Infrastructure Quality and the Subsidy Trap

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Online Appendix

A Derivation of Likelihood Function

In this appendix, I provide the details of the derivation of the likelihood function based on the demand model in Section II.A and the econometric assumptions in Section II.C.

Let $\nu_{jt} = \eta_{jt} + \varepsilon_{jt}$. From equation (3), write the probability of observing the consumption q_{jt} for household j in period t as equation (A.1).

$$\begin{aligned}
\Pr(q_{jt}) = & \Pr(\nu_{jt} = q_{jt} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .)\ ,\ \eta_{jt} < Q_{sub} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .)) \\
& + \Pr(\nu_{jt} = q_{jt} - \bar{q}_{jt}(p^H_{jt}, y^H_{jt}, .)\ ,\ \eta_{jt} > Q_{sub} - \bar{q}_{jt}(p^H_{jt}, y^H_{jt}, .)) \\
& + \Pr(\epsilon_{jt} = q_{jt} - Q_{sub}\ ,\ Q_{sub} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .) < \eta_{jt} < Q_{sub} - \bar{q}_{jt}(p^H_{jt}, y^H_{jt}, .)) \quad \text{(A.1)}
\end{aligned}$$

Calculation of the first two probabilities in this expression requires the joint distribution of ν_{jt} and η_{jt}. Since ν_{jt} and η_{jt} are normal, their joint distribution $h(\nu_{jt}, \eta_{jt})$ can be written as the product of the conditional distribution of η_{jt} given ν_{jt}, and the marginal distribution of ν_{jt}. Both the marginal and the conditional distributions are normal:

$$\begin{aligned}
\nu_{jt} &\sim N(0, \sigma^2_{\nu_{jt}}) \\
\eta_{jt}|\nu_{jt} &\sim N(\frac{\rho_{jt}\sigma_{\eta_{jt}}}{\sigma_{\nu_{jt}}}\nu_{jt}, \sigma^2_{\eta_{jt}}(1-\rho^2_{jt})) \quad \text{(A.2)}
\end{aligned}$$

where ρ_{jt} is the correlation between η_{jt} and ν_{jt} for household j in period t:

$$\rho_{jt} \equiv \text{corr}(\nu, \eta) = \frac{\sigma_{\eta_{jt}}}{\sigma_{\nu_{jt}}}$$

Consider the first term in equation (A.1):

$$\begin{aligned}
&\Pr(\nu_{jt} = q_{jt} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .)\ ,\ \eta_{jt} < Q_{sub} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .)) \\
&= \int_{-\infty}^{Q_{sub} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .)} h(\nu_{jt} = q_{jt} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .)\ ,\ \eta_{jt})\, d\eta_{jt} \\
&= \int_{-\infty}^{Q_{sub} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .)} \frac{1}{\sigma_{\nu_{jt}}}\phi\left(\frac{q_{jt} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .)}{\sigma_{\nu_{jt}}}\right)\phi\left(\frac{\eta_{jt} - \frac{\rho\sigma_{\eta_{jt}}}{\sigma_{\nu_{jt}}}(q_{jt} - \bar{q}_{jt}(p^L_{jt}, y_{jt}, .))}{\sigma_{\eta_{jt}}\sqrt{1-\rho^2_{jt}}}\right) d\eta_{jt}
\end{aligned}$$

$$= \frac{1}{\sigma_{\nu_{jt}}}\phi\left(\frac{q_{jt}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}{\sigma_{\nu_{jt}}}\right)\int_{-\infty}^{Q_{sub}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}\phi\left(\frac{\eta_{jt}}{\sigma_{\eta_{jt}}\sqrt{1-\rho^2_{jt}}}-\frac{\rho(q_{jt}-\bar{q}_{jt}(p^L_{jt},y_{jt},.))}{\sigma_{\nu_{jt}}\sqrt{1-\rho^2_{jt}}}\right)d\eta_{jt}$$

$$= \frac{1}{\sigma_{\nu_{jt}}}\phi\left(\frac{q_{jt}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}{\sigma_{\nu_{jt}}}\right)\Phi\left(\frac{Q_{sub}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}{\sigma_{\eta_{jt}}\sqrt{1-\rho^2_{jt}}}-\frac{\rho_{jt}(q_{jt}-\bar{q}_{jt}(p^L_{jt},y_{jt},.))}{\sigma_{\nu_{jt}}\sqrt{1-\rho^2_{jt}}}\right) \tag{A.3}$$

where $\phi()$ and $\Phi()$ represent the standard normal pdf and cdf respectively. The derivation of the expression for the second term in equation (A.1) is similar.

The third term in equation (A.1) is simpler because it requires the joint distribution of the two independent random variables, η_{jt} and ε_{jt}:

$$\Pr(\epsilon_{jt}=q_{jt}-Q_{sub}\ ,\ Q_{sub}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)<\eta_{jt}<Q_{sub}-\bar{q}_{jt}(p^H_{jt},y^H_{jt},.))$$
$$= \frac{1}{\sigma_\epsilon}\phi\left(\frac{q_{jt}-Q_{sub}}{\sigma_\epsilon}\right)\int_{Q_{sub}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}^{Q_{sub}-\bar{q}_{jt}(p^H_{jt},y^H_{jt},.)}\phi\left(\frac{\eta_{jt}}{\sigma_{\eta_{jt}}}\right)d\eta_{jt}$$
$$= \frac{1}{\sigma_\epsilon}\phi\left(\frac{q_{jt}-Q_{sub}}{\sigma_\epsilon}\right)\left(\Phi\left(\frac{Q_{sub}-\bar{q}_{jt}(p^H_{jt},y^H_{jt},.)}{\sigma_{\eta_{jt}}}\right)-\Phi\left(\frac{Q_{sub}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}{\sigma_{\eta_{jt}}}\right)\right) \tag{A.4}$$

Equation (A.1) for the probability of observing q_{jt} can now be rewritten in terms of the previous expressions involving the normal pdf and cdf:

$$\Pr(q_{jt}) = \frac{1}{\sigma_{\nu_{jt}}}\phi\left(\frac{q_{jt}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}{\sigma_{\nu_{jt}}}\right)\Phi\left(\frac{Q_{sub}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}{\sigma_{\eta_{jt}}\sqrt{1-\rho^2_{jt}}}-\frac{\rho_{jt}(q_{jt}-\bar{q}_{jt}(p^L_{jt},y_{jt},.))}{\sigma_{\nu_{jt}}\sqrt{1-\rho^2_{jt}}}\right)$$
$$+\frac{1}{\sigma_{\nu_{jt}}}\phi\left(\frac{q_{jt}-\bar{q}_{jt}(p^H_{jt},y^H_{jt},.)}{\sigma_{\nu_{jt}}}\right)\left(1-\Phi\left(\frac{Q_{sub}-\bar{q}_{jt}(p^H_{jt},y^H_{jt},.)}{\sigma_{\eta_{jt}}\sqrt{1-\rho^2_{jt}}}-\frac{\rho_{jt}(q_{jt}-\bar{q}_{jt}(p^H_{jt},y^H_{jt},.))}{\sigma_{\nu_{jt}}\sqrt{1-\rho^2_{jt}}}\right)\right)$$
$$+\frac{1}{\sigma_\epsilon}\phi\left(\frac{q_{jt}-Q_{sub}}{\sigma_\epsilon}\right)\left(\Phi\left(\frac{Q_{sub}-\bar{q}_{jt}(p^H_{jt},y^H_{jt},.)}{\sigma_{\eta_{jt}}}\right)-\Phi\left(\frac{Q_{sub}-\bar{q}_{jt}(p^L_{jt},y_{jt},.)}{\sigma_{\eta_{jt}}}\right)\right) \tag{A.5}$$

Equation (A.5) can then be used to give the log-likelihood function:

$$\ln L = \sum_{j,t}\ln[\Pr(q_{jt})] \tag{A.6}$$

Note that in the likelihood function the expression for $\sigma_{\eta_{jt}}$ depends on the appliance holdings of household j, as shown in equation (5).

B Analysis of Distribution Upgrades

In this appendix, I provide additional details for the model of the combined retail and distribution firms and how their profit is determined from the household demand for electricity and the price and subsidy parameters set by the government. I also provide details of the computation of the profitability of network upgrades.

As described in Section I, for residential customers, the combined distribution and retail firms are monopolists within their distribution areas. Price schedules are set by the government and the cost of wholesale electricity purchases is passed through, with a lag, into these prices. The retailing and distribution costs are assumed to be fixed. The decision considered in this paper is the choice to upgrade the distribution infrastructure for a small geographical area such as a neighborhood or village.

Firms bill each of their customers for a consumption quantity $\hat{q}_{jt}$. For households with upgraded connections that include a meter, the billed amount corresponds to actual consumption: $\hat{q}_{jt} = q_{jt}$. For households with the low-quality, unmetered connections, it is impossible to measure true consumption. As described in Section I, the electricity sector regulations contain several procedures that can be used to compute the consumption for the unmetered household's bill. The firm can use a communal meter to measure the total amount of electricity flowing into a settlement, and then divide this equally among households. This means that households are billed not only for the average consumption, but also for the average losses between the meter and the dwelling. An alternative method for computing the bill is to use the average billed consumption for all households in the same stratum in the distribution firm's service territory. I show results for both methods of determining the pre-upgrade billed quantity.

Revenue for the firm is the sum of subsidy transfers from the government and bill payments by households, both of which depend on the quantity that the firm bills to the household, $\hat{q}_{jt}$. Figure 1 shows the general price schedule for households in informal settlements, including both the regular Stratum 1 subsidy and the Social Energy subsidy. s_L includes both the 50 percent Stratum 1 subsidy and the 2 cents/kWh Social Energy subsidy, while s_H is only the Social Energy subsidy. Because there is no cap on the consumption quantity for the Social Energy subsidy, consumption above Q_{sub} is billed for less than P_{ft}.

As shown in Figure 1, if the household pays their bill, then the firm receives the base price P_{ft} for the entire billed consumption, so that the firm's revenue is the area $A+B+E+F+G$. In the case of non-payment by the household, the firm receives only the subsidy transfer: $A+E$ for the first Q_{sub} units billed, and B for subsequent units. Revenue for the firm from household j in period t is given by equation (B.1).[1]

$$\text{Rev}_{jt} = \text{Pay}_{jt}\, P_{ft}\hat{q}_{jt} + \max\left[0, (1-\text{Pay}_{jt})((s_L - s_H)\min(\hat{q}_{jt}, Q_{sub}) + s_H\hat{q}_{jt})P_{ft}\right] \tag{B.1}$$

Pay_{jt} is equal to 1 if household j pays their bill for period t, and 0 otherwise.

Figure 1 illustrates the important asymmetry between paying and non-paying cases. If the household pays its bill each month, the size of the subsidy has no direct effect on the firm's revenue and profit: a larger subsidy means that the government pays more but the household pays less, so that the firm's total revenue remains constant.[2] However, this is not true for non-paying households. For non-payers, the only revenue for the firm is the fiscal transfer to cover the subsidy component of the unpaid bill. Unlike the case of the paying user, an extra dollar of subsidy provided for a non-paying household results in an extra dollar of revenue (and profit) for the firm. Therefore the firm has an incentive to increase the value of the subsidy for non-paying households, such as by lobbying the government to expand the subsidy program for these users, or by optimally choosing the calculation method of $\hat{q}_{jt}$ for unmetered households.

The major cost for the firm is the variable cost of the electricity used by the household, which the firm is required to buy from the wholesale market at a price c_{ft} per unit. Electrical line losses mean that if the firm pays for 1 unit of electricity at the entry point to its network, only $(1 - l_{jt})$ units would reach household j. Equivalently, if household j consumes q_{jt} units in period t, the firm will pay for $\frac{q_{jt}}{1-l_{jt}}$ units. The size of the line losses l_{jt} may be reduced as the firm upgrades its network.

The expression for the customer-level profit excludes fixed costs, such as maintenance and administrative expenses. These do not vary with the level of the customer's consumption and are assumed not to change as a result of upgrading a customer's connection.

The firm's profit for customer j, before fixed and capital costs, is given by equation (B.2).

$$\begin{aligned}\Pi_{jt} = {} & \text{Pay}_{jt}\, P_{ft}\hat{q}_{jt} + \max\left[0, (1-\text{Pay}_{jt})((s_L - s_H)\min(\hat{q}_{jt}, Q_{sub}) + s_H\hat{q}_{jt})P_{ft}\right] \\ & - c_{ft}\left(\frac{q_{jt}}{1-l_{jt}}\right)\end{aligned} \tag{B.2}$$

[1] s_L is negative for the households in Strata 5 and 6 who pay more than P_{ft} for their electricity consumption to partially fund the subsidies for the poorer households. The firm still receives $P_{ft}\hat{q}_{jt}$ if the household pays, since the contribution is returned to the government. However, if the household does not pay, the firm's revenue is zero rather than negative: the firm does not have to fund the household's contribution in the event of non-payment.

[2] However, as shown by the estimates from the structural model of household demand, the subsidy has an indirect effect on revenue from paying users, by changing the marginal price face by households and so the quantity demanded.

Investment in the network upgrade affects the firm's profits through five channels. First, the installation of meters enables monitoring of the consumption of individual households, so that the billed amount $\hat{q}_{jt}$ corresponds to actual consumption q_{jt}. Second, the "normalization" of services reduces the size of the subsidy if the subsidy is conditional on the household having low-quality infrastructure.[3] Third, the installation of individual metered connections increases the ability of the firm to enforce payment by end users, so that Pay_{jt} is more likely to be 1 rather than 0.[4] Fourth, the improved infrastructure reduces the number of outages and so increases demand for most households by the results in Section II. Finally, the improvements to the distribution network reduce the amount of electrical line losses, l_{jt}.

The firm makes its decision whether to upgrade the network supplying customer j's neighborhood by comparing Π_{jt} with the existing low-quality infrastructure to the value of Π_{jt} with the upgraded network. If the increase in profit exceeds the firm's required rate of return on the capital cost of the upgrade, then the upgrade would take place. Otherwise, the firm would maintain its existing low level of service.

For the analysis of the decision to upgrade informal distribution networks, I selected a sample of households that are representative of those living in areas with low-quality electricity supply. I chose the 100 counties in Colombia with the most hours of outage during 2005.[5] The minimum level of outages of the 100 counties in the sample is 29 outage hours per month, or slightly less than one hour per day, averaged across all electricity users in the county.

The firm's decision to upgrade an informal distribution network will typically be made at the level of a group of adjoining neighborhoods. For example, in the PRONE data for upgrades funded by the government, the median number of households for each project is

[3] As mentioned in Section I, the Social Energy subsidy is provided to households in informal settlements, which are defined as areas that obtain electricity through unauthorized connections to the distribution network. Upgrading the infrastructure in the informal settlement may result in the loss of eligibility for this subsidy transfer.

[4] In general, the upgrade provides the firm with the ability to disconnect a non-paying household. (Rojas and Lallement, 2007) discuss technologies that are being developed in Brazil and South Africa in order to facilitate payment enforcement. These include the use of prepaid meters, combined with the shortening of the low voltage distribution network and the installation of small transformers for each customer, possibly even incorporated into the meter. Any attempt to bypass the meter would result in damage to electrical appliances.

[5] For each of the 495,000 service transformers in the data, I calculated the monthly mean outage hours and the monthly mean number of outages in 2005. In the ArcMap software I used the transformer coordinates to match each transformer to the county in which it is located. Next, for each of the 860 counties which are connected to the national transmission network and for which I have 2005 transformer data, I calculated the monthly mean outage hours as the weighted average of the transformer-level mean outage hours, where the weights are the number of users connected to each transformer. I rank the 860 counties by the outage hours and use the 100 counties with the greatest outage duration. Counties with less than 50 electricity users were excluded.

359. I approximate this level of decision making by using Stratum 1 households in the urban areas of small counties in Colombia. Typically each county has a single urban center surrounded by rural areas that may contain smaller settlements. The median number of Stratum 1 households in the 100 counties in the sample is 1067.[6]

For the counties in the sample I extracted dwelling and household characteristics from the 2005 long-form Census, selecting a random sample of 250 households for each of the 100 counties. I assigned each household to a stratum based on household characteristics and the county-level proportions of households in each stratum. The analysis is based on those households assigned to Stratum 1.[7] Retail prices and wholesale costs for each county, for each month of 2005, were obtained from CREG.[8]

With this data set, I used the demand parameters described in Section II to simulate monthly household-level consumption for 2005 under two scenarios. First, I assumed that the households were unmetered, that they faced the county-level mean number and duration of outages, and that they did not pay for their consumption of electricity. This scenario corresponds to the situation of households living in informal settlements. Second, I assumed that the households received an upgrade that reduced their outages by 75 percent, that a meter was installed, and that the household began paying their electricity bill. Although the analysis is hypothetical—I do not know which of the households in the sample are unmetered and non-paying—it is likely that the households in the sample have similar characteristics to those living in informal settlements.

Consumption before and after the upgrade is predicted as follows. First, for each household in the data, I draw 50 values of ε_{jt} and η_{jt} from the estimated distribution of these unobservables. The distribution of η_{jt} depends on the appliance holdings of household j. For each of these 50 draws of ε_{jt} and η_{jt}, I use the estimated demand parameters and the observable characteristics of the household to calculate electricity consumption before the upgrade, based on equation (3). The calculation of consumption before the upgrade assumes

[6] If there is heterogeneity across the Stratum 1 neighborhoods within a county, then the firm may decide to upgrade some neighborhoods but not others. However, as long as the hypothetical upgrade is unprofitable at the overall county level, there will be some subgroup of the Stratum 1 households for which it is unprofitable and will not take place.

[7] The Census data do not identify the stratum of the household. I estimated a probit model for membership in Stratum 1 for the households in the main estimation sample, using income and expenditure categories, dwelling characteristics (number of rooms, wall and floor materials), and appliance ownership dummies. Using the estimates from this model, I predicted the probability of belonging to Stratum 1 for the households in the upgrade sample. I selected the pN households with the highest probability of being in Stratum 1, where p is the observed proportion of households in that county in Stratum 1, and N is the number of households in the Census data for that county. As a robustness check, I also used data for a sample of all households in each county without assignment to strata. These results are reported in Table 7.

[8] I use the post-2007 values of Q_{sub} for informal settlements, which are lower than in 2005. Using the values of Q_{sub} for 2005 would increase the profitability of firms before the upgrade.

a price of zero and the county-level mean number and duration of outages. The predicted monthly consumption quantity before the upgrade is the mean of these 50 calculated values. Next, I use the same 50 draws of ε_{jt} and η_{jt} to calculate electricity consumption after the upgrade, with the lower duration of outages and the regulated price schedule. The predicted monthly consumption quantity after the upgrade is the mean of these 50 calculated values. I assume that the improvement in reliability from the upgrade only affects contemporaneous outages and not the historical outage averages. Table 7 includes a summary of the results for an alternative specification in which the historical averages also change.

Using these predictions for consumption before and after the upgrade, I then use equation (B.2) to calculate the firm's monthly profit before and after the upgrade for each household in the sample. These were aggregated to the county level to give the mean change in profit as a result of the upgrade for each county. If the incremental change in profit as a result of the investment exceeds the monthly capital cost of the upgrade, based on an annual cost of capital of 13.4 percent, then the upgrade will be profitable for the firm and the firm will be willing to make the investment.[9] Otherwise, the firm will prefer not to upgrade and instead continue supplying non-paying, unmetered customers in informal settlements.[10]

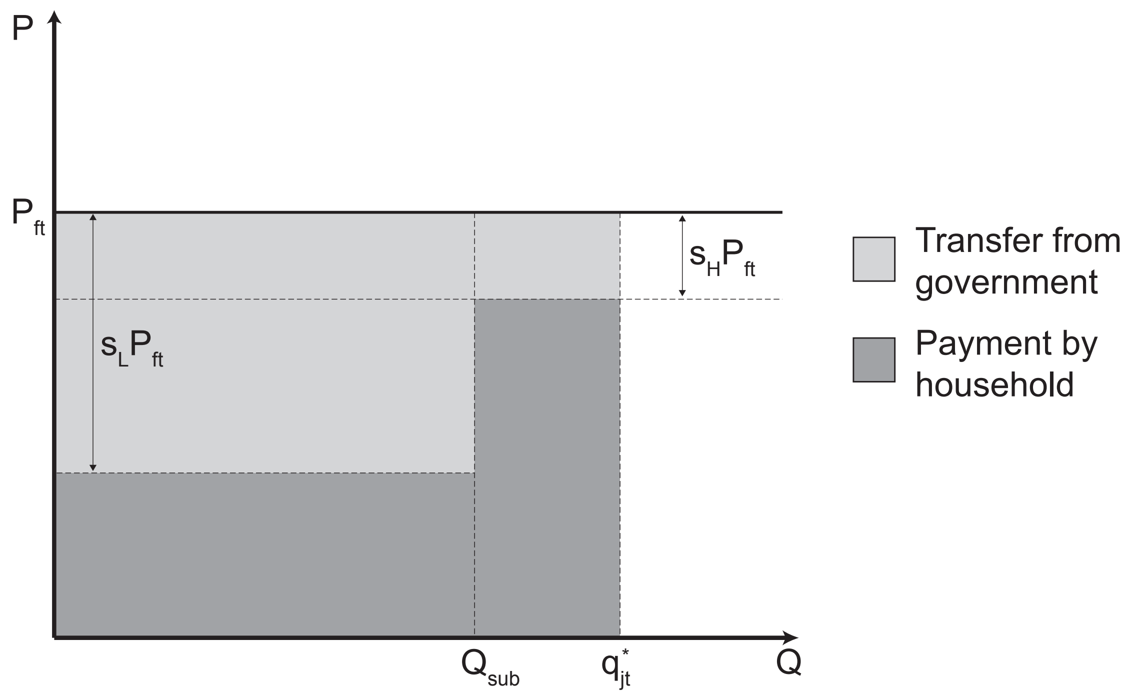
References

Rojas, Juan Manuel, and Dominique Lallement. 2007. "Meeting the Energy Needs of the Urban Poor: Lessons from Electrification Practitioners."

[9] The cost of capital used is the real, pre-tax weighted average cost of capital for electricity distribution activities in Colombia that are subject to a price cap, as calculated by the regulator for 2008 (CREG, "Costo de Capital para Remunerar la Actividad de Distribución de Energía Eléctrica", Document CREG-001, Jan 17, 2008). Assumptions in their calculation include a debt/equity ratio of 66 percent, an unlevered beta of 0.58, a market risk premium of 7.13 percent, and a country risk premium of 3.38 percent.

[10] Parameter assumptions for the calculation of the firm's profit are as follows. The firm receives the Social Energy Fund subsidy of 2 cents/kWh for the amount $\hat{q}$. Distribution losses after the upgrade are assumed to be 17.3 percent, the level of losses incorporated into the regulatory price schedules in December 2005. Before the upgrade distribution losses are assumed to be twice that amount: 34.6 percent. As a robustness check, in Table 7 I show the results based on the assumption that the pre-upgrade losses are only 50 percent higher. The cost of the upgrade is assumed to be US$510 per household. This is the mean upgrade cost per household for 99 network normalization projects submitted to PRONE (data from the Ministry of Mines and Energy). 67 percent of the projects have a cost per household within 20 percent of $510.

Figure 1: Computation of Firm's Revenue with Increasing Block Pricing Schedule



Notes: If the household does not pay their electricity bill then the firm's revenue is the subsidy transfer from the government, corresponding to the area $A + B + E$. If the household does pay their bill then the firm's revenue is the entire rectangle $P_{ft}\hat{q}_{jt}$, the sum of the household payment $F + G$ and the subsidy transfer $A + B + E$.