Risk Matters: A Comment - Technical Appendix

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I. Technical Appendix - For Online Publication

Appendix I documents the coding issues in the Matlab implementation of the model simulation in more detail by showing the associated computer code of the Jesús Fernández-Villaverde, Pablo A. Guerrón-Quintana, Juan F. Rubio-Ramírez and Martín Uribe (2011) (FGRU) replication files posted at http://www.aeaweb.org/articles.php?doi=10.1257/aer.101.6.2530. Fernández-Villaverde et al. (2011) (FGRU) calibrate their model at monthly frequency, perform a variable substitution to obtain a log-linearization, and use third-order perturbation techniques to simulate the model. We make use of the third order perturbation capacities of Dynare (Stéphane Adjemian, Houtan Bastani, Fréderic Karamé, Michel Juillard, Junior Maih, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto and Sébastien Villemot, 2011) to simulate the model.[1] Appendix III presents the corrected version of Figure 6 in FGRU. Appendices IV and V document the simulation and pruning schemes used for impulse response function (IRF) generation and moment computation. Appendix VI compares the numerical convergence behavior of the standard deviation of the Isabel Correia, Joao C. Neves and Sergio Rebelo (1995)-approximation and of the net export to output ratio. Appendix VII compares the deterministic steady state, the ergodic mean in the absence of shocks (EMAS),[2] and the ergodic mean.

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[1] The resulting policy functions are identical up to the 8th digit to the ones derived from Mathematica by FGRU.

[2] We use the term EMAS for FGRU's concept of "[s]tarting from the ergodic mean and in the absence of shocks" (p. 10 in their technical appendix). The EMAS is the fixed point of the third order approximated policy functions in the absence of shocks. It can be obtained by simulating the system with all shocks set to 0 for all time periods, starting at the deterministic steady state, and iterating it forward until convergence. Sometimes, it is referred to as the "stochastic steady state" (e.g. Michel Juillard and Ondra Kamenik, 2005), because it is the point of the state space where, in absence of shocks in that period, agents would choose to remain although they are taking future volatility into account.

I. The Coding Issues in the Published Replication Files

A. Variable Substitution and Calibration to Monthly Frequency

As can be seen in Listing 1,[3] a variable substitution is performed to obtain log-linearized decision rules.

LISTING 1 - THE MODEL IN EMERGING.NB

```
func1 = (Exp[c])^-ups - Exp[\[Lambda]];
func2 = betabeta*Exp[\[Lambda]p] - Exp[\[Lambda]]/(1 + Exp[r]) +
   Exp[\[Lambda]] (dp - ds) dtheta;
func3 = -Exp[pi] +
    betabeta ((1 - delta)*Exp[pip] +
       alpha Exp[yp]/Exp[kp] Exp[\[Lambda]p]);
func4 = thetheta (Exp[h])^(
     omega + 1) - (1 - alpha) Exp[y] Exp[\[Lambda]];
func5 = -Exp[\[Lambda]] +
   Exp[pi] (1 - phi/2 (Exp[invest]/Exp[investlag] - 1)^2 -
       phi Exp[invest]/
        Exp[investlag] (Exp[invest]/Exp[investlag] - 1)) +
    betabeta*
     Exp[pip] phi (Exp[investp]/
      Exp[invest])^2 (Exp[investp]/Exp[invest] - 1);
func6 = Exp[y] - (Exp[k])^alpha (Exp[g] Exp[h])^(1 - alpha);
func8 = dp/(1 + Exp[r]) - d + Exp[y] - Exp[c] -
   Exp[invest] - (dp - ds)^2 dtheta/2;
func9 = Exp[r] - Exp[rs] - er - etb;
func7 = -Exp[kp] + (1 - delta) Exp[
      k] + (1 - phi/2 (Exp[invest]/Exp[investlag] - 1)^2) Exp[invest];
```

Listing 2 shows that the decision rules are for a model calibrated to monthly frequency ($\bar{r} = 0.02$).

LISTING 2 - THE MODEL CALIBRATION IN EMERGING.NB

```
parmrule2 = {omega -> 1000, dtheta -> 0.001, thetheta -> 1, ups -> 5,
    delta -> 0.014, alpha -> 0.32, phi -> 95, rlib -> 0.02, ecap -> 0,
    rhosigmar -> 0.94, rhor -> 0.97, sigmag -> Log[0.015],
    eeta -> 0.46, meansigmar -> -5.71,
```

[3]Listing 1 and 2 are Mathematica code, all others Matlab.

```
rhosigmatb -> 0.94, rhotb -> 0.95, eetb -> 0.13, rhog -> 0.95,
meansigmatb -> -8.06};
```

B. Time Aggregation: Moments and IRFs

LISTING 3 - VARIABLE AGGREGATION IN `IRF_MOMENTS.M`

```
206     xaux = x_c' ;
        xauxtrimestral=zeros(size(xaux,1)/3,3);

        % Transform simulation from monthly to quarterly
210     for i=1:3;
            for j=1:size(xauxtrimestral,1);
212             xauxtrimestral(j,i)=sum(xaux((j-1)*3+1:3*j,i));
            end
214     end

216     % c_d order: consump invest output
        % Compute net exports using transformation in Correia, Neves,
218     % Rebelo: Business Cycle in Small Open Economies (European Economic
    Review, 1995)
        net_exp = xauxtrimestral(:,3) - xauxtrimestral(:,2) - xauxtrimestral
    (:,1) ;
220     net_exp = (net_exp/abs(mean(net_exp)) - 1)/100 ;

222     % HP filter data
        % NOTE: HPF computes HP trend
224     %       Please use your preferred HP filter in the next lines

226     for i = 1:3
            c_d(:,i) = xauxtrimestral(:,i) - HPF(xauxtrimestral(:,i),1600) ;
228     end

230     c_d(:,4) = net_exp - HPF(net_exp,1600) ;
```

Line 206 of Listing 3 assigns the matrix of simulated control variables at monthly frequency `x_c` (with the first three entries being consumption, investment, and output) into `xaux`. The loop in lines 210-214 then aggregates monthly percentage deviations from steady state into quarterly data for output, consumption, and investment (contained in the columns of `xaux`).

Instead of averaging the percentage deviations of these flow variables, they are accumulated. As lines 226-230 show, these are the quarterly variables that are HP-filtered and used to compute the moments.

Listing 4 - IRF aggregation in irf_moments.m

```
% Unit of time in model is month
% IRFs are accumulated to transform to quarters

figure

for i = 1:nc-3
    if i == 6 % Average quarterly debt
        aux = 100/(bss+xlast(6))*(x_c(i,2:Tplot)-xlast(i)*ones(1,Tplot-1)) ;
        for j = 1:(Tplot-2)/3
            argenq(i,j) = sum(aux((j-1)*3+1:3*j)) ;
        end
        subplot(3,2,i); plot(0:(Tplot-2)/3-1,argenq(i,:)/3,'LineWidth',1.5) ; axis tight; grid on
        title(varnm(i,:),'fontsize',13) ;
    elseif i == 5 % Annualized interest rate
        aux = 10000*x_s(8,2:Tplot) ;
        for j = 1:(Tplot-2)/3
            argenq(i,j) = sum(aux((j-1)*3+1:3*j)) ;
        end
        subplot(3,2,i); plot(0:(Tplot-2)/3-1,12*argenq(i,:)/3,'LineWidth',1.5) ; axis tight; grid on
        title(varnm(i,:),'fontsize',13) ;
    else
        aux = 100*(x_c(i,2:Tplot)-xlast(i)*ones(1,Tplot-1)) ;
        for j = 1:(Tplot-2)/3
            argenq(i,j) = sum(aux((j-1)*3+1:3*j)) ;
        end
        subplot(3,2,i); plot(0:(Tplot-2)/3-1,argenq(i,:),'LineWidth',1.5) ; axis tight; grid on
        title(varnm(i,:),'fontsize',13) ;
    end
end
```

Listing 4 shows the time aggregation from monthly model IRFs to the quarterly IRFs reported in FGRU. As can be seen in lines 313-319, the stock of debt is first expressed as a percentage deviation from the EMAS (line 314) and then aggregated by taking the mean response over three subsequent quarters: the subsequent three months are summed up (line 316) and then divided by 3 before plotting (line 318). Similarly, lines 321-325 take the interest rate spread $\varepsilon_{r,t}$, average it and multiply it by $12 \times 100 \times 100 = 120{,}000$ to transform it into annualized basis points.

Line 328 computes the difference between the log of the monthly model variables (output, consumption, investment, and hours) and the EMAS times 100, which has the interpretation of a percentage deviation. Lines 329 to 331 sum up the monthly percentage deviations over the quarter, before line 332 plots them without dividing by three as was the case in line 318. As a result, the percentage deviations from steady state of the flow variables like output, consumption, investment, and hours are overstated by a factor of three when plotting the IRFs.

To see this, consider e.g. quarterly output Y_q as the sum of the three monthly outputs $Y_{m,i}$:

$$Y_q = Y_{m,1} + Y_{m,2} + Y_{m,3} \; . \tag{1}$$

Performing a log-linearization around the deterministic steady state yields:

$$\bar{Y}_q \hat{Y}_q = \bar{Y}_m \hat{Y}_{m1} + \bar{Y}_m \hat{Y}_{m2} + \bar{Y}_m \hat{Y}_{m3} \; , \tag{2}$$

where bars denote steady state values and hats percentage deviations from steady state. Divide by $\bar{Y}_q = 3\bar{Y}_m$ to obtain quarterly percentage deviations:

$$\hat{Y}_q = \frac{\bar{Y}_m}{\bar{Y}_q}\hat{Y}_{m1} + \frac{\bar{Y}_m}{\bar{Y}_q}\hat{Y}_{m2} + \frac{\bar{Y}_m}{\bar{Y}_q}\hat{Y}_{m3} = \frac{1}{3}\left(\hat{Y}_{m1} + \hat{Y}_{m2} + \hat{Y}_{m3}\right) \; . \tag{3}$$

Thus, the mean of the percentage deviations from steady state is appropriate, not the sum.

C. Computing Net Exports

Line 219 of Listing 3 computes net exports as

$$NX_t - \overline{NX} = \hat{Y}_t - \hat{C}_t - \hat{I}_t \;, \tag{4}$$

where $\hat{X}_t \approx \frac{X_t - \bar{X}}{\bar{X}}$ denotes a variable in percentage deviations from steady state obtained from summing up the percentage deviations (line 212 of Listing 3). The correct approximation is

$$NX_t - \overline{NX} = \bar{Y}\hat{Y}_t - \bar{C}\hat{C}_t - \bar{I}\hat{I}_t \;. \tag{5}$$

Line 220 performs a Correia, Neves and Rebelo (1995)-approximation of the net exports:

$$\widehat{NX}_t = \frac{NX_t}{|\,\text{mean}(NX_t)|} - 1 \;, \tag{6}$$

but additionally divides by 100. For consistency reasons, the same is done when reporting the empirical standard deviation (see line 36 of Listing 5). This implies that both the empirical and theoretical moments for net exports are underreported by a factor of 100 in the paper. Thus, the relative volatility of net exports, σ_{nx}/σ_y is not 0.39 for Argentinean data and 0.48 for the model as reported in FGRU, but 39 and 48, respectively. In the main paper, we report the values in the form originally reported in FGRU.

Listing 5 - Net Export display in empirical_moments.m

```
   disp('Moments Argentina: vol c/vol y  vol invt/vol y  vol net export/vol y')
36 [std(cd) std(id) std(nd)/100]/std(yd)
```

D. Computing the Net Exports Share

Line 247 of Listing 6 computes the net exports to output share from the national income accounting identity:

$$NX_t = Y_t - C_t - I_t = D_t - \frac{D_{t+1}}{1+r_t} + \frac{\Phi_D}{2}\left(D_{t+1} - \bar{D}\right)^2 \tag{7}$$

at the EMAS as

$$\frac{\widetilde{NX}}{\widetilde{Y}} = \frac{\left[\bar{D} + \left(\widetilde{D} - \bar{D}\right)\right]\frac{\bar{r}-1}{\bar{r}}}{e^{\log\left(\bar{Y}\right)+\left(\log\left(\widetilde{Y}\right)-\log\left(\bar{Y}\right)\right)}}, \tag{8}$$

where the respective deviations of the EMAS from the deterministic steady state are stored in `xlast`. But in the EMAS $\widetilde{D} \neq \bar{D}$. Thus, the adjustment cost term in equation (7) is not zero. As a consequence, the permanent portfolio holdings costs paid at the EMAS are not accounted for when computing the net exports required to finance the debt stock. For the original calibration this coding issue is inconsequential due to the low debt adjustment costs. But when recalibrating the model, the debt holding costs need to be taken into account as one cannot know a priori if they are substantial.

Listing 6 - Net-Export Share Calibration in irf_moments.m

```
% 2.2.1 Compute moments in Table 7, Column M1

disp('Ratio net exports/output')
((bss+xlast(6))*(Irate-1)/Irate)/(exp(adyss+xlast(3)))
```

II. Net Exports Keeping the Structural Parameters at the Values Calibrated in FGRU

From Table 1 it can also be seen that for Ecuador, Venezuela, and Brazil, the difference between the corrected relative volatility of net exports and the relative volatility reported in FGRU is sometimes larger than for the benchmark case of Argentina and sometimes smaller. The reason turns out to be the poor numerical convergence behavior of FGRU's measure of net export volatility.[4] As we know the seed FGRU use for Argentina, but not for Ecuador, Venezuela, and Brazil, this numerical instability drives a further wedge between the relative volatility for Ecuador, Venezuela, and Brazil reported in FGRU and the corrected relative volatilities we report in Table 1.

Table 1—Net Exports Keeping the Structural Parameters at the Values Calibrated in FGRU

	Argentina				Ecuador			
	FGRU	TA	TA+NX	Data	FGRU	TA	TA+NX	Data
$\rho_{NX,Y}$	0.05	0.05	0.43	-0.76	-0.04	-0.04	0.24	-0.60
σ_{NX}/σ_Y	0.48	1.43	1.63	0.39	1.77	9.15	1.38	0.39
	Venezuela				Brazil			
	FGRU	TA	TA+NX	Data	FGRU	TA	TA+NX	Data
$\rho_{NX,Y}$	-0.10	-0.10	0.47	-0.11	0.18	0.17	0.78	-0.26
σ_{NX}/σ_Y	1.60	13.33	1.87	0.18	0.60	1.95	3.87	0.18

Note: first and fifth column: moments reported in FGRU. Second and sixth column: moments obtained using the FGRU simulation, but correcting the time aggregation (TA). Third and seventh column: moments obtained using the FGRU simulation, but correcting the time aggregation and net export computation (TA+NX). Fourth and eighth column: moments obtained from HP-filtered data. Simulations are conducted with 200 repetitions of 96 periods using the FGRU pruning scheme. For Argentina, the same set of pseudo-random numbers as in FGRU was used, while the simulation for the other countries had to rely on a different pseudo-random number generator seed.

[4]This behavior can also be seen in FGRU's official replication code. Changing the pseudo-random number seed from the 2 they used to 20, leaving aggregation, net export computation, simulation length, and the number of replications unchanged, leads to a tripling of σ_{NX}/σ_Y from 0.48 to 1.46.

III. Figure FGRU6: IRFs Debt/Output, Current Account, Net Exports

Figure 6 in FGRU, reproduced here as Figure 1 due to non-availability of replication codes, depicts the responses of the debt to output ratio, the current account, and net exports to a risk shock.

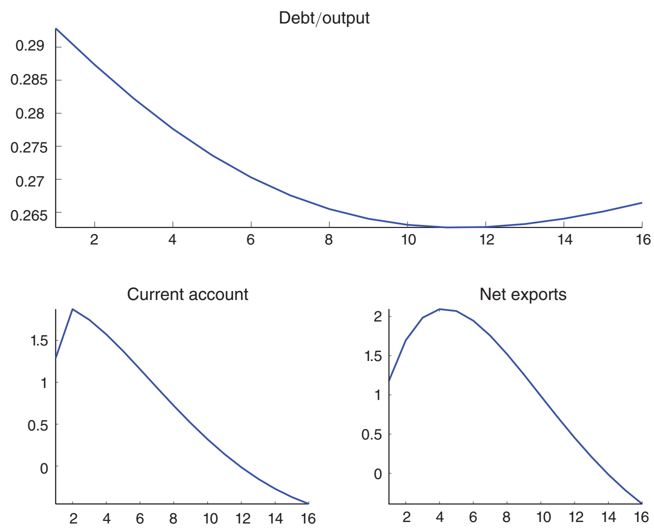

FIGURE 1. IRFs DEBT/OUTPUT, CURRENT ACCOUNT, NET EXPORTS

Note: Reproduced Figure 6 from Fernández-Villaverde et al. (2011), p. 2553.

FGRU report the debt to output ratio IRF in that figure not in percentage deviations from the EMAS, but as the absolute value. Figure 2 displays that the debt to output ratio dropped from about 0.293 by about 2.9 percentage points to 0.263 after 11 periods. But this is inconsistent with the FGRU-IRFs in Figure 3, which show that debt dropped by 3.873 percent while output dropped by 0.1907 percent. Thus, the new debt-to-output-ratio should be up to first order $(1 - 0.03873)D/((1 - 0.001907)Y) \approx 0.963D/Y$, i.e. it should drop by about 3.7 percent (not percentage points). The net export IRF is affected by from the incorrect weighting used in their computation as shown in section **??**. Regarding the current account, FGRU state that it is in "percentage points of [its] ergodic mean". But this is not possible for it is defined as $CA_t = D_t - D_{t-1}$ and thus has ergodic mean zero. Thus, it is unclear what the lower left panel of Figure 2 depicts.

The left column of Figure 2 shows the corrected version of Figure 6 in FGRU that uses output to normalize net exports and the current account, giving them the interpretation of

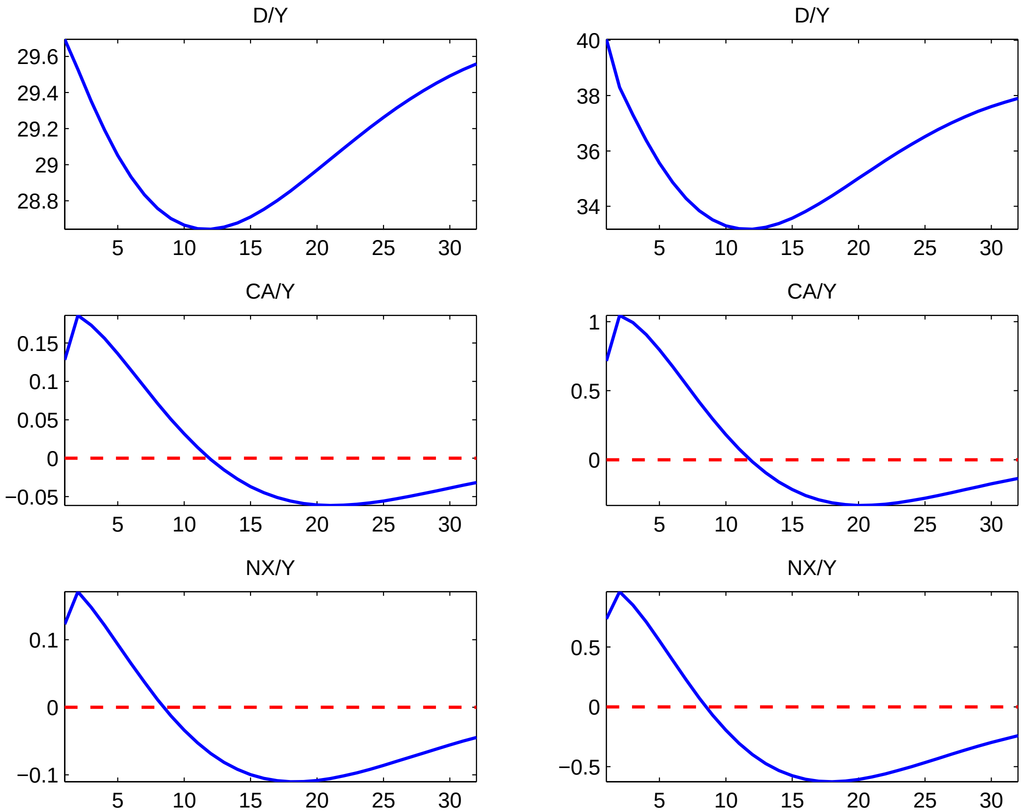

FIGURE 2. DEBT/OUTPUT, CURRENT ACCOUNT, AND NET EXPORTS DYNAMICS.

Note: Left column: IRFs from the corrected FGRU model; Right column: IRFs from the recalibrated corrected model. Row 1: Debt-to-quarterly-GDP ratio in percent of quarterly GDP. Row 2: Current account to GDP ratio in absolute deviation from the EMAS. Row 3: Net exports to GDP ratio in absolute deviation from the EMAS.

being in output units. As can be seen, in the corrected model with the original calibration the current account implications of risk shocks are rather muted.

The right column of Figure 2 displays the IRFs from the recalibrated corrected model quantifying the central "debt reduction mechanism". After a one standard deviation risk shock, the debt to quarterly output ratio falls by about 6.5 percentage points after three years, while current account and net exports increase by about 1 percent of output on impact in order to finance the deleveraging.

IV. IRFs at the Ergodic Mean

A. IRF Generation

The use of higher-order perturbation techniques to solve the model implies that the model solution is not linear anymore. Thus, the IRFs will depend on both the sequence of future shocks, u_t, and the point in the state space at which the IRFs are started, i.e. the past history of shocks, Ω_t. To circumvent this problem, Gary Koop, M. Hashem Pesaran and Simon M. Potter (1996) suggested the concept of Generalized Impulse Response Functions (GIRFs) that e.g. allow considering "representative" IRFs at the ergodic mean. The GIRF at time $t+n$ after a shock u_t is given by

$$GIRF_n\left(u_t, \Omega_{t-1}\right) = E\left[Y_{t+n} | u_t, \Omega_{t-1}\right] - E\left[Y_{t+n} | \Omega_{t-1}\right] \ , \tag{9}$$

that is, given a point in the state space, the future shock realizations are averaged out.

In contrast, FGRU also condition on future shocks by setting them to 0 when generating their IRFs and start the IRFs at the EMAS. Denote the future realization of shocks with Ω^{fut}. FGRU effectively use the definition

$$\begin{aligned} IRF_n\left(\nu_t, \Omega_t\right) = & E\left[Y_{t+n} | u_t, \Omega_{t-1} = \left\{\ldots, 0\right\}, \Omega_{t+1}^{fut} = \left\{0, \ldots\right\}\right] \\ & - E\left[Y_{t+n} | 0, \Omega_{t-1} = \left\{\ldots, 0\right\}, \Omega_{t+1}^{fut} = \left\{0, \ldots\right\}\right] \ , \end{aligned} \tag{10}$$

where the expected values can be dropped as everything is deterministic.

This choice of computing the IRFs at the EMAS has two important implications. First, computing the non-linear IRFs not as the expected difference in responses as in (9) but also conditioning on future shocks and setting them to 0, only allows capturing part of the economic effects of risk shocks. To see this, inspect the particular pruning algorithm[5] used

[5] As first noted in Jinill Kim, Sunghyun Kim, Ernst Schaumburg and Christopher A. Sims (2008), higher order perturbation solutions tend to explode due to the accumulation of terms of increasing order. For example, in a second order approximated solution, the quadratic term at time t will be raised to the power of two in the quadratic term at $t+1$, thus resulting in a quartic term, which will become a term of order 8 at $t+2$ and so on. As a solution, Kim et al. (2008) proposed "pruning" all terms of higher order, i.e. computing the quadratic term at $t+1$ by only squaring the first-order term from time t. This procedure, however, is not easily generalized to third order as there are several potential ways of pruning.

in FGRU for IRF-generation.[6] Consider a generic model solution of the form

$$x_t = g\left(x_{t-1}^{states}, u_t, \sigma\right) \ , \tag{11}$$

where x_t is an $n_x \times 1$ vector of endogenous variables, x_{t-1}^{states} is the vector of states contained in x_t,[7] u_t is an $n_u \times 1$ vector of mean zero disturbances, and σ is the perturbation parameter. Denote partial derivatives with subscripts. The pruned third order solution for the endogenous variables' deviations from their steady state, $\hat{x}_t^{3rd} = x_t^{3rd} - \bar{x}$, used by FGRU, is computed from the recursion

$$\begin{aligned}
\hat{x}_t^{3rd} =& g_x \hat{x}_{t-1}^{3rd,states} + g_u u_t \\
&+ \frac{1}{2}\left[g_{xx}\left(\hat{x}_{t-1}^{1st,states} \otimes \hat{x}_{t-1}^{1st,states}\right) + 2g_{xu}\left(\hat{x}_{t-1}^{1st,states} \otimes u_t\right) + g_{uu}\left(u_t \otimes u_t\right) + g_{\sigma\sigma}\sigma^2\right] \\
&+ \frac{1}{6}\begin{bmatrix} g_{xxx}\left(\hat{x}_{t-1}^{1st,states} \otimes \hat{x}_{t-1}^{1st,states} \otimes \hat{x}_{t-1}^{1st,states}\right) + g_{uuu}\left(u_t \otimes u_t \otimes u_t\right) \\ +3g_{xxu}\left(\hat{x}_{t-1}^{1st,states} \otimes \hat{x}_{t-1}^{1st,states} \otimes u_t\right) + 3g_{xuu}\left(\hat{x}_{t-1}^{1st,states} \otimes u_t \otimes u_t\right) \\ +3g_{x\sigma\sigma}\sigma^2 \hat{x}_{t-1}^{1st,states} + 3g_{u\sigma\sigma}\sigma^2 u_t \end{bmatrix}
\end{aligned} \tag{12}$$

$$\hat{x}_t^{1st} = g_x \hat{x}_{t-1}^{1st,states} + g_u u_t \ . \tag{13}$$

That is, all higher order terms are based on the first-order terms.[8] The recursion in equations (12)-(13) is completed by an initial condition[9] of:

$$\hat{x}_0^{3rd} = \tilde{x} - \bar{x} \tag{14}$$

$$\hat{x}_0^{1st} = 0 \ . \tag{15}$$

Because $\hat{x}_0^{1st} = 0$ and all higher order terms in equation (12) are based on it, the effect

[6]The IRF-pruning scheme differs from the scheme used for simulations, see Appendix V.

[7]We use the Dynare notation that stacks the state transition and observation equations (see Adjemian et al., 2011).

[8]This choice results in an inferior performance compared to e.g. the pruning scheme by Martin M. Andreasen, Jesús Fernández-Villaverde and Juan F. Rubio-Ramírez (2013) that augments the state space to keep track of first to third order terms and uses the Kronecker product of the first and second order terms to compute the third order term (see Hong Lan and Alexander Meyer-Gohde, 2013*b*, for more details).

[9]As shown in Lan and Meyer-Gohde (2013*b*) there are infinitely many different past shock realizations that can lead to being at a particular point in the state-space at time 0, all of them associated with particular values for $\hat{x}_0^{3rd}$ and $\hat{x}_0^{1st}$. Equations (14) to (15) are consistent with the EMAS in that one particular shock combination giving rise to these values is the total absence of past shocks.

of the initial condition $\tilde{x} - \bar{x}$ will mostly be neglected. Equation (13) effectively is a first-order policy function, which is known not to react to risk shocks, except for the state σ_{t-1}. Considering (12), this and the conditioning on all shocks being $0 \; \forall \; t+i, \; i > 0$ implies that, in the terminology of Hong Lan and Alexander Meyer-Gohde (2013*a*), only the "risk adjustment channel" is present (via the constant term $1/2 \times g_{\sigma\sigma} \times \sigma^2$ and the time-varying risk-adjustment $1/2 \times g_{u\sigma\sigma} \times \sigma^2 \times u_t$ in period t where $u_t \neq 0$). But the difference in "amplification effects" introduced by (risk) shocks and embedded in the other higher order terms is totally absent. Thus, the difference in the interaction between the location in the state space and future shocks, introduced by the risk shock, is not captured.

Second, the IRFs are computed at a particular point in the pruned state space where agents factor in the uncertainty of the system, but where there has been an infinite absence of shocks. Due to the absence of shocks and thus of "amplification effects" embedded in the higher order terms, agents will dare to incur a relatively high amount of debt. As shown in Table 3, the difference between the EMAS and the unconditional mean amounts to 20 percent.[10]

B. IRF Generation

Figure 3 compares the responses after a one-standard deviation interest rate risk shock reported in FGRU (red dashed lines) with the responses when the time aggregation error is corrected (blue solid lines). It can be seen that correcting the time aggregation error mechanically results in the size of the shock response dropping to one third of the value reported in FGRU. For example, instead of dropping by 0.19 percent, output falls by a 0.06 percent at its trough.

[10]An alternative would be to compute GIRFs at the true ergodic mean using the methods proposed in Andreasen, Fernández-Villaverde and Rubio-Ramírez (2013).

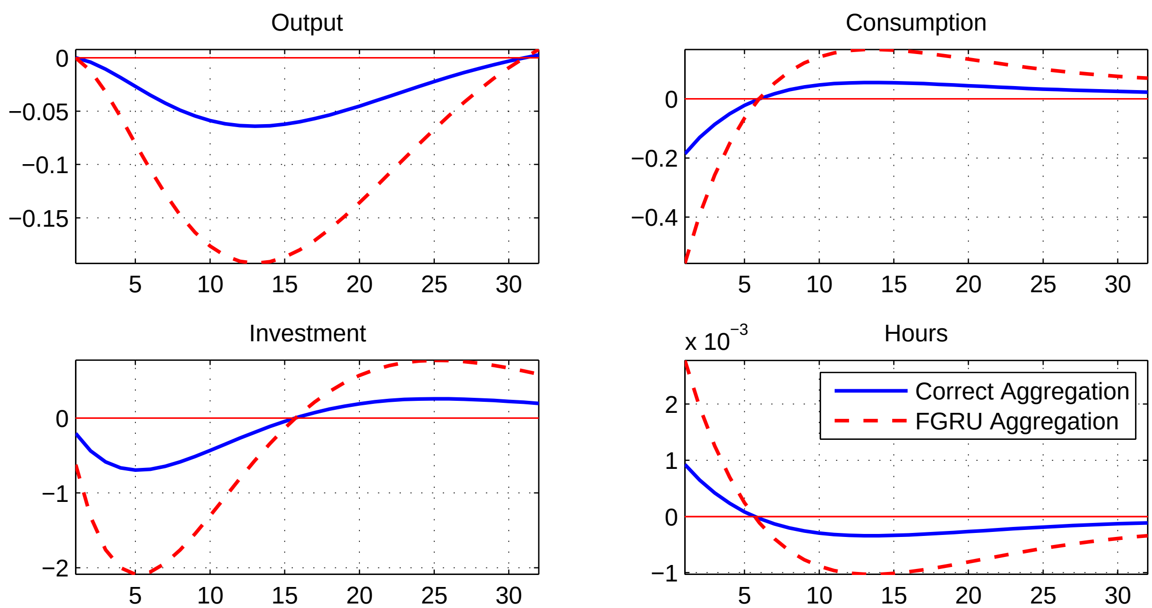

Figure 3. Comparison of Quarterly IRFs for Different Aggregation Schemes.

Note: IRFs to a one-standard deviation shock to interest rate risk premium uncertainty. Blue solid line: correct aggregation by averaging percentage deviations of monthly flow variables; red dashed line: aggregation by summing up monthly percentage deviations of flow variables.

V. Starting Simulations at the Ergodic Mean in the Absence of Shocks

The simulations conducted in FGRU use a different pruning scheme than the IRF-generation. Denote the time periods of the simulations with $t = 1, \ldots, 96$, the simulation repetition with $i = 1, \ldots, 200$, and a generic variable with $y_{t,i}$.

1) At time $t = 1$
   - set the third order term of the states $x_{1,i}^{3rd,states}$ to the EMAS and the non-state elements of $x_{1,i}^{3rd}$ to the deterministic steady state

   If $i = 1$
   - set the first-order term $x_{1,1}^{1st}$ to the deterministic steady state
   - set the shock term used in the first-order term to $u_{2,1}^{1st} = 0$
   - draw a random shock vector $u_{2,1}$

   else if $i \neq 1$
   - set the first-order state term $x_{1,i}^{1st,states}$ to $x_{96,i-1}^{1st,states}$
   - set the shock term used in the first-order term to $u_{2,i}^{1st} = u_{97,i-1}^{1st}$
   - set $u_{2,i} = u_{2,1}$
2) for $t = 2$ to 96:
   - Use the unpruned state space representation to compute the time t values of the exogenous state variables
   - To compute the time t values of the endogenous states, use the recursion

$$\begin{aligned}\hat{x}_{t,i}^{3rd} =& g_x \hat{x}_{t-1,i}^{3rd,states} + g_u u_{t,i} \\ &+ \frac{1}{2}\left[g_{xx}\left(\hat{x}_{t-1,i}^{1st,states} \otimes \hat{x}_{t-1,i}^{1st,states}\right) + 2g_{xu}\left(\hat{x}_{t-1}^{1st,states} \otimes u_{t,i}^{1st}\right) + g_{uu}\left(u_{t,i}^{1st} \otimes u_{t,i}^{1st}\right) + g_{\sigma\sigma}\sigma^2\right] \\ &+ \frac{1}{6}\begin{bmatrix} g_{xxx}\left(\hat{x}_{t-1,i}^{1st,states} \otimes \hat{x}_{t-1,i}^{1st,states} \otimes \hat{x}_{t-1,i}^{1st,states}\right) + g_{uuu}\left(u_{t,i}^{1st} \otimes u_{t,i}^{1st} \otimes u_{t,i}^{1st}\right) \\ +3g_{xxu}\left(\hat{x}_{t-1,i}^{1st,states} \otimes \hat{x}_{t-1,i}^{1st,states} \otimes u_{t,i}^{1st}\right) + 3g_{xuu}\left(\hat{x}_{t-1,i}^{1st,states} \otimes u_{t,i}^{1st} \otimes u_{t,i}^{1st}\right) \\ +3g_{x\sigma\sigma}\sigma^2 \hat{x}_{t-1,i}^{1st,states} + 3g_{u\sigma\sigma}\sigma^2 u_{t,i}^{1st} \end{bmatrix}\end{aligned} \tag{16}$$

(17)

$$\hat{x}_{t,i}^{1st} = g_x \hat{x}_{t-1,i}^{1st,states} + g_u u_{t,i}^{1st}$$

- Draw a random shock vector $u_{t+1,i}$
- Set $u^{1st}_{t+1,i} = u_{t+1,i}$
- Use $\hat{x}^{3rd}_{t,i}$ as the simulated variable

Four things are noteworthy. First, the simulations for the exogenous laws of motion for TFP, the T-bill rate, the country risk premium, and the two volatility processes do not use pruning. They are instead based on iterating the full third-order approximated policy function forward. This seems to pose no practical problems in the simulations we conducted as we encountered no explosive behavior. But using the full higher-order polynomial approximation to the true stationary exogenous law of motion implies that the stability properties of the underlying policy function are not necessarily inherited (see e.g. Wouter J. Den Haan and Joris De Wind, 2012). Thus, the exogenous laws of motion may suffer from exactly the problem for which using a pruning algorithm was advocated. Second, the actual simulations only start at time $t = 2$, because for $t = 1$ the endogenous variables are assumed to be at the deterministic steady state. Nevertheless, this first time point with zero deviations from steady state is included in the 96 time periods used to compute simulated moments. As the simulated system will on average transition to the ergodic mean, this introduces an initial jump from $t = 1$ to $t = 2$, which even the subsequent HP-filtering will not completely smooth out. Third, for the first actual simulation period, i.e. $t = 2$, the simulated first and third order terms are based on different structural shocks, $u^{1st}_{2,i}$ and $u_{2,i}$, respectively. Hence, agents in the model are assumed to react to two different shock realizations at the same time. Fourth, the first shock $u_{2,i}$ at $t = 2$ is always equal to the one of the first simulation, i.e. $u_{2,1}$.

One important implication of this particular simulation scheme is that due to starting at the EMAS for the third order term and then hitting the equilibrium system with shocks, the simulations will slowly transition to the ergodic distribution. As the simulations are always restarted at this point after 96 periods and there is no burnin, most draws will not yet come from the ergodic distribution. Put differently, the moments from 10,000 simulations of 96 periods and the ones from one simulation of 960,000 periods considerably differ, as shown in Table 2.

Table 2—Second Moments of Long vs. Short Simulations

	Argentina			Ecuador		
	Data	Short Sim.	Long Sim.	Data	Short Sim.	Long Sim.
σ_Y	4.77	1.78	2.06	2.46	0.73	0.94
σ_C/σ_Y	1.31	1.52	1.72	2.48	2.22	2.19
σ_I/σ_Y	3.81	4.06	5.56	9.32	9.89	11.86
σ_{NX}/σ_Y	0.39	4.72	5.80	0.65	2.41	0.98
$\rho_{NX,Y}$	-0.76	0.41	0.38	-0.60	0.24	0.23
$\widetilde{NX}/\widetilde{Y}$	1.78	1.75	1.75	3.86	3.95	3.95
	Venezuela			Brazil		
	Data	Short Sim.	Long Sim.	Data	Short Sim.	Long Sim.
σ_Y	4.72	1.51	1.70	4.64	1.50	1.67
σ_C/σ_Y	0.87	0.51	0.51	1.10	0.45	0.46
σ_I/σ_Y	3.42	3.94	4.57	1.65	1.73	2.16
σ_{NX}/σ_Y	0.18	0.34	0.31	0.23	6.23	2.00
$\rho_{NX,Y}$	-0.11	0.45	0.37	-0.26	0.77	0.71
$\widetilde{NX}/\widetilde{Y}$	4.07	4.14	4.14	0.10	0.52	0.52

Note: first and fourth column: moments obtained from HP-filtered data. Second and fifth column: moments of the FGRU model with corrected aggregation and net export computation, based on 10,000 replications of 96 periods. Third and sixth column: moments of the FGRU model with corrected aggregation and net export computation, based on 1 replication of 960,000 periods

VI. Convergence Behavior of the Net Exports to Output Ratio

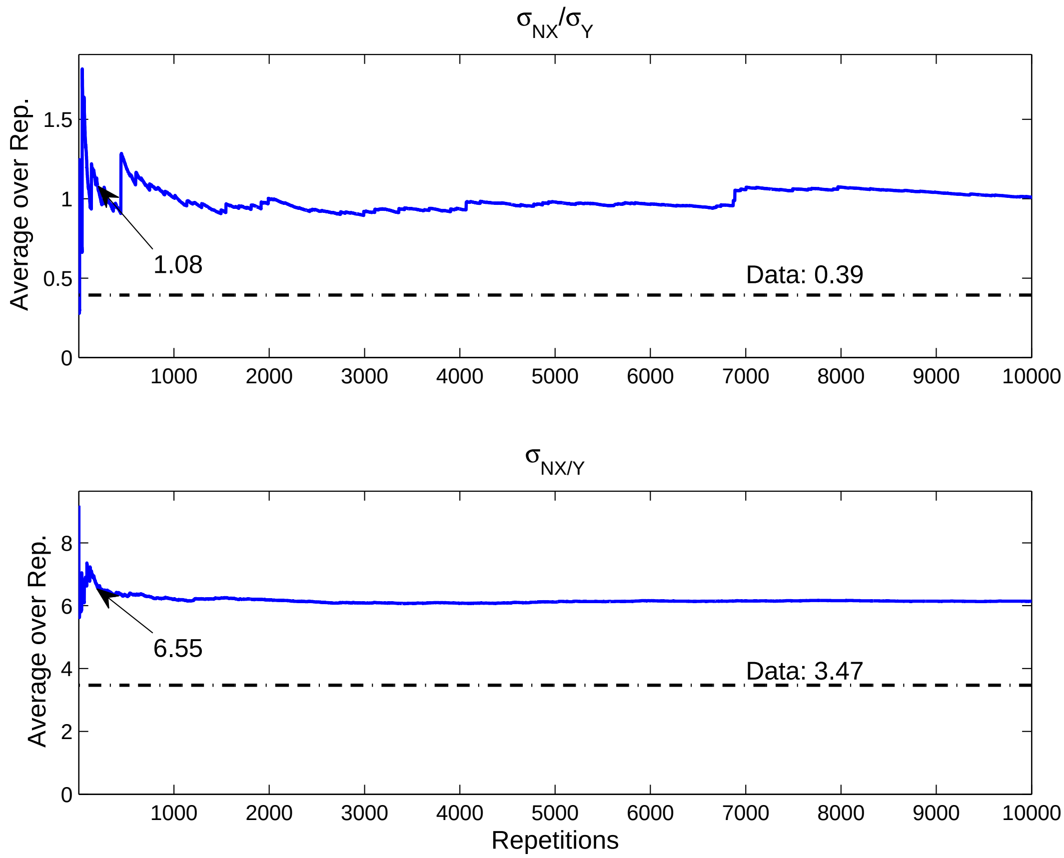

Figure 4. Convergence Behavior of Different Net Export Volatility Statistics in the Recalibrated Model

Note: top panel: relative volatility of net exports to output σ_{NX}/σ_Y. Net exports transformed to percentage deviations using the Correia, Neves and Rebelo (1995)-approximation. Bottom panel: standard deviation of the net exports to output ratio $\sigma_{NX/Y}$. The blue solid line shows the mean standard deviation (y-axis) over the up to 10,000 samples (x-axis) of simulating 96 months of data. The black dashed dotted line shows the actual data moments. The data are based on the corrected aggregation and net export computation. The black arrow indicates the value after 200 replications.

Using one long simulation for the Correia, Neves and Rebelo (1995) (CNR)-approximation instead of averaging over many short ones is no alternative. It does not allow for capturing small sample biases potentially present in the data and, due to the particular pruning scheme and simulation scheme used in FGRU, leads to results that are not comparable to the short simulations. See Appendix V for details.

VII. Steady State, EMAS, and Ergodic Mean

TABLE 3—STEADY STATE, EMAS, AND ERGODIC MEAN: FGRU CALIBRATION

	Argentina			Ecuador		
	Steady State	EMAS	Erg. Mean	Steady State	EMAS	Erg. Mean
D	4.000	2.551	2.090	13.000	12.040	12.072
K	3.293	3.287	3.309	3.745	3.757	3.757
C	0.878	0.888	0.905	0.945	0.951	0.951
H	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
Y	1.051	1.049	1.056	1.196	1.200	1.200
I	-0.975	-0.982	-0.969	-0.523	-0.512	-0.518
NX/Y	0.027	0.018	0.005	0.043	0.039	0.038
CA	0.000	0.000	0.000	0.000	0.000	0.000
	Venezuela			Brazil		
	Steady State	EMAS	Erg. Mean	Steady State	EMAS	Erg. Mean
D	22.000	21.422	21.445	3.000	2.709	2.651
K	4.002	4.009	4.010	4.001	4.003	4.005
C	0.982	0.985	0.985	1.030	1.031	1.032
H	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
Y	1.278	1.280	1.280	1.278	1.278	1.279
I	-0.267	-0.260	-0.265	-0.267	-0.266	-0.265
NX/Y	0.043	0.041	0.041	0.006	0.005	0.005
CA	0.000	0.000	-0.000	0.000	0.000	-0.000

Note: first column: deterministic steady state, second column: ergodic mean in the absence of shocks (EMAS); third column: theoretical mean based on the third-order pruned state space of Andreasen, Fernández-Villaverde and Rubio-Ramírez (2013). D, NX/Y, and CA are reported in levels, while all other variables are in logs. The model is at monthly frequency.

Table 3 implies that $D/Y_{annual} = 2.09/(12 \times exp(1.056)) \approx 0.0606$. In the recalibrated model, $D/Y_{annual} = 4.251/(12 \times exp(1.106)) \approx 0.1172$.

Table 4—Steady State, EMAS, and Ergodic Mean: Recalibration

	Argentina		
	Steady State	EMAS	Ergodic Mean
D	18.802	3.595	4.251
K	3.294	3.461	3.463
C	0.750	0.933	0.919
H	-0.003	-0.004	-0.004
Y	1.052	1.105	1.106
I	-0.975	-0.807	-0.860
NX/Y	0.129	0.018	0.022
CA	0.000	0.000	0.000

Note: first column: deterministic steady state, second column: ergodic mean in the absence of shocks (EMAS); third column: theoretical mean based on the third-order pruned state space of Andreasen, Fernández-Villaverde and Rubio-Ramírez (2013). D, NX/Y, and CA are reported in levels, while all other variables are in logs. The model is at monthly frequency.

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