Online Appendix for “Correlation Neglect, Voting Behavior and Information Aggregation”

By GILAT LEVY AND RONNY RAZIN

In this online appendix we show that the qualitative nature of our results still holds in a model with finite voters who vote strategically. In particular, we show that the results of Lemma 2 imply that behavioural electorates will induce more information aggregation than rational electorates for a wide range of environments.

As is standard in the strategic voting literature, consider a model with a finite population, n (odd) voters. Each voter i has an ideal policy υ_i distributed according to some distribution $F(\upsilon_i)$ with density $f(\upsilon_i)$ on $[-1, 1]$. The ideal policy υ_i is private information of voter i. One specification of the model that we now need to take a stand on, is what behavioural voters believe about the information structure of other voters. Below we assume that they believe that other voters are like themselves. That is, they believe that other voters have independent information sources. This specific assumption will not affect the qualitative results reported here.[1] All the other assumptions about the information structure are as in the paper.

Our first result shows that voting behaviour, although strategic, follows a similar pattern to that in the paper. One difference is that in Lemma 1, when voters were assumed to vote sincerely, one set of cutoffs characterised the equilibrium both for the rational and the behavioural electorates. In contrast, when voters learn from the pivotal event, the equilibrium cutoffs will generally be different between the model with a rational electorate and a model with a behavioural electorate. Lemma A1 below replicates the results of Lemma 1 to the case of strategic voters:

Lemma A1: *In any symmetric equilibrium, there exists values $\upsilon_l^{J,2}, \upsilon_l^{J,1}, \upsilon_r^{J,1}, \upsilon_r^{J,2}$ such that $\upsilon_l^{J,2} < \upsilon_l^{J,1} < \upsilon_r^{J,1} < \upsilon_r^{J,2}$ and (where $J = R, B$ relates to the rational and behavioural electorates respectively): (i) One signal: A voter with $\upsilon \in [\upsilon_l^{J,1}, \upsilon_r^{J,1}]$ who observes only one signal votes for r(l) if the signal is 1(-1). A voter with $\upsilon > \upsilon_r^{J,1}$ ($\upsilon < \upsilon_l^{J,1}$) who observes only one signal votes for r (l). (ii) Two signals: A voter with $\upsilon \in [0, \upsilon_r^{J,2}]$ ($[\upsilon_l^{J,2}, 0]$) who believes he has two independent signals votes for r (l) unless the two signals are -1 (1). A voter with $\upsilon > \upsilon_r^{J,2}$ ($\upsilon < \upsilon_l^{J,2}$) always votes for r (l).*

Proof: The proof follows from noting: (i) that conditioning on being pivotal, all voters learn the same information, (ii) the monotonicity assumed in the utilities and signal structure, and (iii) that the model with behavioural voters is equivalent to a model with rational voters, up to the voters putting too much weight on their own signal and on what they learn from the pivotal event.

Specifically, consider the cutoff $\upsilon_l^{J,2}$ which is given by the indifference condition of a

[1] It will affect the calculations of the examples below.

voter who has received two 1 signals and conditions on being pivotal,

$EU(\text{voting for } r|piv,1,1) =$
$(-1+v_l^{J,2}-r)^2 + \Pr(1|piv,1,1)((1+v_l^{J,2}-r)^2 - (-1+v_l^{J,2}-r)^2) =$
$EU(\text{voting for } l|piv,1,1) =$
$(-1+v_l^{J,2}-l)^2 + \Pr(1|piv,1,1)((1+v_l^{J,2}-l)^2 - (-1+v_l^{J,2}-l)^2)$
$\Leftrightarrow 4\Pr(1|piv,-1) = \left(l+r-2v_l^{J,2}+2\right) \Leftrightarrow$
$v_l^{J,2} = \frac{l+r}{2} + 1 - 2\Pr(1|piv,1,1) = 1 - 2\Pr(1|piv,1,1)$ given the symmetry of the platforms.

Similarly, we get:
$v_l^{J,2} = 1-2\Pr(1|piv,1,1) < v_l^{J,1} = 1-2\Pr(1|piv,1) <$
$v_r^{J,1} = 1-2\Pr(1|piv,-1) < v_r^{J,2} = 1-2\Pr(1|piv,-1,-1).$■

In the next result we focus on symmetric distributions, $f(v) = f(-v)$. Note that in any symmetric equilibrium in which strategies are symmetric around the median, the pivotal event is uninformative. In these cases, strategic voting and sincere voting are the same. The next proposition shows that in these environments, behavioural electorates will aggregate strictly more information than rational electorates. Moreover, this result is directly related to Lemma 2 in the paper.

Proposition A1: *Consider a symmetric f. In both the rational and the behavioural electorates, there exists a unique symmetric equilibrium in which strategies are symmetric around the median. In this equilibrium: (i) The cutoffs from Lemma A1 are the same for the two types of electorates, i.e., $v_l^{R,k} = v_l^{B,k}$ and $v_r^{R,k} = v_r^{B,k}$ for $k = 1,2$. (ii) The probability that the optimal policy is chosen in any state of the world is strictly higher in a behavioural electorate than in a rational one.*

Proof of Proposition A1: We first prove (i). Note that given the symmetry of voting behaviour, it will be the case that $v_l^{J,k} = -v_r^{J,k}$ for any $J = R, B$ and $k = 1,2$. Assume then some cutoffs $v' \equiv v_r^{J,1} > 0$ and $v'' \equiv v_r^{J,2} > 0$ for the case of one and two signals respectively (with their symmetric mirror images $-v', -v''$).

Note that vote shares in this model are stochastic. Specifically, the probability a random (Rational R, behavioural B) voter votes for r when the state is 1 is given by:

$$\begin{aligned}
\gamma_r^B(1) &= (1-F(-v'')) + (F(v'')-\frac{1}{2})(\rho q + (1-\rho)(1-(1-q)^2) + (\frac{1}{2}-F(-v''))(\rho q + (1-\rho)q^2) \\
\gamma_r^R(1) &= (1-F(-v'')) + (F(v'')-F(v'))(\rho + (1-\rho)(1-(1-q)^2)) + \\
&\quad (F(v')-\frac{1}{2})(\rho q + (1-\rho)(1-(1-q)^2) \\
&\quad +(\frac{1}{2}-F(-v'))(\rho q + (1-\rho)q^2) + (F(-v')-F(-v''))((1-\rho)q^2)
\end{aligned}$$

Similarly one can define $\gamma_r^B(-1)$ and $\gamma_r^R(-1)$.

The probability of vote share $V_r^J(1) = \frac{m}{n}$ (for platform r in state $\omega = 1$ in electorate

J) is given by,

$$\Pr(V_r^J(1) = \frac{m}{n}) = \binom{m}{n} (\gamma_r^J(1))^m (1 - \gamma_r^J(1))^{n-m}$$

We now define, for a voter of type υ_i and with information Ω, the difference in expected utility from voting to $r > 0$ and $-r$, denoted by $\Delta(\upsilon_i|\Omega)$. Note below that $(r-(\upsilon+1))^2 - (-r-(\upsilon+1))^2 = -4r(\upsilon+1)$ and $(r-(\upsilon-1))^2 - (-r-(\upsilon-1))^2 = -4r(\upsilon-1)$.

$$\Delta(\upsilon_i|\Omega) = -\Pr(w=1|\Omega)\binom{\frac{n-1}{2}}{n}(\gamma_r^J(1))^{\frac{n-1}{2}}(1-\gamma_r^J(1))^{\frac{n-1}{2}}(-4r(\upsilon_i+1))$$
$$-\Pr(w=-1|\Omega)\binom{\frac{n-1}{2}}{n}(\gamma_r^J(-1))^{\frac{n-1}{2}}(1-\gamma_r^J(-1))^{\frac{n-1}{2}}(-4r(\upsilon_i-1)) =$$
$$\binom{\frac{n-1}{2}}{n}(\gamma_r^J(1))^{\frac{n-1}{2}}(1-\gamma_r^J(1))^{\frac{n-1}{2}}[-\Pr(w=1|\Omega)(-4r(\upsilon_i+1))-\Pr(w=-1|\Omega)(-4r(\upsilon_i-1))]$$

where the last equality follows by symmetry as $\gamma_r^J(-1) = 1-\gamma_r^J(1)$.

Consider now an electorate with type $J \in \{R, B\}$ voters. Suppose that a voter of type υ_i has received $k \in \{1, 2\}$ signals $s \in \{-1, 1\}$. The cutoffs in lemma A1 are determined by the equation $\Delta(\upsilon_l^{J,k}|\Omega) = 0$ if $s = 1$ or $\Delta(\upsilon_r^{J,k}|\Omega) = 0$ if $s = -1$, where Ω includes the signals as well as the information gleaned by focusing on the pivotal event. By symmetry, the pivotal event is completely uninformative and so we have that without loss of generality Ω can include only the signals. Moreover, the two conditions can be simplified by dividing by the expression $\binom{\frac{n-1}{2}}{n}(\gamma_r^J(1))^{\frac{n-1}{2}}(1-\gamma_r^J(1))^{\frac{n-1}{2}}$ implying a new condition,

$$\begin{aligned}\upsilon_l^{J,k} &= (\Pr(w=-1|k \text{ signals of } 1) - \Pr(w=1|k \text{ signals of } 1))\\ \upsilon_r^{J,k} &= (\Pr(w=-1|k \text{ signals of } -1) - \Pr(w=1|k \text{ signals of } -1))\end{aligned}$$

But as these only depend on the signals and not on equilibrium behaviour we conclude the proof of (i).

To see (ii), note that as the cutoffs are the same for the two types of electorates, the voting behaviour follows Lemma 1. As F is symmetric, $\lambda = \frac{1}{2}$ and we fall under the case of $1-q < \lambda < q$. This implies that the probability a voter votes for the correct policy (the expected vote share for the correct policy) is strictly higher in the behavioural electorate in each state. Therefore, as votes are stochastic, the probability that the correct policy is chosen is strictly higher in the behavioural electorate.

Finally, the existence and uniqueness of symmetric equilibria in which strategies are symmetric around the median obviously follows from the above equations for $\upsilon_l^{J,k}$ and $\upsilon_r^{J,k}$. ■

Robustness to asymmetric distributions: In the above symmetric model, strategic voting behaviour implies sincere voting. More generally strategic voting will prescribe voting behaviour that is different than sincere voting. However, the above sincere equilibrium is a strict equilibrium and thus as $1-q < \frac{1}{2} < q$, if we perturb the model, the equilibrium will involve voting strategies that are not sincere but will still have $1-q < \lambda < q$. This implies that Proposition A1 extends to an open set of distribution including asymmetric ones.

As mentioned above, when voters are strategic, in general the equilibrium cutoffs will be different in the model with a rational electorate and in the model with a behavioural electorate. This implies that comparing the two electorates becomes harder (and is exacerbated by potential multiplicity of equilibria). Below we provide an example that shows that our results can also hold in such equilibria.

An example with an asymmetric distribution of voters: We now illustrate that behavioural electorates might still aggregate more information even for large levels of asymmetries in the underlying distribution.

For simplicity consider a case in which there are three voters and the distribution F is a truncated uniform distribution $[-0.6, 1)$ with an atom on 1 with mass 0.4. Also for simplicity, assume that $\rho = 1$ so that information sources that voters are exposed to are always correlated. This implies that in each model, rational and behavioural, we have to consider only two cutoffs (remember that $l = -r$ but that voting behaviour will not be necessarily symmetric):

$$\begin{aligned} \upsilon_r^{J,2} &= 1 - 2\Pr(1|piv^J, -1), \\ \upsilon_l^{J,2} &= 1 - 2\Pr(1|piv^J, 1), \end{aligned}$$

where $J = R$ or B depending on whether the electorate is rational or behavioural.

We consider different values of q (the accuracy of the signal), and compute the cutoffs above (which are a solution to a fixed point equation as the information voters learn from being pivotal depends on the cutoff themselves).[2] Note that when the electorate is behavioural, they behave as if they had received (and others have received) one signal with precision $\frac{q^2}{q^2+(1-q)^2}$, while the computation of information aggregation demands that we consider the true precision of the signal. Specifically, let IA^J denote the probability of the electorate choosing the optimal policy (averaging over both states). We then have:

	$\upsilon_l^{R,2}, \upsilon_r^{R,2}$	IA^R	$\upsilon_l^{B,2}, \upsilon_r^{B,2}$	IA^B
$q = 0.6$	$-0.16, 0.23$	0.526	$-0.29, 0.47$	0.55
$q = 0.7$	$-0.3, 0.489$	0.6	$-0.49, 0.817$	0.68
$q = 0.8$	$-0.43, 0.72$	0.73	$-0.59, 0.97$	0.81
$q = 0.9$	$-0.56, 0.916$	0.87	$-0.6, 0.99$	0.89

We find that for these different precisions of the signal, information aggregation is strictly higher in the behavioural electorate (that is, the probability of choosing the optimal policy). Note that the interval of informative voting increases with the precision of

[2]For these parameter values the equilibrium is unique.

the signal, and that the interval of informative voting in the behavioural electorate contains the respective interval in the rational electorate. Indeed, the behavioural electorate chooses the correct policy with a higher probability.