Mismatch Unemployment
Online Appendix Not for Publication

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Appendix Not For Publication

A Theoretical Appendix

This Appendix formally derives all the theoretical results of Sections 1 and 5. In what follows, we adopt a recursive formulation for all the planner's problems, and state them as dynamic-programming problems where the arguments of the planner's value function V are the relevant state variables. The prime symbol $(')$ is used to denote next-period values.

A.1 Benchmark environment

We solve the planner's problem of Section 1.1. The efficient allocation at any given date is the solution of the following planner's problem that we write in recursive form:

$$\begin{aligned} V\left(\mathbf{e};\mathbf{v},\boldsymbol{\phi},Z,\Delta,\Phi\right) &= \max_{\{u_i\geq 0\}}\sum_{i=1}^{I} Z\left(e_i+h_i\right)+\beta\mathbb{E}\left[V\left(\mathbf{e}';\mathbf{v}',\boldsymbol{\phi}',Z',\Delta',\Phi'\right)\right] \\ s.t. &: \\ \sum_{i=1}^{I}\left(e_i+u_i\right) &= 1 \qquad \text{(A1)} \\ h_i &= \Phi\phi_i m\left(u_i,v_i\right) \qquad \text{(A2)} \\ e_i' &= \left(1-\Delta\right)\left(e_i+h_i\right) \qquad \text{(A3)} \\ &\Gamma_{Z,\Delta,\Phi}\left(Z',\Delta',\Phi';Z,\Delta,\Phi\right),\ \Gamma_{\mathbf{v}}\left(\mathbf{v}';\mathbf{v},Z',\Delta',\Phi'\right),\Gamma_{\phi}\left(\boldsymbol{\phi}';\boldsymbol{\phi}\right) \qquad \text{(A4)} \end{aligned}$$

The per period output for the planner is equal to $Z\left(e_i+h_i\right)$ in each market i. The first constraint $(A1)$ states that the planner has $1-\sum_{i=1}^{I} e_i$ unemployed workers available to allocate across sectors. Equation $(A2)$ states that, once the allocation $\{u_i\}$ is chosen, the frictional matching process in each market yields $\Phi\phi_i m\left(u_i,v_i\right)$ new hires which add to the existing e_i active matches. Equation $(A3)$ describes separations and the determination of next period's distribution of active matches $\{e_i'\}$ in all sectors. Line $(A4)$ in the problem collects all the exogenous stochastic processes the planner takes as given.

It is easy to see that this is a concave problem where first-order conditions are sufficient for optimality. At an interior solution ($u_i>0$ for all i), the choice of how many unemployed workers u_i to allocate in market i yields the first-order condition

$$Z\Phi\phi_i m_{u_i}\left(\frac{v_i}{u_i}\right)+\beta\mathbb{E}\left[V_{e_i}\left(\mathbf{e}';\mathbf{v}',\boldsymbol{\phi}',Z',\Delta',\Phi'\right)\right]\left(1-\Delta\right)\Phi\phi_i m_{u_i}\left(\frac{v_i}{u_i}\right)=\mu, \qquad \text{(A5)}$$

where μ is the multiplier on constraint $(A1)$. The right-hand side (RHS) of this condition is the shadow value of an additional worker in the unemployment pool available to search. The left-hand side (LHS) is the expected marginal value of an additional unemployed worker allocated to sector i. The derivative of the sector-specific matching function m is written as a function of local market tightness only (with a slight abuse of notation) because of its CRS specification.

The Envelope condition with respect to the state e_i yields:

$$V_{e_i}\left(\mathbf{e};\mathbf{v},\boldsymbol{\phi},Z,\Delta,\Phi\right)=Z-\mu+\beta(1-\Delta)\mathbb{E}\left[V_{e_i}\left(\mathbf{e}';\mathbf{v}',\boldsymbol{\phi}',Z',\Delta',\Phi'\right)\right], \tag{A6}$$

from which it is immediate to see, by iterating forward, that $\mathbb{E}\left[V_{e_i}\left(\mathbf{e}';\mathbf{v}',\boldsymbol{\phi}',Z',\Delta',\Phi'\right)\right]$ is independent of i, since productivity and the job destruction rate are common across all sectors.[1] Using this result into $(A5)$, the optimal rule for the allocation of unemployed workers across sectors can be written as equation (1) in the main text.

A.2 Heterogenous productivities and destruction rates

We extend the baseline model of Section 1.1 as follows. Individuals (still in measure one) can be either employed in sector i (e_i), or unemployed and searching in sector i (u_i), or out of the labor force. The aggregate labor force is $\ell=\sum_{i=1}^{I}(e_i+u_i)\leq 1$.

Labor productivity in sector i is given by Zz_i, where each idiosyncratic component z_i is strictly positive, i.i.d. across sectors and independent of the aggregate state Z. The non-employed individuals produce output $\zeta Z>0$ (which can also be interpreted as the value of additional leisure), and the unemployed incur in an extra disutility cost of search $\xi>0$.

Let the conditional distribution of the vector $\mathbf{z}=\{z_i\}$ be $\Gamma_z(\mathbf{z}',\mathbf{z})$. The idiosyncratic component of the exogenous destruction rate in sector i is δ_i, i.i.d. across sectors and independent of Δ, Z and z_i. Let the conditional distribution of the vector $\boldsymbol{\delta}=\{\delta_i\}$ be $\Gamma_\delta(\boldsymbol{\delta}',\boldsymbol{\delta})$. The survival probability of a match is then $(1-\Delta)(1-\delta_i)$. The vector $\{Z,\Delta,\Phi,\mathbf{z},\mathbf{v},\boldsymbol{\phi},\boldsymbol{\delta}\}$ takes strictly positive values.

It is convenient to impose additional structure on some conditional distributions: as specified in the text, we assume that $(Z,1-\Delta,z_i,1-\delta_i)$ are all positive martingales. The timing of events is exactly as before, with the decision on the size of the labor force for next period taken at the end of the current period. The recursive formulation of the planner's problem has three additional states compared to the problem of Section 1.1: the current number of unemployed workers u, the vector of productive efficiencies $\mathbf{z}$, and the vector of destruction rates $\boldsymbol{\delta}$. The

[1] We are also using the transversality condition $\lim_{t\to\infty}\beta^t(1-\Delta)^t\mathbb{E}\left[V_{e_{it}}\right]=0$.

planner solves the problem:

$$
\begin{aligned}
V\left(u,\mathbf{e};\mathbf{z},\mathbf{v},\boldsymbol{\phi},\boldsymbol{\delta},Z,\Delta,\Phi\right) &= \max_{\{u_i,\ell'\}}\sum_{i=1}^{I} Zz_i\left(e_i+h_i\right)-\xi u+Z\zeta\left[1-\sum_{i=1}^{I}\left(e_i+h_i\right)\right] \\
&+ \beta\mathbb{E}\left[V\left(u',\mathbf{e}';\mathbf{z}',\mathbf{v}',\boldsymbol{\phi}',\boldsymbol{\delta}',Z',\Delta',\Phi'\right)\right] \quad \text{(A7)}\\
s.t. &: \\
\sum_{i=1}^{I} u_i &\leq u \quad \text{(A8)}\\
h_i &= \Phi\phi_i m\left(u_i,v_i\right) \quad \text{(A9)}\\
e_i' &= \left(1-\Delta\right)\left(1-\delta_i\right)\left(e_i+h_i\right) \quad \text{(A10)}\\
u' &= \ell'-\sum_{i=1}^{I} e_i' \quad \text{(A11)}\\
u_i &\in \left[0,u\right],\ell'\in\left[0,1\right], \quad \text{(A12)}
\end{aligned}
$$

$$
\Gamma_{Z,\Delta,\Phi}\left(Z',\Delta',\Phi';Z,\Delta,\Phi\right),\ \Gamma_{\mathbf{v}}\left(\mathbf{v}';\mathbf{v},Z',\Delta',\Phi',\mathbf{z}'\right),\Gamma_{\phi}\left(\boldsymbol{\phi}';\boldsymbol{\phi}\right),\Gamma_{\mathbf{z}}\left(\mathbf{z}';\mathbf{z}\right),\Gamma_{\delta}\left(\boldsymbol{\delta}',\boldsymbol{\delta}\right) \quad \text{(A13)}
$$

The choice of how many unemployed workers u_i to allocate in the i market yields the first-order condition

$$
Z\left(z_i-\zeta\right)\Phi\phi_i m_{u_i}\left(\frac{v_i}{u_i}\right)+\beta\mathbb{E}\left[-V_u'\left(\cdot\right)+V_{e_i}'\left(\cdot\right)\right]\left(1-\Delta\right)\left(1-\delta_i\right)\Phi\phi_i m_{u_i}\left(\frac{v_i}{u_i}\right)=\mu, \quad \text{(A14)}
$$

where μ is the multiplier on constraint $(A8)$. The Envelope conditions with respect to the states u and e_i yield:

$$
\begin{aligned}
V_u\left(u,\mathbf{e};\mathbf{z},\mathbf{v},\boldsymbol{\phi},\boldsymbol{\delta},Z,\Delta,\Phi\right) &= \mu-\xi \quad \text{(A15)}\\
V_{e_i}\left(u,\mathbf{e};\mathbf{z},\mathbf{v},\boldsymbol{\phi},\boldsymbol{\delta},Z,\Delta,\Phi\right) &= Z\left(z_i-\zeta\right)+\beta(1-\Delta)\left(1-\delta_i\right)\mathbb{E}\left[V_{e_i}'-V_u'\right]. \quad \text{(A16)}
\end{aligned}
$$

According to the first Envelope condition, the marginal value of an unemployed to the planner equals the shadow value of being available to search (μ) net of the disutility of search ξ. The second condition states that the marginal value of an employed worker is its flow output this period, net of the foregone output from non-employment, plus its discounted continuation value net of the value of search, conditional on the match not being destroyed.

The optimal decision on the labor force size next period ℓ' requires

$$
\mathbb{E}\left[V_u\left(u',\mathbf{e}';\mathbf{z}',\mathbf{v}',\boldsymbol{\phi}',\boldsymbol{\delta}',Z',\Delta',\Phi'\right)\right]=0. \quad \text{(A17)}
$$

By combining $(A17)$ with $(A15)$, we note that the planner will choose the size of the labor force so that the expected shadow value of an unemployed worker $\mathbb{E}\left[\mu'\right]$ equals search disutility ξ (note that ζ does not feature in this equality because both unemployed job-seekers and non-participants produce ζZ).[2]

[2]It is clear that our result is robust to allowing ξ to be stochastic and correlated with (Z,Δ,Φ).

Using $(A17)$ into the Envelope condition $(A16)$, and exploiting the additional assumption that all the elements of the vector $\mathbf{x} = (Z, 1-\Delta, z_i, 1-\delta_i)$ are independent martingales, iterating forward we arrive at:

$$\mathbb{E}\left[V'_{e_i}\right] = \frac{Z\left(z_i - \zeta\right)}{1 - \beta\left(1-\Delta\right)\left(1-\delta_i\right)} \tag{A18}$$

which, substituted into equation $(A14)$ yields

$$Z\left(z_i - \zeta\right)\Phi\phi_i m_{u_i}\left(\frac{v_i}{u_i}\right) + \frac{\beta\left(1-\Delta\right)\left(1-\delta_i\right)}{1-\beta\left(1-\Delta\right)\left(1-\delta_i\right)} Z\left(z_i - \zeta\right)\Phi\phi_i m_{u_i}\left(\frac{v_i}{u_i}\right) = \mu. \tag{A19}$$

Rearranging, we conclude that the planner allocates idle labor to equalize

$$\frac{z_i - \zeta}{1-\beta\left(1-\Delta\right)\left(1-\delta_i\right)}\phi_i m_{u_i}\left(\frac{v_i}{u_i^*}\right) \tag{A20}$$

across sectors, which is expression (2) in Section (1.2) in the main text. Finally, we note that to guarantee and interior solution, i.e., a positive measure of unemployed workers in each sector, we must impose that the lower bound of the distribution of z_i exceeds ζ.

A.3 Endogenous separations

We now allow the planner to move workers employed in sector i into unemployment (or out of the labor force) at the end of the period, before choosing the size of the labor force for next period. There are two changes to the planner's problem of Section $A.2$. First, the law of motion for employment becomes

$$e'_i = \left(1-\Delta\right)\left(1-\delta_i\right)\left(e_i + h_i\right) - \sigma_i. \tag{A21}$$

Second, the planner has another vector of choice variables $\{\sigma_i\}$, with $\sigma_i \in \left[0, \left(1-\Delta\right)\left(1-\delta_i\right)\left(e_i+h_i\right)\right]$.

The decision of how many workers to separate from sector i employment into unemployment is:

$$\mathbb{E}\left[V'_u\left(\cdot\right) - V'_{e_i}\left(\cdot\right)\right] \begin{cases} < 0 & \to \sigma_i = 0 \\ = 0 & \to \sigma_i \in \left(0, \left(1-\Delta\right)\left(1-\delta_i\right)\left(e_i+h_i\right)\right) \\ > 0 & \to \sigma_i = \left(1-\Delta\right)\left(1-\delta_i\right)\left(e_i+h_i\right) \end{cases} \tag{A22}$$

depending on whether at the optimum a corner or interior solution arises. If the first-order conditions $(A17)$ hold with equality, then the optimality condition $(A22)$ holds with the " $<$ " inequality and $\sigma_i = 0$. As a result, the planner's allocation rule (2) remains unchanged.

A.4 Heterogeneous sensitivities to the aggregate shock

Let productivity in sector i be Z^{η_i} and let $\log Z$ follow a unit root process with innovation ϵ independent of Δ and distributed as $N(-\sigma_\epsilon/2, \sigma_\epsilon)$. Note that $\mathbb{E}\left[(Z')^{\eta_i}\right] = Z^{\eta_i} \exp\left(\eta_i(\eta_i - 1)\frac{\sigma_\epsilon}{2}\right)$. denote $\Omega_i \equiv \exp\left(\eta_i(\eta_i - 1)\frac{\sigma_\epsilon}{2}\right)$. We maintain that $(1-\Delta, 1-\delta_i)$ follow unit root processes. Using $(A17)$ into the Envelope condition $(A16)$ yields

$$V_{e_i} = Z^{\eta_i} - \zeta Z + \beta(1-\Delta)(1-\delta_i)\mathbb{E}\left[V'_{e_i}\right]. \tag{A23}$$

Solving (A23) forward by using the unit root assumption, we obtain

$$\mathbb{E}\left[V'_{e_i}\right] = \frac{Z^{\eta_i}\Omega_i}{1-\beta(1-\Delta)(1-\delta_i)\Omega_i} - \frac{\zeta Z}{1-\beta(1-\Delta)(1-\delta_i)}.$$

Substituting this expression for $\mathbb{E}\left[V'_{e_i}\right]$ into equation $(A14)$ and rearranging, we conclude that the planner allocates unemployed workers so to equalize

$$\left[\frac{Z^{\eta_i}}{1-\beta(1-\Delta)(1-\delta_i)\Omega_i} - \frac{\zeta Z}{1-\beta(1-\Delta)(1-\delta_i)}\right]\phi_i m_{u_i}\left(\frac{v_i}{u_i^*}\right),$$

across sectors, which is expression (3) in Section (1.3) in the main text. Since η_i could be larger than one, a necessary additional technical condition we must impose is $\beta(1-\Delta)(1-\delta_i)\Omega_i < 1$ for all i.

A.5 Properties of the mismatch index

First, we prove that $0 \leq \mathcal{M}_{\phi t} \leq 1$. Since all the components of the sum in (8) are positive, $\mathcal{M}_{\phi t} \leq 1$. Under maximal mismatch (no markets where unemployment and vacancies coexist), the index is exactly equal to one. To show that $\mathcal{M}_{\phi t} \geq 0$, note that

$$\begin{aligned}
1 - \mathcal{M}_{\phi t} &= \frac{1}{v_t^\alpha u_t^{1-\alpha}} \frac{1}{\left[\sum_{i=1}^I \phi_{it}^{\frac{1}{\alpha}}\left(\frac{v_{it}}{v_t}\right)\right]^\alpha} \sum_{i=1}^I \left(\phi_{it}^{\frac{1}{\alpha}} v_{it}\right)^\alpha (u_{it})^{1-\alpha} \\
&\leq \frac{1}{v_t^\alpha u_t^{1-\alpha}} \frac{1}{\left[\sum_{i=1}^I \phi_{it}^{\frac{1}{\alpha}}\left(\frac{v_{it}}{v_t}\right)\right]^\alpha} \left[\sum_{i=1}^I \left(\phi_{it}^{\frac{1}{\alpha}} v_{it}\right)\right]^\alpha \left(\sum_{i=1}^I u_{it}\right)^{1-\alpha} \\
&= 1
\end{aligned}$$

where the $\leq$ sign follows from Hölder's inequality. It is easy to show that the index becomes exactly zero in absence of mismatch by substituting the allocation rule (7) into the index.

By inspecting (8), it is also easy to see that the $\mathcal{M}_{\phi t}$ index is invariant to "pure" aggregate shocks that shift the total number of vacancies and unemployed up or down, but leave the vacancy and unemployment shares across markets unchanged.

To show that the mismatch index is increasing in the level of disaggregation, consider an economy where the aggregate labor market is described by two dimensions indexed by (i, j), e.g., I regions $\times$ J occupations. Let $\mathcal{M}_{\phi It}$ be the mismatch index over the I sectors and $\mathcal{M}_{\phi IJt}$ be the one over the $I \times J$ sectors. From the disaggregated matching function, we have $h_{ijt} = \Phi_t \phi_{ijt} v_{ijt}^{\alpha} u_{ijt}^{1-\alpha}$. Summing this expression over j yields

$$h_{it} = \sum_{j=1}^{J} \Phi_t \phi_{ijt} v_{ijt}^{\alpha} u_{ijt}^{1-\alpha} = \Phi_t \left[\sum_{j=1}^{J} \phi_{ijt} \left(\frac{v_{ijt}}{v_{it}} \right)^{\alpha} \left(\frac{u_{ijt}}{u_{it}} \right)^{1-\alpha} \right] v_{it}^{\alpha} u_{it}^{1-\alpha}. \tag{A24}$$

At the aggregated level, we have $h_{it} = \Phi_t \phi_{it} v_{it}^{\alpha} u_{it}^{1-\alpha}$ and therefore (A24) implies that

$$\phi_{it} = \sum_{j=1}^{J} \phi_{ijt} \left(\frac{v_{ijt}}{v_{it}} \right)^{\alpha} \left(\frac{u_{ijt}}{u_{it}} \right)^{1-\alpha}. \tag{A25}$$

Now consider the disaggregated matching index. We have

$$1 - \mathcal{M}_{\phi IJt} = \sum_{i=1}^{I} \sum_{j=1}^{J} \frac{\phi_{ijt}}{\bar{\phi}_{IJt}} \left(\frac{v_{ijt}}{v_t} \right)^{\alpha} \left(\frac{u_{ijt}}{u_t} \right)^{1-\alpha} \tag{A26}$$

for

$$\bar{\phi}_{IJt} = \left[\sum_{i=1}^{I} \sum_{j=1}^{J} \phi_{ijt}^{\frac{1}{\alpha}} \left(\frac{v_{ijt}}{v_t} \right) \right]^{\alpha}. \tag{A27}$$

Manipulating the above expression yields

$$\begin{aligned} 1 - \mathcal{M}_{\phi IJt} &= \frac{1}{\bar{\phi}_{IJt} v_t^{\alpha} u_t^{1-\alpha}} \sum_{i=1}^{I} \sum_{j=1}^{J} \phi_{ijt} v_{ijt}^{\alpha} u_{ijt}^{1-\alpha} \\ &= \frac{1}{\bar{\phi}_{IJt} v_t^{\alpha} u_t^{1-\alpha}} \sum_{i=1}^{I} v_{it}^{\alpha} u_{it}^{1-\alpha} \sum_{j=1}^{J} \phi_{ijt} \left(\frac{v_{ijt}}{v_{it}} \right)^{\alpha} \left(\frac{u_{ijt}}{u_{it}} \right)^{1-\alpha} \\ &= \frac{1}{\bar{\phi}_{IJt}} \sum_{i=1}^{I} \phi_{it} \left(\frac{v_{it}}{v_t} \right)^{\alpha} \left(\frac{u_{it}}{u_t} \right)^{1-\alpha} \end{aligned}$$

where the third step above follows from (A25). Next, manipulating (A27) delivers

$$\begin{aligned} \bar{\phi}_{IJt} &= \left\{ \frac{1}{v_t} \sum_{i=1}^{I} v_{it} \left(\left[\sum_{j=1}^{J} \phi_{ijt}^{\frac{1}{\alpha}} \left(\frac{v_{ijt}}{v_{it}} \right) \right]^{\alpha} \right)^{\frac{1}{\alpha}} \right\}^{\alpha} \\ &= \left\{ \frac{1}{v_t} \sum_{i=1}^{I} v_{it} \left(\left[\sum_{j=1}^{J} \phi_{ijt}^{\frac{1}{\alpha}} \left(\frac{v_{ijt}}{v_{it}} \right) \right]^{\alpha} \cdot \left[\sum_{j=1}^{J} \frac{u_{ijt}}{u_{it}} \right]^{1-\alpha} \right)^{\frac{1}{\alpha}} \right\}^{\alpha} \end{aligned}$$

where the second step above follows from the identity $\sum_{j=1}^{J} u_{ijt} = u_{it}$. Applying Holder's inequality yields

$$\begin{aligned}\bar{\phi}_{IJt} &\geq \left\{ \frac{1}{v_t} \sum_{i=1}^{I} v_{it} \left(\sum_{j=1}^{J} \phi_{ijt} \left(\frac{v_{ijt}}{v_{it}} \right)^{\alpha} \left(\frac{u_{ijt}}{u_{it}} \right)^{1-\alpha} \right)^{\frac{1}{\alpha}} \right\}^{\alpha} \\ &= \left\{ \sum_{i=1}^{I} \phi_{it}^{\frac{1}{\alpha}} \left(\frac{v_{it}}{v_t} \right) \right\}^{\alpha} = \bar{\phi}_{It}\end{aligned}$$

where $\bar{\phi}_{It}$ is an expression equivalent to $\bar{\phi}_{IJt}$ in (A27) for the case where the $(I \times J)$ sectors are collapsed into I sectors. Combining results, we have shown that

$$1 - \mathcal{M}_{\phi IJt} \leq \sum_{i=1}^{I} \frac{\phi_{it}}{\bar{\phi}_{It}} \left(\frac{v_{it}}{v_t} \right)^{\alpha} \left(\frac{u_{it}}{u_t} \right)^{1-\alpha} = 1 - \mathcal{M}_{\phi It}$$

and so we must have $\mathcal{M}_{\phi IJt} \geq \mathcal{M}_{\phi It}$.

A.6 Planner's problem with endogenous vacancies

Optimal vacancy creation Consider the planner's problem of Section 1.2 solved in Appendix A.2, the most general of our environments. To simplify the notation, without loss of generality, let z_i denote output in sector i net of the flow output from nonemployment ζ. If we let the creation of vacancies $\{v_i\}$ be under the control of the planner, we have:

$$\begin{aligned}
V(u, \mathbf{e}; \mathbf{z}, \phi, \delta, \kappa, Z, \Delta, \Phi) &= \max_{\{u_i, v_i, \ell'\}} \sum_{i=1}^{I} Z z_i (e_i + h_i) - K_i(v_i) - \xi u \\
&+ \beta \mathbb{E}\left[V(u', \mathbf{e}'; \mathbf{z}', \phi', \delta', \kappa', Z', \Delta', \Phi')\right] && \text{(A28)} \\
s.t. &: \\
\sum_{i=1}^{I} u_i &\le u && \text{(A29)} \\
h_i &= \Phi \phi_i m(u_i, v_i) && \text{(A30)} \\
e_i' &= (1-\Delta)(1-\delta_i)(e_i + h_i) && \text{(A31)} \\
u' &= \ell' - \sum_{i=1}^{I} e_i' && \text{(A32)} \\
u_i &\in [0, u], \ell' \in [0,1], v_i \ge 0 && \text{(A33)}
\end{aligned}$$

$$\Gamma_{Z,\Delta,\Phi}(Z', \Delta', \Phi'; Z, \Delta, \Phi),\ \Gamma_\phi(\phi'; \phi), \Gamma_{\mathbf{z}}(\mathbf{z}'; \mathbf{z}), \Gamma_\delta(\delta', \delta), \Gamma_\kappa(\kappa', \kappa) \qquad \text{(A34)}$$

The optimality condition for vacancy creation is

$$K_{v_i}(v_i^*) = \Phi \phi_i m_{v_i}\left(\frac{v_i^*}{u_i^*}\right)\left\{Z z_i + \beta (1-\Delta)(1-\delta_i) \mathbb{E}\left[V_{e_i}'(\cdot)\right]\right\}.$$

Using the expression for $\mathbb{E}\left[V_{e_i}'(\cdot)\right]$ obtained in $(A18)$ and the functional forms for K_i and m specified in the main text, we obtain expression (16).

Calculation of planner's vacancies We now lay out an algorithm to compute the planner's optimal allocation of vacancies across sectors. Rearranging condition $(A20)$ dictating the optimal allocation of unemployed workers across sectors, given the distribution of vacancies $\{v_i^*\}$, yields

$$\frac{v_i^*}{u_i^*} = \left[\frac{\mu}{1-\alpha} \frac{1}{\frac{Z z_i \Phi \phi_i}{1-\beta(1-\Delta)(1-\delta_i)}}\right]^{\frac{1}{\alpha}} \qquad \text{(A35)}$$

where μ is the multiplier on the resource constraint $\sum_{i=1}^{I} u_i \le u$. Substituting $(A35)$ into (16) yields an equation for the optimal number of vacancies in sector i which reads

$$v_i^* = \frac{1}{\kappa_i}\left(\frac{\alpha}{1-\alpha}\right)^{1/\varepsilon}\left(\frac{1}{\mu}\right)^{\frac{(1-\alpha)/\varepsilon}{\alpha}}\left[(1-\alpha)\frac{Z z_i \Phi \phi_i}{1-\beta(1-\Delta)(1-\delta_i)}\right]^{\frac{1/\varepsilon}{\alpha}}. \qquad \text{(A36)}$$

Summing over all $i's$, we arrive at the optimal share of vacancies in sector i

$$\frac{v_i^*}{v_t^*} = \frac{\frac{1}{\kappa_i}\left[\frac{z_i\phi_i}{1-\beta(1-\Delta)(1-\delta_i)}\right]^{\frac{1/\varepsilon}{\alpha}}}{\sum_{i=1}^{I}\frac{1}{\kappa_i}\left[\frac{z_i\phi_i}{1-\beta(1-\Delta)(1-\delta_i)}\right]^{\frac{1/\varepsilon}{\alpha}}} \tag{A37}$$

only as a function of parameters, which is quite intuitive: the higher is productive, matching and job creation efficiency in sector i, relative to the other sectors, the larger its share of vacancies. However, to solve the model, we need to determine the *level* of v_i^* which requires eliminating μ from $(A36)$. Combining again the two first order conditions, and summing across all sectors, we arrive at

$$u^* = \left(\frac{\alpha}{1-\alpha}\right)^{1/\varepsilon}\left[Z\Phi\left(1-\alpha\right)\right]^{\frac{1+1/\varepsilon}{\alpha}}\left(\frac{1}{\mu}\right)^{\frac{1+(1-\alpha)/\varepsilon}{\alpha}}\cdot\sum_{i=1}^{I}\frac{1}{\kappa_i}\left[\frac{z_i\phi_i}{1-\beta\left(1-\Delta\right)\left(1-\delta_i\right)}\right]^{\frac{1+1/\varepsilon}{\alpha}} \tag{A38}$$

which establishes a unique inverse relationship between μ and u^*: the higher the number of idle workers, the lower the shadow value of the constraint.

Equation $(A38)$ suggests an algorithm to solve for v_i^*. At any date, before choosing how to allocate vacancies and unemployed workers, the total number of idle workers is a state variable for the planner, i.e., u^* is known. One can therefore back out μ from $(A38)$, and then v_i^* from $(A36)$ and u_i^* from $(A35)$.

Counterfactual unemployment To perform the counterfactual on unemployment with endogenous vacancies, we use the same iterative procedure described in Section 2.2, with the caveat that the relationship between the planner's job-finding rate and the empirical job-finding rate at date t is now given by

$$f_t^* = \frac{h_t^*}{u_t^*} = \Phi_t\bar{\phi}_{xt}^*\left(\frac{v_t^*}{u_t^*}\right)^{\alpha} = f_t\cdot\frac{1}{1-\mathcal{M}_{xt}}\cdot\left(\frac{u_t}{u_t^*}\right)^{\alpha}\cdot\left[\left(\frac{\bar{\phi}_{xt}^*}{\bar{\phi}_{xt}}\right)\cdot\left(\frac{v_t^*}{v_t}\right)^{\alpha}\right], \tag{A39}$$

where $\bar{\phi}_{xt}$ is given by equation (11), and $\bar{\phi}_{xt}^*$ is the same aggregator with shares (v_{it}^*/v_t^*) instead of (v_{it}/v_t). When $v_{it}^* = v_{it}$ (i.e., $\varepsilon\to\infty$), equation $(A39)$ collapses to the relationship $f^* = \left[f/\left(1-\mathcal{M}_{xt}\right)\right]\left(u_t/u_t^*\right)^{\alpha}$ that we have used in our baseline counterfactual with exogenous vacancies.

A.7 Model with heterogeneous α

We now extend the model of Section 1.2 and introduce sector-specific matching functions m_i. We retain the constant-return Cobb-Douglas specification, but we allow the vacancy share α (and hence the unemployment share $1-\alpha$) to vary across sectors, i.e., hires in sector i at date t are now given by the matching function

$$h_{it} = \Phi_t\phi_{it}v_{it}^{\alpha_i}u_{it}^{1-\alpha_i}. \tag{A40}$$

By replicating all the steps outlined in Section A.2, we arrive at the set of I first-order conditions (one for each sector i):

$$(1-\alpha_i)\,\frac{(z_{it}-\zeta)}{1-\beta\,(1-\Delta_t)\,(1-\delta_{it})}\left(\frac{v_{it}}{u^*_{it}}\right)^{\alpha_i}=\mu_t \tag{A41}$$

which, together with the adding-up constraint $\sum_{i=1}^{I} u_{it}=u_t$, yields a system of $(I+1)$ equations in $(I+1)$ unknowns $\left\{\{u^*_{it}\}_{i=1}^{I},\mu_t\right\}$ at every date t, which can be solved numerically.

Since optimal hires are $h^*_{it}=\Phi_t\phi_{it}v_{it}^{\alpha_i}\left(u^*_{it}\right)^{1-\alpha_i}$, the mismatch index at t is

$$\mathcal{M}_{xt}=1-\frac{h_t}{h^*_t}=1-\frac{\sum_{i=1}^{I}\phi_{it}v_{it}^{\alpha_i}u_{it}^{1-\alpha_i}}{\sum_{i=1}^{I}\phi_{it}v_{it}^{\alpha_i}\left(u^*_i\right)^{1-\alpha_i}}.$$

Even if this mismatch index has no longer a closed form, it is easy to compute once we have the vector of planner's allocations of unemployed workers across sectors $\{u^*_{it}\}$. The counterfactual unemployment rate is still obtained as described in Section 3.2 of the paper.

B Data Appendix

B.1 Comparison between JOLTS and HWOL vacancies

Vacancies recorded in JOLTS are derived from a sample of about 16,000 business establishments. JOLTS vacancies represent "all unfilled, posted positions available at an establishment on the last day of the month. The vacancy must be for a specific position where work can start within thirty days, and an active recruiting process must be underway for the position." (Faberman, 2009, p. 86). As noted in Section 3, the HWOL database collects ads from job listings posted by employers on thousands of internet job boards and online newspapers. The HWOL program uses a mid-month survey reference period. For example, data for October would be the sum of all posted ads from September 14th through October 13th. This reference period is aligned to the BLS unemployment "job search" time period. The monthly vacancy counts that we use in our calculations are total monthly unduplicated ads appearing in the reference period. This figure therefore includes both newly posted ads and ads reposted from the previous months.

Sampled establishments in the JOLTS only report their own direct employees and exclude "employees of temporary help agencies, employee leasing companies, outside contractors, and consultants," which are counted by their employer of record, not by the establishment where they are working.[3] Thus, this approach captures temp-help and leasing workers as long as their employers are sampled in the JOLTS, but does not capture the self-employed contract workforce (these workers typically receive a 1099-MISC form instead of a W-2 form to report payments received for services they provide). On the other hand, the HWOL series includes postings for contract work. In what follows, we often also report HWOL vacancy counts excluding contract work, to make the series more comparable to the JOLTS measure of vacancies, but in our empirical analyses of mismatch with HWOL data we consider all ads, including those for contract work.

We perform two exercises to compare the vacancy counts we get from each data source, one at the regional level and one at the industry level—region and industry are the only dimensions available in both JOLTS and HWOL. First, we compare total vacancies by Census region in Figure C1. The HWOL series tend to be lower than the JOLTS series before 2008 (especially in the South), and higher from 2008 onwards (especially in the Northeast). The two series are closest in the West: here the correlation between the HWOL and JOLTS series is 0.94. In the other three regions the correlation is lower: 0.27 in the Midwest, 0.40 in the South, and 0.54 in the Northeast. Our re-weighing strategy in Section 6 enables us to correct for the possibility that online ads penetration may differ across regions.

For about 57% of the job listings, we observe the NAICS code of the employer. There-

[3]See the JOLTS Technical Note at http://www.bls.gov/news.release/jolts.tn.htm.

fore, we are able to directly compare vacancy counts by industry from HWOL to those in the JOLTS. We report in Figure C2 scatterplots of vacancy shares by industry from JOLTS and from HWOL—for the latter, we report both total vacancies, as well as vacancies without contract work. The top panel of the figure reports average vacancy shares over the sample period under consideration. Most data points are close to the 45-degree line, indicating that the vacancy shares by industry in the two series line up fairly well, especially when we omit contract work from HWOL to make it more comparable to the JOLTS. The only two sectors where JOLTS and HWOL show significant differences in vacancy shares are "Public Administration" and "Accommodation and Food Services." The bottom panel reports the change in average vacancy shares between 2006 and the 12 month period around December 2009 for each series. Again, the JOLTS and HWOL series are quite close to each other, with the exception of "Public Administration."

We have investigated whether the missing industry information in HWOL exhibits any systematic patterns that may have skewed our analysis. For robustness, we re-weighted the industry observations in HWOL as follows: first, we dropped observations from individual Job Boards with the highest rates of missing NAICS codes. Then, we re-weighted the remaining observations to correct for any correlation between NAICS missing values and Job Board, occupation or Census region. In other words, if vacancies for specific (Job Board, SOC, Census region) combinations are more likely to have missing NAICS codes, the vacancies that do have NAICS information in those cells are assigned a larger weight in computing total vacancies by industry.[4] The resulting vacancy shares are almost identical to those based on the raw data.

To sum up, the comparison between JOLTS and HWOL vacancy counts suggests that there are some discrepancies in the behavior of two series. The main concerns are (i) the possible over- or under-use of online advertisement in certain sectors (regions and/or industries) and (ii) the presence of an upward trend in the use of online recruitment that could artificially mitigate the drop in job advertisements around the last recession (and inflate the subsequent recovery). We address these issues in Section 6 and show that our quantitative results on mismatch measures are robust.

B.2 Matching function estimation

Throughout our analysis we assume matching functions are constant returns to scale. We begin by imposing a Cobb-Douglas specification. At the end of this section we show that, when we

[4]For example, suppose a (Job Board, SOC, Census region) cell has four observations. Observation one is in NAICS code 11, observations two and three are in NAICS code 13, and observation four has a missing NAICS. Thus, the missing NAICS rate is 0.25. Then, a weight of $1/(1-0.25) = 1.333$ is applied to each observation with non-missing NAICS. So we find 1.333 job vacancies in NAICS code 11, and 2.667 job vacancies in NAICS code 13.

allow for a more general CES specification, our results point towards an elasticity of substitution statistically close to one.

To compute market-specific matching efficiency parameters, ϕ_i, and vacancy share α, we use various data sources. At the industry level, we use vacancies and hires from JOLTS, and unemployment counts from the CPS. At the occupation level, we use vacancies from HWOL but do not have a direct measure of hires as in JOLTS. Therefore, we construct hires from the CPS using flows from unemployment into a given occupation i for people who are surveyed in adjacent months. Because these monthly flows are quite noisy, we use a 3-month moving average of the data, and aggregate occupations into five broad occupation groups. For comparison purposes, we replicate the analysis at the industry level using the constructed "CPS hires" as well. At the aggregate level, we perform the estimation using both JOLTS and HWOL vacancies, and both JOLTS and CPS hires.

The estimation of matching functions is subject to an endogeneity problem, as shocks to unobserved matching efficiency may affect the number of vacancies posted by firms—much like TFP shocks affect firm's choice of labor input. To deal with this issue, we follow two strategies suggested by Borowczyk-Martins, Jolivet, and Postel-Vinay (2012). First, they recognize that some of the major movements in matching efficiency inducing a bias in the OLS estimator are low-frequency ones. As a result, modeling explicitly the dynamics of matching efficiency through time-varying polynomials and structural breaks goes a long way towards solving the problem even with the simple OLS estimator. This is the first route we take. At the aggregate level, we estimate:

$$\log\left(\frac{h_t}{u_t}\right) = const + \gamma'\mathbf{QTT_t} + \alpha\log\left(\frac{\mathbf{v_t}}{\mathbf{u_t}}\right) + \epsilon_{\mathbf{t}}, \tag{B1}$$

where $\mathbf{QTT_t}$ is a vector of four elements for the quartic time trend which is meant to capture shifts in aggregate matching efficiency (i.e., Φ_t in the model).

At the sectoral level, we are interested in the sector-specific component of matching efficiency orthogonal to common aggregate movements in aggregate matching efficiency. Therefore, at the industry and 2-digit occupation level, we perform the following panel regression:

$$\log\left(\frac{h_{it}}{u_{it}}\right) = \gamma'\mathbf{QTT_t} + \chi_{\{\mathbf{t}\leq\mathbf{07}\}}\log\left(\phi_{\mathbf{i}}^{\mathbf{pre}}\right) + \chi_{\{\mathbf{t}>\mathbf{07}\}}\log\left(\phi_{\mathbf{i}}^{\mathbf{post}}\right) + \alpha\log\left(\frac{\mathbf{v_{it}}}{\mathbf{u_{it}}}\right) + \epsilon_{\mathbf{it}}, \tag{B2}$$

where $\chi_{\{t>07\}}$ is an indicator for months after December 2007, the official start of the recession, to absorb sector-specific shifts in matching efficiency.

Borowczyk-Martins, Jolivet and Postel-Vinay (2012) also propose a GMM estimator to take care of the simultaneity bias. This method requires imposing an ARMA(p,q) structure on the matching efficiency process: we follow their model selection protocol and set $p = 3$ and $q = 3$. We use an over-identified GMM estimator implemented with four lags of market tightness and one lag of the job-finding rate as instruments, as they argue it is the one delivering the most precise parameter estimates.

Table C4 displays the full set of estimates of the vacancy share parameter α. In the aggregate regressions, the estimated vacancy share varies between 0.32 and 0.67; in the panel regressions, the estimates are somewhat lower varying between 0.24 and 0.53. To construct our mismatch indices, and in our calculation of mismatch unemployment, we pick a value of $\alpha = 0.5$ for two reasons. First, it is the midpoint of our estimates with aggregate data. Second, our mismatch indices are typically highest for $\alpha = 0.5$; therefore, in the spirit of reporting an upper bound for mismatch unemployment, we use this value.

The estimated quartic time trend (not shown) drops during the recession in all our OLS specifications, consistent with a deterioration of aggregate matching efficiency. With regard to sectoral matching efficiency, in our baseline calculations we use the estimates obtained with JOLTS hires for the industry level mismatch analysis, and those with CPS hires for the occupation level analysis. In all cases, we use the *pre-recession* matching efficiency parameter estimates, and verify the robustness of our findings to this choice. The estimated matching efficiency parameters ϕ_i pre- and post-recession are reported in Tables C6-C8. Beyond movements in the common component Φ_t, the quartic in time, changes over time in sector-specific matching efficiencies are small.

Finally, in order to examine the plausibility of the Cobb-Douglas specification, we generalize (B2) and estimate the following CES specification via minimum distance:

$$\log\left(\frac{h_{it}}{u_{it}}\right) = \gamma'\mathbf{QTT_t} + \chi_{\{\mathbf{t\leq 07}\}}\log\left(\phi_{\mathbf{i}}^{\mathbf{pre}}\right) + \chi_{\{\mathbf{t>07}\}}\log\left(\phi_{\mathbf{i}}^{\mathbf{post}}\right) + \frac{\mathbf{1}}{\sigma}\log\left[\alpha\left(\frac{\mathbf{v_{it}}}{\mathbf{u_{it}}}\right)^{\sigma} + (\mathbf{1}-\alpha)\right] + \epsilon_{\mathbf{it}}. \tag{B3}$$

Recall that $\sigma \in (-\infty, 1)$ with $\sigma = 0$ being the Cobb-Douglas case. A simulated annealing algorithm is used to ensure that we attain a global minimum. 95% confidence intervals are computed via bootstrap methods. The estimation results are reported in Table C5. The point estimates of σ range from -0.11 to 0.18 depending on the specification, implying an elasticity between 0.9 and 1.2. In the specification with HWOL vacancies and CPS hires, we cannot reject the null that $\sigma = 0$ at the 5% significance level. In the other specifications with JOLTS data, $\sigma = 0$ lies just outside the 95% confidence interval, but the point estimates are close to zero, implying values close to unity for the elasticity of the matching function.

B.3 Adjustment in sectoral unemployment count

Let u_{it} be the number of unemployed workers at date t whose last job is in sector i, and U_{it} be the true number of unemployed actually searching in sector i at date t. Also let u_{it}^j be the number of unemployed whose last job is in sector i and who are searching in sector j. By definition, we have $u_{it} = \sum_{j=1}^{I} u_{it}^j$. The key unknown at each date t is the vector $\{U_{it}\}$.

From the panel dimension of CPS we observe h^j_{it}, the number of unemployed workers hired in sector j in period t whose last job was in sector i. Let the total number of hires in sector j in period t be h^j_t. Assume that the job-finding rate in sector j is the same for all unemployed, independent of the sector of provenance, with the sole exception if their previous job was in that same sector, in which case their job-finding rate is higher by a factor $\gamma_t \geq 1$, or:

$$\frac{h^j_{jt}}{u^j_{jt}} = (1+\gamma_t)\,\frac{h^j_{it}}{u^j_{it}}, \text{ for } i \neq j. \tag{B4}$$

The average hiring rate of sector j is the total number of hires for j divided by the total number of unemployed looking in sector j or:

$$\frac{h^j_t}{U_{jt}} = \sum_{i\neq j}\left(\frac{h^j_{it}}{u^j_{it}}\right)\left(\frac{u^j_{it}}{U_{jt}}\right) + \left(\frac{h^j_{jt}}{u^j_{jt}}\right)\left(\frac{u^j_{jt}}{U_{jt}}\right).$$

Substituting $(B4)$ into the above equation delivers:

$$\frac{h^j_t}{U_{jt}} = \sum_{i\neq j}\left(\frac{h^j_{it}}{u^j_{it}}\right)\left(\frac{u^j_{it}}{U_{jt}}\right) + (1+\gamma_t)\,\frac{h^j_{it}}{u^j_{it}}\left(\frac{u^j_{jt}}{U_{jt}}\right).$$

Because the ratio h^j_{it}/u^j_{it} is the same across all $i \neq j$, we can pull it out of the sum above and obtain, after rearranging:

$$\frac{h^j_{it}}{u^j_{it}} = \begin{cases} \left(\frac{h^j_t}{U_{jt}}\right)\left[1+\gamma_t\left(\frac{u^j_{jt}}{U_{jt}}\right)\right]^{-1} & \text{if } i \neq j \\ (1+\gamma_t)\left(\frac{h^j_t}{U_{jt}}\right)\left[1+\gamma_t\left(\frac{u^j_{jt}}{U_{jt}}\right)\right]^{-1} & \text{if } i = j \end{cases} \tag{B5}$$

Since we do not observe u^j_{jt}/U_{jt}, we want to substitute it out. Note that

$$\frac{u^j_{jt}}{U_{jt}} = \frac{\frac{h^j_{jt}}{h^j_t}\left(\frac{1}{1+\gamma_t}\right)}{1-\frac{h^j_{jt}}{h^j_t}\left(\frac{\gamma_t}{1+\gamma_t}\right)}$$

and using this expression in $(B5)$, we arrive at a relationship between the hiring rate from i to j and the average hiring rate in j:

$$\frac{h^j_{it}}{u^j_{it}} = \xi^j_{it}\cdot\frac{h^j_t}{U_{jt}} \tag{B6}$$

where

$$\xi^j_{it} = \begin{cases} 1-\frac{h^j_{jt}}{h^j_t}\left(\frac{\gamma_t}{1+\gamma_t}\right) & \text{if } i \neq j \\ (1+\gamma_t)\left[1-\frac{h^j_{jt}}{h^j_t}\left(\frac{\gamma_t}{1+\gamma_t}\right)\right] & \text{if } i = j \end{cases}$$

Rearranging equation $(B6)$ and summing across all j yields, at every t, the I equations:

$$u_{it} = \sum_{j=1}^{N} \frac{1}{\xi_{it}^{j}} \left(\frac{h_{it}^{j}}{h_{t}^{j}} \right) U_{t}^{j}$$

in the $(I+1)$ unknowns $\{U_{jt}\}, \gamma_t$. The last equation needed is the "aggregate consistency" condition

$$\sum_{j=1}^{I} U_{jt} = \sum_{j=1}^{I} u_{jt} \tag{B7}$$

stating that the true distribution of unemployed across sectors must sum to the observed total number of unemployed. We therefore have a system of $(I+1)$ equations in $(I+1)$ unknowns.

In our calculation of unemployment counts, to guarantee a non-negative solution to the linear system, we set to zero all entries in the transition matrices h_{it}^{j} which account for less than 5% of hires h_{t}^{j} in any given sector at any date t. We find that the estimated values of γ_t are all close to one.

B.4 Reweighting of HWOL vacancies

Let v_{irt}^{H} be the vacancies in the HWOL data for industry $i = 1, ..., I$ and region $r = 1, ..., R$ in month t. Let v_{irt}^{J} be the corresponding count for JOLTS vacancies. The objective is to reweigh monthly vacancies in HWOL to match those in JOLTS by industry and region (the only two common variables across data sets). We therefore solve, at every t, the following set of $(I \times R)$ equations

$$\begin{aligned} \sum_{i=1}^{I} v_{irt}^{H} \cdot \omega_{it} \cdot \omega_{rt} &= v_{rt}^{J} \\ \sum_{r=1}^{R} v_{irt}^{H} \cdot \omega_{it} \cdot \omega_{rt} &= v_{it}^{J} \end{aligned}$$

for the $(I \times R)$ vector of weights $\{\omega_{it}, \omega_{rt}\}$. Our solution algorithm imposes that weights must be positive, but this constraint is never binding in practice. Table C13 reports the average estimates of these weights over 2005-2006 and 2010-2011. We then compute reweighed vacancy counts by occupation o in month t as

$$v_{ot}^{H} = \sum_{i=1}^{I} \sum_{r=1}^{R} \omega_{it} \cdot \omega_{rt} \cdot v_{oirt}^{H}.$$

Our reweighed occupational mismatch index of Figure C26 is based on this revised vacancy count.

C Additional figures and tables

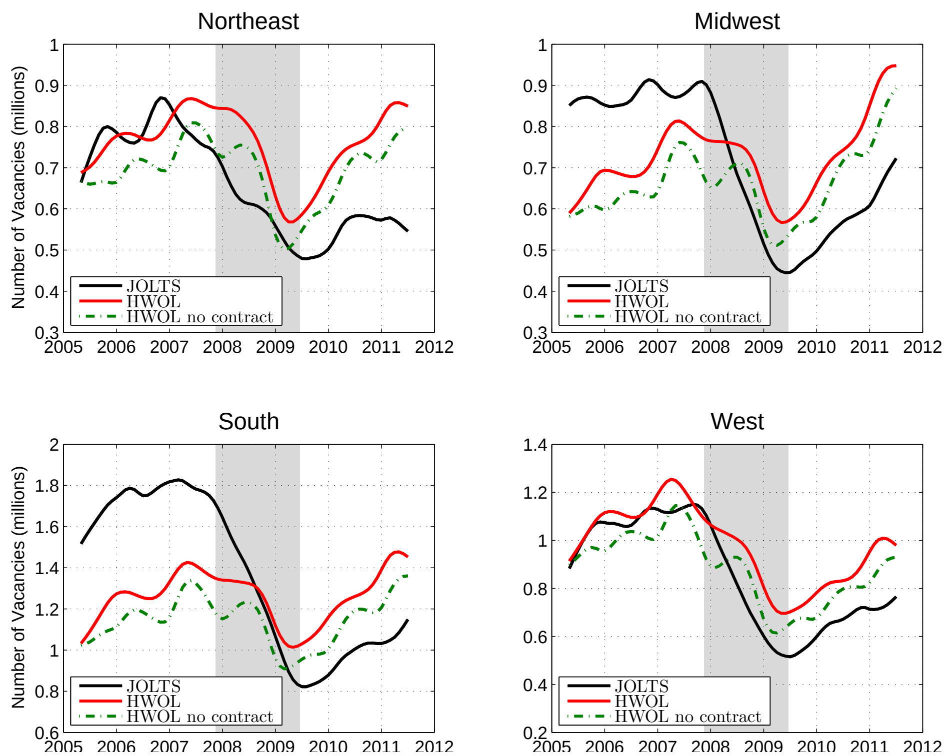
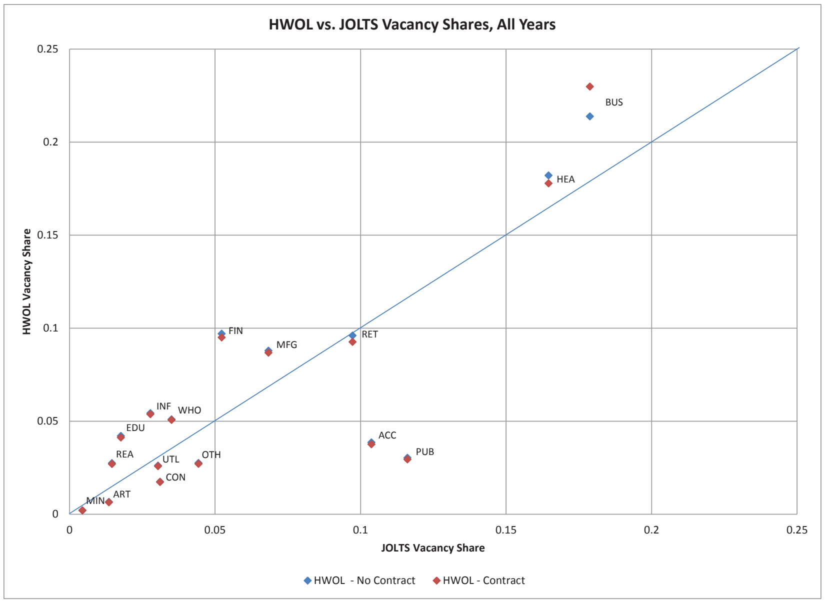


Figure C1: Comparison between the JOLTS and the HWOL (The Conference Board Help Wanted OnLine Data Series). Top-left panel: Northeast, Top-right panel: Midwest, Bottom-left panel: South, Bottom-right panel: West.



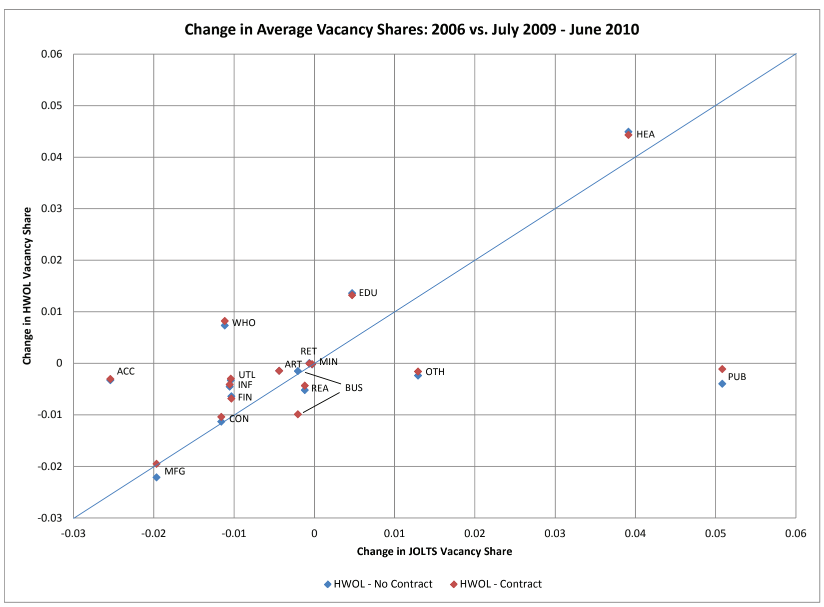


Figure C2: Top panel: comparison between vacancy shares in the JOLTS and HWOL (The Conference Board Help Wanted OnLine Data Series) for the May 2005 to June 2011 period. Bottom panel: change in average vacancy shares from 2006 to July 2009-June 2010 in the JOLTS and the HWOL. See Table C1 for an explanation of industry labels.

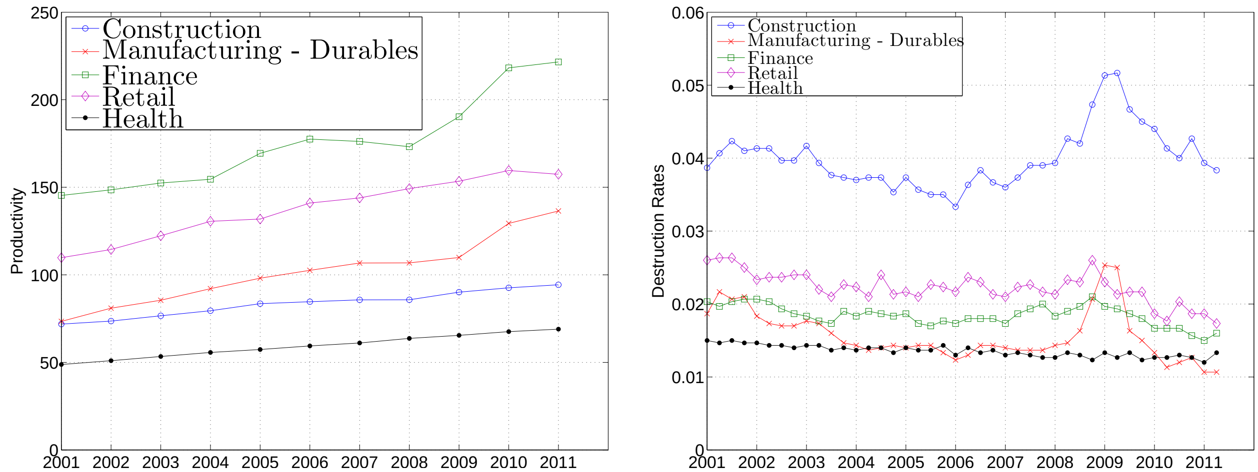


Figure C3: Productivity levels (left panel) and job destruction rates (right panel) for selected industries. Source: BEA and BLS for productivity levels and BED for job destruction rates.

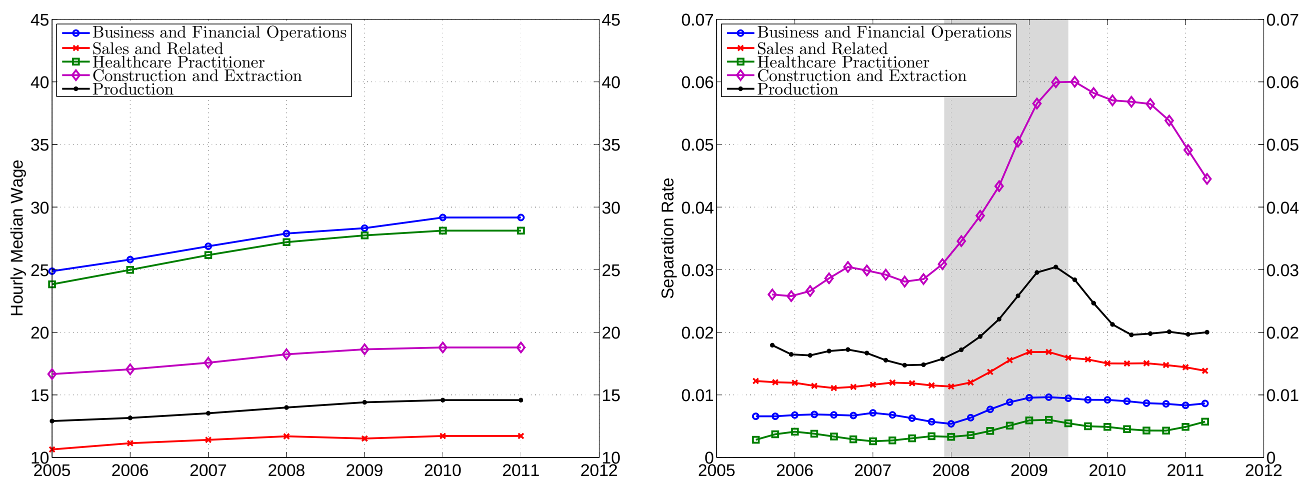


Figure C4: Wages (left panel) and job separation rates (right panel) for selected occupations. Source: OES for wages and CPS for job separation rates.

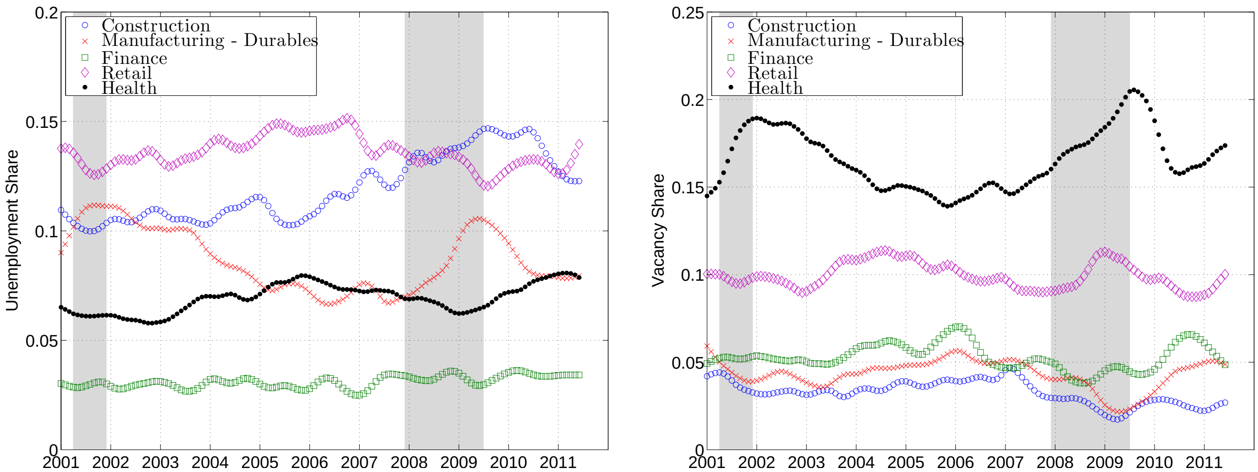


Figure C5: Unemployment and vacancy shares by selected industry.

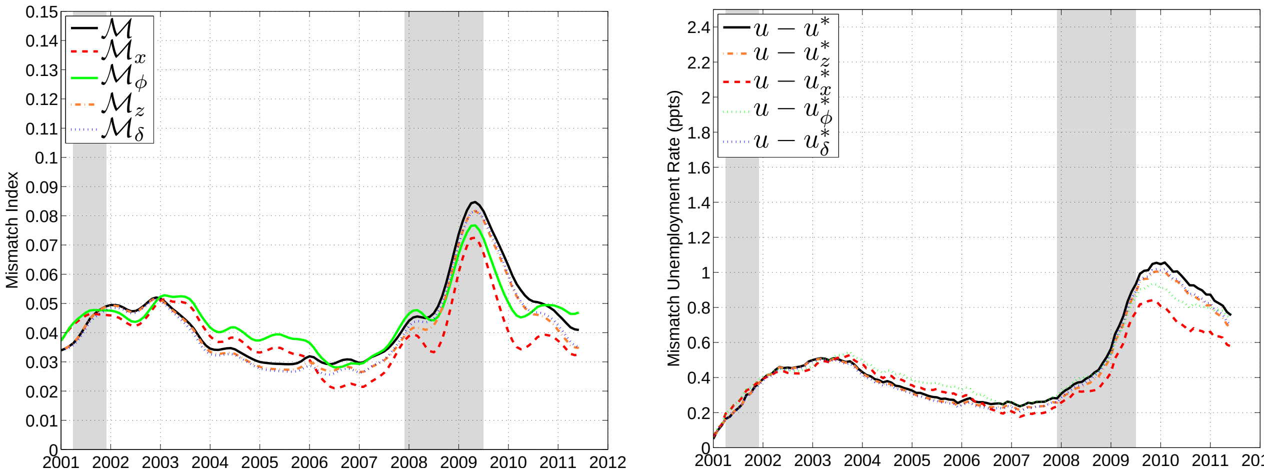


Figure C6: Mismatch indexes $\mathcal{M}_t$, $\mathcal{M}_{xt}$, $\mathcal{M}_{\phi t}$, $\mathcal{M}_{zt}$, and $\mathcal{M}_{\delta t}$ by industry (left panel) and the corresponding mismatch unemployment rates (right panel).

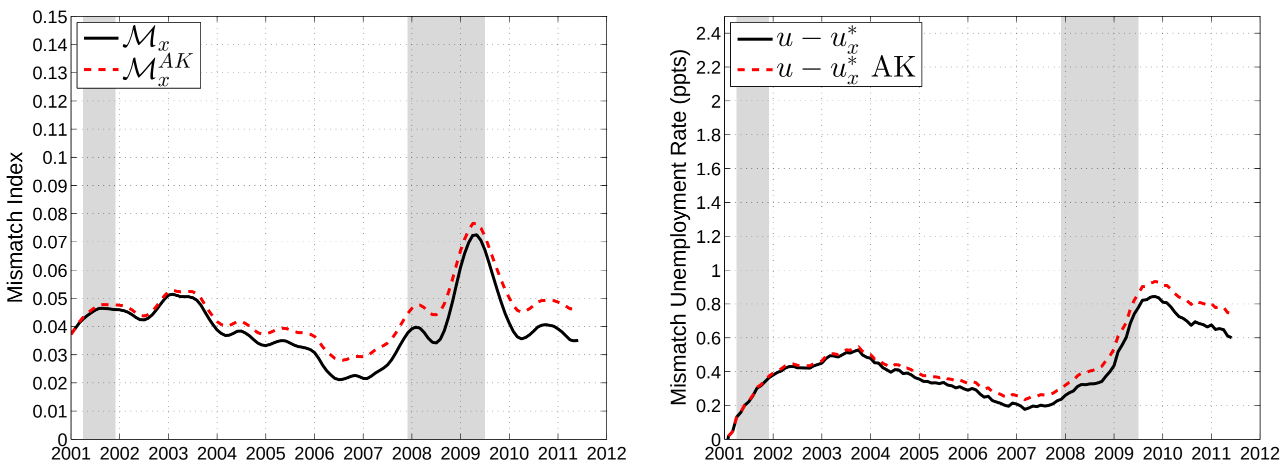


Figure C7: Mismatch indexes $\mathcal{M}_{xt}$ by industry (left panel) and corresponding mismatch unemployment rates (right panel) for the baseline specification and with the Abraham-Katz (AK) specification with heterogenous sensitivities to aggregate shocks.

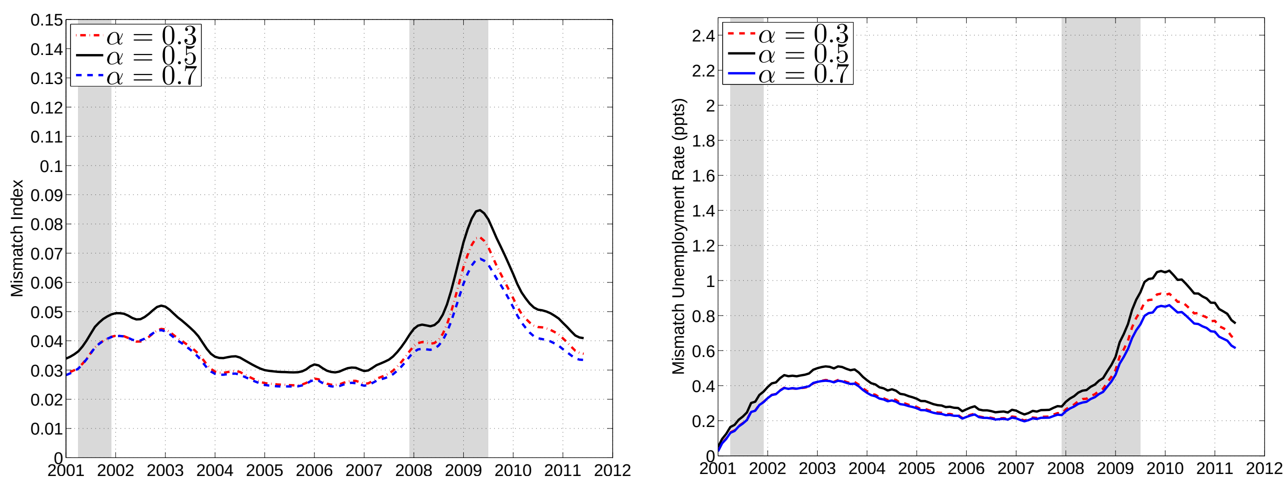


Figure C8: Mismatch index $\mathcal{M}_t$ by industry (left panel) and the corresponding mismatch unemployment rates (right panel) for various values of α, the vacancy share parameter in the matching function

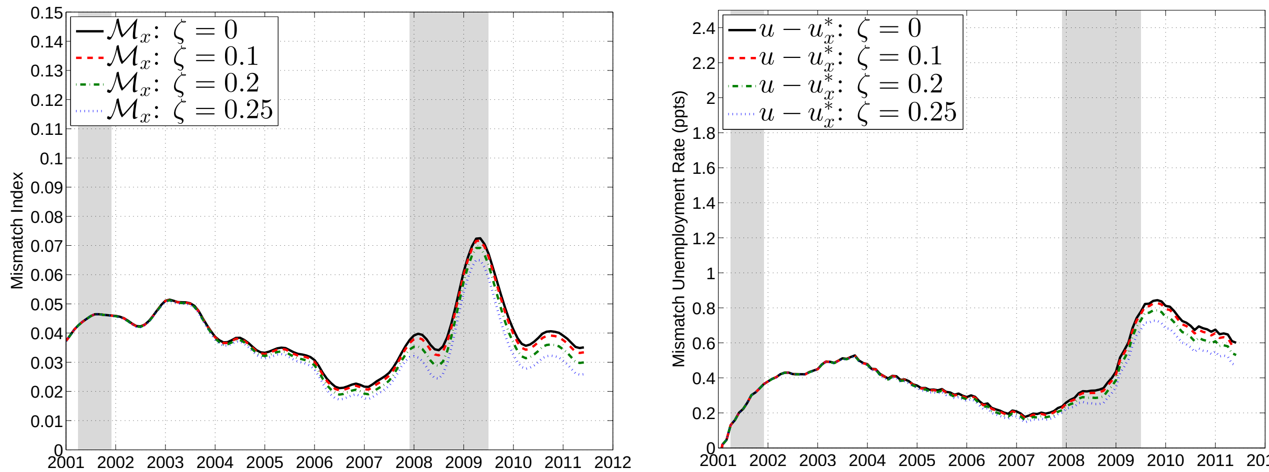


Figure C9: Mismatch index $\mathcal{M}_t$ by industry for different values of the utility flow from non-employment ζ (left panel) and the corresponding mismatch unemployment rates (right panel).

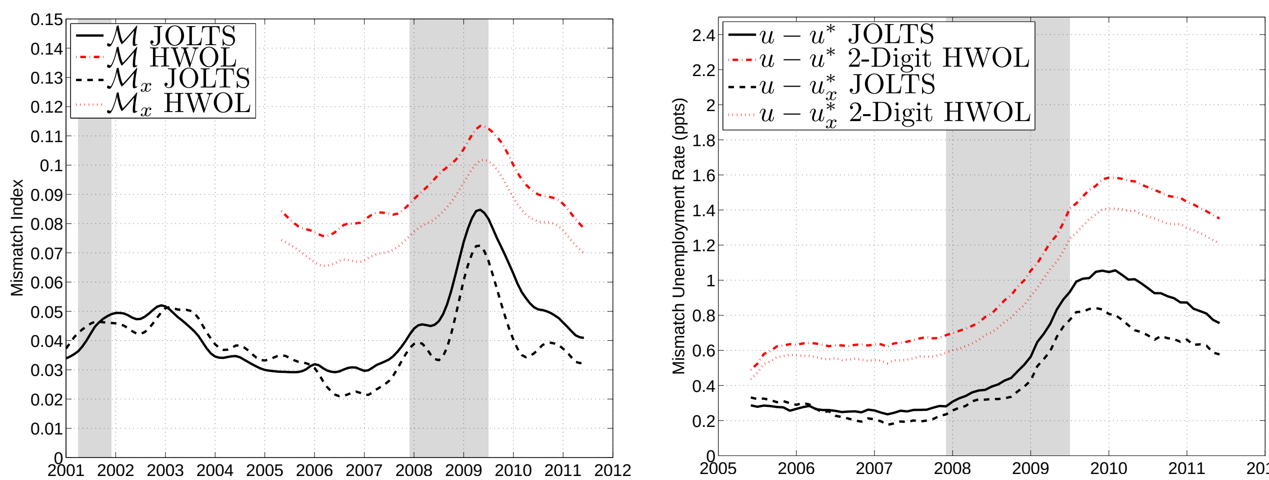


Figure C10: Mismatch indexes $\mathcal{M}_t$ (left panel) and the corresponding mismatch unemployment rates (right panel) across industries using the industry classification in JOLTS and the 2-digit industry classification in HWOL (The Conference Board Help Wanted OnLine Data Series).

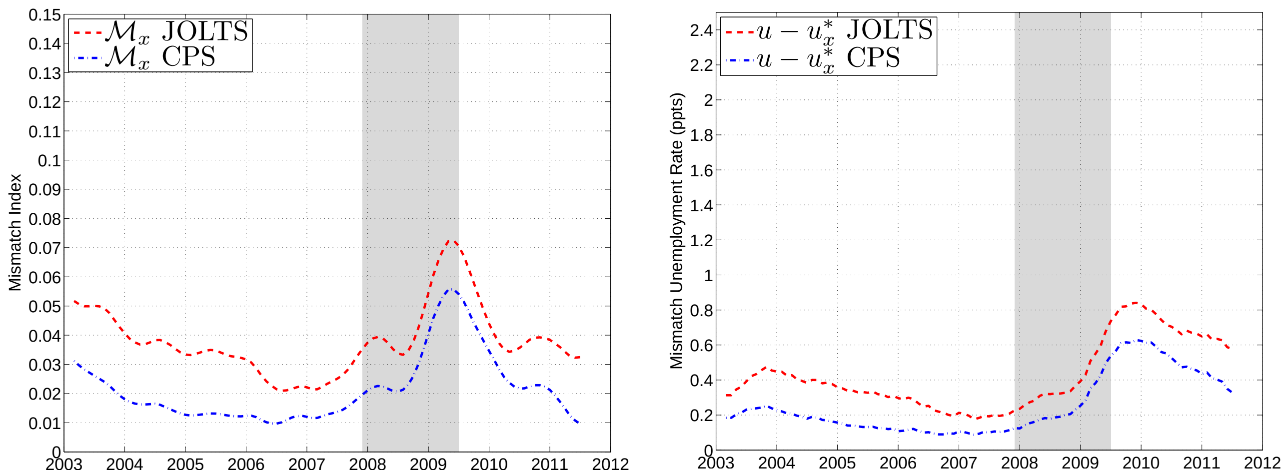


Figure C11: Mismatch index $\mathcal{M}_{xt}$ by industry (left panel) and the corresponding mismatch unemployment rates (right panel) using the JOLTS measure of hires and an estimate of of hires from unemployment based on the CPS.

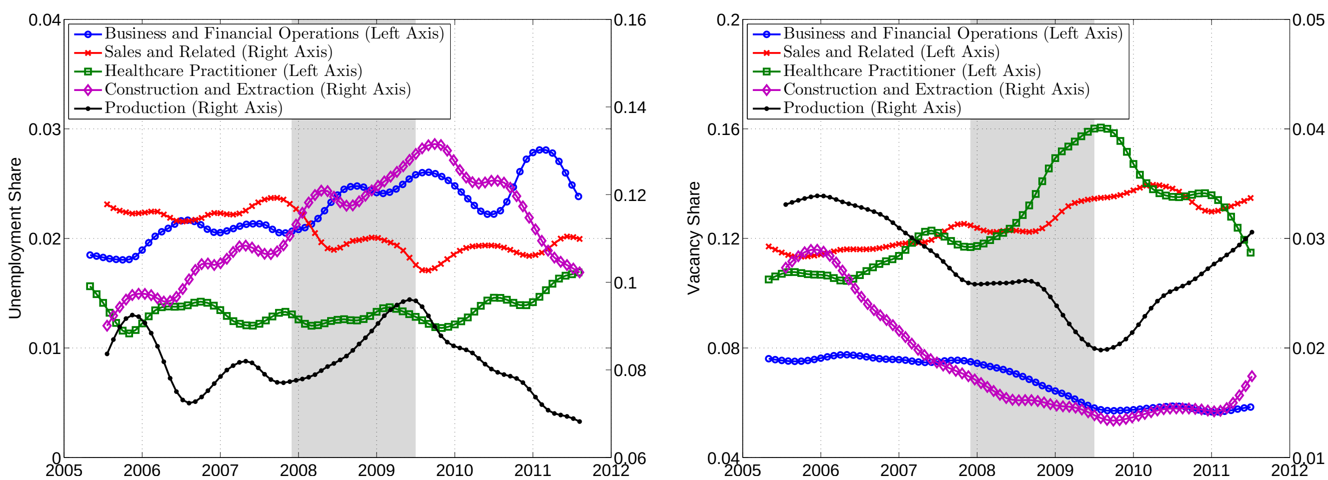


Figure C12: Unemployment and vacancy shares by selected occupation.

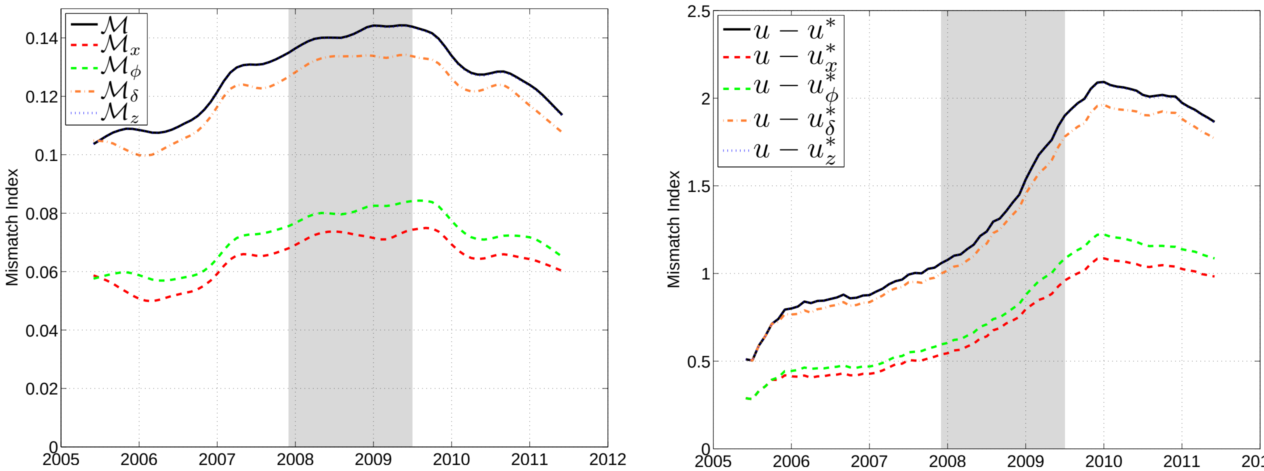


Figure C13: Mismatch indexes $\mathcal{M}_t$, $\mathcal{M}_{xt}$, $\mathcal{M}_{\phi t}$, $\mathcal{M}_{zt}$, and $\mathcal{M}_{\delta t}$ by occupation (left panel) and the corresponding mismatch unemployment rates (right panel).

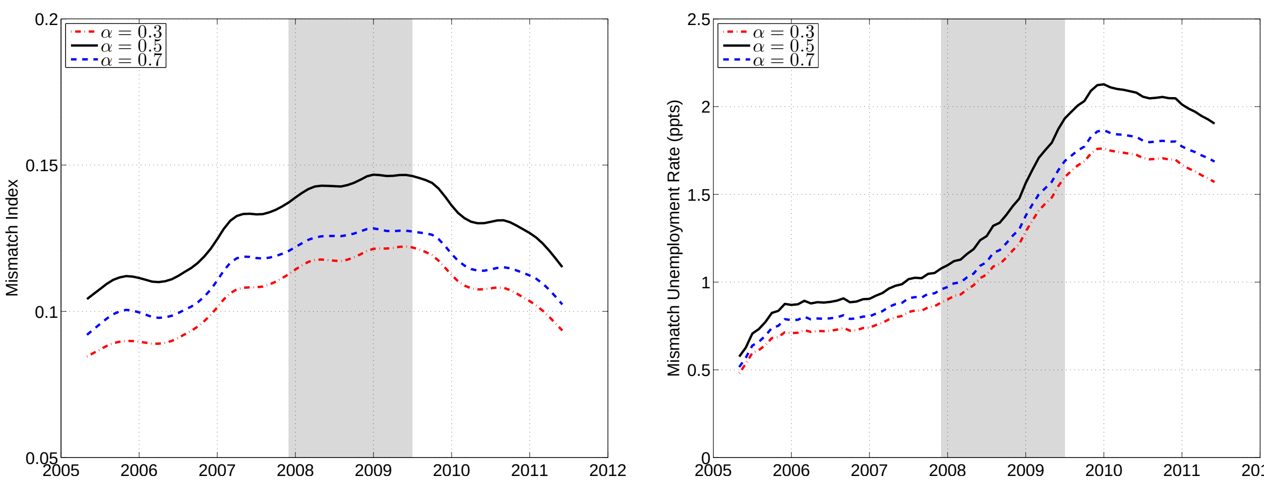


Figure C14: Mismatch index $\mathcal{M}_t$ by occupation (left panel) and the corresponding mismatch unemployment rates (right panel) for various values of α, the vacancy share parameter of the matching function.

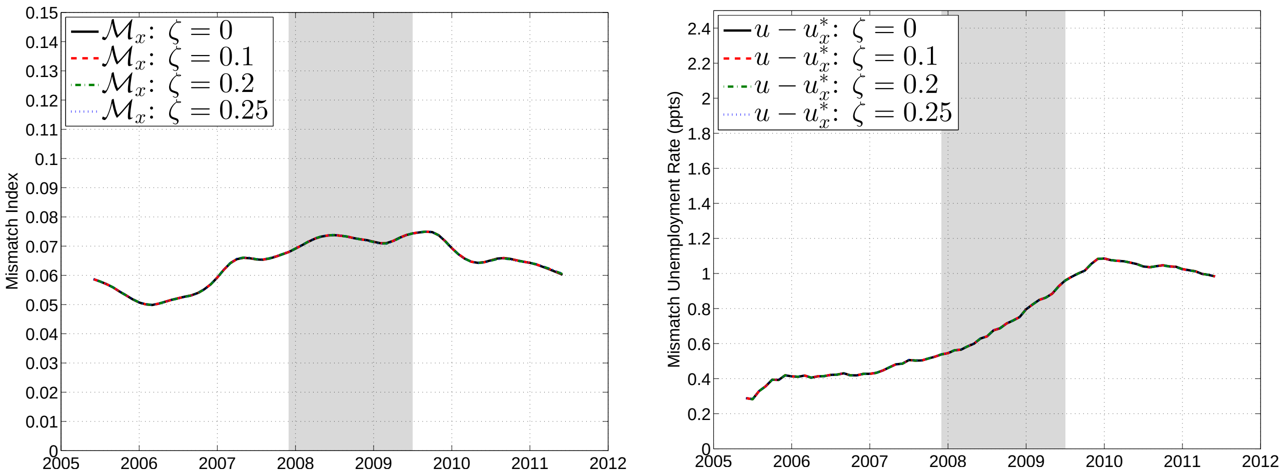


Figure C15: Mismatch index $\mathcal{M}_t$ by occupation for different values of the flow utility from nonemployment ζ (left panel), and the corresponding mismatch unemployment rates (right panel).

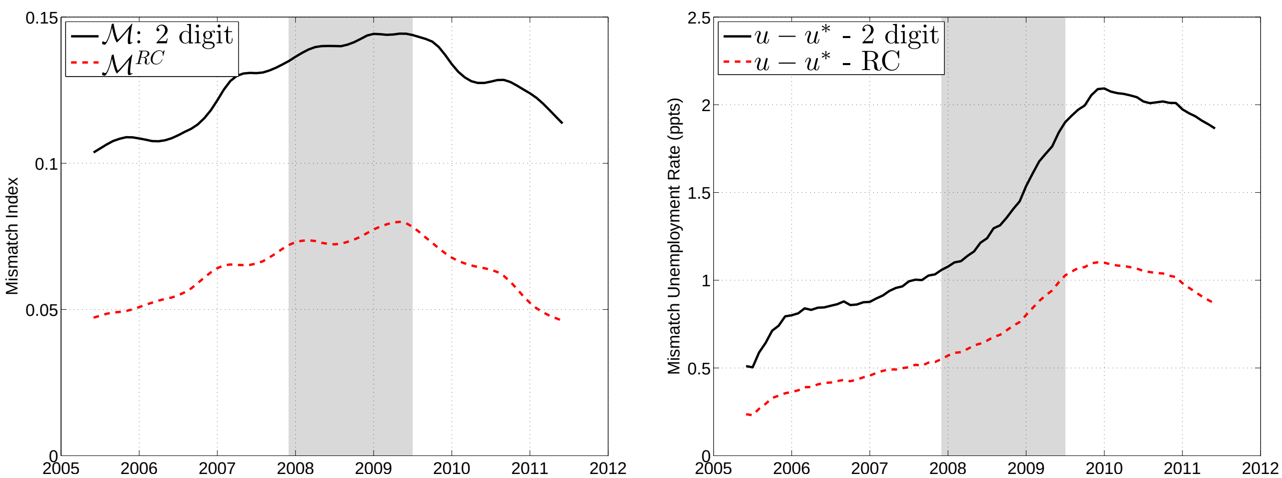


Figure C16: Mismatch indexes $\mathcal{M}$ across four occupations groups (routine/cognitive, manual/non-manual, and across 2-digit occupations (left panel). Corresponding mismatch unemployment rates (right panel).

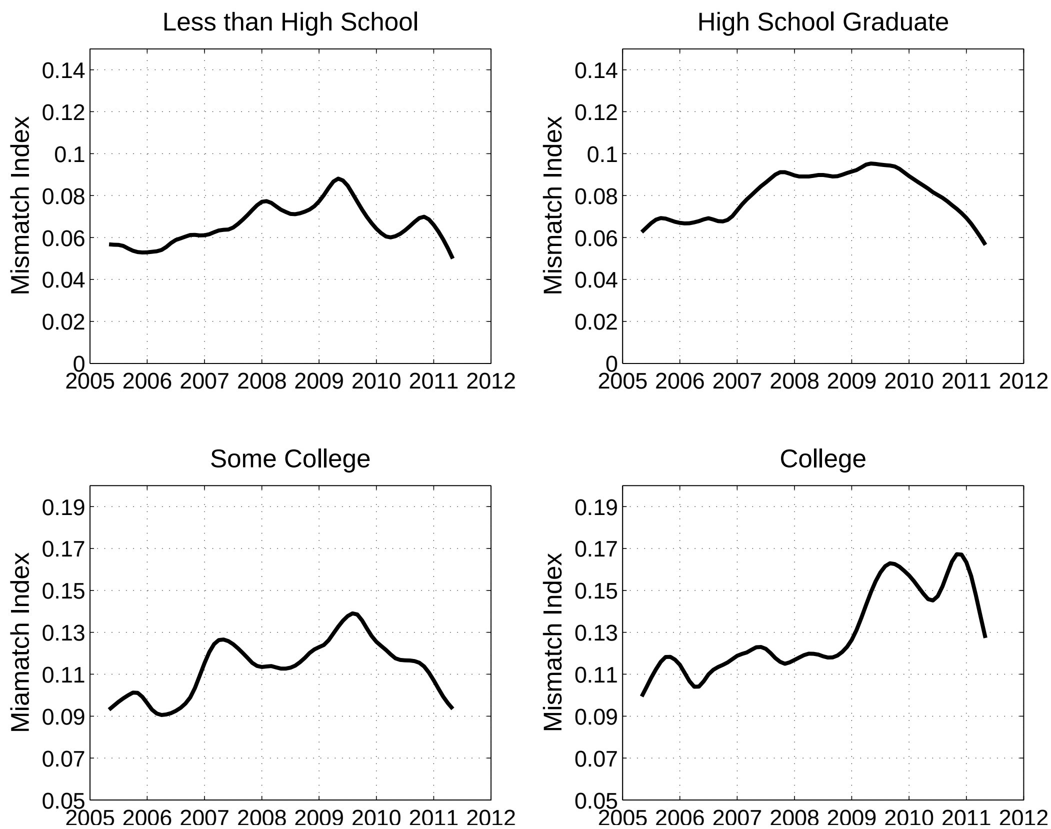


Figure C17: Mismatch indexes ($\mathcal{M}_t$) by occupation for different education groups.

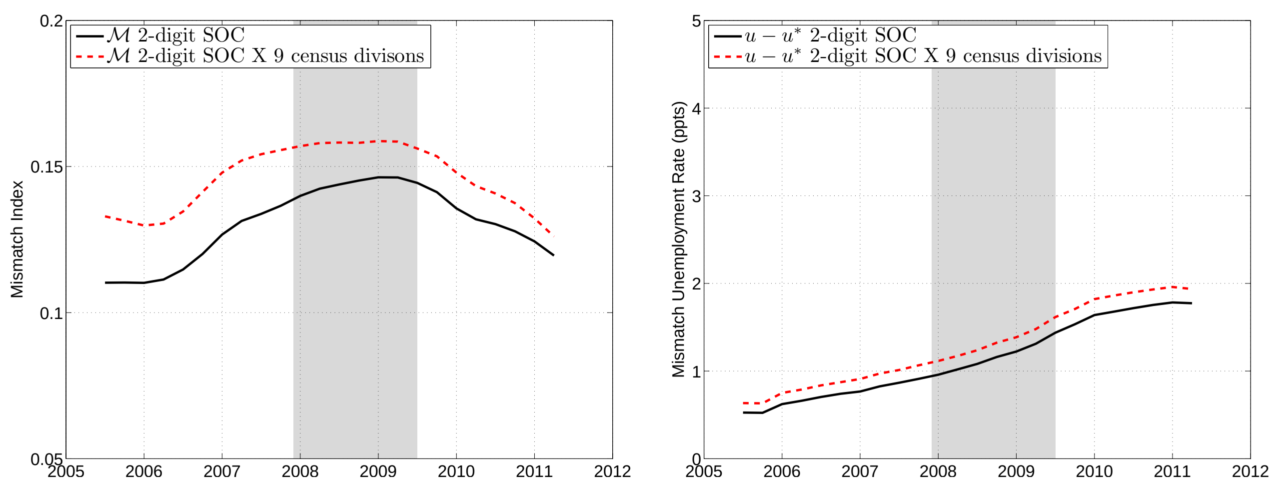


Figure C18: Mismatch index $\mathcal{M}_t$ by occupation and location (left panel) and the corresponding mismatch unemployment rates (right panel).

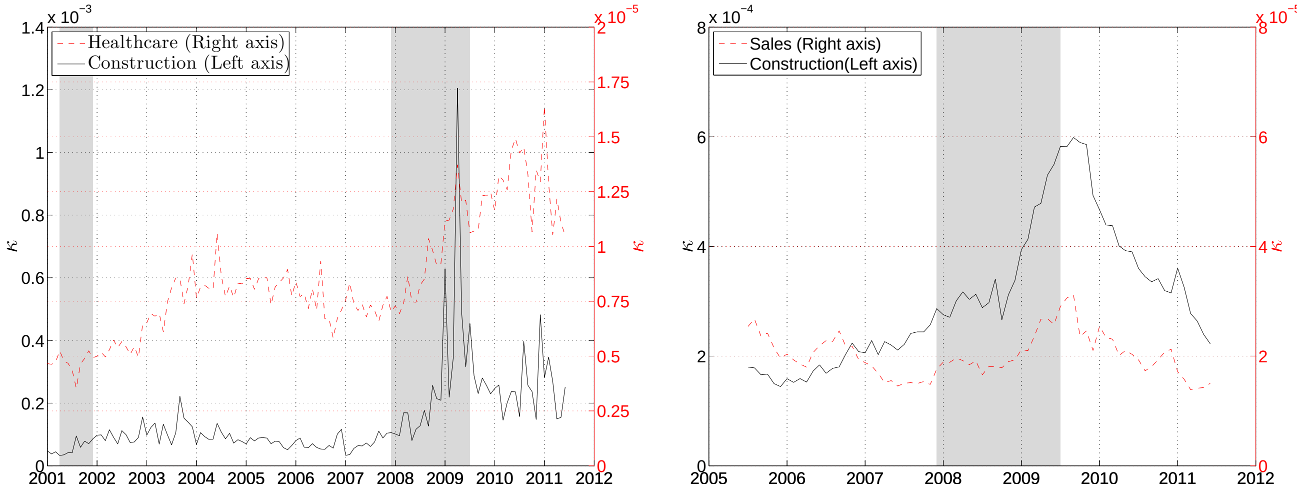


Figure C19: Time series of κ estimated with $\varepsilon = 1$ for two selected industries: construction and health care (left panel) and two selected occupations: construction and extraction occupations, and sales and related occupations (right panel). The cost is normalized by average annual labor productivity of the industry (annual wage for the occupation).

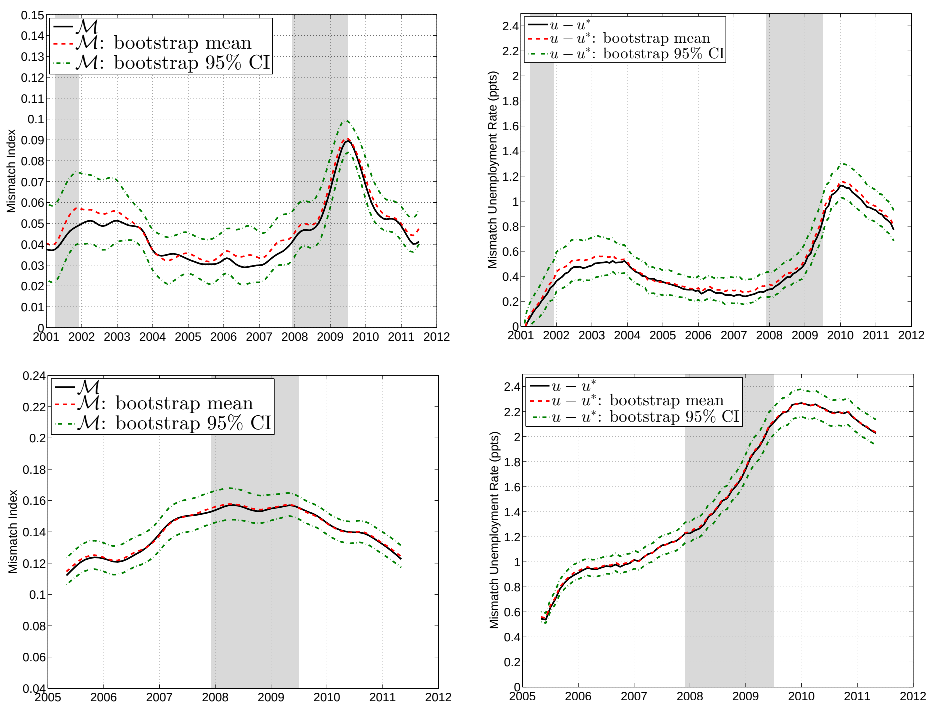


Figure C20: Mismatch index $\mathcal{M}_t$ by 2-digit industry (top-left panel) and occupation (bottom-left), mismatch unemployment rate by industry (top-right panel) and by occupation (bottom-left panel) with the corresponding 95% bootstrapped confidence intervals.

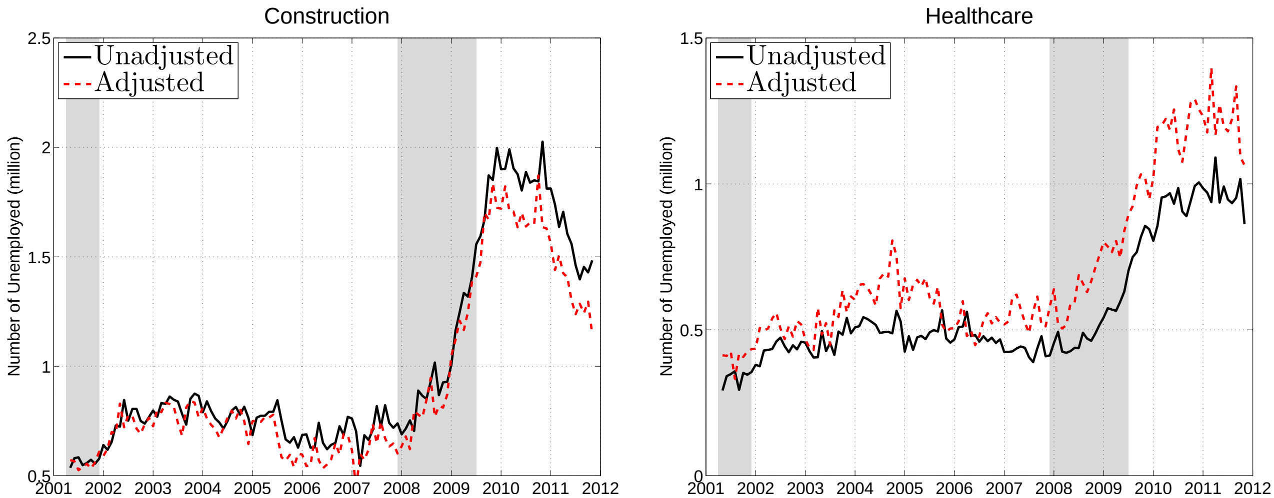


Figure C21: Adjusted unemployment counts for selected industries.

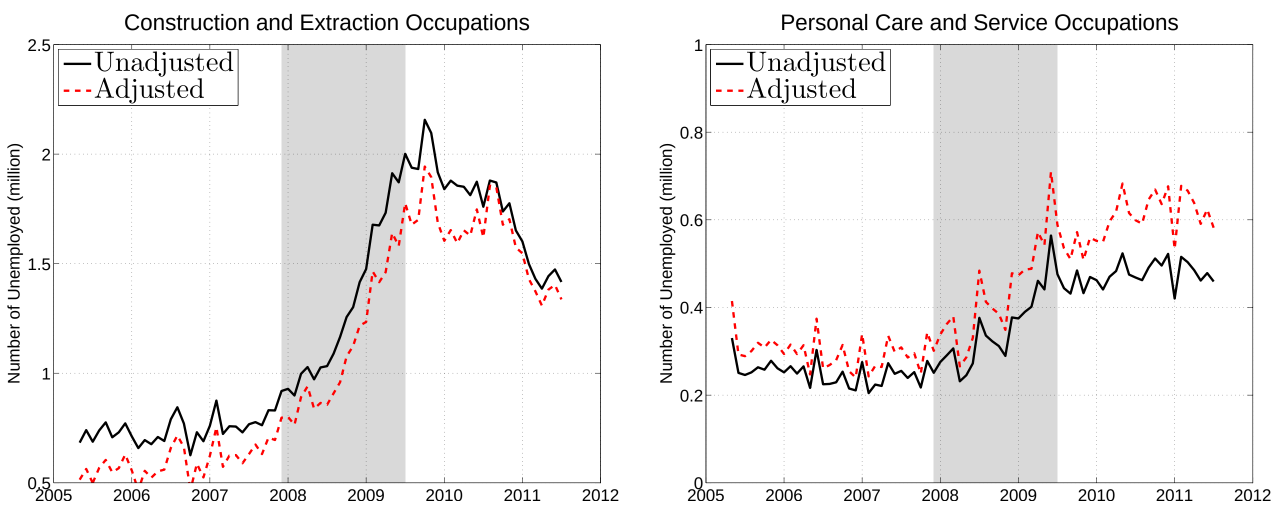


Figure C22: Adjusted unemployment counts for selected occupations.

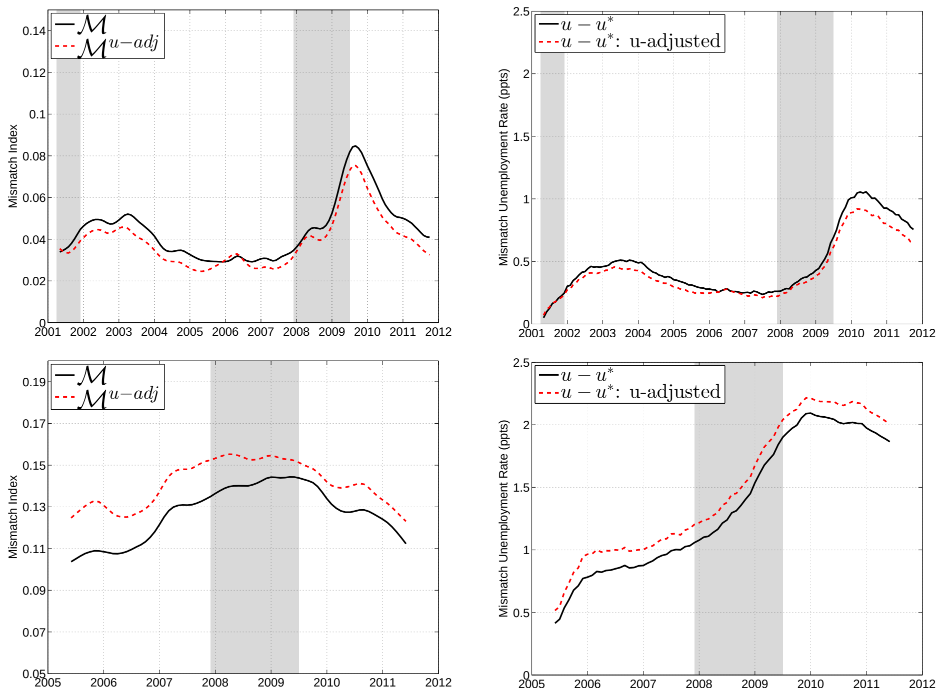


Figure C23: Mismatch index with unadjusted ($\mathcal{M}$) and adjusted ($\mathcal{M}^{u-adj}$) unemployment counts by industry (top-left panel) and corresponding mismatch unemployment (top-right panel). Mismatch index with adjusted and unadjusted unemployment counts by occupation (bottom-left panel) and corresponding mismatch unemployment (bottom-right panel).

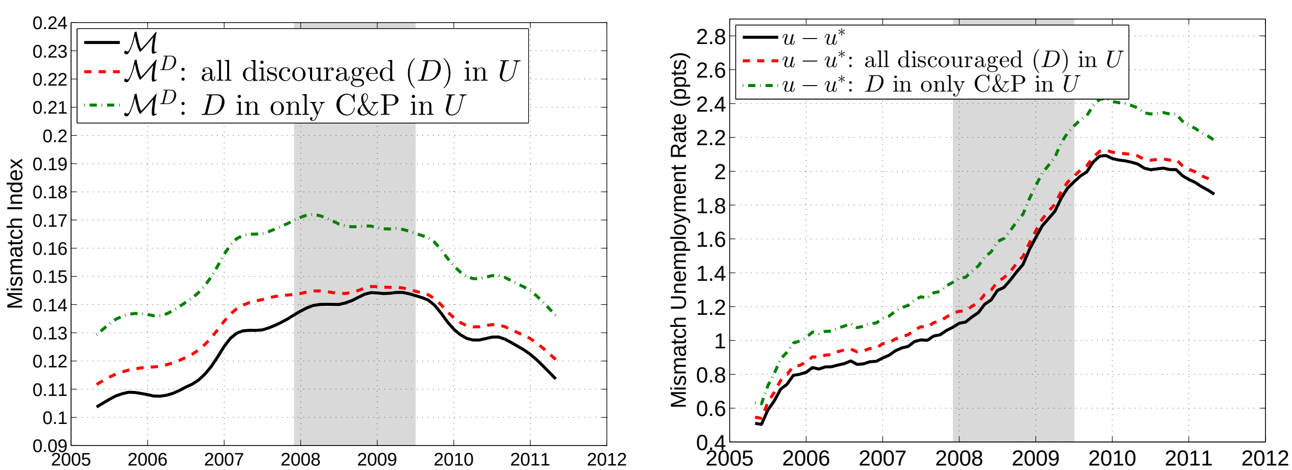


Figure C24: Mismatch indexes $\mathcal{M}_t$ by occupation (left panel) and the corresponding mismatch unemployment rates (right panel) including discouraged workers. The first correction includes all discouraged workers (D) among the unemployed (U). The second is a correction only for Construction (C) and Production-related (P) occupations.

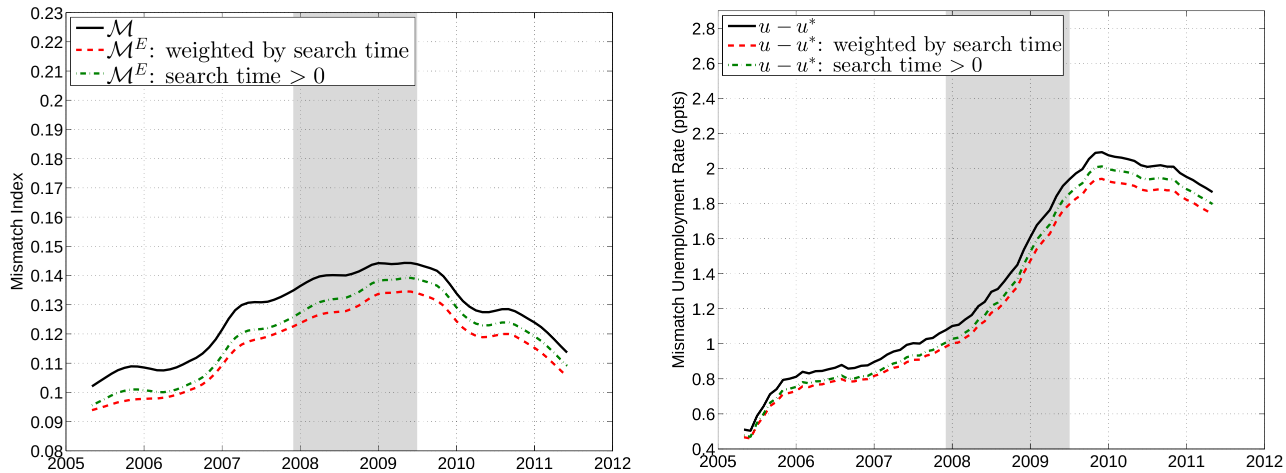


Figure C25: Mismatch indexes $\mathcal{M}_t$ by occupation (left panel) and the corresponding mismatch unemployment rates (right panel) including employed job seekers. The first correction weights employed workers by their reported search time in ATUS (relative to the search time of the unemployed), the second does not.

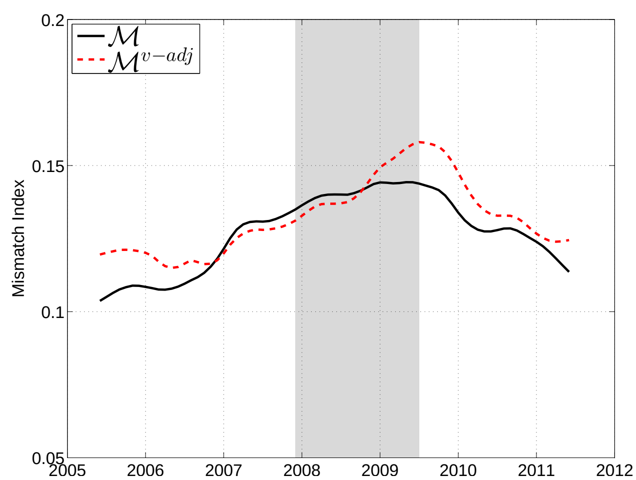


Figure C26: Mismatch index by 2-digit occupation: unadjusted index and index computed with reweighted HWOL vacancies

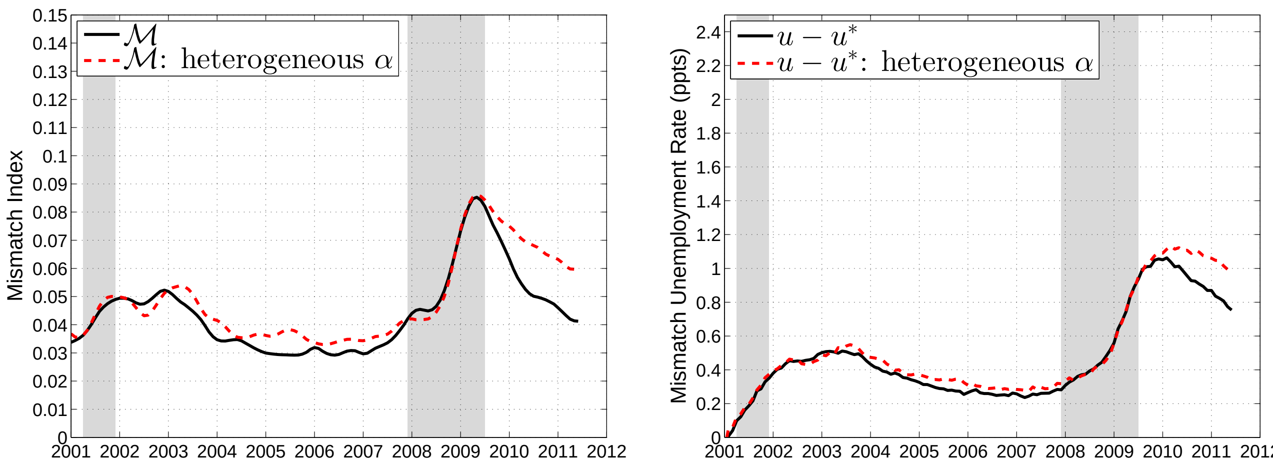


Figure C27: Mismatch indexes $\mathcal{M}_t$ by industry (left panel) and the corresponding mismatch unemployment rates (right panel) in the model with heterogenous α.

Code	Industry
ACC	Accomodation and Food Services
ART	Arts, Entertainment and Recreation
CON	Construction
EDU	Education Services
FIN	Finance and Insurance
PUB	Government
HEA	Health Care and Social Assistance
INF	Information
MFG	Manufacturing-Durable Goods
MFG	Manufacturing-Nondurable Goods
MIN	Mining
OTH	Other Services
BUS	Professional and Business Services
REA	Real Estate and Rental and Leasing
RET	Retail Trade
UTL	Transportation, Warehousing and Utilities
WHO	Wholesale Trade

Table C1: Industry classification in the JOLTS. The codes in the left column are those used in Figure C2.

Code	Occupation	Classification
110000	Management Occupations	Cognitive/Non-routine
130000	Business and Financial Operations Occupations	Cognitive/Non-routine
150000	Computer and Mathematical Occupations	Cognitive/Non-routine
170000	Architecture and Engineering Occupations	Cognitive/Non-routine
190000	Life, Physical, and Social Science Occupations	Cognitive/Non-routine
210000	Community and Social Service Occupations	Cognitive/Non-routine
230000	Legal Occupations	Cognitive/Non-routine
250000	Education, Training, and Library Occupations	Cognitive/Non-routine
270000	Arts, Design, Entertainment, Sports, and Media Occupations	Cognitive/Non-routine
290000	Healthcare Practitioners and Technical Occupations	Cognitive/Non-routine
310000	Healthcare Support Occupations	Manual/Non-routine
330000	Protective Service Occupations	Manual/Non-routine
350000	Food Preparation and Serving Related Occupations	Manual/Non-routine
370000	Building and Grounds Cleaning and Maintenance Occupations	Manual/Non-routine
390000	Personal Care and Service Occupations	Manual/Non-routine
410000	Sales and Related Occupations	Cognitive/Routine
430000	Office and Administrative Support Occupations	Cognitive/Routine
470000	Construction and Extraction Occupations	Manual/Routine
490000	Installation, Maintenance, and Repair Occupations	Manual/Routine
510000	Production Occupations	Manual/Routine
530000	Transportation and Material Moving Occupations	Manual/Routine

Table C2: 2-digit SOC Codes used in our empirical analysis. The classification in the right column is that used in Figure C16.

Code	Occupation
111000	Top Executives
113000	Operations Specialties Managers
119000	Other Management Occupations
131000	Business Operations Specialists
132000	Financial Specialists
151000	Computer Occupations
211000	Counselors, Social Workers, and Other Community and Social Service Specialists
252000	Preschool, Primary, Secondary, and Special Education School Teachers
272000	Entertainers and Performers, Sports and Related Workers
291000	Health Diagnosing and Treating Practitioners
311000	Nursing, Psychiatric, and Home Health Aides
339000	Other Protective Service Workers
352000	Cooks and Food Preparation Workers
353000	Food and Beverage Serving Workers
359000	Other Food Preparation and Serving Related Workers
372000	Building Cleaning and Pest Control Workers
373000	Grounds Maintenance Workers
399000	Other Personal Care and Service Workers
411000	Supervisors of Sales Workers
412000	Retail Sales Workers
413000	Sales Representatives, Services
419000	Other Sales and Related Workers
433000	Financial Clerks
434000	Information and Record Clerks
435000	Material Recording, Scheduling, Dispatching, and Distributing Workers
436000	Secretaries and Administrative Assistants
439000	Other Office and Administrative Support Workers
452000	Agricultural Workers
472000	Construction Trades Workers
493000	Vehicle and Mobile Equipment Mechanics, Installers, and Repairers
499000	Other Installation, Maintenance, and Repair Occupations
512000	Assemblers and Fabricators
514000	Metal Workers and Plastic Workers
519000	Other Production Occupations
533000	Motor Vehicle Operators
537000	Material Moving Workers

Table C3: 3-digit SOC Codes used in our empirical analysis.

	Aggregate regressions				Panel regressions	
	JOLTS		HWOL		Industry (JOLTS)	Occupation (HWOL)
	OLS	GMM	OLS	GMM	OLS	OLS
JOLTS Hires	0.654	0.661	–	–	0.532	–
	(0.010)	(0.037)	–	–	(0.013)	–
Sample Size	126	126	–	–	2,142	–
CPS Hires	0.318	0.298	0.332	0.536	0.241	0.279
	(0.017)	(0.136)	(0.038)	(0.059)	(0.014)	(0.016)
Sample Size	126	126	72	72	404	370

Table C4: OLS and GMM estimates of the vacancy share α using the JOLTS and HWOL datasets. S.E. in parenthesis. See Section B.2 for details.

	JOLTS		HWOL	
	α	σ	α	σ
JOLTS Hires	0.576 [0.542,0.603]	0.152 [0.051,0.242]	-	-
CPS Hires	0.301 [0.267,0.350]	0.18 [0.08,0.303]	0.239 [0.194,0.291]	-0.108 [-0.226,0.004]

Table C5: Estimates of the vacancy share α and CES substitutability parameter σ, using industry and occupation level data. 95-5 confidence intervals computed via bootstrap. Sample sizes are the same as in Table C4.

Industry	ϕ^{pre}	ϕ^{post}
Mining	1.71	1.36
Arts	1.69	1.87
Construction	1.66	1.73
Accommodations	1.53	1.60
Retail	1.47	1.46
Professional and Business Services	1.43	1.45
Real Estate	1.41	1.22
Wholesale	1.21	1.35
Other	1.14	1.16
Transportation and Utilities	1.14	1.16
Manufacturing - Nondurables	0.96	1.00
Education	0.94	1.02
Health	0.93	1.05
Government	0.87	0.89
Finance	0.85	0.73
Manufacturing - Durables	0.84	0.78
Information	0.76	0.70

Table C6: Estimates of industry-specific match efficiencies using hires from the JOLTS.

Industry Groups	Industry	ϕ^{pre}	ϕ^{post}
Group 1	Construction Mining	0.50	0.55
Group 2	Manufacturing Other Transportation and Utilities	0.42	0.44
Group 3	Accommodations Arts Professional and Business Services Retail Wholesale	0.38	0.39
Group 4	Education Finance Government Health Information Real Estate	0.33	0.33

Table C7: Estimates of industry-specific match efficiencies using hires from the CPS.

Occupation Groups	Occupation	ϕ^{pre}	ϕ^{post}
Service	Protective Service Occupations Food Preparation and Serving Related Occupations Building and Grounds Cleaning and Maintenance Occupations Personal Care and Service Occupations	0.58	0.63
Natural Resources, Construction and Maintenance	Construction and Extraction Occupations Installation, Maintenance, and Repair Occupations	0.56	0.63
Production, Transportation and Material Moving	Production Occupations Transportation and Material Moving Occupations	0.48	0.52
Sales and Office	Sales and Related Occupations Office and Administrative Support Occupations	0.37	0.35
Management, Professional and Related	Management Occupations Business and Financial Operations Occupations Computer and Mathematical Occupations Architecture and Engineering Occupations Life, Physical, and Social Science Occupations Community and Social Service Occupations Legal Occupations Education, Training, and Library Occupations Arts, Design, Entertainment, Sports, and Media Occupations Healthcare Practitioners and Technical Occupations Healthcare Support Occupations	0.32	0.33

Table C8: Estimates of occupation-specific match efficiencies using hires from the CPS.

	Index	$u_{06} - u^*_{06}$	$u_{10.09} - u^*_{10.09}$	$\Delta(u - u^*)$	$\Delta(u - u^*)/\Delta u$
JOLTS Hires	$\mathcal{M}$	0.26	1.01	0.75	13.9%
	$\mathcal{M}_x$	0.24	0.84	0.59	11.0%
	$\mathcal{M}_x^{AK}$	0.28	0.89	0.61	11.2%
	$\mathcal{M}_x^{v^*}(\varepsilon = 0.5)$	0.67	1.90	1.22	22.5%
	$\mathcal{M}_x^{v^*}(\varepsilon = 1.0)$	0.35	1.24	0.90	16.6%
	$\mathcal{M}_x^{v^*}(\varepsilon = 2.0)$	0.27	0.95	0.69	12.7%
	$\mathcal{M}_\phi$	0.29	0.92	0.63	11.7%
	$\mathcal{M}_z$	0.24	0.96	0.72	13.4%
	$\mathcal{M}_\delta$	0.23	0.98	0.74	13.7%
	$\mathcal{M}^{u-adj}$	0.25	0.89	0.65	11.9%
	$\mathcal{M}(\alpha = 0.3)$	0.22	0.89	0.67	12.4%
	$\mathcal{M}(\alpha = 0.5)$	0.26	1.01	0.75	13.9%
	$\mathcal{M}(\alpha = 0.7)$	0.22	0.82	0.60	11.1%
	$\mathcal{M}_x^{break}$	0.25	0.92	0.67	12.4%
	$\mathcal{M}_x(\zeta = 0.10)$	0.24	0.82	0.59	10.8%
	$\mathcal{M}_x(\zeta = 0.20)$	0.23	0.79	0.56	10.3%
	$\mathcal{M}_x(\zeta = 0.25)$	0.22	0.73	0.51	9.4%
CPS Hires	$\mathcal{M}$	0.27	1.03	0.77	12.4%
	$\mathcal{M}_x$	0.10	0.61	0.51	9.4%
HWOL	$\mathcal{M}$	0.63	1.51	0.88	16.3%
	$\mathcal{M}_x$	0.56	1.35	0.79	14.7%

Table C9: Changes in mismatch unemployment at the industry level. All the changes are calculated as the difference between October 2009 and the average of 2006. Note that $\Delta u = 5.4$ percentage points.

	Index	$u_{06} - u^*_{06}$	$u_{10.09} - u^*_{10.09}$	$\Delta(u - u^*)$	$\Delta(u - u^*)/\Delta u$
2-digit	$\mathcal{M}$	0.85	2.00	1.15	21.3%
	$\mathcal{M}_x$	0.42	1.02	0.60	11.1%
	$\mathcal{M}_x^{v^*}(\varepsilon = 0.5)$	1.08	2.60	1.52	28.1%
	$\mathcal{M}_x^{v^*}(\varepsilon = 1.0)$	0.75	1.81	1.07	19.7%
	$\mathcal{M}_x^{v^*}(\varepsilon = 2.0)$	0.58	1.41	0.83	15.3%
	$\mathcal{M}^{u-adj}$	0.84	2.00	1.16	21.4%
	$\mathcal{M}^{v-adj}$	0.92	2.12	1.19	22.1%
	$\mathcal{M}^D$ (all discouraged in U)	0.92	2.03	1.11	20.6%
	$\mathcal{M}^D$ (D in C&P in U)	1.06	2.33	1.27	23.4%
	$\mathcal{M}^E$ (E: weighted by search time)	0.78	1.90	1.13	20.9%
	$\mathcal{M}^E$ (E: fraction searching)	0.79	1.97	1.18	21.8%
	$\mathcal{M}_\phi$	0.46	1.15	0.69	12.8%
	$\mathcal{M}_z$	0.85	2.00	1.15	21.2%
	$\mathcal{M}_\delta$	0.80	1.86	1.05	19.5%
	$\mathcal{M}(\alpha = 0.3)$	0.72	1.69	0.96	17.8%
	$\mathcal{M}(\alpha = 0.5)$	0.85	2.00	1.14	21.3%
	$\mathcal{M}(\alpha = 0.7)$	0.79	1.77	0.98	18.1%
	$\mathcal{M}_x^{break}$	0.42	0.98	0.56	10.4%
	$\mathcal{M}(\zeta = 0.10)$	0.42	1.02	0.60	11.1%
	$\mathcal{M}(\zeta = 0.20)$	0.42	1.02	0.60	11.1%
	$\mathcal{M}(\zeta = 0.25)$	0.42	1.02	0.60	11.1%
3-digit	$\mathcal{M}$	1.33	2.91	1.58	29.3%
	$\mathcal{M}_x$	0.79	1.73	0.94	17.4%
	$\mathcal{M}_\phi$	0.83	1.85	1.02	18.8%
	$\mathcal{M}_z$	1.33	2.91	1.58	29.2%
	$\mathcal{M}_\delta$	1.29	2.80	1.50	27.8%

Table C10: Changes in mismatch unemployment at the occupation level. All the changes are calculated as the difference between October 2009 and the average of 2006. Note that $\Delta u = 5.4$ percentage points.

Index	$u_{Q1.01} - u^*_{Q1.01}$	$u_{06.03} - u^*_{06.03}$	$\Delta(u - u^*)$	$\Delta(u - u^*)/\Delta u$
$\mathcal{M}$	0.09	0.50	0.41	22.8%
$\mathcal{M}_x$	0.10	0.50	0.41	21.7%
$\mathcal{M}^{u-adj}$	0.11	0.43	0.32	17.8%
$\mathcal{M}_x^{v^*}(\varepsilon = 1.0)$	0.20	0.70	0.50	26.8%

Table C11: Changes in mismatch unemployment at the industry level for the 2001 recession. All the changes are calculated as the difference between June 2003 (month in which the unemployment rate peaked for the 2001 recession) and the average of 2001Q1. Note that $\Delta u = 1.8$ percentage points.

		2005-2007		2008-2011	
	Occupation	D	U	D	U
11	Management	3.86	4.44	4.24	5.47
13	Business and Financial	2.23	2.26	2.24	2.70
15	Computer and Math	0.90	1.22	1.17	1.36
17	Architecture and Engineering	0.72	0.77	0.84	1.30
19	Life, Physical, and Social Science	0.58	0.45	0.40	0.46
21	Community and Social Service	0.79	0.80	0.79	0.84
23	Legal	0.43	0.45	0.81	0.46
25	Education, Training, and Library	4.85	3.22	5.31	2.84
27	Arts, Design, Entertainment, Sports, and Media	1.94	1.95	2.81	1.92
29	Healthcare Practitioners and Technical	1.90	1.48	2.26	1.45
31	Healthcare Support	2.29	2.34	1.85	1.94
33	Protective Service	1.47	1.76	1.96	1.44
35	Food Preparation and Serving Related	10.62	9.47	9.99	8.19
37	Building and Grounds Cleaning and Maintenance	6.78	6.18	6.22	5.71
39	Personal Care and Service	6.06	3.85	6.27	3.50
41	Sales and Related	15.01	12.94	12.62	11.75
43	Office and Administrative Support	12.91	13.18	12.79	12.60
45	Fishing and Farming	1.63	1.48	1.93	1.41
47	Construction and Extraction	8.40	11.04	8.93	13.34
49	Installation, Maintenance, and Repair	2.48	3.04	2.64	3.42
51	Production	6.13	8.94	5.80	9.23
53	Transportation and Material Moving	8.02	8.75	8.11	8.69

Table C12: Distribution of discouraged and unemployed workers across occupations; percent of D and U in each occupation.

	Weight 2005-2006	Weight 2010-2011
Industry		
Accomodation and Food Services	2.25	2.43
Arts, Entertainment and Recreation	1.07	1.03
Construction	1.42	1.32
Education Services	0.44	0.55
Finance and Insurance	0.49	0.56
Government	2.94	2.35
Health Care and Social Assistance	0.79	0.83
Information	0.49	0.58
Manufacturing-Durable Goods	0.81	0.64
Manufacturing-Nondurable Goods	0.75	0.63
Mining	0.82	1.23
Other Services	1.34	1.14
Professional and Business Services	0.34	0.35
Real Estate and Rental and Leasing	0.56	0.52
Retail Trade	0.92	1.04
Transportation, Warehousing and Utilities	1.00	1.07
Wholesale Trade	0.61	0.73
Region		
Northeast	0.90	0.99
West	1.18	0.97
Southwest	0.68	0.92
South	1.17	1.23

Table C13: Estimated weights which equalize monthly JOLTS and HWOL (The Conference Board Help Wanted OnLine Data Series) vacancy counts by industry and region (average weight is normalized to one each month).

	α	ϕ
Mining	0.5549	1.4503
	(0.056)	(0.110)
Construction	0.3999	1.1542
	(0.040)	(0.083)
Durable goods manufacturing	0.5757	0.7565
	(0.026)	(0.026)
Nondurable goods manufacturing	0.5381	0.8250
	(0.030)	(0.033)
Wholesale trade	0.5126	1.0329
	(0.029)	(0.020)
Retail trade	0.6488	1.3904
	(0.042)	(0.051)
Transportation and warehousing	0.4174	0.8851
	(0.037)	(0.030)
Information	0.5103	0.6210
	(0.030)	(0.018)
Financial activities	0.6485	0.6936
	(0.053)	(0.014)
Real estate	0.3528	1.0877
	(0.055)	(0.044)
Professional & business services	0.5922	1.2406
	(0.028)	(0.018)
Education	0.401	0.7213
	(0.056)	(0.036)
Healthcare	0.6932	0.7459
	(0.026)	(0.011)
Arts, entertaiment, and recreation	0.3511	1.2342
	(0.051)	(0.068)
Accommodation & food services	0.5543	1.3247
	(0.024)	(0.025)
Other	0.3836	0.9120
	(0.044)	(0.029)
Government	0.7891	0.7454
	(0.042)	(0.012)

Table C14: Estimates of α and ϕ by industry. Standard errors are in parentheses.