa_T \leq \kappa < \min(\kappa_{SNB}, \kappa_{NT})$ in the Proposition.

Proof of Lemma 2

For part (i), if $\theta = H$, the expected posterior is given by:

$$\begin{aligned}
 q^H &= (1 - \lambda) \left[\frac{1}{3}q(-1) + \frac{1}{3}q(0) + \frac{1}{3}q(1) \right] + \lambda \left(\frac{1}{3}q(0) + \frac{1}{3}q(1) + \frac{1}{3}q(2) \right) \\
 &= \frac{1 - \lambda}{3}q(-1) + \frac{1}{3}q(0) + \frac{1}{3}q(1) + \frac{\lambda}{3}q(2) \\
 &= \frac{(1 - \lambda)^2}{6 - 3\lambda} + \frac{1}{3} + \frac{\lambda}{3}.
 \end{aligned} \tag{4}$$

We have:

$$\begin{aligned}
 \frac{\partial q^H}{\partial \lambda} &= \frac{1}{3} + \frac{1}{3} \left[\frac{-2(1 - \lambda)(2 - \lambda) + (1 - \lambda)^2}{(2 - \lambda)^2} \right] \\
 &= \frac{1}{3} \left[1 + \left(\frac{1 - \lambda}{2 - \lambda} \right)^2 - 2 \frac{1 - \lambda}{2 - \lambda} \right] \\
 &= \frac{1}{3} \left[1 - \left(\frac{1 - \lambda}{2 - \lambda} \right) \right]^2 \\
 &> 0.
 \end{aligned}$$

The expected posterior is increasing in λ : if the speculator is more likely to be present, she is more likely to impound her information into prices by trading.

Moving to part (ii), if $\theta = L$, we have:

$$\begin{aligned}
 q^L &= \frac{1}{3} (q(-1) + q(0) + q(1)) \\
 &= \frac{1 - \lambda}{6 - 3\lambda} + \frac{1}{3}.
 \end{aligned} \tag{5}$$

This quantity is decreasing in λ . Even though the speculator does not trade upon $\theta = L$ if she is present, her information is still partially incorporated into prices. With $\theta = L$, there is a $\frac{1}{3}$ probability that the order flow is $X = -1$. This is consistent with the speculator being absent (in which case the state may be either *H* or *L*) or her being present and observing $\theta = L$; it

is not consistent with the speculator observing $\theta = H$. The greater the likelihood that the speculator is present, the greater the likelihood that $X = -1$ stems from $\theta = L$, and thus the greater the decrease in the market maker's posterior. Part (iii) follows from simple calculations.

Proof of Proposition 2

For parts (i) and (ii), we have:

$$\begin{aligned} q^{H,spec} &= \frac{1}{3}(q(0) + q(1) + q(2)) \\ &= \frac{2}{3}, \end{aligned} \tag{6}$$

$$\begin{aligned} q^{L,spec} &= \frac{1}{3}(q(-1) + q(0) + q(1)) \\ &= \frac{1-\lambda}{6-3\lambda} + \frac{1}{3}. \end{aligned} \tag{7}$$

Note that $q^{H,spec}$ is independent of λ , but $q^{L,spec}$ is decreasing in λ . The variable λ can affect the expected posterior in two ways: first, it can change the relative likelihood of the different order flows, and second, it can change the actual posterior given a certain order flow. Since we are conditioning on the speculator being present, the first channel is ruled out: conditional on the speculator being present and $\theta = H$, $X \in \{0, 1, 2\}$ with uniform probability regardless of λ ; conditional on the speculator being present and $\theta = L$, $X \in \{-1, 0, 1\}$ with uniform probability regardless of λ . Turning to the second channel, the only posterior that depends on λ is $q(-1)$: since $X = -1$ is inconsistent with the speculator being present and seeing $\theta = H$, it has a particularly negative impact on the likelihood of $\theta = H$ if the speculator is more likely to be present. In contrast, $X \in \{-2, 2\}$ is fully revealing and so the posterior is independent of λ ; $X \in \{0, 1\}$ is completely uninformative and so the posterior is again independent of λ . Since $X = -1$ can only occur in the presence of a speculator if she has received bad news, only $q^{L,spec}$ depends on λ but $q^{H,spec}$ does not. Part (iii) follows from simple calculations.

Proof of Lemma 3

We start by calculating p_0 . With probability $\frac{1}{2}$, the state will be $\theta = L$ and there is no trade, regardless of whether the speculator is present. Thus, $X \in \{-1, 0, 1\}$ with equal probability. With probability $\frac{1}{2}$, the state will be $\theta = H$. If the speculator is absent (w.p. $(1 - \lambda)$), there is no trade and we again have $X \in \{-1, 0, 1\}$. If the speculator is present, $X \in \{0, 1, 2\}$. Letting $p(X)$ denote the stock price set by the market maker after observing order flow X at $t = 1$,

the price at $t = 0$ will be the expectation over all possible future prices at $t = 1$, and is given as follows:

$$\begin{aligned}
p_0 &= \frac{\lambda}{2} \left(\frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{1}{3}p(2) \right) + \left(1 - \frac{\lambda}{2} \right) \left(\frac{1}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1) \right) \\
&= \frac{1}{3} \left(\left(1 - \frac{\lambda}{2} \right) p(-1) + p(0) + p(1) + \frac{\lambda}{2}p(2) \right) \\
&= \frac{1}{6} [3R_H + 3R_L - 2c + 2\lambda x].
\end{aligned} \tag{8}$$

Even though the initial belief y is independent of λ , the initial stock price p_0 is increasing in λ , because the speculator provides information to improve the manager's decision.

For part (i), if $\theta = H$ is realized, the expected price at $t = 1$ is given by:

$$\begin{aligned}
p_1^H &= (1 - \lambda) \left[\frac{1}{3}p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1) \right] + \lambda \left(\frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{1}{3}p(2) \right) \\
&= \frac{1 - \lambda}{3} p(-1) + \frac{1}{3}p(0) + \frac{1}{3}p(1) + \frac{\lambda}{3}p(2) \\
&= \frac{(3 - \lambda)R_H + (3 - 2\lambda)(R_L + \lambda x)}{3(2 - \lambda)} - \frac{c}{3}.
\end{aligned} \tag{9}$$

Note that:

$$\begin{aligned}
\frac{\partial p_1^H}{\partial \lambda} &= \frac{1}{3}p(2) - \frac{1}{3}p(-1) + \frac{1 - \lambda}{3} \frac{\partial p(-1)}{\partial \lambda} \\
&= \frac{R_H - R_L + 2(3 - 4\lambda + \lambda^2)x}{3(2 - \lambda)^2} > 0,
\end{aligned}$$

i.e., p_1^H is increasing in λ , since the speculator impounds information about the high state into prices.

Turning to part (ii), if $\theta = L$ is realized, the expected price at $t = 1$ is given by:

$$p_1^L = \frac{1}{3} (p(-1) + p(0) + p(1)). \tag{10}$$

We have $\frac{\partial p_1^L}{\partial \lambda} = \frac{R_L - R_H + 2x}{3(2 - \lambda)^2}$. If the speculator is more likely to be present, then $X = -1$ is more likely to result from $\theta = L$. Thus, the price is higher if and only if firm value is higher in this state, i.e., $R_L + x > R_H - x$ (Case 2).

The calculations of $p_1^H - p_0$ and $p_1^L - p_0$ follow automatically.

Proof of Proposition 3

For part (i), if the speculator receives positive information, she will buy one share and so the expected price becomes:

$$p_1^{H,spec} = \frac{1}{3} (p(0) + p(1) + p(2)). \quad (11)$$

Unlike p_1^H (equation (9)), this quantity is independent of λ , for the same reasons that $q^{H,spec}$ (equation (6)) is independent of λ . The stock return realized when the speculator receives good information is thus given by:

$$\begin{aligned} p_1^{H,spec} - p_0 &= \frac{1}{3} (p(0) + p(1) + p(2)) - \frac{1}{3} \left(\left(1 - \frac{\lambda}{2}\right) p(-1) + p(0) + p(1) + \frac{\lambda}{2} p(2) \right) \\ &= \frac{1}{3} \left(1 - \frac{\lambda}{2}\right) (p(2) - p(-1)) \\ &= \frac{1}{6} (R_H - R_L + 2(1 - \lambda)x) > 0, \end{aligned} \quad (12)$$

and we have

$$\frac{\partial (p_1^{H,spec} - p_0)}{\partial \lambda} = -\frac{1}{3}x < 0.$$

Equation (12) is decreasing in λ , whereas the stock return not conditioning on the speculator's presence, $p_1^H - p_0$, was increasing in λ . This reversal is because p_0 is increasing in λ , but $p_1^{H,spec}$ is independent of λ .

For part (ii), if the speculator is present and receives negative information, we have:

$$p_1^{L,spec} = \frac{1}{3} (p(-1) + p(0) + p(1)) = p_1^L, \quad (13)$$

and

$$\begin{aligned} p_1^{L,spec} - p_0 &= \frac{1}{3} (p(-1) + p(0) + p(1)) - \frac{1}{3} \left(\left(1 - \frac{\lambda}{2}\right) p(-1) + p(0) + p(1) + \frac{\lambda}{2} p(2) \right) \\ &= \frac{\lambda}{6} (p(-1) - p(2)) = p_1^L - p_0 < 0. \end{aligned}$$

Parts (iii) and (iv) follow from simple calculations.

Dropping constants, both equation (4) (the asymmetry between the price impact of good and bad news) and equation (5) (the average return, conditional on the speculator being present)

in the main content of the paper become:

$$(1 - \lambda) \left(\frac{R_H - R_L + 2(1 - \lambda)x}{2 - \lambda} \right).$$

Differentiating with respect to λ gives:

$$\frac{R_L - R_H - 2(3 - 4\lambda + \lambda^2)x}{(2 - \lambda)^2} < 0.$$

Thus, both equations (4) and (5) in the paper are decreasing in λ .

Proof of Corollary 2

We start with part (i). If the speculator receives good news, she will buy and the investment will be undertaken only if the noise trader buys. We thus have $p_2^{H,spec} = \frac{1}{3}(R_H + x - c) + \frac{2}{3}R_H$. This observation yields:

$$\begin{aligned} p_2^{H,spec} - p_1^{H,spec} &= R_H + \frac{1}{3}(x - c) - \frac{1}{3}(p(0) + p(1) + p(2)) \\ &= \frac{1}{3}(R_H - R_L). \end{aligned}$$

Moving to part (ii), if the speculator receives bad news, she will not trade. The firm reduces investment only if the noise trader sells. We thus have $p_2^{L,spec} = \frac{1}{3}(R_L + x - c) + \frac{2}{3}R_L$. This yields:

$$\begin{aligned} p_2^{L,spec} - p_1^{L,spec} &= R_L + \frac{1}{3}(x - c) - \frac{1}{3}(p(-1) + p(0) + p(1)) \\ &= \frac{(3 - 2\lambda)(R_L - R_H) + 2(1 - \lambda)x}{3(2 - \lambda)}, \end{aligned}$$

which is negative in Case 1, but can be positive or negative in Case 2. Part (iii) follows from simple calculations. For part (iv), we first calculate the expected firm value at $t = 2$ if the speculator is present, not conditioning on the state. If $\theta = H$, investment depends on the order flow: if $X = 2$, we have $d = 1$ and so firm value is $v = R_H + x - c$; if $X \in \{0, 1\}$, we have $d = 0$ and so $v = R_H$. If $\theta = L$, disinvestment depends on the order flow: if $X = -1$, we have $d = -1$ and so $v = R_L + x - c$; if $X \in \{0, 1\}$, we have $d = 0$ and so $v = R_L$. Expected firm value at $t = 2$ is thus given by:

$$p_2^{spec} = \frac{1}{2}(R_H + R_L) + \frac{1}{3}(x - c),$$

and so we have

$$p_2^{spec} - p_1^{spec} = \frac{1}{6} \frac{1 - \lambda}{2 - \lambda} (R_L - R_H + 2x),$$

which is positive if we are in Case 2 and negative if we are in Case 1.

B Full Analyses and Proofs of Alternative Models

B.1 Equilibria when firm value is non-monotonic in states: Full analysis

In this subsection, we consider the case where, if the firm disinvests, its value is higher in state $\theta = L$ ($R_H - x < R_L + x$). Hence, disinvestment is sufficiently powerful to outweigh the effect of the state on firm value and lead to a higher value in the low state.

The analysis of equilibrium outcomes becomes more complicated in the case of non-monotonicity. In the core model, where firm value is monotone in the state, a positively-informed speculator always loses money by selling and a negatively-informed speculator always loses money by buying, since firm value is always higher in state H than in state L . However, now that firm value may be higher in state L , a positively-informed speculator may find it optimal to sell and a negatively-informed speculator may find it optimal to buy. Hence, there are nine possible pure-strategy equilibria (each type of speculator – positively-informed and negatively-informed – may either buy, sell, or not trade). The following Lemma simplifies the equilibrium analysis, moving us closer to the analysis conducted in the core model.

Lemma 1 (*No equilibrium with trading against information*). *Suppose that $R_H - x < R_L + x$. Then:*

(i) *The trading game has no pure-strategy equilibrium where the speculator sells when she knows that $\theta = H$.*

(ii) *The trading game has no pure-strategy equilibrium where the speculator buys when she knows that $\theta = L$.*

Proof. See Appendix B.3. ■

Following the Lemma, there are four possible pure-strategy equilibria, just as in the previous subsection: NT , T , SNB , and BNS . However, the conditions for these equilibria to hold are now tighter. The reason that the positively-informed speculator never sells *in equilibrium* is

that if the market maker and the manager believe that she sells, she cannot make a profit from selling. However, she still might be tempted to *deviate* to selling in any of the four equilibria mentioned above. When she sells, she potentially misleads the market maker and the manager to believe that the negatively-informed speculator is present, and so to disinvest. Since disinvestment is suboptimal if $\theta = H$, this decision reduces firm value and causes the speculator to make a profit on her short position. Hence, for any of the above four equilibria to hold, an additional condition must be satisfied to ensure that the positively-informed speculator does not have an incentive to deviate to selling. Interestingly, the same issue does not arise with the negatively-informed speculator, as she never has an incentive to deviate to buying. If she does so, she misleads the market maker and the manager to believe that the positively-informed speculator is present, and so to (incorrectly) take the investment. This decision reduces firm value, causing the speculator to incur a loss from selling.¹

In analyzing deviations from the equilibrium, another issue that arises in this subsection is the specification of off-equilibrium beliefs. In Case 1, due to monotonicity, the only assumption that satisfied the Intuitive Criterion was that an off-equilibrium order flow of $X = 2$ is due to the positively-informed speculator (and so the posterior is $q = 1$), while an off-equilibrium order flow of $X = -2$ is due to the negatively-informed speculator (and so the posterior is $q = 0$). In this subsection, however, the Intuitive Criterion is not sufficient to rule out other off-equilibrium beliefs. We nevertheless retain this assumption regarding off-equilibrium beliefs, which is reasonable given the possible equilibria in our model. Our results remain the same for any other off-equilibrium beliefs that are monotone in the order flow.²

Proposition 4 provides the characterization of equilibrium outcomes.

Proposition 1 *4 (Equilibrium, firm value is non-monotone in the state). Suppose that $R_H - x < R_L + x$, and suppose that the belief of the market maker and the manager is that an off-equilibrium order flow of $X = -2$ ($X = 2$) is associated with the presence of negatively-informed (positively-informed) speculator. Then, if $R_H - R_L$ is sufficiently high compared to x (formally, $R_H - R_L > \frac{4}{3}x$), the characterization of equilibrium outcomes is identical to that in Lemma 1 for the case of feedback and Proposition 1 for the case of no feedback.*

¹Goldstein and Guembel (2008) also derive conditions to ensure that the speculator does not deviate from the equilibrium to trade against her information.

²Other papers that use similar monotonicity assumptions for off-equilibrium beliefs include Gul and Sonnenschein (1988) and Bikhchandani (1992).

More specifically, the following **additional** conditions are required for the various equilibria to hold:

Equilibrium NT: $\kappa \geq \frac{2}{3}(R_L - R_H + x)$.

Equilibrium SNB: in the case of feedback, $\kappa \geq \frac{2}{3}(R_L - R_H + x)$; in the case of no feedback, $\kappa \geq \frac{2}{3}(R_L - R_H + x)$.

Equilibrium BNS: in the case of feedback, $\frac{6-2\lambda}{2-\lambda}x < \frac{12-5\lambda}{4-2\lambda}(R_H - R_L)$; in the case of no feedback, $\frac{2}{3}x < \frac{12-5\lambda}{12-6\lambda}(R_H - R_L)$.

Equilibrium T: in the case of feedback, $\frac{4}{2-\lambda}x < 3(R_H - R_L)$; in the case of no feedback, $\frac{2}{3}x < (R_H - R_L)$.

The condition $R_H - R_L > \frac{4}{3}x$ is sufficient for all of the above conditions to be satisfied.

Proof. The calculations of the posterior q , the manager's decision d and the price p for different order flows X in the various possible equilibria are identical to those provided in the proof of Proposition 1. Hence, the conditions for the positively-informed speculator to choose between buying and not trading and for the negatively-informed speculator to choose between selling and not trading are identical to those derived in the proof of Proposition 1. Analyzing the possible trading profits for the negatively-informed speculator from deviating to buying in each of the four possible equilibria, it is straightforward to see that she always loses from buying and hence will never deviate. Appendix B.3 calculates the possible trading profits for the positively-informed speculator from deviating to selling in each of the four possible equilibria, which yields the additional conditions stated in the body of the proposition. These conditions are automatically satisfied when $R_H - R_L > \frac{4}{3}x$. ■

As Proposition 4 demonstrates, the main force identified in the previous subsection for the case where $R_H - x > R_L + x$, exists also in the case where $R_H - x < R_L + x$. That is, the feedback effect deters informed selling relative to informed buying. In this subsection, this force is even stronger because the minimum transaction cost required to deter the negatively-informed speculator from selling in the *BNS* equilibrium, $\kappa_T \equiv \frac{1}{3} \left[\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x) + \frac{1}{2}(R_H - R_L) \right]$, is lower when $R_H - x < R_L + x$: the first term in the expression for κ_T is negative. A strong feedback effect, in which disinvestment not only mitigates the effect of the low state but also overturns it, implies that the negatively-informed speculator can make a loss from selling – even before transaction costs. This result is in contrast to standard informed trading models where a speculator can never make a loss (before transactions costs) if she trades in the direction of her

information. This loss occurs at the $X = -1$ node. As in the core model, the key to this result is $\lambda < 1$. Even though both the speculator and market maker know that disinvestment will occur if $X = -1$, they have differing views on firm value conditional on disinvestment. The speculator knows that disinvestment will occur, *and* that disinvestment is desirable for firm value (since she knows that $\theta = L$), and so firm value is $R_L + x - c$. In contrast, the market maker knows the disinvestment will occur but is not certain that it is optimal, because she is unsure of the underlying state θ . Order flow $X = -1$ is consistent with a negatively-informed speculator, but also with an absent speculator and selling by the noise trader. Hence, it is possible that $\theta = H$, in which case disinvestment is undesirable, leading to firm value of $R_H - x - c$. Therefore, the price set by the market maker is only $\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c$, since he puts weight on the possibility that disinvestment may be undesirable, and so the speculator loses $\frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$ before transaction costs.

Moreover, Proposition 4 also shows that the feedback effect generates an additional force in this subsection: the desire of the positively-informed speculator to deviate and manipulate the price by selling. She can potentially profit from leading the manager to divest incorrectly, which enables her to gain on her short position. The manipulation incentive is not strong enough to interfere with equilibrium conditions as long as $R_H - R_L$ is sufficiently high relative to x ; a sufficient condition is $R_H - R_L > \frac{4}{3}x$. In this case, the loss from trading against good news (which is proportional to $R_H - R_L$) is high relative to the benefit from manipulation (which is proportional to x , the value destroyed by inducing the manager to divest incorrectly). Otherwise, additional conditions are required to sustain the various possible equilibria.

B.2 Positive initial position: Full analysis

We now assume that the speculator starts off with a stake of α and can trade $s \in \{-1, 0, 1\}$. Thus, her final position becomes $\{\alpha - 1, \alpha, \alpha + 1\}$. Our main result from the core model is that, under feedback, the range of κ that supports the *BNS* equilibrium is strictly greater than that which supports the *SNB* equilibrium. Thus, for conciseness, we consider the case of feedback ($\frac{1}{2-\lambda} > \gamma_1 = \frac{1}{2} + \frac{c}{2x} \iff \frac{1-\lambda}{2-\lambda} < \gamma_{-1} = \frac{1}{2} - \frac{c}{2x}$) and analyze only the *BNS* and *SNB* equilibria, rather than the *T* and *NT* equilibria.

Buy Not Sell Equilibrium BNS.

Under the positively-informed speculator's equilibrium strategy of buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is $(\alpha+1)(R_H + x - c) - (R_H + x - c) = \alpha(R_H + x - c)$.
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $(\alpha + 1)R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \alpha R_H + \frac{1}{2}(R_H - R_L)$.

If she deviates to not trading:

- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and her payoff is αR_H .
- W.p. $\frac{1}{3}$, $X = -1$, and her payoff is $\alpha(R_H - x - c)$.

Her expected gross gain from deviating to not trading is:

$$\frac{1}{3} [2\alpha x + (R_H - R_L)] \equiv \kappa_{NT}^\alpha.$$

Under the negatively-informed speculator's equilibrium strategy of not trading:

- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and her payoff is αR_L .
- W.p. $\frac{1}{3}$, $X = -1$ and her payoff is $\alpha(R_L + x - c)$.

If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$ and she is fully revealed. Thus her payoff is $(\alpha - 1)(R_L + x - c) + (R_L + x - c) = \alpha(R_L + x - c)$.
- W.p. $\frac{1}{3}$, $X = -1$ and she receives $\frac{1-\lambda}{2-\lambda}(R_H - x - c) + \frac{1}{2-\lambda}(R_L + x - c)$ per share. Her payoff is $(\alpha - 1)(R_L + x - c) + (\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c) = \alpha(R_L + x - c) + \frac{1-\lambda}{2-\lambda}(R_H - R_L - 2x)$.
- W.p. $\frac{1}{3}$, $X = 0$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $(\alpha - 1)R_L + (\frac{1}{2}R_H + \frac{1}{2}R_L) = \alpha R_L + \frac{1}{2}(R_H - R_L)$.

Her expected gross gain from deviating to selling is:

$$\frac{1}{3} \left[\alpha(x - c) + \frac{1 - \lambda}{2 - \lambda} (R_H - R_L - 2x) + \frac{1}{2} (R_H - R_L) \right] \equiv \kappa_T^\alpha.$$

The *BNS* equilibrium holds for $\kappa \in [\kappa_T^\alpha, \kappa_{NT}^\alpha]$.

Sell Not Buy Equilibrium SNB.

Under the positively-informed speculator's equilibrium strategy of not trading:

- W.p. $\frac{1}{3}$, $X = 1$ and her payoff is $\alpha(R_H + x - c)$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$ and her payoff is αR_H .

If she deviates to buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is $(\alpha+1)(R_H + x - c) - (R_H + x - c) = \alpha(R_H + x - c)$.
- W.p. $\frac{1}{3}$, $X = 1$, and she pays $\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c$ per share. Her payoff is $(\alpha + 1)(R_H + x - c) - (\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c) = \alpha(R_H + x - c) + \frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x)$
- W.p. $\frac{1}{3}$, $X = 0$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $(\alpha + 1)R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \alpha R_H + \frac{1}{2}(R_H - R_L)$.

Thus, her expected gross gain from deviating to buying is:

$$\frac{1}{3} \left[\alpha(x - c) + \frac{1 - \lambda}{2 - \lambda} (R_H - R_L + 2x) + \frac{1}{2} (R_H - R_L) \right] \equiv \kappa_{SNB}^\alpha.$$

Under the negatively-informed speculator's equilibrium strategy of selling:

- W.p. $\frac{1}{3}$, $X = -2$, and she is fully revealed. Her payoff is $(\alpha - 1)(R_L + x - c) + (R_L + x - c) = \alpha(R_L + x - c)$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $(\alpha - 1)R_L + (\frac{1}{2}R_H + \frac{1}{2}R_L) = \alpha R_L + \frac{1}{2}(R_H - R_L)$.

If she deviates to not selling:

- W.p. $\frac{1}{3}$, $X = 1$, and her payoff is $\alpha(R_L - x - c)$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and her payoff is αR_L .

Thus, her expected gross gain from deviating to not trading is:

$$\frac{1}{3} [2\alpha x + (R_H - R_L)] \equiv \kappa_{NT}^\alpha.$$

The *SNB* equilibrium holds for $\kappa \in [\kappa_{SNB}^\alpha, \kappa_{NT}^\alpha]$.

The maximum value of κ that supports the *BNS* and *SNB* equilibria is the same for both equilibria (κ_{NT}^α). The transaction cost must be sufficiently small to deter the positively-informed speculator from deviating to not trading in *BNS*, and the negatively-informed speculator from deviating to not trading in *SNB*. Under *BNS*, the positively-informed speculator's motive to play her equilibrium strategy of buying is that doing so leads to correct investment. Under *BNS*, the negatively-informed speculator's motive to play her equilibrium strategy of selling is that doing so leads to correct disinvestment. Since the value created by correct investment equals the value created by correct disinvestment, these motives are equally strong, thus leading to the same threshold.

While the maximum value of κ must be sufficiently low not to deter the positively-informed speculator from deviating to not buying (under *BNS*) and the negatively-informed speculator from deviating not selling (under *SNB*), the minimum value of κ must be sufficiently high to deter the negatively-informed speculator from deviating to selling (under *BNS*) and positively-informed speculator from deviating to buying (under *SNB*). While this minimum is κ_T^α for *BNS*, it is κ_{SNB}^α for *SNB*. We have

$$\begin{aligned} \kappa_{SNB}^\alpha - \kappa_T^\alpha &= \frac{1}{3} \left\{ \alpha[(x - c) - (x - c)] + \frac{1 - \lambda}{2 - \lambda} [(R_H - R_L + 2x) - (R_H - R_L - 2x)] \right\} \quad (14) \\ &= \frac{1 - \lambda}{3} \frac{4x}{2 - \lambda} > 0 \end{aligned}$$

Thus, the minimum value is strictly smaller for *BNS*, and so the range of κ that support *BNS* is a strict superset of the range that supports *SNB* – just as in the core model where the speculator has a zero initial stake.

To understand the intuition, the first term in the difference (14) is zero, since the value created by correct investment equals the value created by correct disinvestment. The second term is positive due to the feedback effect: investment increases the sensitivity of firm value to the state of nature (which becomes now $R_H - R_L + 2x$), and disinvestment decreases the sensitivity of firm value to the state of nature (which becomes now $R_H - R_L - 2x$). Due to the

feedback effect, the negatively-informed speculator's motive to deviate to selling under BNS is relatively low, as it may induce the manager to efficiently disinvest and thus reduces the profits on the share she sells. In contrast, due to the same feedback effect, the positively-informed speculator's motive to deviate to buying under SNB is relatively high, as it may induce the manager to efficiently invest and thus increase the profits on the share she buys. Thus, the minimum transaction cost to deter deviation to trading is higher in SNB than BNS , and the SNB equilibrium is strictly more difficult to sustain.

Note that the difference $\kappa_{SNB}^\alpha - \kappa_T^\alpha$ is *independent* of the initial stake α . While a higher α increases the speculator's incentives to sell on negative information, since doing so increases the value of her block, it equally increases her incentives to buy on positive information for the same reason. These two effects cancel out, and so the only difference is the gain on the one share that the speculator trades. Thus, her initial position does not matter.

B.3 Additional proofs

Proof of Lemma 4

For part (i), suppose that the speculator sells when she knows that $\theta = H$: then $X \in \{-2, -1, 0\}$ when $\theta = H$. In each of these nodes, the posterior probability q of state H is at least $\frac{1}{2}$ (since these nodes are consistent with the action of the positively-informed speculator and may or may not be consistent with the action of the negatively-informed speculator, depending on her equilibrium action). Then the manager will choose $d \in \{0, 1\}$ and so firm value is either R_H or $R_H + x - c$. The price, however, will incorporate the possibility that $\theta = L$. For $d \in \{0, 1\}$, firm value is lower under $\theta = L$ than under $\theta = H$. Thus, the price the speculator receives will be lower than firm value, and so the speculator makes a loss from selling.

For part (ii), suppose that the speculator buys when she knows that $\theta = L$: then $X \in \{0, 1, 2\}$ when $\theta = L$. Given that the positively-informed speculator does not sell, the posterior probability q is $\frac{1}{2}$ at $X \in \{0, 1\}$. Thus, the manager chooses $d = 0$ and so firm value is R_L . Since the price is $\frac{1}{2}R_H + \frac{1}{2}R_L$, the speculator will lose money on these nodes. When $X = 2$, there are two possibilities. If the positively-informed speculator buys in equilibrium, then the outcome is the same as on the other nodes. If she does not trade in equilibrium, then the negatively-informed speculator is revealed, buying a security worth $R_L + x - c$ for a price of $R_L + x - c$. Thus, in expectation she makes a loss, given she loses at $X \in \{0, 1\}$.

Proof of Proposition 4

This proof only provides material supplementary to Appendix B.1. As discussed in the main text, it is straightforward to show that the negatively-informed speculator will not deviate to buying. Here, we calculate the profits made if the positively-informed speculator deviates to selling, to derive the necessary conditions to prevent such a deviation.

No Trade Equilibrium NT. Under the positively-informed speculator's equilibrium strategy of not trading, her payoff is 0. If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and her (gross) payoff is $-(R_H - x - c) + (R_L + x - c) = R_L - R_H + 2x$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_H + \frac{1}{2}R_H + \frac{1}{2}R_L = \frac{1}{2}(R_L - R_H)$.

Thus, her overall gross gain from deviating to selling is given by:

$$\frac{2}{3}(R_L - R_H + x) \equiv \kappa_{NT}^M.$$

Thus, if and only if $\kappa \geq \kappa_{NT}^M$, she will not deviate to selling. The above calculations apply both in the case of feedback and no feedback. The sufficient condition $R_H - R_L > \frac{4}{3}x$ implies $R_L - R_H + x < 0$, and hence the additional equilibrium condition is satisfied.

Partial Trade Equilibrium BNS. Under the positively-informed speculator's equilibrium strategy of buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{2}{3}$, $X \in \{0, 1\}$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and her payoff is $-(R_H - x - c) + R_L + x - c = R_L - R_H + 2x$.
- W.p. $\frac{1}{3}$, $X = -1$, and her payoff is $-(R_H - x - c) + \frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c = \frac{1}{2-\lambda}(R_L - R_H + 2x)$ in the case of feedback and $-R_H + \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L = \frac{1}{2-\lambda}(R_L - R_H)$ in the case of no feedback.

- W.p. $\frac{1}{3}$, $X = 0$, and her payoff is $-R_H + \frac{1}{2}R_H + \frac{1}{2}R_L = \frac{1}{2}(R_L - R_H)$.

Thus, she will not deviate if

$$\frac{6 - 2\lambda}{2 - \lambda}x < \frac{12 - 5\lambda}{4 - 2\lambda}(R_H - R_L)$$

in the case of feedback and

$$\frac{2}{3}x < \frac{12 - 5\lambda}{12 - 6\lambda}(R_H - R_L)$$

in the case of no feedback.

In the case of feedback, the condition is equivalent to

$$R_H - R_L > \frac{12 - 4\lambda}{12 - 5\lambda}x.$$

It is straightforward to show that

$$\frac{4}{3} > \frac{12 - 4\lambda}{12 - 5\lambda},$$

and hence the sufficient condition $R_H - R_L > \frac{4}{3}x$ implies the additional equilibrium condition in the case of feedback. A similar argument shows that the additional equilibrium condition is also satisfied in the case of no feedback.

Partial Trade Equilibrium SNB. Under the positively-informed speculator's equilibrium strategy of not trading, her payoff is 0. If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and her payoff is $-(R_H - x - c) + R_L + x - c = R_L - R_H + 2x$.
- W.p. $\frac{2}{3}$, $X \in \{-1, 0\}$, and her payoff is $-R_H + \frac{1}{2}R_H + \frac{1}{2}R_L = \frac{1}{2}(R_L - R_H)$.

Thus, her expected gross gain from deviating to selling is given by:

$$\frac{2}{3}[R_L - R_H + x] \equiv \kappa_{SNB}^M.$$

Thus, she will not deviate to selling if and only if $\kappa \geq \kappa_{SNB}^M$. It is straightforward to verify that the condition $R_H - R_L > \frac{4}{3}x$ is sufficient for κ_{SNB}^M to be negative and for the additional equilibrium conditions to be satisfied.

Trade Equilibrium T. Under the positively-informed speculator's equilibrium strategy of buying:

- W.p. $\frac{1}{3}$, $X = 2$, and she is fully revealed. Her payoff is 0.
- W.p. $\frac{1}{3}$, $X = 1$. In the case of feedback, she pays $\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c$ per share, and so her payoff is $R_H + x - c - (\frac{1}{2-\lambda}(R_H + x) + \frac{1-\lambda}{2-\lambda}(R_L - x) - c) = \frac{1-\lambda}{2-\lambda}(R_H - R_L + 2x)$. In the case of no feedback, she pays $\frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L$ per share, and so her payoff is $R_H - (\frac{1}{2-\lambda}R_H + \frac{1-\lambda}{2-\lambda}R_L) = \frac{1-\lambda}{2-\lambda}(R_H - R_L)$.
- W.p. $\frac{1}{3}$, $X = 0$, and she pays $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $R_H - (\frac{1}{2}R_H + \frac{1}{2}R_L) = \frac{1}{2}(R_H - R_L)$.

If she deviates to selling:

- W.p. $\frac{1}{3}$, $X = -2$, and her payoff is $-(R_H - x - c) + R_L + x - c = R_L - R_H + 2x$.
- W.p. $\frac{1}{3}$, $X = -1$. In the case of feedback, she receives $\frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c$ per share, and so her payoff is $-(R_H - x - c) + \frac{1-\lambda}{2-\lambda}(R_H - x) + \frac{1}{2-\lambda}(R_L + x) - c - \kappa = \frac{1}{2-\lambda}(R_L - R_H + 2x)$. In the case of no feedback, she receives $\frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L$ per share, and so her payoff is $-R_H + \frac{1-\lambda}{2-\lambda}R_H + \frac{1}{2-\lambda}R_L = \frac{1}{2-\lambda}(R_L - R_H)$.
- W.p. $\frac{1}{3}$, $X = 0$, and she receives $\frac{1}{2}R_H + \frac{1}{2}R_L$ per share. Her payoff is $-R_H + \frac{1}{2}R_H + \frac{1}{2}R_L = \frac{1}{2}(R_L - R_H)$.

Thus, she will not deviate if

$$\frac{4}{2-\lambda}x < 3(R_H - R_L)$$

in the case of feedback and

$$\frac{2}{3}x < (R_H - R_L)$$

in the case of no feedback. The condition $R_H - R_L > \frac{4}{3}x$ implies $3(R_H - R_L) > 4x\frac{1}{2-\lambda}$ and $R_H - R_L > \frac{2}{3}x$, and thus the additional equilibrium conditions.

References

Bikhchandani, Sushil (1992): “A Bargaining Model with Incomplete Information.” *Review of Economic Studies* 59, 187–203.

Gul, Frank and Hugo F. Sonnenschein (1988): “On Delay in Bargaining with One-Sided Uncertainty.” *Econometrica* 56, 601–611.