Wages and Informality in Developing Countries

Online Appendix*

Costas Meghir, Renata Narita and Jean-Marc Robin

November 14, 2014

*Costas Meghir: Yale University, IFS and NBER, Department of Economics, 37 Hillhouse Avenue, New Haven CT 06511, USA (email: c.meghir@yale.edu); Renata Narita: University of Sao Paulo, Department of Economics, Avenida Professor Luciano Gualberto 908, Sao Paulo, SP, Brazil, 05508-010 (email: rnarita@usp.br); Jean Marc Robin: Sciences Po and UCL, Office E410, 28 rue des Saints-Pres, 75007 Paris, France (email:jeanmarc.robin@sciencespo.fr). We thank three anonymous referees as well as Joe Altonji, Carlos Corseuil , Corina Mommaerts, Ana Reynoso, Chris Taber and seminar participants at the NBER Labor Studies group, the University of Wisconsin Madison, Yale University for comments. Costas Meghir thanks the ESRC for funding under the Professorial Fellowship RES-051-27-0204 as well as the Cowles Foundation and the ISPS at Yale. Renata Narita thanks the Brazilian Ministry of Education CAPES for financial support from through the scholarship BEX 1960-03-2. Robin gratefully acknowledges financial support from the Economic and Social Research Council through the ESRC Centre for Microdata Methods and Practice grant RES-589-28-0001, and from the European Research Council (ERC) grant ERC-2010-AdG-269693-WASP. We also thank the ESRC Centre for Microeconomic Analysis of Public Policy at the Institute for Fiscal Studies. Responsibility for any errors is ours.

ONLINE APPENDIX

I. Equilibrium Offer and Accepted Contract Distributions

In this section, we derive G_1 and G_2 from F_1 and F_2.

From the main paper recall the equilibrium flow condition for the formal sector

$$\left[\lambda_{10} + \lambda_{11}\overline{F}_1(W)\right] m_1 G_1(W) + \lambda_{12} m_1 \int_{\underline{W}_1}^{W} \overline{F}_2(x)\, dG_1(x) \\ = \lambda_{01} u F_1(W) + \lambda_{21} m_2 \int_{\underline{W}_2}^{W} \left[F_1(W) - F_1(x)\right] dG_2(x). \quad (.1)$$

By equation (.1), for any $W \in [\underline{W}_1, \overline{W}_1]$,

$$\left[\lambda_{10} + \lambda_{11}\overline{F}_1(W)\right] m_1 G_1(W) + \lambda_{12} m_1 \int_{\underline{W}_1}^{W} \overline{F}_2(x)\, dG_1(x) \\ = \lambda_{01} u F_1(W) + \lambda_{21} m_2 \int_{\underline{W}_2}^{W} \left[F_1(W) - F_1(x)\right] dG_2(x).$$

Making use of the identities (integration by parts):

$$\int_{\underline{W}_1}^{W} \overline{F}_2(x)\, dG_1(x) = \overline{F}_2(W) G_1(W) + \int_{\underline{W}_1}^{W} G_1(x)\, dF_2(x),$$

$$\int_{\underline{W}_2}^{W} \left[F_1(W) - F_1(x)\right] dG_2(x) = \int_{\underline{W}_2}^{W} G_2(x) dF_1(x),$$

we can rewrite this equation as

$$d_1(W) \frac{m_1}{u} G_1(W) = \lambda_{01} F_1(W) - \Phi(W), \quad (.2)$$

where $d_1(W) = \lambda_{10} + \lambda_{11}\overline{F}_1(W) + \lambda_{12}\overline{F}_2(W)$, and

$$\Phi(W) = \lambda_{12} \int_{\underline{W}_1}^{W} \frac{m_1}{u} G_1(x)\, dF_2(x) - \lambda_{21} \int_{\underline{W}_2}^{W} \frac{m_2}{u} G_2(x)\, dF_1(x). \quad (.3)$$

Turning to the informal sector, the equivalent equation

$$\left[\lambda_{20} + \lambda_{22}\overline{F}_2(W)\right] m_2 G_2(W) + \lambda_{21} m_2 \int_{\underline{W}_2}^{W} \overline{F}_1(x)\, dG_2(x) \\ = \lambda_{02} u F_2(W) + \lambda_{12} m_1 \int_{\underline{W}_1}^{W} \left[F_2(W) - F_2(x)\right] dG_1(x). \quad (.4)$$

indicates that for $W \in [\underline{W}_2, \overline{W}_2]$,

$$\left[\lambda_{20} + \lambda_{22}\overline{F}_2(W)\right] m_2 G_2(W) + \lambda_{21} m_2 \int_{\underline{W}_2}^{W} \overline{F}_1(x)\, dG_2(x)$$
$$= \lambda_{02} u F_2(W) + \lambda_{12} m_1 \int_{\underline{W}_1}^{W} \left[F_2(W) - F_2(x)\right] dG_1(x).$$

Using the same integrations by part, we obtain that

$$d_2(W)\frac{m_2}{u} G_2(W) = \lambda_{02} F_2(W) + \Phi(W), \tag{.5}$$

where $d_2(W) = \lambda_{20} + \lambda_{21}\overline{F}_1(W) + \lambda_{22}\overline{F}_2(W)$.

Next, multiplying equation (.2) by $\frac{\lambda_{12} f_2(W)}{d_1(W)}$ (with $f_2 = F_2'$) and equation (.5) by $-\frac{\lambda_{21} f_1(W)}{d_2(W)}$, and adding the two resulting equations, we obtain the first-order differential equation

$$\Phi' = A - B\Phi, \tag{.6}$$

where

$$A = \lambda_{01} F_1 \tfrac{\lambda_{12} f_2}{d_1} - \lambda_{02} F_2 \tfrac{\lambda_{21} f_1}{d_2},$$
$$B = \tfrac{\lambda_{12} f_2}{d_1} + \tfrac{\lambda_{21} f_1}{d_2},$$

with boundary condition $\Phi(U) = 0$ (in fact $\Phi(W) = 0, \forall W \leq \max\{\underline{W}_1, \underline{W}_2\}$).

The solution of differential equation (.6) is given by

$$\Phi(W) = \frac{\int_U^W e^{\int_U^x B(x')dx'} A(x)\, dx}{e^{\int_U^W B(x)\, dx}}. \tag{.7}$$

Substituting this solution back into equations (.2) and (.5) we obtain the equilibrium relationship between the distribution of offered (F) and accepted (G).

II. Computing the Equilibrium

In this section we describe the computation of the equilibrium. We assume known the number of workers M, the potential number of firms N and the potential distribution of productivity Γ.

A. Exogenous Transition Rates

Assume that rates λ_{ij} are fixed.

1. Define contract value offer distribution F_1 and F_2, with supports bounds $\underline{W}_2 = U < \underline{W}_1 < \overline{W}_2 < \overline{W}_1$. Recall that

   $$rU = b + \lambda_{01}(\mu_1 - U) + \lambda_{02}(\mu_2 - U), \tag{.8}$$

   where $\mu_1 = \int_{\underline{W}_1}^{\overline{W}_1} x\, dF_1(x), \mu_2 = \int_{\underline{W}_2}^{\overline{W}_2} x\, dF_2(x)$ denote the mean contract values offered in the formal and the informal sector respectively, and b is the flow-value of leisure. Hence

   $$\underline{W}_2 = U = \frac{b + \lambda_{01}\mu_1 + \lambda_{02}\mu_2}{r + \lambda_{01} + \lambda_{02}}.$$

   Define the shares of firms in each sector $n_i, i = 1, 2$, with $n_1 + n_2 \leq 1$.

2. Use steady-state flow conditions in Appendix () to derive m_1, m_2, u and G_1, G_2 from F_1, F_2.

3. Profit maximization then implies that optimal decision rules satisfy

$$p = K_1^{-1}(W) = (1 + \tau + \lambda_{10}s)[w_1(W) + w_1'(W)\frac{\ell_1(W)}{\ell_1'(W)}],$$
$$p = K_2^{-1}(W) = w_2(W) + w_2'(W)\frac{\ell_2(W)}{\ell_2'(W)} + c\gamma\ell_2(W)^{\gamma-1},$$

with

$$(1 + \lambda_{10}s)w_1(W) = (r + \lambda_{10})W - \lambda_{10}(U + UI) - \lambda_{11}\int_W^{\overline{W}_1}\overline{F}_1(x)\,dx - \lambda_{12}\int_W^{\overline{W}_2}\overline{F}_2(x)\,dx,$$
$$(1 + \lambda_{10}s)w_1'(W) = r + \lambda_{10} + \lambda_{11}\overline{F}_1(W_1(w)) + \lambda_{12}\overline{F}_2(W_1(w)),$$
$$\ell_i(W) = \frac{M}{N}\frac{1}{n_i}\frac{h_i(W)}{d_i(W)},\ i = 1, 2,$$
$$d_i(W) = \lambda_{i0} + \lambda_{i1}\overline{F}_1(W) + \lambda_{i2}\overline{F}_2(W),$$
$$h_i(W) = \lambda_{0i}u + \lambda_{1i}m_1G_1(W) + \lambda_{2i}m_2G_2(W),$$
$$G_i'(W) = \frac{1}{m_i}\frac{h_i(W)}{d_i(W)}F_i'(W).$$

4. Then calculate productivity distributions

$$\Gamma_i(K_i^{-1}(W)) = n_iF_i(W),\ i = 1, 2.$$

5. Consistency with the predetermined distribution of productivity Γ requires that

$$\Gamma(p) = \begin{cases} \Gamma(\underline{p}_2) + \Gamma_2(p), & \forall p \in [\underline{p}_2, \underline{p}_1], \\ \Gamma(\underline{p}_2) + \Gamma_1(p) + \Gamma_2(p), & \forall p \in [\underline{p}_1, \overline{p}_2], \\ \Gamma(\underline{p}_2) + n_2 + \Gamma_1(p), & \forall p \in [\overline{p}_2, \overline{p}_1], \end{cases}$$

with $\Gamma(\underline{p}_2) + n_1 + n_2 = 1$.

6. If this consistency restriction is not satisfied, reiterate that sequence with another guess of F_1, F_2 and n_1, n_2.

In practice we discretize functions and approximate integrals as described in the estimation section, and we search for discrete approximations of F_1 and F_2, as well as shares n_1, n_1 so as to minimize a distance between Γ and its prediction. The dimensionality of the optimization problem can be reduced by using simple parametric approximations for F_1, F_2 such as the beta distribution used in the estimation section.

B. Endogenous Transition Rates

If the transition rates are endogenous then

$$\lambda_{i1} = \frac{n_1}{(n_1 + \alpha n_2)}s_if(\xi); \quad \lambda_{i2} = \frac{\alpha n_2}{(n_1 + \alpha n_2)}s_if(\xi); \quad \xi = \frac{n_1 + \alpha n_2}{u + s_1m_1 + s_2m_2}.$$

So step 2 cannot be calculated because tightness ξ is a function of m_1, m_2. Let $m_i(\xi)$ and $u(\xi)$ denote the shares of workers in formal/informal sectors and unemployed that can be calculated in step 2 given ξ.

Then ξ is the fixed point

$$\xi = \frac{n_1 + \alpha n_2}{u(\xi) + s_1 m_1(\xi) + s_2 m_2(xi)}.$$

Solving for ξ must be embedded in step 2 if rates are endogenous.

III. Estimation

Let F_1 and F_2 be two candidate offer distributions, defined on the spaces of contract present values. Although we could implement this procedure nonparametrically, we use non standard beta distributions as approximations:

$$F_i(x) = \text{betacdf}\left(\frac{x - \underline{W}_i}{\overline{W}_i - \underline{W}_i}; \alpha_i, \beta_i\right) \;\; i = 1, 2;\; \underline{W}_i \leq x \leq \overline{W}_i,$$

where betacdf$(\cdot; \alpha, \beta)$ is the CDF of a beta distribution with parameters α and β. An important practical reason why a (flexible) parametric specification is useful is that, in order to calculate the function Φ and other transition rates (see below) we need to calculate offer densities $f_1 = F_1'$ and $f_2 = F_2'$. Assuming a parametric specification guarantees the smoothness of both the distribution function and its derivative.

To estimate the parameters we use the method of moments. We match the distribution of wages for each sector and the transition rates implied by the model to those observed in the data. Given the above specification, we need to estimate the six arrival rates and the two job destruction rates all denoted by $\boldsymbol{\lambda} = (\lambda_{ij})_{i,j=0,1,2}$ and six further parameters $\boldsymbol{\theta} = (\underline{W}_1, \overline{W}_1, \underline{W}_2, \overline{W}_2, \alpha_1, \beta_1, \alpha_2, \beta_2)$ characterizing the offer distribution. Our algorithm estimates $\boldsymbol{\theta}$ given the $\boldsymbol{\lambda}$, then $\boldsymbol{\lambda}$ given $\boldsymbol{\theta}$, and iterates until convergence. Although we could estimate all parameters at the same time, this turned out to be a very quick procedure in practice.

A. Contract Offer Distributions

We start by taking the arrival rates $\boldsymbol{\lambda}$ as given to estimate $\boldsymbol{\theta}$ as follows. Let $z_k = \cos(k\pi/K), k = 0, ..., K$, be $K + 1$ Chebychev nodes on $[-1, 1]$. These nodes allow to define grids on $[\underline{W}_1, \overline{W}_1]$ and $[\underline{W}_2, \overline{W}_2]$ as

$$W_{ik} = \frac{\underline{W}_i + \overline{W}_i}{2} + \frac{\underline{W}_i - \overline{W}_i}{2} z_k,\;\; i = 1, 2,\;\; k = 0, ..., K.$$

Recall that

$$(1 + \lambda_{10} s) w_1(W) = (r + \lambda_{10})W - \lambda_{10}\left(U + UI\right) - \lambda_{11} \int_W^{\overline{W}_1} \overline{F}_1(x)\, dx - \lambda_{12} \int_W^{\overline{W}_2} \overline{F}_2(x)\, dx, \qquad (.9)$$

and

$$w_2(W) = (r + \lambda_{20})W - \lambda_{20} U - \lambda_{21} \int_W^{\overline{W}_1} \overline{F}_1(x)\, dx - \lambda_{22} \int_W^{\overline{W}_2} \overline{F}_2(x)\, dx. \qquad (.10)$$

For each point on the grids, one can calculate a corresponding wage w_{ik} using equations (.9) and (.10), and replacing integrals by quadrature approximations. The appropriate quadrature for Chebychev nodes is the Clenshaw-Curtis (CC) quadrature, whose weights ω_k can be easily calculated using Fast Fourier

Transform (FFT) (see Waldvogel, 2006). For example, we have

$$(1+\lambda_{10}s)w_{1k} = (r+\lambda_{10})W_{1k} - \lambda_{10}\left(\underline{W}_2 + UI\right) - \lambda_{11}\frac{\underline{W}_1 - \overline{W}_1}{2}\sum_{\ell=0}^{K}\omega_\ell \mathbf{1}_{(W_{1\ell}>W_{1\ell})}\overline{F}_1(W_{1\ell}) - \lambda_{12}\frac{\underline{W}_2 - \overline{W}_2}{2}\sum_{\ell=0}^{K}\omega_\ell \mathbf{1}_{(W_{2\ell}>W_{1\ell})}\overline{F}_2(W_{2\ell}),$$

where $\mathbf{1}_{(\cdot)}$ is the indicator function. A similar expression can be obtained to determine wage nodes for the informal sector, w_{2k}.

Then we search for $\boldsymbol{\theta}$ minimizing

$$Q_1(\boldsymbol{\theta}|\boldsymbol{\lambda}) = \sum_{i=1,2}\sum_{k=0}^{K}\left(\widehat{G}_i^*(w_{ik}) - G_i(W_{ik})\right)^2,$$

where $G_i(W_{ik})$ is calculated using equation (.11),

$$m_i G_i(W) = \frac{\lambda_{0i}F_i(W) - \Phi(W)}{d_i(W)}u, \quad i = 1,2, \tag{.11}$$

and replacing integrals by CC-quadrature approximations, and $\widehat{G}_i^*$ is an estimate of wage distribution functions.

Note that, assuming that $U = \underline{W}_2 \leq \underline{W}_1$ and $\overline{W}_2 \leq \overline{W}_1$, we have

$$(1+\lambda_{10}s)\underline{w}_1 = (r+\lambda_{10})\underline{W}_1 - \lambda_{10}\left(\underline{W}_2 + UI\right) - \lambda_{11}\left(\mu_1 - \underline{W}_1\right) - \lambda_{12}\int_{\underline{W}_1}^{\overline{W}_2}\overline{F}_2(x)\,dx, \tag{.12}$$

$$(1+\lambda_{10}s)\overline{w}_1 = (r+\lambda_{10})\overline{W}_1 - \lambda_{10}\left(\underline{W}_2 + UI\right), \tag{.13}$$

$$\underline{w}_2 = r\underline{W}_2 - \lambda_{21}(\mu_1 - \underline{W}_2) - \lambda_{22}(\mu_2 - \underline{W}_2), \tag{.14}$$

$$\overline{w}_2 = (r+\lambda_{20})\overline{W}_2 - \lambda_{20}\underline{W}_2 - \lambda_{21}\int_{\overline{W}_2}^{\overline{W}_1}\overline{F}_1(x)\,dx, \tag{.15}$$

where $[\underline{w}_1, \overline{w}_1]$ and $[\underline{w}_2, \overline{w}_2]$ are the observed wage supports in the formal and informal sectors, respectively, and with

$$\mu_i = \underline{W}_i + (\overline{W}_i - \underline{W}_i)\frac{\alpha_i}{\alpha_i + \beta_i}, \; i = 1,2.$$

Hence, we can simplify the estimation problem slightly by using equations (.13) and (.14) to substitute observed wage bounds $\underline{w}_2$ and $\overline{w}_1$ for $\underline{W}_2 = U$ and $\overline{W}_1$ (given the α, β and $\underline{W}_1, \overline{W}_2$).

B. Transition Rates

In a similar way as we estimate $\boldsymbol{\theta}$ given $\boldsymbol{\lambda}$, we can estimate $\boldsymbol{\lambda}$ given $\boldsymbol{\theta}$. Natural counterparts to the theoretical transition rates can be calculated from observed flows between states (0: unemployment; 1: working in the formal sector; and 2: working in the informal sector).

From the labor force survey, we calculate the intensity of transitions from unemployment to job ($\widehat{D}_{0j}$; $j = 1,2$), from a formal sector job to unemployment, to another job in the same sector or to the informal sector ($\widehat{D}_{1j}$; $j = 0,1,2$) and similar ones for a workers initially in the informal sector ($\widehat{D}_{2j}; j = 0,1,2$).

We then estimate our transition parameters using the method of moments. We choose the parameters so as to match the observed transition rates between sectors.

Consider first the workers who are unemployed at the date of the first interview, that we follow over T periods. Workers are not heterogeneous in this model and hence the remaining unemployment duration is exponentially distributed. Thus the implied proportion of those who move out of unemployment and into a job in sector j over the time period of observation T is

$$D_{0j} = \frac{\lambda_{0j}}{\lambda_{01} + \lambda_{02}}(1 - e^{-(\lambda_{01}+\lambda_{02})T}),\ j = 1, 2 \tag{.16}$$

Now consider workers in the formal sector. Over T periods the proportion making a transition to an alternative job in the same sector, to a job in the informal sector or to unemployment is, respectively, for $i = 1, 2$,

$$\begin{aligned} D_{ij} &= \int_{\underline{W}_i}^{\overline{W}_i} \frac{\lambda_{ij}\overline{F}_i(x)}{d_i(x)}(1 - e^{-d_i(x)T})dG_i(x),\ j = 1, 2, \\ D_{i0} &= \int_{\underline{W}_i}^{\overline{W}_i} \frac{\lambda_{i0}}{d_i(x)}(1 - e^{-d_i(x)T})dG_i(x). \end{aligned} \tag{.17}$$

where $d_i(W) = \lambda_{i0} + \lambda_{i1}\overline{F}_1(W) + \lambda_{i2}\overline{F}_2(W)$. Now, in equilibrium,

$$\frac{dG_i(x)}{dF_i(x)} = \frac{1}{m_i}\frac{h_i(x)}{d_i(x)}. \tag{.18}$$

with $h_i(W) = \lambda_{0i}u + \lambda_{1i}m_1G_1(W) + \lambda_{2i}m_2G_2(W)$. This allows to replace the derivative of G_i by that of F_i inside the integral. This is useful as we have parameterized F_1, F_2 using a continuous distribution. Then CC-quadrature can be used to approximate the integral.

These are the model counterparts for these empirical moments as functions of the arrival rates, the job destruction rates, the offers distributions F_i and as a function of the equilibrium contract values distributions G_i ($i = 1, 2$). Contract offers and equilibrium distributions are related by a complex function as explained in Appendix .

We can thus estimate $\boldsymbol{\lambda}$ given $\boldsymbol{\theta}$ by minimizing the criterion

$$Q_2(\boldsymbol{\lambda}|\boldsymbol{\theta}) = \sum_{i,j=0,1,2} \left(\widehat{D}_{ij} - D_{ij}\right)^2,$$

where $\widehat{D}_{ij}$ is the empirical counterpart of D_{ij}.

Alternating between the minimization of Q_1 and Q_2, given respectively the latest update on the parameters is just a standard and simple numerical optimization algorithm for minimizing both functions jointly, taking into account that they both depend on common parameters. Since the model is just identified (for nonparametric specification of F_1 and F_2) weighing the moments is irrelevant for efficiency. Although we could implement this procedure nonparametrically (using sieve estimators), we use instead the non standard beta distribution as an approximation.

C. Productivity Distribution

Up to this point, there has been no need to use the firm profit functions, or indeed the distribution of productivities. To complete estimation we need to estimate the cost function of informality. This will allow us to characterize the choice of firms to locate in either sector and ultimately to carry out counterfactual simulations.

We specify the cost function as $C = c\ell_2(W)^\gamma$, with c and γ being the parameters to be estimated. Given values for c and γ, and $n_2 \leq 1$, we solve for the labor force size in the formal sector ($\ell_1(W) = \frac{M}{N}\frac{1}{n_1}\frac{h_1(W)}{d_1(W)}$) and in the informal sector ($\ell_2(W) = \frac{M}{N}\frac{1}{n_2}\frac{h_2(W)}{d_2(W)}$). From the firm's maximization problem in each sector, we next derive the support of the distribution of formal and informal productivities, i.e. $p_1 = K_1^{-1}(W)$ and $p_2 = K_2^{-1}(W)$ respectively.

Recall the firms optimization problem:

$$\pi_1(p) = \max_{W \geq \max\{U, W_1(w_{\min})\}} (1-t)\left[p - (1+\tau+\lambda_{10}s)w_1(W)\right]\ell_1(W), \tag{.19}$$

$$\pi_2(p) = \max_{W \geq U} \left[p - w_2(W)\right]\ell_2(W) - C(\ell_2(W)), \tag{.20}$$

The first order conditions gives

$$p_1 = K_1^{-1}(W) = (1+\tau+\lambda_{10}s)\left[w_1(W) + w_1'(W)\frac{\ell_1(W)}{\ell_1'(W)}\right], \tag{.21}$$

$$p_2 = K_2^{-1}(W) = w_2(W) + w_2'(W)\frac{\ell_2(W)}{\ell_2'(W)} + c\gamma\ell_2(W)^{\gamma-1}, \tag{.22}$$

where the expressions for $w_1'(W)$, $i = 1, 2$, are given by

$$\begin{aligned} w_1'(W) &= \frac{r + \lambda_{10} + \lambda_{11}\overline{F}_1(W_1(w)) + \lambda_{12}\overline{F}_2(W_1(w))}{1+\lambda_{10}s}, \\ w_2'(W) &= r + \lambda_{20} + \lambda_{21}\overline{F}_1(W_2(w)) + \lambda_{22}\overline{F}_2(W_2(w)), \end{aligned}$$

and where firm sizes can be differentiated using

$$\frac{\ell_i}{\ell_i'} = \frac{h_i/d_i}{(h_i/d_i)'} = \frac{h_i d_i}{h_i' d_i - h_i d_i'}, \quad h_i' = \lambda_{0i}u + \lambda_{1i}\frac{h_1}{d_1}F_1' + \lambda_{2i}\frac{h_2}{d_2}F_2', \quad d_i' = \lambda_{i1}\overline{F}_1' + \lambda_{i2}\overline{F}_2'.$$

For each point of the contract grids, W_{ik}, one can thus calculate a point p_{ik} on a productivity grid, with $\underline{p}_2 = p_{20}$, $\underline{p}_1 = p_{10}$, $\overline{p}_2 = p_{2N}$ and $\overline{p}_1 = p_{1N}$:

$$p_{1k} = (1+\tau+\lambda_{10}s)\left[w_1(W_{1k}) + w_1'(W_{1k})\left.\frac{h_1^2}{h_1'd_1 - h_1 d_1'}\right|_{W=W_{1k}}\right], \; k = 0, ..., K,$$

and

$$p_{2\ell} = w_2(W_{2\ell}) + w_2'(W_{2\ell})\left.\frac{h_2 d_2}{h_2' d_2 - h_2 d_2'}\right|_{W=W_{2\ell}} + \widetilde{c} \times \gamma\left(\frac{1}{\widetilde{n}_2}\frac{h_2(W_{2\ell})}{d_2(W_{2\ell})}\right)^{\gamma-1}, \; \ell = 0, ..., K,$$

for $\widetilde{n}_2 = \frac{n_2}{n_1+n_2}$ and $\widetilde{c} = c\left(\frac{M}{(n_1+n_2)N}\right)^{\gamma-1} = c\left(\frac{M}{N_1+N_2}\right)^{\gamma-1}$ (where N_1, N_2 are the total numbers of formal and informal firms operating in the economy at the equilibrium).

Moreover, we have

$$\begin{aligned} \pi_1(p_{1k}) &= (1-t)(1+\tau+\lambda_{10}s)w_1'(W_{1k})\frac{\ell_1(W_{1k})^2}{\ell_1'(W_{1k})} \\ &= \frac{M}{N}\frac{1}{n_1+n_2}\left[\frac{1}{\widetilde{n}_1} \times (1-t)(1+\tau+\lambda_{10}s)w_1'(W_{1k})\left.\frac{h_1^2}{h_1'd_1 - h_1 d_1'}\right|_{W=W_{1k}}\right], \end{aligned}$$

for $\widetilde{n}_1 = \frac{n_1}{n_1+n_2}$, and

$$\begin{aligned}
\pi_2(p_{2\ell}) &= w_2'(W_{2\ell})\frac{\ell_2(W_{2\ell})^2}{\ell_2'(W_{2\ell})} + c(\gamma-1)\ell_2(W_{2\ell})^\gamma \\
&= \frac{M}{N_1+N_2}\left[\frac{1}{\widetilde{n}_2} \times w_2'(W_{2\ell}) \left.\frac{h_2^2}{h_2' d_2 - h_2 d_2'}\right|_{W=W_{2\ell}} + \widetilde{c} \times (\gamma-1)\left(\frac{1}{\widetilde{n}_2}\frac{h_2(W_{2\ell})}{d_2(W_{2\ell})}\right)^\gamma\right].
\end{aligned}$$

Note that both profit functions are proportional to $\frac{M}{N_1+N_2}$, which is not necessarily known at this stage if the number of informal firms in the economy is not directly observed.

Equilibrium conditions require that $\pi_2(\underline{p}_2) = 0$, and $\pi_1(p) = \pi_2(p) > 0$ for $p \in [\underline{p}_1, \overline{p}_2]$. We thus estimate $\widetilde{c}$ and γ, and $\widetilde{n}_1 \geq 0$ and $\widetilde{n}_2 \geq 0$ such that $\widetilde{n}_1 + \widetilde{n}_2 = 1$, so as to minimize

$$\pi_2(p_{20})^2 + \sum_{k,\ell=0}^{K} \mathcal{K}(p_{1k} - p_{2\ell})[\pi_1(p_{1k}) - \pi_2(p_{2\ell})]^2,$$

where $\mathcal{K}$ is a kernel density. Notice that a direct estimate of $\widetilde{n}_1, \widetilde{n}_2$ immediately follows if estimates of the numbers of formal and informal firms (N_1 and N_2) are available:

$$\widetilde{n}_i = \frac{N_i}{N_1+N_2}.$$

Then define

$$\widetilde{\Gamma}_i(p_{ik}) = \widetilde{n}_i F_i(W_{ik}),\ k = 0, ..., K,$$

and interpolate for all $p \in [p_{i0}, p_{iK}]$, and set $\widetilde{\Gamma}(p) = 0$ for $p < p_{i0}$ and $\widetilde{\Gamma}(p) = \widetilde{n}_i$ for $p > p_{iK}$. The unconditional productivity distribution Γ satisfies

$$\Gamma(p) = \begin{cases} \Gamma(p_{20}) + (n_1+n_2)\widetilde{\Gamma}_2(p), & \forall p \in [\underline{p}_2, \underline{p}_1], \\ \Gamma(p_{20}) + (n_1+n_2)\left[\widetilde{\Gamma}_1(p) + \widetilde{\Gamma}_2(p)\right], & \forall p \in [\underline{p}_1, \overline{p}_2], \\ \Gamma(p_{20}) + (n_1+n_2)\left[\widetilde{n}_2 + \widetilde{\Gamma}_1(p)\right], & \forall p \in [\overline{p}_2, \overline{p}_1]. \end{cases}$$

By postulating a parametric form for Γ, say a log-normal with parameters μ and σ^2, one can then estimate $n_1 + n_2, \mu, \sigma^2$ by maximizing the fit on a grid for $[p_{20}, p_{1K}]$.

Finally, $\frac{M}{N_1+N_2}$ can be estimated by fitting the average firm size in the informal sector. The cost parameter c is then $c = \widetilde{c}\left(\frac{M}{N_1+N_2}\right)^{1-\gamma}$.

IV. Comparing transitions estimated from the PME data to those implied from the RAIS data

Our data set, the PME (2008), is the only source including workers in both the formal and the informal sector. An interesting question is how the transitions we compute over the short window of observation offered by the PME compare to those we observe if we had a longer window of observation. Table C.1 below shows annualised transition rates estimated based on the quarterly transitions rates we obtain from the PME and compares them to annual transitions computed from RAIS, an administrative matched employer employee data set. Since the latter includes only formal workers and firms, the comparisons that can be made are limited. Moreover, as we explain below sampling issues make the comparison imperfect.

Most obviously we can compare the annualised transition rates between formal jobs (D_{11}). These are

Table .1
Comparing some transitions between PME and RAIS data

		Sao Paulo		Salvador		All	
		Males	Females	Males	Females	Males	Females
D_{11}	*PME*	6.2	4.7	10.2	8.2	7.4	5.6
		(0.4)	(0.6)	(0.8)	(1.3)	(0.4)	(0.5)
	RAIS	6.3	7.6	7.6	4.1	6.4	7.4
$D_{10} + D_{12} + D_{10'}$	*PME*	24.0	36.7	20.6	22.4	23.0	33.1
		(0.9)	(1.8)	(1.2)	(2.3)	(0.7)	(1.4)
	RAIS	33.7	31.6	35.8	29.8	33.9	31.4

Note: Transitions are percentages per year. The transitions from RAIS data (Census of registered firms) are averages obtained from month j in year t to month j in year t+1, over the period 2002-2007. D_{11}: annualised formal to formal transition rates. $D_{10} + D_{12} + D_{10'}$ for the PME is exit from the formal sector to unemployment, informal jobs and inactivity. For RAIS it is the rate of exit from the data base, which includes only formal employment. Standard errors for the PME annualied transition rates in parentheses.

of the same order of magnitude and there is no systematic difference between the data sets: PME implies sometimes larger and sometimes smaller transitions, reflecting some noise. When we average across regions the numbers are much closer. Once sampling variation is accounted for there is not much difference left between the two.

We can also compare the annualized exit from the formal sector (to unemployment, informal and inactivity) estimated from the PME to the exit rate from the RAIS (2014) data base, which should correspond, assuming all formal workers are correctly reported by firms once they stay within the formal sector. Note that the inactive are not part of the model and have been removed from the data we use for estimation. However, to make the transitions more comparable between the two data sets we add them back in for the purposes of this exercise. We compare this to the rate at which people disappear from the RAIS data. On average for females the numbers are almost the same, and once we allow for sampling error in the PME they are indistinguishable. For males PME implies a lower overall exit rate from the formal sector, consistent with the effects of stock sampling.

Thus, overall the two data sets give similar albeit not identical transition rates. The main difference is for males. Nevertheless one needs to remember that the sampling structure of the two data sets is different. Formally the two sets of transitions would be the same if there was no unobserved heterogeneity across jobs, which is both unlikely and inconsistent with our model. In particular, the PME stock sampling (which is fully accounted for by the estimation of the model) will skew the sample towards individuals employed in high productivity firms from which exit rates will be lower. On the other hand the RAIS includes the universe of formal firms.

References

[1] PME (2008), "Pesquisa Mensal de Emprego" Instituto Brasileiro de Geografia e Estatistica (IBGE) 2008 Rio de Janeiro, Data for years 2002-2007.

[2] RAIS (2014)- Relacao Anual de Informacoes Sociais Ministerio do Trabalho e Emprego Brazil, 2014 Brasilia, 2002/2007.

[3] Waldvogel, J. (2006): "Fast Construction of the Fejér and Clenshaw-Curtis Quadrature Rules," BIT *Numerical Mathematic*, 46, 195–202.