

Consumer Search and Double Marginalization

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ONLINE APPENDIX

This online appendix consists of the following parts. Appendix A provides the proofs of Propositions 4-6. Appendix B deals with nonlinear prices in the form of two-part tariffs. In Appendix C we give a treatment of retail oligopoly, while in Appendix D we provide an overview of the issues we raise in the main paper in a search model with product differentiation.

Appendix A: Proofs for Propositions 4, 5 and 6

PROOF OF PROPOSITION 4:

From Theorem 1 we know that for any given λ it is the case that $\bar{p} = \rho$ for all $s < \bar{s}_\lambda$. As we know that an equilibrium exists for $s \geq \bar{s}_\lambda$ we focus on $s < \bar{s}_\lambda$. Denote by $w^*(\rho)$ the set of optimal wholesale prices given a reservation price ρ . This set is not empty because of the continuity of $\pi(w, \rho)$ on a compact support $[0, P]$. Also, denote by $\rho^*(w)$, the reservation price for a given wholesale price w and given retailers' optimal behavior. An equilibrium exists, if there is a pair $\{w, \rho\}$ such that $\rho = \rho^*(w)$ and $w \in w^*(\rho)$.

Note that for $\lambda = 1$ we have that the manufacturer's profit equals $D(w)w$ for any ρ , and by Assumption 1, this is strictly concave and maximized at $p^m(0)$. It is also the case that $\lim_{\lambda \rightarrow 1} \pi(w, \rho) = D(w)w$ and thus, by continuity, that $\lim_{\lambda \rightarrow 1} w^*(\rho) = p^m(0)$. For all $s < \bar{s}_\lambda$ we have that $\rho^*(p^m(0)) < p^m(p^m(0)) < P$ and as $\lim_{\lambda \rightarrow 1} \rho^*(w) = w + s$ and $\rho^*(w)$ is a continuous function for all λ , we have that for λ values close to 1, we can find $\varepsilon_\lambda > 0$ such that $\rho^*(w) < p^m(0) + s$ for all $w < p^m(0) - \varepsilon_\lambda$ and $\rho^*(w) > p^m(0) + s$ for all $w > p^m(0) + \varepsilon_\lambda$. Thus, for λ values close to 1 equilibrium may fail to exist only if there is a discontinuity in $w^*(\rho)$. To prove that there is no such discontinuity, we prove that $\pi(w, \rho)$ is strictly concave for high enough λ so that the set $w^*(\rho)$ has only one element.

The proof that $\pi(w, \rho)$ is strictly concave for high enough λ essentially shows that the second-order derivative is strictly lower than 0. In the proof of Theorem 2 we have derived that the derivative of the manufacturer's profit can be written as

$$\frac{\partial \pi}{\partial w} = \frac{-(1-\lambda)^2}{D(\rho)} + \frac{(1+\lambda)^2 D(p)}{D^2(\rho)} + (1-\lambda)^2 \int_{p(w)}^{\rho} \frac{D'(p)}{D^2(p)} \left[\frac{(\rho-w)^2}{(p-w)^2} + \frac{2w(\rho-w)(\rho-p)}{(p-w)^3} \right] dp$$

so that the second-order derivative is equal to

$$\begin{aligned} \frac{\partial^2 \pi}{\partial w^2} &= \frac{(1+\lambda)^2 D'(\underline{p})}{D^2(\rho)} \frac{\partial \underline{p}}{\partial w} - (1+\lambda)^2 \frac{D'(\underline{p})}{D^2(\rho)} \left[1 + \frac{2(\rho-\underline{p})w}{(\underline{p}-w)(\rho-w)} \right] \frac{\partial \underline{p}}{\partial w} \\ &+ (1-\lambda)^2 \int_{\underline{p}(w)}^{\rho} \frac{D'(\underline{p})}{D^2(\underline{p})} \left[\frac{4(\rho-w)(\rho-\underline{p})}{(\underline{p}-w)^3} - \frac{2(\rho-\underline{p})w}{(\underline{p}-w)^3} + \frac{6(\rho-w)(\rho-\underline{p})w}{(\underline{p}-w)^4} \right] d\underline{p} = \\ &- \frac{2(1+\lambda)^2(\rho-\underline{p})w D'(\underline{p})}{(\underline{p}-w)(\rho-w)D^2(\rho)} \frac{\partial \underline{p}}{\partial w} + (1-\lambda)^2 \int_{\underline{p}(w)}^{\rho} \frac{D'(\underline{p})}{D^2(\underline{p})} \left[\frac{2(2\rho\underline{p}-3\underline{p}w+\rho w)(\rho-w)}{(\underline{p}-w)^4} \right] d\underline{p}. \end{aligned}$$

We develop first the integral part of this expression by using integration by parts:

$$\begin{aligned} \int_{\underline{p}(w)}^{\rho} \frac{D'(\underline{p})}{D^2(\underline{p})} \left[\frac{2(2\rho\underline{p}-3\underline{p}w+\rho w)(\rho-w)}{(\underline{p}-w)^4} \right] d\underline{p} &= -\frac{D'(\rho)}{D^2(\rho)} \frac{2\rho(\rho-\rho)^2}{(\rho-w)^3} + \frac{D'(\underline{p})}{D^2(\underline{p})} \frac{2\underline{p}(\rho-\underline{p})^2}{(\underline{p}-w)^3} \\ &- \int_{\underline{p}(w)}^{\rho} \frac{-2(D'(\underline{p}))^2 + D(\underline{p})D''(\underline{p})}{D^3(\underline{p})} \left[\frac{2\underline{p}(\rho-\underline{p})^2}{(\underline{p}-w)^3} \right] d\underline{p}. \end{aligned}$$

Using again integration by parts on the integral part, a second time yields

$$\begin{aligned} \int_{\underline{p}(w)}^{\rho} \frac{-2(D'(\underline{p}))^2 + D(\underline{p})D''(\underline{p})}{D^3(\underline{p})} \left[\frac{2\underline{p}(\rho-\underline{p})^2}{(\underline{p}-w)^3} \right] d\underline{p} &= 2 \left(\rho - \frac{w(\rho-w)^2}{2(\rho-w)^2} - \frac{(\rho-3w)(\rho-w)}{\rho-w} \right. \\ &+ \left. (-2\rho + 3w) \ln(\rho-w) \right) \frac{-2(D'(\rho))^2 + D(\rho)D''(\rho)}{D^3(\rho)} - 2 \left(\underline{p} - \frac{w(\rho-w)^2}{2(\underline{p}-w)^2} - \frac{(\rho-3w)(\rho-w)}{\underline{p}-w} \right. \\ &+ \left. (-2\rho + 3w) \ln(\underline{p}-w) \right) \frac{-2(D'(\underline{p}))^2 + D(\underline{p})D''(\underline{p})}{D^3(\underline{p})} \\ &- 2 \int_{\underline{p}(w)}^{\rho} \left(\left(\underline{p} - \frac{w(\rho-w)^2}{2(\underline{p}-w)^2} - \frac{(\rho-3w)(\rho-w)}{\underline{p}-w} + (-2\rho + 3w) \ln(\underline{p}-w) \right) \times \right. \\ &\quad \left. \times \frac{6(D'(\underline{p}))^3 - 6D(\underline{p})D'(\underline{p})D''(\underline{p}) + D^2(\underline{p})D'''(\underline{p})}{D^4(\underline{p})} \right) d\underline{p} \end{aligned}$$

Using the fact that $\frac{1-\lambda}{\underline{p}-w} = \frac{(1+\lambda)D(\underline{p})}{(\rho-w)D(\rho)}$ is finite for any $w < \rho$ we know that for λ close to 1, $\frac{(1-\lambda)^2}{(\underline{p}-w)^k}$ is close to 0 for $k < 2$. As in the last integral the fraction with different derivatives of the demand function is finite, the integral can be approximated with terms that only involve $\frac{1}{(\underline{p}-w)^k}$ for $k < 2$. Hence, the second-order derivative of the manufacturer's profit function for λ close to 1 (dropping also all other terms that are close to 0) will be approximately equal to

$$\begin{aligned} &- \frac{2(1+\lambda)^2(\rho-\underline{p})w D'(\underline{p})}{(\underline{p}-w)(\rho-w)D^2(\rho)} \frac{\partial \underline{p}}{\partial w} + \frac{D'(\underline{p})}{D^2(\underline{p})} \frac{2(1-\lambda)^2 \underline{p}(\rho-\underline{p})^2}{(\underline{p}-w)^3} \\ &+ \frac{(1-\lambda)^2 w(\rho-w)^2 - 2(D'(\underline{p}))^2 + D(\underline{p})D''(\underline{p})}{(\underline{p}-w)^2 D^3(\underline{p})}. \end{aligned} \tag{1}$$

Using $\frac{(1-\lambda)^2(\rho-w)^2}{(\underline{p}-w)^2} = \frac{(1+\lambda)^2 D^2(\underline{p})}{D^2(\rho)}$, $\frac{(1+\lambda)^2}{D^2(\rho)}$ times (1) can be rewritten as

$$- \frac{2(\rho-\underline{p})w D'(\underline{p})}{(\underline{p}-w)(\rho-w)} \frac{\partial \underline{p}}{\partial w} + \frac{2\underline{p}(\rho-\underline{p})^2 D'(\underline{p})}{(\underline{p}-w)(\rho-w)^2} + \frac{-2w(D'(\underline{p}))^2 + wD(\underline{p})D''(\underline{p})}{D(\underline{p})}.$$

We now develop an approximation for $\frac{\partial p}{\partial w}$. Differentiating (7) with respect to w yields

$$(1 + \lambda) \left(\left(\frac{\partial p}{\partial w} - 1 \right) D(\underline{p}) + (\underline{p} - w) D'(\underline{p}) \frac{\partial p}{\partial w} \right) = -(1 - \lambda) D(\rho),$$

which can be written as

$$\frac{\partial p}{\partial w} = \frac{1 - \frac{(1-\lambda) D(\rho)}{(1+\lambda) D(\underline{p})}}{1 + (\underline{p} - w) \frac{D'(\underline{p})}{D(\underline{p})}} = \frac{D(\underline{p})(\rho - \underline{p})}{(\rho - w) (D(\underline{p}) + (\underline{p} - w) D'(\underline{p}))}.$$

So we have that

$$\frac{(1 + \lambda)^2}{D^2(\rho)} \frac{\partial^2 \pi}{\partial w^2} = \frac{2(\rho - \underline{p})^2 D'(\underline{p})}{(\underline{p} - w)(\rho - w)^2} \frac{(\underline{p} - w)(\underline{p} D'(\underline{p}) + D(\underline{p}))}{(D(\underline{p}) + (\underline{p} - w) D'(\underline{p}))} + \frac{-2w(D'(\underline{p}))^2 + wD(\underline{p})D''(\underline{p})}{D(\underline{p})},$$

which for λ close to 1 is approximately equal to

$$\frac{2D'(\underline{p}) (\underline{p} D'(\underline{p}) + D(\underline{p}))}{D(\underline{p})} + \frac{-2w (D'(\underline{p}))^2 + wD(\underline{p})D''(\underline{p})}{D(\underline{p})},$$

which for λ close to 1 is approximately equal to

$$2D'(w) + wD''(w)$$

which is the second-order derivative in case $\lambda = 1$. As we assume $pD(p)$ is twice differentiable and strictly concave, this is strictly lower than 0.

PROOF OF PROPOSITION 5:

In case of linear demand the manufacturer's profits can be written as

$$\left(\frac{(1-\lambda)^2(1-w)^4}{32\lambda} \int_{\underline{p}(w)}^{\bar{p}(w)} \frac{1+w-2p}{(p-w)^3(1-p)^2} dp \right) w = \frac{(1-\lambda)^2(1-w)^4 w}{32\lambda(1-w)^3} \int_{\underline{p}(w)}^{\bar{p}(w)} \left(\frac{A}{(p-w)^3} + \frac{B}{(p-w)^2} + \frac{C}{(1-p)^2} + \frac{D}{p-w} + \frac{E}{1-p} \right) dp.$$

where $A = (1 - w)^2$, $B = 0$, $C = -(1 - w)$ and $D = E = -1$. Thus, manufacturer's profits can be written as

$$\pi(w, \bar{p}(w)) = \frac{(1 - \lambda)^2}{32\lambda} (1 - w) w \left[\frac{(1 - w)^2}{-2(p - w)^2} - \frac{1 - w}{1 - p} - \ln \frac{p - w}{1 - p} \right]_{\underline{p}(w)}^{p^m(w)}.$$

Using $p^m(w) = \frac{1+w}{2}$ and $1 - p^m(w) = p^m(w) - w = \frac{1-w}{2}$, we have that $\underline{p}(w) = \frac{1+w-(1-w)\sqrt{\frac{2\lambda}{1+\lambda}}}{2}$. Plugging these into the profit function and simplifying, manufacturer's profits can be written as

$$\pi(w, \bar{p}(w)) = g(\lambda)(1 - w)w$$

for some function $g(\lambda)$. It follows that manufacturer's profits are concave in w and that the optimal choice of w equals $1/2$.

PROOF OF PROPOSITION 6:

We first concentrate on the part dealing with the weighted expected retail price. This part of the proposition is true if the retail price $\frac{1}{1+\lambda}$ at $s = 0$ is higher than the expected retail price for high s . The weighted average of the expected price can be written as

$$\begin{aligned}\tilde{p} &= (1-\lambda) \int_{\underline{p}(w)}^{\bar{p}(w)} pf(p) dp + 2\lambda \int_{\underline{p}(w)}^{\bar{p}(w)} pf(p)(1-F(p)) dp \\ &= \frac{(1-\lambda)^2 \pi_r(\bar{p}(w))^2}{2\lambda} \int_{\underline{p}(w)}^{\bar{p}(w)} \frac{p\pi_r'(p)}{\pi_r(p)^3} dp,\end{aligned}$$

which for linear demand yields

$$\frac{(1-\lambda)^2 \pi_r(\bar{p}(w))^2}{2\lambda} \int_{\underline{p}(w)}^{\bar{p}(w)} \frac{p(1+w-2p)}{(p-w)^3(1-p)^3} dp.$$

The expression under the integral can be written as

$$\frac{A}{(p-w)^3} + \frac{B}{(1-p)^3} + \frac{C}{(p-w)^2} + \frac{D}{(1-p)^2} + \frac{E}{p-w} + \frac{F}{1-p},$$

where $A = \frac{w}{(1-w)^2}$, $B = -\frac{1}{(1-w)^2}$, $C = E = F = \frac{1}{(1-w)^3}$, and $D = -\frac{w}{(1-w)^3}$. The integral is then given by

$$\left[\frac{-\frac{1-w}{(1-p)^2} - \frac{w(1-w)}{(p-w)^2} - \frac{2w}{1-p} - \frac{2}{p-w} + 2 \ln \left(\frac{p-w}{1-p} \right)}{2(1-w)^3} \right]_{\underline{p}(w)}^{\bar{p}(w)}.$$

Using $\underline{p}(w) = \frac{1+w-(1-w)\sqrt{\frac{2\lambda}{1+\lambda}}}{2}$, $w = 1/2$ and $\bar{p}(w) = 3/4$, the weighted average price can be written as

$$\frac{-6\lambda\sqrt{2\lambda(1+\lambda)} + 48\lambda - 2\sqrt{2\lambda(1+\lambda)} - (1-\lambda)^2 \ln \left(\frac{1-\sqrt{\frac{2\lambda}{1+\lambda}}}{1+\sqrt{\frac{2\lambda}{1+\lambda}}} \right)}{64\lambda}.$$

It is readily verified numerically that for the relevant λ , i.e., $\lambda > 0.47$, the above is lower than $\frac{1}{1+\lambda}$. Thus average weighted price for any $s > \bar{s}_\lambda$ is lower than at $s \rightarrow 0$.

We now concentrate on the part dealing with the manufacturer's profits. When $s \rightarrow 0$, manufacturer's profit is given by $\frac{1}{1+\lambda} \left(1 - \frac{1}{1+\lambda}\right) = \frac{\lambda}{(1+\lambda)^2}$. From Proposition 5, we know that for high s manufacturer's profits can be written as $g(\lambda)(1-w)w$, where $w = 1/2$ and

$$g(\lambda) = \frac{(1-\lambda)^2}{32\lambda} \left[2 \left(\frac{1}{\sqrt{\frac{2\lambda}{1+\lambda}} + 1} + \frac{1}{\left(1 - \sqrt{\frac{2\lambda}{1+\lambda}}\right)^2} - 2 \right) + \ln \left(\frac{1 - \sqrt{\frac{2\lambda}{1+\lambda}}}{1 + \sqrt{\frac{2\lambda}{1+\lambda}}} \right) \right].$$

Comparing this expression with $\frac{\lambda}{(1+\lambda)^2}$ one can either simply plot the two expressions to verify that $\frac{\lambda}{(1+\lambda)^2}$ is higher for all $0.47 < \lambda < 1$, or one can follow a more tedious route and verify the following points. First, $\frac{1}{4}g(\lambda)$ and $\frac{\lambda}{(1+\lambda)^2}$ are both equal to $\frac{1}{4}$ for $\lambda = 1$ and for $\lambda \approx 0.47$ the manufacturer's profit $\frac{\lambda}{(1+\lambda)^2}$ for $s \rightarrow 0$ is higher. Second, by using the substitution $x = \sqrt{\frac{2\lambda}{1+\lambda}}$, one can show that both expressions are decreasing in λ and the derivative of $\frac{\lambda}{(1+\lambda)^2}$ is lower at $\lambda = 1$ whereas it is higher at $\lambda \approx 0.47$, and for only one value of λ the derivatives are equal to each other. It follows that manufacturer's profits are higher when $s \rightarrow 0$ and $\lambda > 0.47$.

Finally, we deal with total industry profits. When $s \rightarrow 0$, industry profit equals manufacturer's profit and is thus equal to $\frac{\lambda}{(1+\lambda)^2}$. Total industry profits for high s equals $\pi(0.5, 0.75) + (1-\lambda)\frac{1}{16}$. As for $\lambda = 1$ total profit equals $1/4$ and the derivative of $\frac{\lambda}{(1+\lambda)^2}$ is higher than that of the expression above for all relevant parameters ($\lambda > .47$), it follows that total profits are higher when s is high than when $s \rightarrow 0$.

Appendix B: Two-Part Tariffs

When the upstream firm engages in two-part tariffs, it charges a price per unit and a fixed fee to maximize profits. As firms are risk neutral, the upstream firm can charge a fixed fee equal to the expected profits of the retailers such that the retailers are still willing to participate. Thus, when charging an optimal two-part tariff, the upstream firm effectively maximizes total industry profits. In this case, the upstream firm has clearly less incentives to squeeze the retailers' profits as it can recover these profits by charging a higher fixed fee. So, the question is whether two-part tariffs completely eliminate the triple marginalization problem (like it eliminates the double marginalization problem) or whether part of the problem remains.

First, note that when the wholesale price arrangement is observed by consumers, the upstream firm wants to set its upstream price per unit such that the retail price distribution centers around the integrated monopoly price of 0.5 to maximize total industry profit. As there is price dispersion at the retail level, the upstream firm will never charge a price per unit that is equal to its marginal cost. To do so, would imply that the upper bound of the retail price distribution is equal to the integrated monopoly price (for large search cost s) or below (for smaller values of s) so that almost surely the retail price is effectively too small to maximize profits. To have the retail price distribution center around the integrated monopoly price, the upstream firm sets a positive price per unit and a fixed fee. This is shown in Figure 1 for the case where $\lambda = 0.5$ and demand is linear and given by $D(p) = 1 - p$. Note that because of the price distribution at the retail level, the upstream firm cannot get the maximal profit of an integrated monopolist.

Next, consider the case where the wholesale price arrangement is not observed by consumers. If the upper bound of the retail price distribution is given by the retail monopoly price, the same consideration apply as above and the outcome of the two

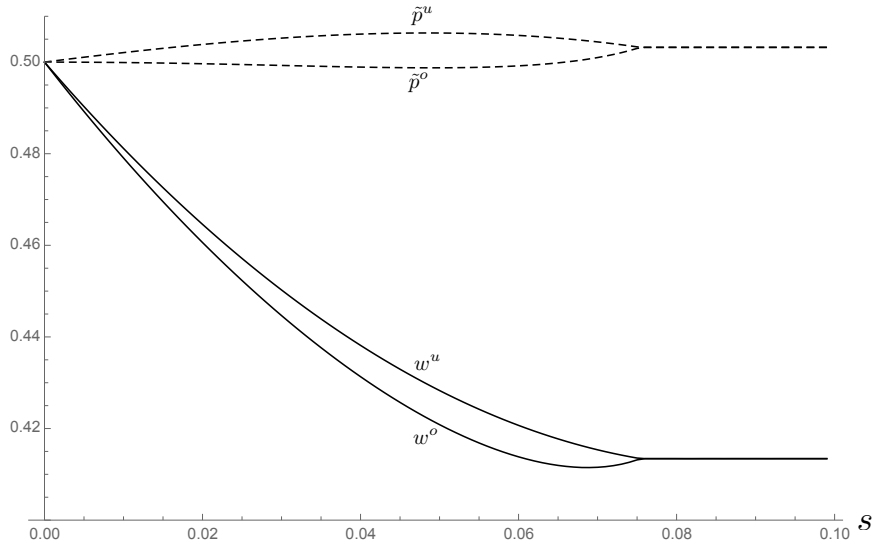


Figure 1: Two-part tariffs. Upstream prices (solid) and weighted average downstream prices (dashed) for the observed (blue) and unobserved (red thick) retailers' costs as functions of s for $\lambda = 0.5$.

models coincide. However, when the search cost s is small, the upper bound of the retail price distribution is given by the non-shoppers' reservation price, which now does not depend on the wholesale price per unit. Thus, like in the baseline model without fixed fee, the retailers' price distribution (and thus the demand for the upstream firm) reacts less to an increase in the price per unit set by the upstream firm. thus, compared to the case where the wholesale price arrangement is observed by consumers, the upstream firm has (again) an incentive to increase the per unit wholesale price. As with two-part tariffs, the upstream firm effectively maximizes total industry profits and the downstream profits decrease with an increase in the per unit wholesale price, the incentives to increase the per unit wholesale price are dampened. The figure shows that the triple marginalization problem continues to exist, but the magnitude is much smaller.

Moreover, the effect analyzed in the previous section that the triple marginalization problem is most severe for small search cost, disappears. The reason why, as explained above, under two-part tariffs the triple marginalization problem continues to exist for positive search cost is because of the price dispersion at the retail level. When the search cost becomes arbitrarily small, however, the retail price dispersion disappears and the upstream firm can extract maximal profits by setting the wholesale price per unit equal to the integrated monopoly price. Thus, under two-part tariffs the two models converge for both large search cost and when the search cost is arbitrarily small.

Appendix B: Retail Oligopoly

In this appendix we show that the qualitative properties of the equilibria under retail duopoly extend to the case where there are more than two retail firms. The effects shown by the numerical analyses we present here are easily interpreted. Roughly speaking, there are two effects. First, the region of parameters where the knowledge consumers have on the wholesale price makes a difference becomes smaller. This is quite intuitive if we recall the result by Stahl (1989) that the reservation price is increasing in the number of firms. In the context of our model, this implies that when the number of firms is larger the upper bound of the retail price distribution is given by the retail monopoly price for smaller values of s . As this is the region where the two models coincide, the region where there is a difference between the two information scenarios becomes smaller. Second, when the search cost is small, the triple marginalization effect becomes stronger. That is, the larger the number of firms, the stronger is the effect that the equilibrium wholesale price is decreasing in s . The reason is that with a larger number of firms, the more probability mass the retail price distribution gives to higher prices and the smaller the effect of an increase in the wholesale price on the demand for the upstream firm.

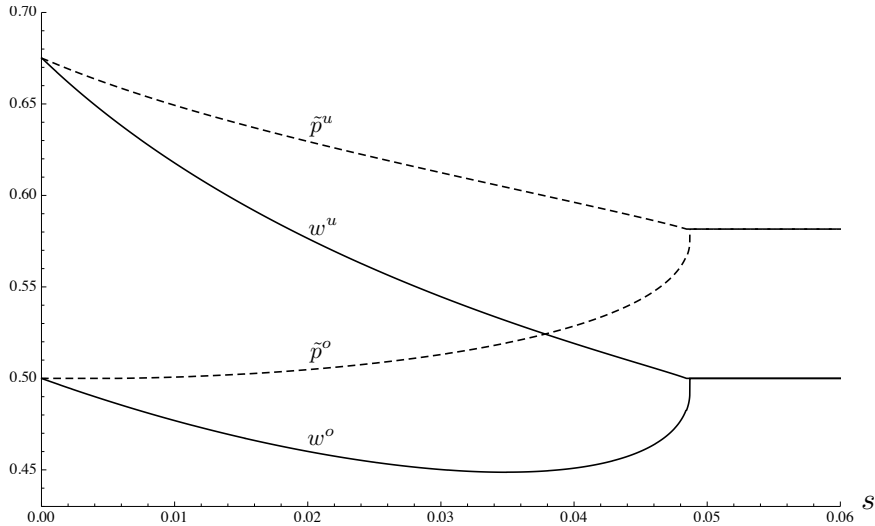


Figure 2: Upstream and weighted average downstream prices for the two models for $N = 3$.

We show these effects in two different ways. For $N = 3$, we show that Figure 2 depicting the relationship between wholesale and expected retail price as a function of s is similar to the corresponding figure for $N = 2$ (apart from the two differences noted above). Next, we also show the relationship between wholesale price and the number of firms when the search cost is small. As explained above, Figure 3 shows that this relationship is increasing.

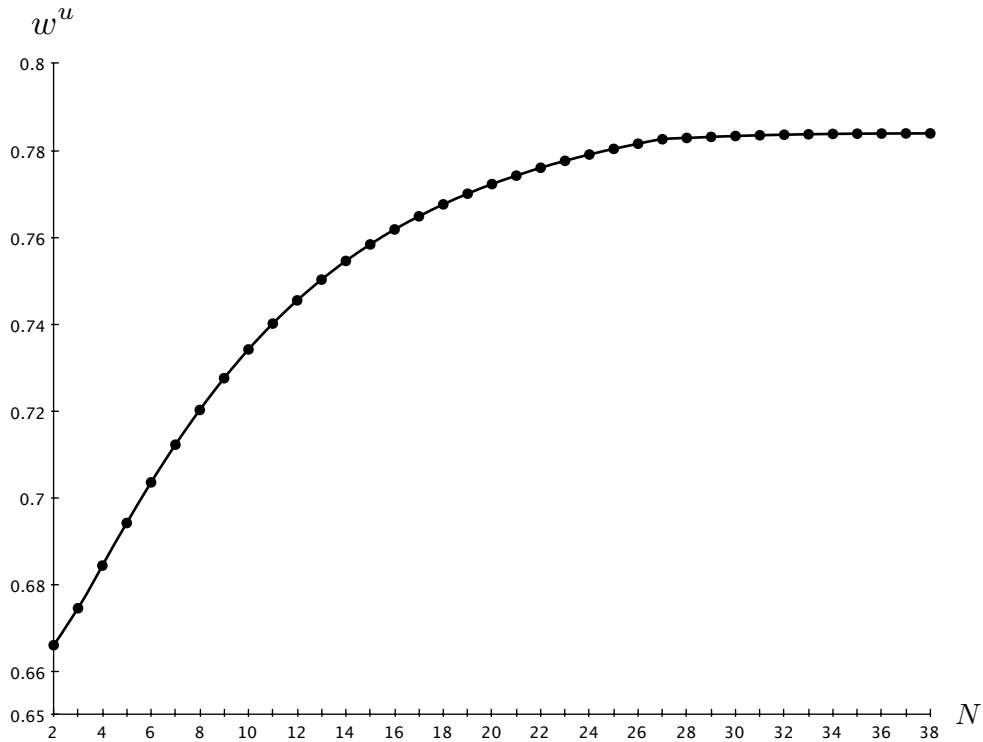


Figure 3: The upstream price for $\lambda = 0.5$ and $s = 0.0001$ as function of N .

Appendix C: Extension to Differentiated Goods

In this appendix we show that our results can be extended to a setting with differentiated goods. To do so, instead of the homogeneous goods model used in the body of the paper, we use a modification of Wolinsky (1986).

Assume an upstream manufacturer produces an essential input at the marginal cost of zero. He charges w for one unit to two competing retailers, 1 and 2. Retailers transform the input costlessly into the final differentiated good one for one. Valuation a consumer attaches to a good sold by retailer i is v_i which is drawn according to $F(v_i)$ over the interval $[\underline{v}, \bar{v}]$. These valuations are independent between the two firms and consumers.

Each consumer costlessly visits one of the retailers with equal probability and finds out v_i and p_i . Consumer can then choose to visit the other retailer at a cost s . He can always go back to the first retailer at no additional cost. There is a unit mass of consumers per firm.

The timing is as follows: the upstream firm charges w which is observed by downstream firms (and possibly by consumers). The downstream firms charge prices, consumers visit one retailer for free, decide whether to visit the other one and make purchases if the best available $v_i - p_i$ is above zero, their outside option. We use Perfect Bayesian Equilibrium as the solution concept, and impose passive beliefs for consumers, thus upon

observing a deviation by a retailer, consumers believe that this and only this retailer has deviated.

Observed Upstream Price

Let us solve the game backwards. At the stage when downstream firms choose prices w is known.

Assume that a consumer visits retailer 1 that charges p_1 and believes the other retailer charges p^* , the symmetric equilibrium price. As standard in this literature, define u^* as the solution to:

$$\int_{u^*}^{\bar{v}} (v - u^*) f(v) dv = s.$$

If such solution does not exist, then $u^* = \underline{v}$.

As is well known in the search literature, u^* is the expected utility of searching another firm including the search cost. This means that a consumer who draws v_1 will search the other firm if $v_1 < u^* - p^* + p_1$. For consumers to ever search we need that $u^* > p^*$, an assumption we make now and impose later.

Expected demand for firm 1 which charges p_1 , while firm 2 charges p^* and all consumers expect the firm they have not visited first to charge p^* is given by:

$$D_1 = (1 - G(u^* - p^* + p_1))(1 + G(u^*)) + 2 \int_{p_1}^{u^* - p^* + p_1} G(p^* - p_1 + v) g(v) dv$$

The first order condition for firm 1's profit maximization, along with the equilibrium condition $p_1 = p^*$ gives the following price setting rule:

$$p^* = c + \frac{1 - G(p^*)^2}{g(u^*)(1 - G(u^*)) + 2 \left(g(p^*)G(p^*) + \int_{p^*}^{u^*} g(v)^2 dv \right)}. \quad (2)$$

Manufacturer's demand for equal retail prices is given by $(1 - G(p)^2)$ and so the upstream firm will set w^* that solves

$$\frac{\partial}{\partial w} (1 - G(p^*(w))^2) w = 0$$

where $p^*(w)$ is implicitly defined by (2). Assuming that the upstream profit is well behaved, and that $p^*(w^*) < u^*$, we have the solution to the model. Condition $p^*(w^*) < u^*$ will hold when s is sufficiently small. We shall assume this from now on.

Unobserved Upstream Price

Now we turn to the case where w is not observed by consumers. As in our baseline model with homogeneous goods, consumers cannot change their belief p^e in response to a change in w . To accommodate this, assume that consumers hold some beliefs w^e

and p^e , that they do not update upon observing a deviation by a retailer they visit first (these are the passive beliefs used in the previous section).

Expected demand for retailer 1 that charges p_1 , while firm 2 charges p_2 and all consumers believe that both retailers charge p^e is given by:

$$D_1 = (1 - G(u^* - p^e + p_1))(1 + G(u^* - p^e + p_2)) + 2 \int_{p_1}^{u^* - p^e + p_1} G(p_2 - p_1 + v)g(v) dv$$

Normally, in the last expression p_2 would be set equal to p^e . But when the manufacturer deviates and its deviation is not observed by consumers, consumers expect both firms to charge p^e and both retailers know that neither will charge p^e , so we have to allow for p_2 to differ from p^e .

To see why misplaced expectations are important note two things. First, the total demand when $p_1 = p_2 = p$ is equal to $1 - G(p)^2$ and does not depend on p^e . So the manufacturer does not *per se* care about misplaced expectations. Second, the derivative of the demand for firm 1 at around $p_1 = p_2$, which is negative, depends on p^e . To see this let us take the derivative of D_1 with respect to p_1 and set $p_1 = p_2 = p$:

$$-2g(p)G(p) - g(p - p^* + w)(1 - G(p - p^* + w)) - 2 \int_p^{p - p^* + w} g(v)^2 dv. \quad (3)$$

When beliefs are correct ($p^e = p$) previous simplifies to:

$$-2g(p)G(p) - g(w)(1 - G(w)) - 2 \int_p^w g(v)^2 dv \quad (4)$$

Both of these expressions are negative. Let us define $x = p - p^*$ and take the derivative of (3) with respect to x and evaluate it at $x = 0$. This derivative is given by

$$-g(w)^2 - (1 - G(w))g'(w). \quad (5)$$

If (5) is negative, then when consumers expect to see lower prices than both firms charge ($x > 0$), derivative of a firm's demand with respect to its own price is larger in absolute value, and so retailers charge lower prices. This means that $\frac{\partial Q}{\partial w^e} < 0$, and so the manufacturer charges a higher price in the unobserved case.

Note that if (5) is negative, it is the condition for log-concavity of $1 - G(w)$. This condition is implied by log-concavity of $g(w)$. It is necessary in the standard Wolinsky model to have that prices increase in search cost (see Anderson and Renault (1999)). It is satisfied for many commonly used distributions such as uniform, normal and logistic, but for exponential (5) is zero, and therefore prices coincide in the observed and unobserved cases.

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