Online Appendix to
"The Cost of Financial Frictions for Life Insurers"

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October 22, 2014

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Appendix A. Calibrating the Insurance Pricing Model

In Section III, we modeled the supply side of insurance markets through an insurance company's maximization problem, given an exogenous demand function. We now endogenize demand through the consumers' maximization problem in the presence of search frictions. We then calibrate and solve for the insurance company's policy and value functions to better understand how optimal insurance pricing is related to firm value and the shadow cost of capital.

1. A Fully Specified Model of Insurance Markets

Our goal is to calibrate and solve the simplest version of the model in Section III that captures the essence of our empirical findings. Therefore, we start with a version in which insurance companies sell only one type of policy. We assume that both reserve and actuarial values are constant and denote them as $\widehat{V}$ and V, respectively.

Consumers

In each period, there is a continuum (normalized to measure one) of ex ante identical consumers with initial wealth W. Each cohort of consumers is present in the insurance market for only one period. Consumers have quasi-linear utility over the quantity of insurance (i.e., face amount) purchased Q_t and the remaining wealth:

(A1) $$U_t(Q_t, W) = \frac{X_t^{1/\epsilon} Q_t^{1-1/\epsilon}}{1 - 1/\epsilon} + W - P_t Q_t,$$

where $\epsilon > 1$ is the elasticity of demand. The demand shock follows a geometric random walk:

(A2) $$\Delta X_t = \frac{X_t}{X_{t-1}} = \exp\left\{x_t - \frac{\sigma^2}{2}\right\},$$

where $x_t \sim \mathbb{N}(0, \sigma^2)$.

The first-order condition with respect to Q_t implies the following demand function:

(A3) $$Q_t = X_t P_t^{-\epsilon}.$$

Substituting out Q_t in the direct utility function, indirect utility is

(A4) $$U_t(P_t, W) = \frac{X_t P_t^{1-\epsilon}}{\epsilon - 1} + W.$$

Consumers know the distribution of prices $G_t(P)$, including its support $[\underline{P}, \overline{P}]$. However, they do not know which insurance company is selling at which price. Consumers pay a search cost s to be randomly matched with an insurance company and sequentially search with recall until the benefit of additional search is less than its cost. For simplicity, we assume that consumers are equally likely to match with all companies, which turns off product market frictions. We also assume that the search cost is sufficiently high so that the consumer optimally stops searching after the first match:

$$W \leq \int_{\underline{P}}^{\overline{P}} U_t(P, W) dG_t(P) - s \leq U_t\left(\overline{P}, W\right). \tag{A5}$$

Insurance Companies

We start with the insurance company's maximization problem in Section III and add a few additional parametric assumptions. We assume that the return on assets and statutory reserves are constant and equal to the riskless interest rate (i.e., $R_t = R$). The stochastic discount factor is constant and equal to the inverse of the riskless interest rate (i.e., $M_t = 1/R$). We parameterize the fixed cost as

$$C_t = c X_{t-1}^{\omega} X_t^{1-\omega} V^{1-\epsilon}. \tag{A6}$$

The parameter $c \in [0, 1)$ determines the size of the fixed cost, and $\omega \geq 1$ determines its sensitivity to demand shocks. The presence of fixed costs creates operating leverage, which causes the leverage constraint to bind for a sufficiently adverse demand shock.

There is a continuum (normalized to measure one) of insurance companies that have different levels of initial statutory capital K_{t-1}, which cause these companies to be differentially constrained. Each insurance company optimally prices its policy according to equation (21). The differences in the shadow cost of capital across insurance companies, combined with search frictions in the demand side, induce equilibrium price dispersion (see Reinganum (1979) for a formal proof). Prices vary from the Bertrand price $\overline{P} = V(1 - 1/\epsilon)^{-1}$ for insurance companies that are unconstrained to $\underline{P} = \phi^{-1}\widehat{V}(1 - 1/\epsilon)^{-1}$ for those companies that are maximally constrained.

2. Solution by Dynamic Programming

Because demand follows a geometric random walk, we must scale both the value function and statutory capital by market size to make the model stationary. We rewrite the value function as

$$j_t = \frac{J_t + C_t}{X_t V^{1-\epsilon}} = \left(\frac{P_t}{V} - 1\right)\left(\frac{P_t}{V}\right)^{-\epsilon} + \mathbb{E}_t\left[\frac{\Delta X_{t+1}}{R}\left(j_{t+1} - \frac{c}{\Delta X_{t+1}^{\omega}}\right)\right]. \tag{A7}$$

We rewrite the law of motion for statutory capital as

$$k_{t+1} = \frac{RK_t - C_{t+1}}{X_{t+1}V^{1-\epsilon}} = \frac{R}{\Delta X_{t+1}}\left[k_t + \left(\frac{P_t}{V} - \phi^{-1}\frac{\widehat{V}}{V}\right)\left(\frac{P_t}{V}\right)^{-\epsilon}\right] - \frac{c}{\Delta X_{t+1}^{\omega}}. \tag{A8}$$

We rewrite the leverage constraint as

$$k_t + \left(\frac{P_t}{V} - \phi^{-1}\frac{\widehat{V}}{V}\right)\left(\frac{P_t}{V}\right)^{-\epsilon} \geq 0. \tag{A9}$$

The insurance company chooses P_t to maximize firm value (A7), subject to the law of motion for statutory capital (A8) and the leverage constraint (A9).

We discretize the state space into 50 grid points, which we denote as $\{k_s\}_{s=1}^{50}$. We also discretize the demand shock into 7 grid points by Gauss-Hermite quadrature, which we denote as $\{\Delta X_n\}_{n=1}^{7}$. Equation (21) implies that $P_t \geq \underline{P}$ when $\widehat{V}/V < \phi$. Hence, the leverage constraint (A9) can be satisfied as long as initial statutory capital, prior to the sale of new policies, satisfies

$$k_t \geq k_1 = -\frac{1}{\epsilon}\left(\phi^{-1}\frac{\widehat{V}}{V}\right)^{1-\epsilon}\left(1-\frac{1}{\epsilon}\right)^{\epsilon-1}. \tag{A10}$$

Equations (A8) and (A9) imply that $k_{t+1} \geq k_1$ if $-c/\Delta X_1^{\omega} \geq k_1$. Hence, we calibrate the sensitivity of the fixed cost to demand shocks to satisfy

$$\omega = \frac{\log(-c/k_1)}{\log(\Delta X_1)}. \tag{A11}$$

This ensures that the lower bound of the state space (i.e., k_1) can be realized with the worst possible demand shock (i.e., ΔX_1).

Starting with an initial guess for the policy function, $P_1(k_s) = \overline{P}$, we solve the model by value iteration.

1. Iterate on equation (A7) to compute the value function $j_i(k_s)$ corresponding to the current policy function $P_i(k_s)$.

2. For each point k_s on the grid, find $P_{i+1}(k_s)$ that maximizes equation (A7) with $j_{t+1} = j_i(k_s)$.

3. If $\max_{k_s} |P_{i+1}(k_s) - P_i(k_s)|$ is less than the convergence criteria, stop. Otherwise, return to step 1.

3. Calibration

We calibrate the parameters of the model, summarized in Table A1, to explain the pricing of life annuities for males aged 60 in December 2008. A riskless interest rate of 0.5 percent is based on the 1-year Treasury yield in December 2008. The ratio of reserve to actuarial value for life annuities for males aged 60 was 0.78 in December 2008, as reported in Figure 6. An elasticity of demand of 15 generates a realistic markup of 7 percent when the leverage constraint does not bind. A standard deviation of 28 percent for demand shocks is based on the standard deviation of the growth rate for annual premiums on individual annuities. We calibrate the size of the fixed cost to match the typical general expense ratio (excluding commissions) of 1 percent for individual annuities. As discussed above, we calibrate the sensitivity of the fixed cost to demand shocks so that the leverage constraint binds for a sufficiently adverse demand shock. The maximum leverage ratio is 97 percent to match the highest leverage ratio for the cross section of insurance companies in Table 4.

Table A1: Parameters in the Calibrated Model

Parameter	Symbol	Value
Riskless interest rate	$R - 1$	0.5%
Ratio of reserve to actuarial value	$\widehat{V}/V$	0.78
Elasticity of demand	ϵ	15
Standard deviation of demand shocks	σ	28%
Size of the fixed cost	c	1%
Sensitivity of the fixed cost to demand shocks	ω	3.66
Maximum leverage ratio	ϕ	97%

4. Policy and Value Functions

Figure A1 reports the optimal insurance price, firm value, and the shadow cost of capital as functions of initial statutory capital. The figure is shown for a -3.71 standard deviation demand shock, which is sufficiently adverse for the leverage constraint to bind even with initial statutory capital being positive. The leverage constraint does not bind when initial statutory capital is greater than 34 percent of firm value. In this region of the state space, the insurance company sells its policies at a markup of 7 percent, and its firm value is \$100. In the region of the state space where the leverage constraint binds, both the optimal insurance price and firm value are decreasing in initial statutory capital.

When initial statutory capital is 3 percent of firm value, the insurance company sells its policies at a markup of -10 percent, and its firm value is \$68. Put differently, the price falls by 17 percent, and the firm value falls by 32 percent relative to when the leverage constraint does not bind. The shadow cost is \$4.75 per dollar of statutory capital. When we decompose this cost through equation (20), the impact of future financial constraints (i.e., $\mathbb{E}_t[M_{t+1}\partial J_{t+1}/\partial K_t]$) accounts for only \$0.01 of the shadow cost. These magnitudes in the calibrated model are consistent with our estimates of the price reductions and the shadow cost in December 2008.

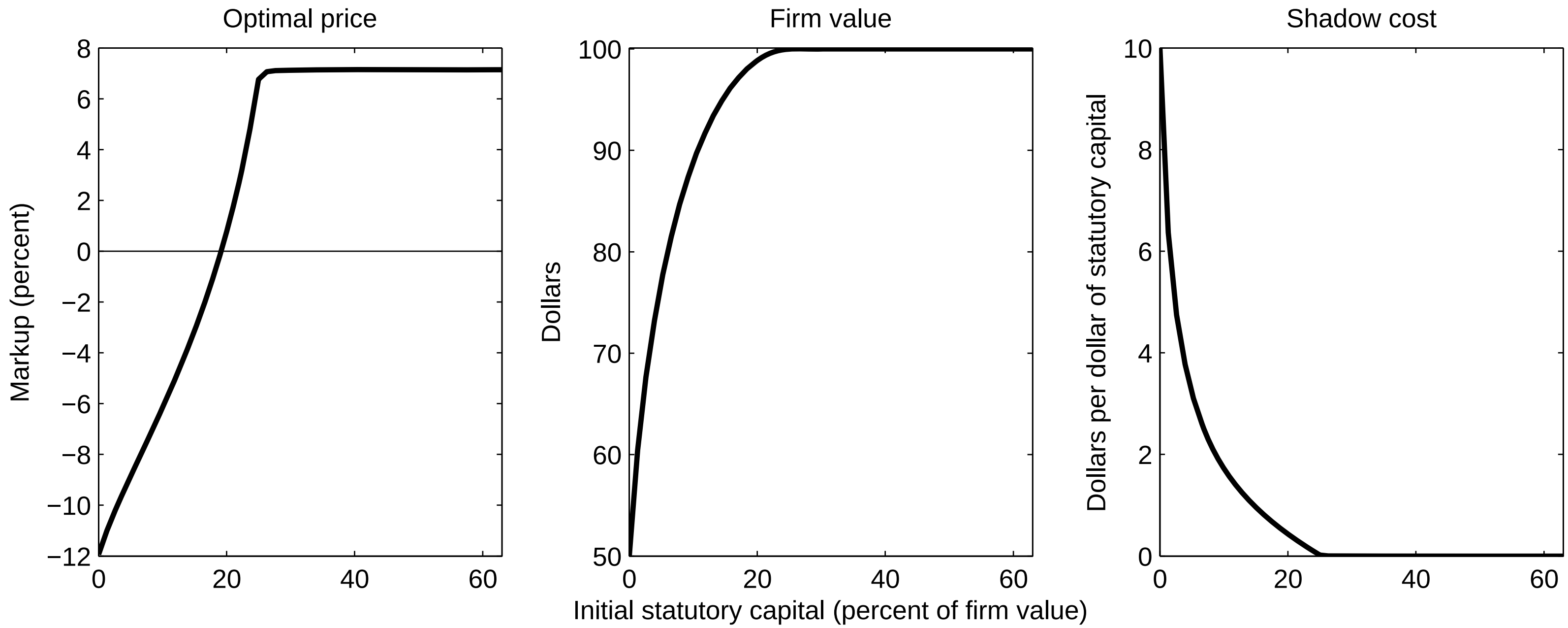

Figure A1: An Insurance Company's Policy and Value Functions

This figure reports the optimal insurance price (i.e., $P_t/V - 1$), firm value (i.e., J_t), and the shadow cost of capital (i.e., $\overline{\lambda}_t$) as functions of initial statutory capital (i.e., K_{t-1}) when the realized demand shock is -3.71 standard deviations. Initial statutory capital is normalized by the firm value corresponding to the highest value of initial statutory capital. Firm value is normalized to \$100 at the highest value of initial statutory capital. Table A1 reports the parameters of the calibrated model.

Appendix B. A Model with Continuous Cost of Financial Frictions

Suppose that the insurance company faces a continuous cost on shortfalls in statutory capital. The cost function $F_t(K_t)$ in period t has the properties that $F_t \geq 0$ and $F_t' \leq 0$. The insurance company's profit in each period is

$$\Pi_t = \sum_{i=1}^{I} (P_{i,t} - V_{i,t}) Q_{i,t} - C_t - F_t. \tag{B1}$$

The insurance company chooses the price $P_{i,t}$ for each type of policy to maximize firm value (17).

The first-order condition for the price of policy i in period t is

$$\frac{\partial J_t}{\partial P_{i,t}} = Q_{i,t} + (P_{i,t} - V_{i,t}) \frac{\partial Q_{i,t}}{\partial P_{i,t}} + f_t \left[Q_{i,t} + \left(P_{i,t} - \phi^{-1} \widehat{V}_{i,t} \right) \frac{\partial Q_{i,t}}{\partial P_{i,t}} \right] = 0, \tag{B2}$$

where

$$f_t = -F_t' + \mathbb{E}_t \left[M_{t+1} \frac{\partial J_{t+1}}{\partial K_t} \right]. \tag{B3}$$

Rearranging equation (B2), the price of policy i in period t is

$$P_{i,t} = V_{i,t} \left(1 - \frac{1}{\epsilon_{i,t}} \right)^{-1} \left(\frac{1 + f_t \phi^{-1} \widehat{V}_{i,t} / V_{i,t}}{1 + f_t} \right). \tag{B4}$$

Note that this equation is equivalent to equation (21) with $-F_t' = \lambda_t$. Therefore, the shadow cost of capital retains the same economic interpretation under this alternative model.

Appendix C. Special Status of Treasury Bonds

Because Treasury bonds are the only assets that perfectly replicate annuities, the law of one price implies that its expected liabilities must be discounted by the Treasury yield curve. However, the low yield on Treasury bonds could reflect their special status as collateral in financial transactions, in addition to being nominally riskless. Although many types of bonds can be used as collateral in financial transactions, Treasury bonds generally have lower haircuts. It is impossible to isolate the impact of special status because other types of bonds that have higher haircuts also have higher credit risk. However, insofar as the yield spread between US agency and Treasury bonds reflects both credit risk and special status, the US agency yield curve serves as a conservative upper bound on the impact of special status.

We first isolate straight US agency bonds (that are not convertible, exchangeable, putable, or redeemable) in the Mergent Fixed Income Securities Database. We limit our sample to bonds rated by Moody's with time to maturity greater than 90 days to avoid abnormal pricing effects for bonds near maturity. We then follow the methodology in Svensson (1994) to estimate the zero-coupon yield curve for each month. Finally, we calculate the actuarial value of policies under the US agency yield curve through equation (2).

Figure C1 reports the time series of the average markup on life annuities for males at various ages. The implied markup on life annuities shifts up relative to Figure 2 because of the relatively high US agency yield curve. In December 2008, the average markup on life annuities was −7.7 percent at age 60, −5.7 percent at age 65, −3.0 percent at age 70, and 0.3 percent at age 75. That is, life insurers sold long-term policies at deep discounts in December 2008, even relative to this conservative upper bound that accounts for the special status of Treasury bonds. More importantly, the variation in implied markups still aligns with the variation in reserve to actuarial value across policies, which is central to our identification strategy as discussed in Section IV.

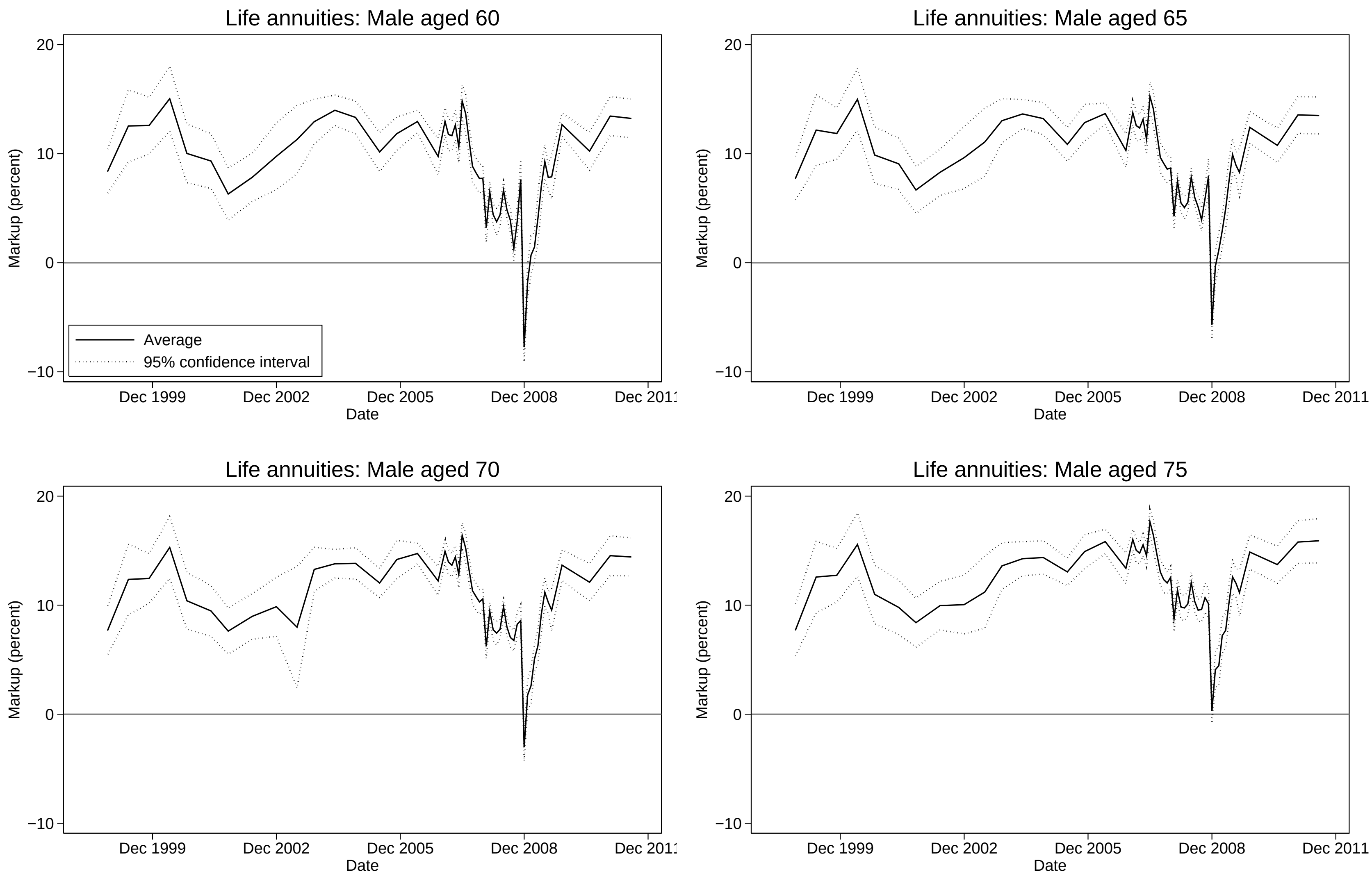

Figure C1: Average Markup on Life Annuities under the US Agency Yield Curve

The markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the appropriate basic mortality table from the American Society of Actuaries and the zero-coupon US agency yield curve. The sample covers life insurers with an A.M. Best rating of A− or higher from May 1998 to July 2011.

Appendix D. Default Probabilities Implied by Credit Default Swaps

We estimate the term structure of risk-neutral default probabilities implied by credit default swaps for six holding companies, whose insurance subsidiaries are in our sample: the Allianz Group, Allstate, MetLife, Genworth Financial, Aviva, and the American International Group. The daily data on credit default swaps and recovery rates are from Markit, and the daily data on the term structure of BBA Libor rates are from OptionMetrics. We focus on 5- and 10-year credit default swaps because they are the most liquid contracts among those that match the maturity of term annuities in our sample.

For a credit default swap originated at time t maturing in M years, the buyer pays a quarterly premium of $p_t(M)/4$ until maturity or default, whichever occurs first. In the event of default, the buyer receives a protection payment of $1 - \Theta$, where Θ is the recovery rate on the underlying bond. Let $R_t(m)$ be the Libor rate at maturity m and time t. Let $S_t(m)$ be the probability at time t that default does not occur prior to time $t + m$, which we parameterize as a step function:

$$\text{(D1)} \qquad S_t(m) = \exp\left\{-\mathbb{1}_{\{m \leq 5\}} D_t(5)m - \mathbb{1}_{\{m>5\}}[D_t(5)5 + D_t(10)(m-5)]\right\},$$

where $D_t(5)$ is the annual default probability at maturity less than five years, and $D_t(10)$ is that for maturity greater than five years.

At origination, the present value of the premiums equals the present value of the protection payment (O'Kane and Turnbull 2003):

$$\text{(D2)} \quad \frac{p_t(M)}{4} \sum_{m=1/4}^{M} \left(\frac{S_t(m)}{R_t(m)^m} + \frac{S_t(m-1) - S_t(m)}{2R_t(m)^m} \right) = (1-\Theta) \sum_{m=1/4}^{M} \frac{S_t(m-1) - S_t(m)}{R_t(m)^m}.$$

On the left-hand side of this equation, the second term in parentheses is an adjustment for the accrued premium if default occurs between two payment dates. The right-hand side is based on an approximation that default can occur at the end of each quarter, which can be refined through a finer grid. Based on equation (D2), we estimate the 5- and 10-year default probabilities for each company at the daily frequency by least squares. We then average the estimated default probabilities over each month.

Appendix E. Life Annuities during the Great Depression

Following Warshawsky (1988), our sample of life annuity prices is from the *Handy Guide* (Spectator Company 1929–1938). We focus on quotes for males aged 60 to 75 (every five years in between). We match the quoted price for each year of the *Handy Guide* to the actuarial value in January of that year. We calculate the actuarial value based on the annuitant mortality table from McClintock (1899) and the zero-coupon Treasury yield curve. We derive the zero-coupon yield curve based on the constant-maturity yield curve from Cecchetti (1988).

Figure E1 reports the time series of the average markup on life annuities for males at various ages, averaged across insurance companies and reported with a 95 percent confidence interval. The key finding is that the markup remained positive throughout the Great Depression. In particular, the average markup for life annuities at age 60 was 22.2 percent in 1932, when the corporate default spread was even higher than the heights reached during the financial crisis.

Prior to the adoption of Standard Valuation Law in the mid-1940s, individual states had their own standards for reserve valuation. However, many states used the annuitant mortality table from McClintock (1899) and a constant discount rate for reserve valuation (e.g., 3.5 percent in California). Figure E2 reports the ratio of reserve to actuarial value for life annuities for males aged 50 to 80 (every ten years in between) at the discount rate of 3.5 percent. The ratio of reserve to actuarial value remained close to or above one throughout the Great Depression. This implies that insurance companies could not raise statutory capital by selling life annuities at a price below actuarial value, which is consistent with the absence of discounts in Figure E1.

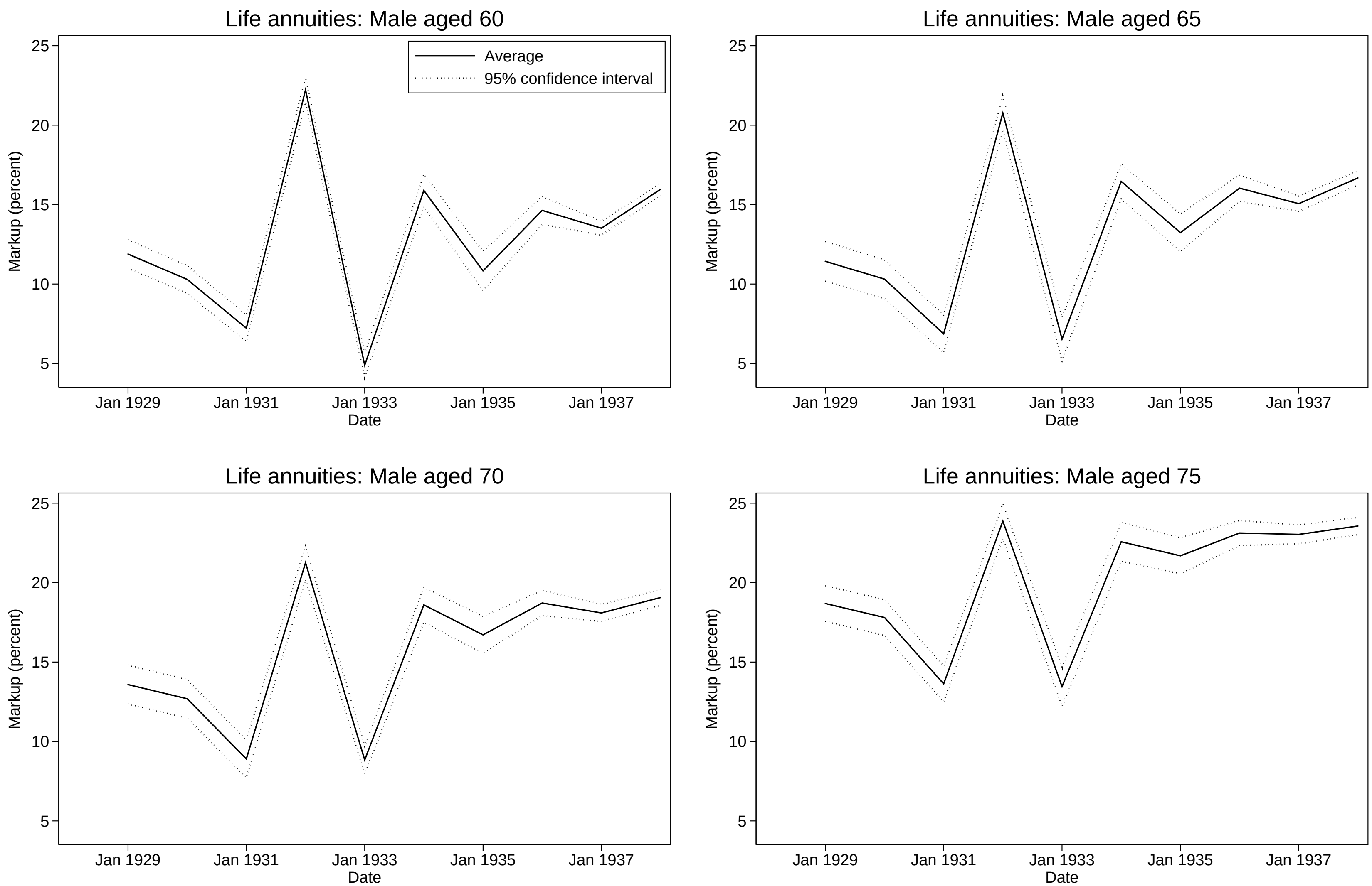

Figure E1: Average Markup on Life Annuities from 1929 to 1938

The markup is defined as the percent deviation of the quoted price from actuarial value. The actuarial value is based on the annuitant mortality table from McClintock (1899) and the zero-coupon Treasury yield curve. The annual sample covers January 1929 to January 1938.

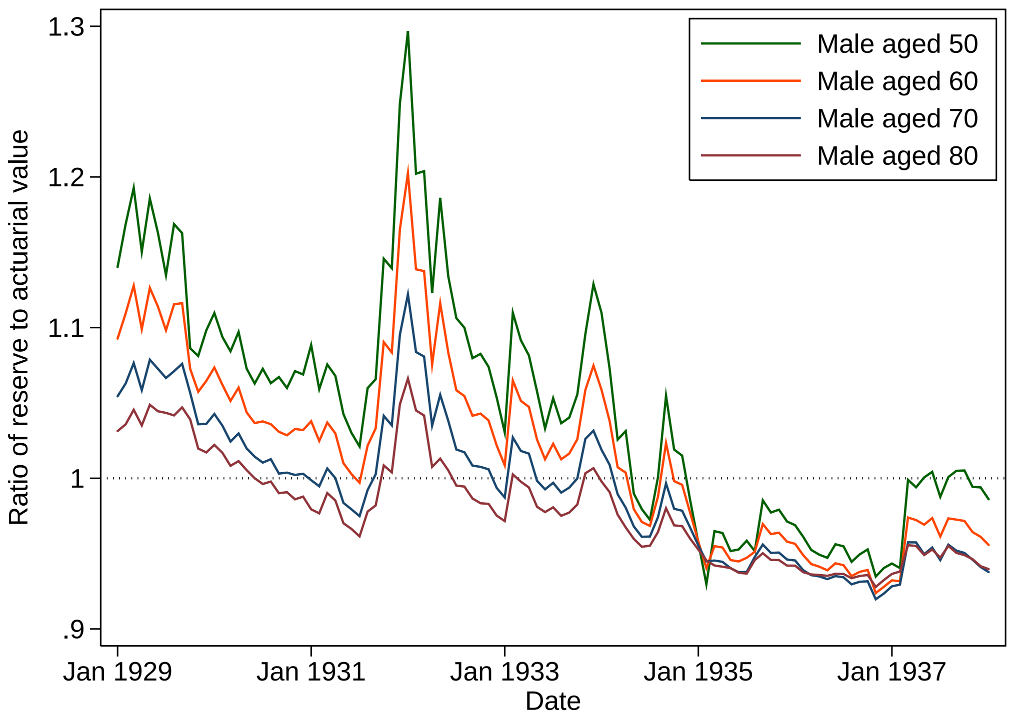

Figure E2: Reserve to Actuarial Value for Life Annuities from 1929 to 1938
The reserve value is based on the annuitant mortality table from McClintock (1899) and a constant discount rate of 3.5 percent. The actuarial value is based on the annuitant mortality table from McClintock (1899) and the zero-coupon Treasury yield curve. The monthly sample covers January 1929 to January 1938.

Appendix F. Additional Tables

Table F1: Estimated Model of Insurance Pricing: Alternative Specification

Explanatory variable	Coefficient	
Panel A. Elasticity of demand		
A.M. Best rating of A or A−	0.114	(0.021)
Log assets	0.175	(0.012)
Asset growth	-0.015	(0.008)
Leverage ratio	0.237	(0.018)
Risk-based capital relative to guideline	-0.003	(0.003)
Current liquidity	0.068	(0.014)
Operating return on equity	0.090	(0.012)
Female	0.013	(0.007)
Age 50	0.059	(0.051)
Age 55	0.112	(0.047)
Age 60	0.169	(0.041)
Age 65	0.245	(0.039)
Age 70	0.353	(0.039)
Age 75	0.435	(0.040)
Age 80	0.442	(0.046)
Age 85	0.563	(0.054)
Interaction effects for life insurance		
A.M. Best rating of A or A−	-2.948	(0.518)
Log assets	-3.585	(0.535)
Asset growth	-1.143	(0.209)
Leverage ratio	-0.286	(0.054)
Risk-based capital relative to guideline	-0.131	(0.042)
Current liquidity	-0.236	(0.104)
Operating return on equity	-0.869	(0.149)
Female	-0.066	(0.045)
Age 30	-6.518	(1.399)
Age 40	-6.992	(1.411)
Age 50	-6.926	(1.411)
Age 60	-6.798	(1.400)
Age 70	-6.438	(1.403)
Age 80	-6.421	(1.409)
Panel B. Shadow cost of capital		
Corporate yield spread	0.661	(0.039)
Log assets	-0.188	(0.070)
Asset growth	-0.195	(0.061)
Leverage ratio	1.514	(0.268)
Risk-based capital relative to guideline	0.455	(0.101)
Net equity inflow	0.007	(0.009)
R^2	0.231	
Observations	45,430	

This table repeats the estimation in Table 3, interacting the insurance company characteristics that enter the shadow cost of capital (27) with a dummy for third quarter of 2008 to first quarter of 2009.

Table F2: Estimated Model of Insurance Pricing: Alternative Sample

Explanatory variable	Coefficient	
Panel A. Elasticity of demand		
A.M. Best rating of A or A−	0.076	(0.020)
Log assets	0.204	(0.013)
Asset growth	0.024	(0.007)
Leverage ratio	0.186	(0.020)
Risk-based capital relative to guideline	-0.020	(0.008)
Current liquidity	0.067	(0.014)
Operating return on equity	-0.004	(0.003)
Female	0.015	(0.008)
Age 50	0.066	(0.049)
Age 55	0.117	(0.046)
Age 60	0.166	(0.040)
Age 65	0.239	(0.039)
Age 70	0.342	(0.039)
Age 75	0.421	(0.040)
Age 80	0.425	(0.047)
Age 85	0.549	(0.055)
Interaction effects for life insurance		
A.M. Best rating of A or A−	-2.860	(0.513)
Log assets	-3.609	(0.479)
Asset growth	-1.181	(0.197)
Leverage ratio	-0.228	(0.062)
Risk-based capital relative to guideline	-0.117	(0.041)
Current liquidity	-0.227	(0.108)
Operating return on equity	-0.775	(0.142)
Female	-0.075	(0.042)
Age 30	-6.546	(1.284)
Age 40	-7.021	(1.298)
Age 50	-6.958	(1.265)
Age 60	-6.826	(1.263)
Age 70	-6.464	(1.278)
Age 80	-6.445	(1.275)
Panel B. Shadow cost of capital		
Corporate yield spread	0.612	(0.059)
Log assets	-0.238	(0.035)
Asset growth	-0.295	(0.036)
Leverage ratio	1.553	(0.152)
Risk-based capital relative to guideline	0.393	(0.048)
Net equity inflow	0.120	(0.031)
R^2	0.234	
Observations	43,838	

This table repeats the estimation in Table 3 with MetLife Investors USA Insurance Company excluded from the sample.

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