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Online Appendix

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The "*Kosher* food" Exchange Economy

The "Kosher Food" exchange economy consists of two goods: meat and dairy. The goods are divisible and a consumer can either consume a quantity of meat or a quantity of dairy but not both.

The Economy: Let $\mathcal{X} = \mathcal{M} \cup \mathcal{D}$ where $\mathcal{M} = \{(a,0) \mid 0 < a \leq 3\}$ and $\mathcal{D} = \{(0,b) \mid 0 < b \leq 3\}$ (for simplicity, we exclude the zero vector from $\mathcal{X}$). The feasibility constraint is $\sum_i x^i = (3,3)$. Convexity is induced by xRy if $x \geq y$. Convexity of preferences is equivalent to monotonicity in both goods.

We take the set of primitive orderings to contain only the two orderings $\geq_M$ and $\geq_D$. The ordering $\geq_M$is the increasing ordering that places all the elements of $\mathcal{M}$ above $\mathcal{D}$ and $\geq_D$is the increasing ordering that places all elements of $\mathcal{D}$ above $\mathcal{M}$.

FWT: Every PE profile is Pareto-optimal as this economy satisfies the condition in Claim 6: Let (x^i) and (y^i) be two feasible profiles such that for every i, $x^i \geq_M y^i$ (an identical argument applies to $\geq_D$). For each agent i, $x_1^i \geq y_1^i$. Feasibility requires that $\sum x_1^i = \sum y_1^i = 3$ and therefore, for each agent, $x_1^i = y_1^i$. Thus, for any agent i who is assigned $x_1^i = y_1^i > 0$, it must be that $x^i = y^i$. For any agent who is assigned $x_1^i = y_1^i = 0$, it must be that $x_2^i \geq y_2^i > 0$. Feasibility again implies that for all i, $x_2^i = y_2^i$ and thus $x^i = y^i$.

SWT: Claim 8 applies and thus any Pareto-optimal profile is a CE profile.

However, a Pareto-optimal profile may not be part of a PE. Suppose, for example, that there are four agents with identical convex preferences satisfying $(2,0) \succ^i (0,3) \succ^i (0,1) \succ^i (1,0)$. The profile $((2,0),(0,2),(0,1),(1,0))$ is not a PE profile although it is Pareto-optimal. (In any Pareto-dominating profile, if either of 2 or 3 is reassigned from dairy to meat, then they must receive more than one unit of meat. Feasibility then requires that agent 1 must be reassigned and he cannot be reassigned to dairy because the total endowment of dairy is inferior for him. Therefore, 2 and 3 cannot be reassigned to meat, nor can they be reassigned within dairy. But, then 1 and 4 are restricted to meat consumption, and no Pareto-improvement is possible for them either.)

Note that, unlike the standard exchange economy, the Kosher food exchange economy allows for Pareto-optimal allocations that are not supported by any linear public ordering. If the above profile were supported by an equilibrium public ordering of the form $p_1x_1 + p_2x_2$, then either: $p_1 \leq p_2$ and agent 2 would deviate from $(0,2)$ to $(2,0)$, or, $p_1 > p_2$ and agent 4 would deviate from $(1,0)$ to $(0,1)$.