

Productivity Losses from Financial Frictions: Can Self-Financing Undo
Capital Misallocation

Online Appendix

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ADDITIONAL PROOFS

B1. Proof of Proposition 3

First, consider the case where capital markets are frictionless, $\lambda = \infty$. In this case, existence and uniqueness is immediate. This is because by Proposition 1 the model collapses to a Solow model with exogenous total factor productivity $Z = \bar{z}^\alpha$. The remainder of the proof is concerned with the case $\lambda < \infty$.

Under assumption 1, the appropriate boundary conditions for (22) are:

$$(B1) \quad -\mu(0)\omega(0) + \frac{1}{2} \frac{d}{dz} [\sigma^2(0)\omega(0)] = -\mu(\bar{z})\omega(\bar{z}) + \frac{1}{2} \frac{d}{dz} [\sigma^2(\bar{z})\omega(\bar{z})] = 0.$$

Useful Preliminary Lemma. The following Lemma will be useful below, and also in the proof of Proposition 4.

LEMMA 4: *If $\omega(z)$ satisfies (22) and (B1), and $\psi(z)$ is the stationary distribution corresponding to (25), then $f(z) = \omega(z)/\psi(z)$ satisfies:*

$$(B2) \quad 0 = s(z)f(z) + \mu(z)f(z) + \frac{1}{2}\sigma^2(z)f''(z), \quad f'(0) = f'(\bar{z}) = 0.$$

The proof of Lemma 4 is at the end of Appendix B.B1.

Main Proof: From (6) and the fact that from Lemma 1 $r = \underline{z}\pi - \delta$

$$(B3) \quad s(z) = \pi[\lambda(z - \underline{z})^+ + \underline{z}] - \rho - \delta,$$

and the cutoff \underline{z} satisfies $\lambda \int_{\underline{z}}^{\bar{z}} z\omega(z)dz = 1$. The key is to write (22) as a continuous eigenvalue problem:

$$(B4) \quad \eta(\pi, \underline{z})\omega(z) = \pi[\lambda(z - \underline{z})^+ + \underline{z}]\omega(z) - \frac{d}{dz}[\mu(z)\omega(z)] + \frac{1}{2} \frac{d^2}{dz^2} [\sigma^2(z)\omega(z)]$$

with boundary conditions (B1) and to solve a system of two equations in two unknowns (π, \underline{z})

$$(B5) \quad \eta(\pi, \underline{z}) = \rho + \delta,$$

$$(B6) \quad \lambda \int_{\underline{z}}^{\bar{z}} \omega(z; \pi) dz = 1.$$

The statement that there exists a unique stationary equilibrium is then equivalent to saying that (B5) and (B6) has a unique solution. Finally, denote by $\underline{z}(\pi)$ the solution to (B6) for future reference.

Step 1: $\partial\eta(\pi, \underline{z}(\pi))/\partial\underline{z} = 0$. **Proof:** integrate (B4)

$$(B7) \quad \eta(\pi, \underline{z}) = \pi \left(\lambda \int_{\underline{z}}^{\bar{z}} (z - \underline{z}) \omega(z) dz + \underline{z} \right),$$

and differentiate with respect to \underline{z} :

$$\frac{\partial\eta(\pi, \underline{z})}{\partial\underline{z}} = \pi \left(1 - \lambda \int_{\underline{z}}^{\bar{z}} \omega(z) dz \right).$$

Evaluating at $\underline{z} = \underline{z}(\pi)$, i.e. imposing that (B6) holds, we have $\partial\eta(\pi, \underline{z}(\pi))/\partial\underline{z} = 0$.

Step 2: any solution to (B5) and (B6) satisfies $\partial\eta/\partial\pi \geq 0$. **Proof:** from Lemma 4 $\eta(\pi, \underline{z})$ also satisfies the transpose eigenvalue problem

$$(B8) \quad \eta(\pi, \underline{z})f = \pi[\lambda(z - \underline{z})^+ + \underline{z}]f + \mu f' + \frac{1}{2}\sigma^2 f'', \quad f'(0) = f'(\bar{z}) = 0$$

Define $u(z) = \log f(z)$ so that $f(z) = e^{u(z)}$, $f' = u'e^u$, $f'' = ((u')^2 + u'')e^u$ and write (B4) as

$$\eta = \pi[\lambda(z - \underline{z})^+ + \underline{z}] + \mu u' + \frac{1}{2}\sigma^2 ((u')^2 + u''), \quad u'(0) = u'(\bar{z}) = 0$$

Differentiate with respect to π and define $v = \partial u/\partial\pi$:

$$\frac{\partial\eta}{\partial\pi} = [\lambda(z - \underline{z})^+ + \underline{z}] + (\mu + \sigma^2 u')v' + \frac{\sigma^2}{2}v''$$

Since $v'(0) = v'(1) = 0$, there is a z_0 such that $v(z_0) = \min_z v(z)$. At this z_0 , $v'(z_0) = 0$ and $v''(z_0) \geq 0$ and therefore

$$\frac{\partial\eta}{\partial\pi} = [\lambda(z_0 - \underline{z})^+ + \underline{z}] + \frac{\sigma^2(z_0)}{2}v''(z_0) \geq 0.$$

Step 3: η is a continuous function of π and \underline{z} . **Proof:** (B8) can be written in operator form as $\eta(\pi, \underline{z})f = \mathcal{A}_{\pi, \underline{z}}f$ where the operator $\mathcal{A}_{\pi, \underline{z}}$ is defined by

$$\mathcal{A}_{\pi, \underline{z}}f \equiv \pi[\lambda(z - \underline{z})^+ + \underline{z}]f + \mu f' + \frac{1}{2}\sigma^2 f''$$

We are interested in the principal eigenvalue $\eta(\pi, \underline{z})$. This can be found through

the so-called maximum principle:⁵³

$$\eta(\pi, \underline{z}) = \max_{\varphi} \langle \mathcal{A}_{\pi, \underline{z}} \varphi, \varphi \rangle \quad \text{s.t.} \quad \langle \varphi, \varphi \rangle \leq 1,$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product, $\langle v, f \rangle = \int_0^{\bar{z}} v(z) f(z) dz$. Since $\mathcal{A}_{\pi, \underline{z}}$ is continuous it follows from Berge's maximum theorem that so is η .

Step 4: conclusion. Denote by $\underline{z}(\pi)$ the solution to (B6) and by $\tilde{\eta}(\pi) = \eta(\pi, \underline{z}(\pi))$. An equilibrium is a solution to $\tilde{\eta}(\pi) = \rho + \delta$. Given Step 3, $\tilde{\eta}(\pi)$ is continuous. From (B7), we have that $\eta(0, \underline{z}) = 0$ and $\lim_{\pi \rightarrow \infty} \eta(\pi, \underline{z}) = \infty$ and so:

$$\tilde{\eta}(0) < \rho + \delta < \lim_{\pi \rightarrow \infty} \tilde{\eta}(\pi),$$

Therefore there exists a solution to $\tilde{\eta}(\pi) = \rho + \delta$ and hence to (B5) and (B6). Given Step 1 and Step 2, we have $\tilde{\eta}'(\pi) = \partial \eta(\pi, \underline{z}(\pi)) / \partial \pi \geq 0$ and hence the solution is unique. \square

Proof of Lemma 4: We know that the stationary productivity distribution $\psi(z)$ satisfies the Kolmogorov forward equation

$$(B9) \quad 0 = -\frac{\partial}{\partial z} [\mu(z)\psi(z)] + \frac{1}{2} \frac{\partial^2}{\partial z^2} [\sigma^2(z)\psi(z)]$$

with boundary conditions

$$-\mu(0)\psi(0) + \frac{1}{2} \frac{d}{dz} [\sigma^2(0)\psi(0)] = -\mu(\bar{z})\psi(\bar{z}) + \frac{1}{2} \frac{d}{dz} [\sigma^2(\bar{z})\psi(\bar{z})] = 0$$

and hence

$$(B10) \quad C = -\mu(z)\psi(z) + \frac{1}{2} \frac{\partial}{\partial z} [\sigma^2(z)\psi(z)]$$

where $C = 0$ from the boundary conditions. Plugging $\omega(z) = \psi(z)f(z)$ into (22) and collecting terms

$$\begin{aligned} 0 &= \left(s(z)f(z) + \mu(z)f'(z) + \frac{1}{2}\sigma^2(z)f''(z) \right) \psi(z) \\ &\quad + 2 \left(-\mu(z)\psi(z) + \frac{1}{2} \frac{\partial}{\partial z} [\sigma^2(z)\psi(z)] \right) f'(z) \\ &\quad + \left(-\frac{\partial}{\partial z} [\mu(z)\psi(z)] + \frac{1}{2} \frac{\partial^2}{\partial z^2} [\sigma^2(z)\psi(z)] \right) f(z). \end{aligned}$$

The last two lines equal zero by (B9) and (B10). Therefore (B2) holds. Next,

⁵³This is the infinite-dimensional analogue of the following fact: the largest eigenvalue of a square matrix \mathbf{A} satisfies $\eta = \max_{\mathbf{x}} \mathbf{x}^T \mathbf{A} \mathbf{x}$ s.t. $\mathbf{x}^T \mathbf{x} \leq 1$ (take first order conditions to see that this implies $\mathbf{A} \mathbf{x} = \eta \mathbf{x}$).

consider the boundary conditions. Using $\omega(z) = f(z)\psi(z)$ at $z = 0$, we have

$$0 = -\mu(0)f(0)\psi(0) + \frac{1}{2}\frac{d}{dz}[\sigma^2(0)f(0)\psi(0)] = -\mu(0)f(0)\psi(0) + \frac{1}{2}\frac{d}{dz}[\sigma^2(0)\psi(0)]f(0) + \frac{1}{2}\sigma^2(0)\psi(0)f'(0).$$

The first two terms equal zero from the boundary condition for ψ so we need $f'(0) = 0$. By the same reasoning $f'(\bar{z}) = 0$. \square

B2. Proof of Proposition 4

With the stochastic process (25), we can write the ODE characterizing the stationary wealth shares as

$$(B11) \quad 0 = \theta s(z)\omega(z) - \frac{d}{dz}[\tilde{\mu}(z)\omega(z)] + \frac{1}{2}\frac{d^2}{dz^2}[\tilde{\sigma}^2(z)\omega(z)]$$

for $z \in (0, \bar{z})$ with boundary conditions

$$(B12) \quad -\tilde{\mu}(0)\omega(0) + \frac{1}{2}\frac{d}{dz}[\tilde{\sigma}^2(0)\omega(0)] = -\tilde{\mu}(\bar{z})\omega(\bar{z}) + \frac{1}{2}\frac{d}{dz}[\tilde{\sigma}^2(\bar{z})\omega(\bar{z})] = 0$$

and where $s(z)$ is given by (B3). Finally, from Lemma 1 and Corollary 1

$$(B13) \quad \pi = \alpha \left(\frac{1-\alpha}{w} \right)^{(1-\alpha)/\alpha} = \alpha Z^{-(1-\alpha)/\alpha} K^\alpha L^{1-\alpha} = (\rho + \delta)Z^{-1/\alpha},$$

and the cutoff \underline{z} satisfies $\lambda(1 - \Omega(\underline{z})) = 1$.

Strategy of Proof: I prove that $\partial\pi/\partial\theta < 0$ which by (B13) is equivalent to $\partial Z/\partial\theta > 0$. I follow a similar strategy as in the proof of Proposition 3 and write (B3), (B11) and (B12) as a continuous eigenvalue problem

$$(B14) \quad \eta(c, \underline{z})\omega(z) = c[\lambda(z - \underline{z})^+ + \underline{z}]\omega(z) - \frac{d}{dz}[\tilde{\mu}(z)\omega(z)] + \frac{1}{2}\frac{d^2}{dz^2}[\tilde{\sigma}^2(z)\omega(z)]$$

with boundary conditions (B1) and where $c = \theta\pi$; and to solve a system of two equations in two unknowns (c, \underline{z})

$$(B15) \quad \eta(c, \underline{z}) = \theta(\rho + \delta)$$

$$(B16) \quad \lambda \int_{\underline{z}}^{\bar{z}} \omega(z; c) dz = 1$$

For future reference, denote by $\underline{z}(c)$ the solution to (B16).

Step 1: $\partial\eta(c, \underline{z}(c))/\partial\underline{z} = 0$. **Proof:** the proof is the same as Step 1 in the proof of Proposition 3, replacing π by c everywhere.

Step 2: any solution to (B15) and (B16) satisfies $c(\partial\eta/\partial c) > \eta$. **Proof:** from

Lemma 4, $\eta(c, \underline{z})$ also satisfies the transpose eigenvalue problem

$$\eta(c, \underline{z})f(z) = c[\lambda(z - \underline{z})^+ + \underline{z}]f(z) + \tilde{\mu}(z)f'(z) + \frac{1}{2}\tilde{\sigma}^2(z)f(z)$$

Define $u(z) = \log f(z)$ so that $f(z) = e^{u(z)}$, $f' = u'e^u$, $f'' = ((u')^2 + u'')e^u$ and write (B14) as

$$(B17) \quad \eta = c[\lambda(z - \underline{z})^+ + \underline{z}] + \tilde{\mu}u' + \frac{1}{2}\tilde{\sigma}^2((u')^2 + u''), \quad u'(0) = u'(\bar{z}) = 0$$

Differentiate with respect to c and multiply by c

$$(B18) \quad c\frac{\partial\eta}{\partial c} = c[\lambda(z - \underline{z})^+ + \underline{z}] + \tilde{\mu}c\frac{\partial u'}{\partial c} + \tilde{\sigma}^2u'c\frac{\partial u'}{\partial c} + \frac{\tilde{\sigma}^2}{2}c\frac{\partial u''}{\partial c}$$

Define $v = c(\partial u/\partial c) - u$ so that

$$v' = c\frac{\partial u'}{\partial c} - u', \quad v'' = c\frac{\partial u''}{\partial c} - u'', \quad v'(0) = v'(\bar{z}) = 0$$

Subtract (B17) from (B18)

$$(B19) \quad c\frac{\partial\eta}{\partial c} - \eta = \hat{\mu}v' + \frac{\tilde{\sigma}^2}{2}v'' + \frac{\tilde{\sigma}^2}{2}(u')^2$$

where $\hat{\mu}(z) = \tilde{\mu}(z) + \tilde{\sigma}^2(z)u'(z)$. Now, define the function m as the solution to the following ODE on $(0, \bar{z})$

$$(B20) \quad 0 = -\frac{d}{dz}[\hat{\mu}(z)m(z)] + \frac{1}{2}\frac{d^2}{dz^2}[\tilde{\sigma}^2(z)m(z)],$$

with boundary conditions $-\hat{\mu}(0)m(0) + \frac{1}{2}\frac{d}{dz}[\tilde{\sigma}^2(0)m(0)] = -\hat{\mu}(\bar{z})m(\bar{z}) + \frac{1}{2}\frac{d}{dz}[\tilde{\sigma}^2(\bar{z})m(\bar{z})] = 0$. Note that this is the Kolmogorov Forward equation corresponding to the stochastic process $dz = \hat{\mu}(z)dt + \tilde{\sigma}(z)dW$. Therefore, m satisfies $\int_0^{\bar{z}} m(z)dz = 1$; and the assumption $\tilde{\sigma}(z) > 0$ (ellipticity) further implies that $m(z) > 0$ for all $z \in (0, \bar{z})$. Next, multiply (B19) by m and integrate

$$\begin{aligned} c\frac{\partial\eta}{\partial c} - \eta &= \int_0^{\bar{z}} \hat{\mu}(z)m(z)v'(z)dz + \frac{1}{2}\int_0^{\bar{z}} \tilde{\sigma}^2(z)m(z)v''(z)dz + \frac{1}{2}\int_0^{\bar{z}} \tilde{\sigma}^2(z)(u'(z))^2m(z)dz \\ &= \int_0^{\bar{z}} \left[-\frac{d}{dz}[\hat{\mu}(z)m(z)] + \frac{1}{2}\frac{d^2}{dz^2}(\tilde{\sigma}^2(z)m(z)) \right] v(z)dz + \frac{1}{2}\int_0^{\bar{z}} \tilde{\sigma}^2(z)(u'(z))^2m(z)dz \\ &= \frac{1}{2}\int_0^{\bar{z}} \tilde{\sigma}^2(z)(u'(z))^2m(z)dz > 0. \end{aligned}$$

The second equality follows from an integration by part; the third equality follows

from (B20); and the inequality follows because $m(z) > 0$ for all z and $(u'(z))^2 \geq 0$ and not identically equal to zero, i.e. strictly positive for *some* z .

Step 3: conclusion. Denote by $\underline{z}(c)$ the solution to (B16) and by $\tilde{\eta}(c) = \eta(c, \underline{z}(c))$. From Step 1, $\tilde{\eta}'(c) = \partial\eta(c, \underline{z}(c))/\partial c$ and from Step 2

$$(B21) \quad \partial \log \tilde{\eta}(c) / \partial \log c > 1$$

An equilibrium is a solution to $\tilde{\eta}(\pi\theta) = \theta(\rho + \delta)$. The solution is a function $\pi(\theta)$ which satisfies

$$\frac{\partial \log \tilde{\eta}}{\partial \log c} \left(1 + \frac{\partial \log \pi}{\partial \log \theta} \right) = 1.$$

From (B21), we have $\partial \log \pi / \partial \log \theta < 0$ and hence from (B13) $\partial Z / \partial \theta > 0$. Finally continuity of $\pi(\theta)$ and hence $Z(\theta)$ can be proved analogously to Step 3 of Proposition 3. \square

B3. Continuity and Differentiability of Wealth Shares

A difficulty arises because $s(z) = \lambda\pi \max\{z - \underline{z}, 0\} + \underline{z}\pi - \rho$ is generally not differentiable at $z = \underline{z}$. I here prove that despite this fact, $\omega(z)$ is continuous and once differentiable everywhere. The proof uses a discrete approximation. Consider a general diffusion

$$dz = \mu(z)dt + \sigma(z)dt.$$

Under certain regularity conditions this diffusion can be approximated by a binomial tree specified as follows.⁵⁴ Divide time into discrete periods of length Δt ; start at some z ; with probability $p(z)$ the process moves up some distance Δz and with probability $q(z) = 1 - p(z)$ it moves down. The step size and probabilities are given by (equations 21-23 in Nelson and Ramaswamy)

$$\Delta z = \sqrt{\Delta t}\sigma(z), \quad p(z) = \frac{1}{2} \left(1 + \frac{\mu(z)}{\sigma(z)}\sqrt{\Delta t} \right), \quad q(z) = 1 - p(z) = \frac{1}{2} \left(1 - \frac{\mu(z)}{\sigma(z)}\sqrt{\Delta t} \right).$$

As $\Delta t \rightarrow 0$, the resulting binomial process converges to the diffusion above.⁵⁵ Using the relationship between the time step and the grid size, the probabilities can also be written as

$$(B22) \quad p(z) = \frac{1}{2} \left(1 + \frac{\mu(z)}{\sigma^2(z)}\Delta z \right), \quad q(z) = 1 - p(z) = \frac{1}{2} \left(1 - \frac{\mu(z)}{\sigma^2(z)}\Delta z \right).$$

Next consider the savings behavior of an entrepreneur with productivity z : if he starts out with wealth a_t at time t , he ends up with $a_{t+\Delta t} = s(z)\Delta t a_t + a_t$ at time

⁵⁴See Nelson and Ramaswamy (1990) who in turn use results from Stroock and Varadhan (1979).

⁵⁵As noted by Nelson and Ramaswamy, the resulting tree does not recombine in the sense that an up move followed by a down move does not bring the process back to the same node. This is a problem for computational approaches but not for the theoretical derivations presented here.

$t + \Delta t$. Next, consider the point $(z, t + \Delta t)$. This could have arisen in either of two ways: $(z - \Delta z, t)$ followed by an up move, or $(z + \Delta z, t)$ followed by a down move. Combining these two observations, the wealth held by entrepreneurs with ability z at time $t + \Delta t$ must satisfy

$$(B23) \quad \omega(z, t + \Delta t) = p(z - \Delta z)[s(z - \Delta z)\Delta t + 1]\omega(z - \Delta z, t) + q(z + \Delta z)[s(z + \Delta z)\Delta t + 1]\omega(z + \Delta z, t).$$

Continuity: Wealth shares at point $(z - \Delta z, t + \Delta t)$ satisfy

$$\omega(z - \Delta z, t + \Delta t) = p(z - 2\Delta z)[s(z - 2\Delta z)\Delta t + 1]\omega(z - 2\Delta z, t) + q(z)[s(z)\Delta t + 1]\omega(z, t).$$

Taking limits as $\Delta z \rightarrow 0$ in (B23) yields $\lim_{x \uparrow z} \omega(x, t) = \frac{1}{2} [\lim_{x \uparrow z} \omega(x, t) + \omega(z, t)]$, or $\lim_{x \uparrow z} \omega(x, t) = \omega(z, t)$. A symmetric argument around the point $(z + \Delta z, t + \Delta t)$ proves that $\lim_{x \downarrow z} \omega(x, t) = \omega(z, t)$, so that $\lim_{x \uparrow z} \omega(x, t) = \lim_{x \downarrow z} \omega(x, t) = \omega(z, t)$. \square

Differentiability: The proof proceeds by taking a first-order approximation in (B23) around z . The approximation of $\omega(z - \Delta z)$ is not straightforward. For any points $z - \Delta z$ and $x < z$,

$$\omega(z - \Delta z, t) \approx \omega(x, t) + \omega_z(x, t)[(z - \Delta z) - x].$$

Taking x to z , we have

$$\omega(z - \Delta z, t) \approx \lim_{x \uparrow z} \omega(x, t) - \lim_{x \uparrow z} \omega_z(x, t)\Delta z = \omega(z, t) - \lim_{x \uparrow z} \omega_z(x, t)\Delta z,$$

where the second equality uses continuity from the first part of the proof. A similar approximation holds for $\omega(z + \Delta z, t)$. Take a first-order approximation to all terms in (B23) *except* to the terms involving $s(z - \Delta z)$ and $s(z + \Delta z)$:

$$(B24) \quad \begin{aligned} \omega(z, t) + \omega_t(z, t)\Delta t &= [p(z) - p'(z)\Delta z][s(z - \Delta z)\Delta t + 1][\omega(z, t) - \lim_{x \uparrow z} \omega_z(x, t)\Delta z] \\ &\quad + [q(z) + q'(z)\Delta z][s(z + \Delta z)\Delta t + 1][\omega(z, t) + \lim_{x \downarrow z} \omega_z(x, t)\Delta z] + o(\Delta z). \end{aligned}$$

Dropping all terms that are of order higher than Δz , and rearranging

$$0 = -p(z) \lim_{x \uparrow z} \omega_z(x, t)\Delta z + q(z) \lim_{x \downarrow z} \omega_z(x, t)\Delta z + [-p'(z) + q'(z)]\omega(z, t)\Delta z + o(\Delta z).$$

Importantly, the terms $s(z - \Delta z)\Delta t$ and $s(z + \Delta z)\Delta t$ always drop because they are of order higher than Δz . Dividing by Δz , taking limits as $\Delta z \rightarrow 0$, and using that $p(z)$ and $q(z)$ tend to $1/2$ while $p'(z)$ and $q'(z)$ tend to zero, we get $0 = \frac{1}{2} [\lim_{x \uparrow z} \omega_z(x, t) - \lim_{x \downarrow z} \omega_z(x, t)]$, which immediately implies the differentiability condition. \square

EQUIVALENCE BETWEEN RENTING AND OWNING/ACCUMULATING CAPITAL

Budget Constraints: I here show that the budget constraint (3) can be derived from a setup in which entrepreneurs own and accumulate capital themselves and trade in risk-free bonds. For sake of clarity, I present the argument for a discrete approximation to the continuous-time framework. Periods are of length Δ . The continuous-time counterparts of the expressions can be obtained by taking Δ to zero. The stock of bonds issued by an entrepreneur, that is his debt, is denoted by d_t . When $d_t < 0$ the entrepreneur is a net lender. In order for there to be an interesting role for credit markets, an entrepreneur's productivity z_t is revealed at the end of period $t - \Delta$, before the entrepreneur issues his debt d_t . That is, entrepreneurs can borrow to finance investment corresponding to their new productivity. The budget constraint and law of motion for capital are

$$0 = \Delta(y_t - w_t l_t - r_t d_t - x_{t+\Delta} - c_t) + d_{t+\Delta} - d_t, \quad k_{t+\Delta} = \Delta x_{t+\Delta} + (1 - \Delta\delta)k_t.$$

where $x_{t+\Delta}$ investment in physical capital. These can be combined as

$$(BC+) \quad k_{t+\Delta} - d_{t+\Delta} = \Delta(y_t - w_t l_t - \delta k_t - r d_t - c) + k_t - d_t.$$

I now argue that by changing slightly the time at which the budget constraint is "recorded", we can derive the budget constraint (3). To this end, define d_t^- and k_t^- as debt and capital *before* investment is made, i.e.

$$d_t^- \equiv d_t - \Delta x_t, \quad k_t^- \equiv k_t - \Delta x_t.$$

The timeline in Figure C illustrates this change in the "recording time" of the budget constraint. Using these definitions we can rewrite (BC+) as

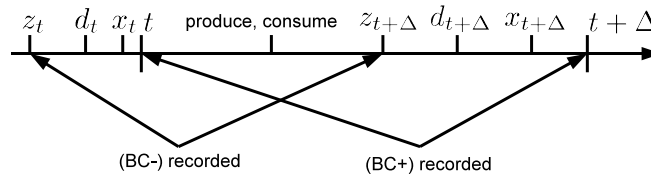


FIGURE C1. TIME LINE

Note: Entrepreneurs first observe their productivity z_t , then issue debt d_t to finance investment x_t . The budget constraints (BC+) is recorded *after* investment x is made; (BC-) is recorded *before* investment is made. This amounts to moving the start of time t forward before investment is made. The two are equivalent.

$$(BC-) \quad k_{t+\Delta}^- - d_{t+\Delta}^- = \Delta[y_t - w_t l_t - \delta k_t - r(d_t^- - k_t^- + k_t)] + k_t^- - d_t^-.$$

Defining total wealth as $a_t \equiv k_t^- - d_t^-$, and collecting terms we get $a_{t+\Delta} = \Delta[y_t - w_t l_t - (r_t + \delta)k_t + r a_t] + a_t$. Rearranging, dividing by Δ and letting Δ tend to zero we obtain (3).

Borrowing Constraint: Similarly, there is a borrowing constraint for the above environment where entrepreneurs own and accumulate capital and trade in bonds, that is equivalent to (4):

$$(BORR+) \quad d_{t+\Delta} \leq \left(1 - \frac{1}{\lambda}\right) k_{t+\Delta}.$$

This constraint says that only a fraction $1 - 1/\lambda$ of next period's capital stock can be externally financed (Note that this fraction is zero when $\lambda = 1$ and one when $\lambda = \infty$). Note the $t + \Delta$ subscripts on both sides of the constraint. This is because both k_t and d_t are state variables and are therefore fixed at time t . However, the constraint determines the next period's capital stock, $k_{t+\Delta}$. Like the budget constraint, the constraint (BORR+) can also be "backdated" to the time *before* investment is made (see the timeline above)

$$(BORR-) \quad k_{t+\Delta} \leq \lambda(k_{t+\Delta} - d_{t+\Delta}) = \lambda(k_{t+\Delta}^- - d_{t+\Delta}^-) = \lambda a_{t+\Delta}.$$

Note that we now also need to include the constraint for period t , $k_t \leq \lambda a_t$, in the constraint set because k_t is not a state variable at time t anymore.

MORE GENERAL FORMULATIONS OF THE CREDIT MARKET FRICTION

Propositions 1 and 2 have been derived under the assumption that financial frictions take the form of the simple collateral constraint (4) that places the same common limit on the leverage ratio of all entrepreneurs. This assumption, which was made for simplicity, is more restrictive than necessary and my results generalize to a number of more general formulations of the credit market friction. For instance, some readers may feel that it is more natural for the borrowing limit to depend on an entrepreneur's productivity so that (4) generalizes to

$$(D1) \quad k \leq \lambda(z)a.$$

I here show that Propositions 1 and 2 go through with slight modification for the case where λ is an arbitrary function (which may even be non-monotonic or discontinuous). To this end, define the following modified credit market quality

and wealth shares

$$(D2) \quad \begin{aligned} \tilde{\lambda}(t) &\equiv \int_0^\infty \lambda(x)\omega(x,t)dx \\ \tilde{\omega}(z,t) &\equiv \frac{\lambda(z)}{\tilde{\lambda}(t)}\omega(z,t). \end{aligned}$$

To cover the case where borrowing constraints take the form (D1), we can modify Proposition 1 as follows: simply replace λ by $\tilde{\lambda}(t)$ and $\omega(z,t)$ by $\tilde{\omega}(z,t)$. This is easy to show following the same steps as in the proof of the original Proposition so stated without proof. Similarly, in Proposition 2 only the definition of the savings rate needs to be changed to

$$(D3) \quad \tilde{s}(z) \equiv \lambda(z) \max\{z\pi - r - \delta, 0\} + r - \rho.$$

The maximum leverage ratio may also depend on the interest rate (or the wage). A simple example, in which it additionally depends on productivity, is as follows. An entrepreneur can avoid paying the interest on the loan, $(r + \delta)(k - a)$, by incurring a cost which equals a fraction $\eta \in (0, 1)$ of firm profits net of wages, $z\pi k$. Then, assuming $r + \delta > \eta\pi z$, investments satisfy $\eta\pi z k \geq (k - a)(r + \delta)$ so that

$$(D4) \quad k \leq \frac{1}{1 - \eta\pi z/(r + \delta)}a = \frac{1}{1 - \eta z/\underline{z}}a \equiv \lambda(z; \underline{z}, \eta)a.$$

Again, using definitions analogous to (D2) and (D3), all results go through with slight modification. This being said, what *is* crucial for my analytic results is the linearity of the constraint (4) in wealth, a . Although my analytic results are robust to these alternative specifications, they may have implications for the quantitative properties of the model.

PARAMETERIZATION OF COMPUTATIONS IN SECTION II

E1. Productivity Process

I assume that productivity follows the Ornstein-Uhlenbeck process (26). An attractive feature of this process is that it is the exact continuous-time equivalent of a discrete-time AR(1) process for which many good estimates are available from the literature (Gourio, 2008; Asker, Collard-Wexler and De Loecker, 2013). I choose the following parameter values. I set the capital share to $\alpha = 1/3$, and the discount and depreciation rates to $\rho = \delta = 0.05$. For the autocorrelation and innovation variance, I use as benchmark values $Corr = \exp(-(1/\theta)) = 0.85$ and $\sigma = 0.56$. This is the average of the country-specific estimates by Asker, Collard-Wexler and De Loecker (2013) for a sample of 33 developing countries (see their Table 7). Though the parameterization of Midrigan and Xu (forthcoming) is

not directly comparable, they use similar parameter values in their benchmark calibration ($Corr = 0.74$ and $\sigma = 0.52$, see their Table 2). Most of my experiments compare this benchmark parameterization to one with different θ .

In line with Assumption 1, I impose an upper bound on productivity in the form of a reflecting barrier. This bound also has the advantage that the first-best, namely allocating all resources to the most productive entrepreneur, is well-defined. I impose an upper bound \bar{z} and assume that the process (26) is reflected at this upper bound. The boundary condition corresponding to such a reflecting barrier is⁵⁶

$$(E1) \quad 0 = -\mu(\bar{z})\omega(\bar{z}, t) + \frac{1}{2} \frac{\partial}{\partial z} [\sigma^2(\bar{z})\omega(\bar{z}, t)], \quad \text{all } t.$$

The exact value for the upper bound on productivity \bar{z} is somewhat arbitrary. I below set it equal to the 95th percentile of a log-normal distribution with mean and variance as in (27), i.e. the 95th percentile of the stationary productivity distribution if there were no upper bound.⁵⁷ Some of the quantitative results will be sensitive to the exact value for the upper bound. One obvious example is the size of TFP losses relative to first-best. But qualitative results, such as the dependence of productivity losses on the persistence of shocks, do not depend on this assumption. See Appendix E.E3 for a discussion of how the size of TFP losses is affected by my choice of the upper bound and in what sense my results can be compared to the existing quantitative literature.

E2. Severity of Financial Frictions

Finally, the parameter λ that governs the degree of financial development can be disciplined with external finance to GDP ratios as in Beck, Demircuc-Kunt and Levine (2000).⁵⁸ This is possible because these external finance to GDP ratios have a direct counterpart in my model. The model predicts that the ratio

⁵⁶See for example Wong (1964). The boundary condition can be motivated from the requirement that wealth shares integrate to one, $\int_0^{\bar{z}} \omega(z, t) dz = 1$. Because this total mass has to be preserved for all t , the law of motion for wealth shares (21) implies that

$$0 = \int_0^{\bar{z}} \frac{\partial \omega(z, t)}{\partial t} dz = -[\mu(z)\omega(z, t)]_0^{\bar{z}} + \frac{1}{2} \left[\frac{\partial}{\partial z} [\sigma^2(z)\omega(z, t)] \right]_0^{\bar{z}}.$$

Using $\omega(0, t) = 0$ for all t , we obtain (E1).

⁵⁷With the reflecting barrier, the stationary distribution is still log-normal but rescaled to integrate to one between zero and the reflecting barrier.

⁵⁸External finance is defined to be the sum of private credit, private bond market capitalization, and stock market capitalization. This definition follows Buera, Kaboski and Shin (2011). See also their footnote 9.

of external finance to *capital* in a given economy equals⁵⁹

$$\frac{D}{K} = 1 - \frac{1}{\lambda}.$$

If there are no capital markets, $\lambda = 1$, there is no external finance: $D/K = 0$. If capital markets are perfect, $\lambda = \infty$, the entire capital stock of the economy is financed externally: $D/K = 1$. Together with the expression for the capital output ratio (19) this implies that the steady state external finance to GDP ratio equals

$$(E2) \quad \frac{D}{Y} = \frac{D}{K} \frac{K}{Y} = \left(1 - \frac{1}{\lambda}\right) \frac{\alpha}{\rho + \delta}.$$

Table E1 lists external-finance to GDP ratios, D/Y , from Beck, Demircuc-Kunt and Levine (2000) and implied λ 's for the US, India, and China. India and China

TABLE E1—EXTERNAL FINANCE TO GDP RATIOS (D/Y) AND IMPLIED λ 'S IN 1997

Country	US	India	China
D/Y	2.53	0.54	0.20
λ	4.15	1.2	1.06

Note: λ is calculated from (E2), assuming that $\alpha = 1/3$, $\rho = 0.05$, $\delta = 0.05$ (implying that $K/Y = 3.33$). External-finance to GDP ratios are from Beck, Demircuc-Kunt and Levine (2000).

are financially considerably less developed than the United States.

E3. Discussion of TFP Losses Relative to First-Best

As just noted, the first-best in my economy is to allocate all resources to the most productive entrepreneur, namely the one whose productivity equals the upper bound \bar{z} . The size of TFP losses relative to first-best is therefore obviously sensitive to the value I choose for this upper bound. The existing literature (Buera and Shin, 2013; Midrigan and Xu, forthcoming; Buera, Kaboski and Shin, 2011) does not face this problem because it assumes that individual production functions display decreasing returns to scale. I here discuss how to interpret the size of TFP losses from financial frictions in my framework, particularly vis-à-vis the literature. In an environment with decreasing returns, the first-best allocation would have all entrepreneurs active but more productive ones using more capital and labor. First-best TFP would depend on the returns to scale and the shape of the productivity distribution. These would need to be carefully calibrated.

⁵⁹To see this note that all active entrepreneurs borrow as much as they can, that is individual borrowing is $d = (\lambda - 1)a$ if $z \geq \bar{z}$ and all inactive entrepreneurs lend. Total borrowing in the economy therefore equals $D = \mathbb{E}[d|d \geq 0] = (\lambda - 1)(1 - \Omega(\bar{z}))K = (\lambda - 1)/\lambda K$ where the last equality uses the market clearing condition $\lambda(1 - \Omega(\bar{z})) = 1$.

Suppose production functions are $y = z(k^\alpha \ell^{(1-\alpha)})^\nu$ with $\nu < 1$. Then first-best TFP is

$$Z^{FB} = \left(\int_0^\infty z^{\frac{1}{1-\nu}} \psi(z) dz \right)^{1-\nu}.$$

Suppose in addition, that productivity is distributed Pareto $\psi(z) = \eta z^{-\eta-1}$. Then $Z^{FB} = \frac{\eta(1-\nu)}{\eta(1-\nu)-1}$. Buera, Kaboski and Shin (2011) calibrate the parameters ν and η to jointly match two moments in data for the US (which they think of as the perfect-credit benchmark): the employment share of the largest 10 percent of establishments, and the earnings share of the top 5 percent of earners. The upper bound for productivity in my framework with constant returns should be thought of as *all this information collapsed into a single number*.

An alternative strategy to the one pursued in the paper, would be to normalize TFP not by its first-best level but by the TFP level for a high value of λ (say ten). This would have the advantage that the allocation in the economy used for normalization is non-degenerate (in contrast to first-best). However, I prefer my strategy because under this alternative the denominator would vary with other parameters, in particular those governing the stochastic process (26) such as its autocorrelation thereby confounding the numerator (note that with decreasing returns, first-best TFP would not vary with those parameters as long as the stationary productivity distribution, $\psi(z)$, is held fixed).

THE MODEL WITH CRRA PREFERENCES

This Appendix spells out the results for the model with CRRA preferences that are used in section III. The coefficient of relative risk aversion is denoted by γ . For sake of brevity, I only spell out those results that are different from the case of logarithmic utility ($\gamma = 1$) considered in the main paper.

F1. Theoretical Results

The analogues of Lemma 2, Proposition 1 and Corollary 1 are:

Lemma D.1 *With CRRA utility with parameter γ , the optimal savings and consumption policy functions are linear in wealth*

(F1)

$$\begin{aligned} \dot{a} &= \tilde{s}(z, t)a, \quad \text{where} \quad \tilde{s}(z, t) = \lambda \max\{z\pi(t) - r(t) - \delta, 0\} + r(t) - \tilde{c}(z, t) \\ c(a, z, t) &= \tilde{c}(z, t)a \end{aligned}$$

where $\tilde{c}(z, t)$ is type z 's marginal propensity to consume out of wealth, which satisfies the second-order partial differential equation (F7) at the end of this Appendix. In the special case of log-utility, $\gamma = 1$, $\tilde{c}(z, t) = \rho$.

Proposition D.1 *Given a time path for wealth shares $\omega(z, t), t \geq 0$, aggregate*

quantities satisfy

$$\begin{aligned} Y &= ZK^\alpha L^{1-\alpha}, \\ \dot{K} &= \alpha ZK^\alpha L^{1-\alpha} - (\tilde{\rho} + \delta)K, \end{aligned}$$

where

$$(F2) \quad \tilde{\rho}(t) = \int_0^\infty \tilde{c}(z, t)\omega(z, t)dz,$$

is the aggregate marginal propensity to consume out of wealth, and all remaining aggregates are as in Proposition 1.

Corollary D.1 *Given stationary wealth shares $\omega(z)$, aggregate steady state quantities solve*

$$\begin{aligned} Y &= ZK^\alpha L^{1-\alpha} \\ \alpha ZK^{\alpha-1} L^{1-\alpha} &= \tilde{\rho} + \delta \end{aligned}$$

where

$$(F3) \quad \tilde{\rho} = \int_0^\infty \tilde{c}(z)\omega(z)dz,$$

and all remaining steady state aggregates are as in Corollary 1.

All remaining expressions are the same simple replacing $s(z, t)$ and ρ by $\tilde{s}(z, t)$ and $\tilde{\rho}(t)$ defined in (F1) and (F2).

F2. Numerical Results

Panel (a) of Figure F1 plots the steady state capital to output ratio $K/Y = \alpha/(\tilde{\rho} + \delta)$ and TFP against λ for three values of $\gamma = 0.5, 1, 2$. The remaining parameter values are those in Appendix E. As stated in the main text, going from $\lambda = 1$ to $\lambda = 4.15$ with $\gamma = 2$, K/Y falls from 4.18 to 3.85 or roughly eight percent; and with $\gamma = 1/2$ it increases from 3.12 to 3.21 or roughly three percent. Panel (b) plots aggregate TFP against $Corr(\log z(t+1), \log z(t)) = e^{-1/\theta}$ for different values of γ and shows that, as in Figure 2 (a), TFP is still a strictly decreasing function of persistence for $\gamma \neq 1$. Figure F2 replicates panel (a) of Figures 4 and K2, showing the transition dynamics of aggregate TFP for $\gamma \in \{0.5, 1, 2\}$ and different initial wealth shares $\omega_0(z)$. Different values of γ have a relatively small effect on transition dynamics and it is still true that transitions are more prolonged if shocks are more persistent.

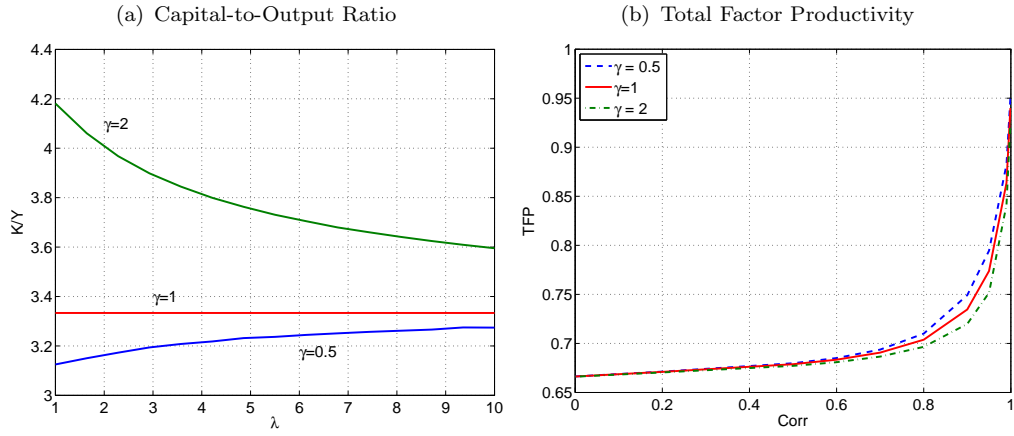


FIGURE F1. STEADY STATE CAPITAL-TO-OUTPUT RATIO AND AGGREGATE TFP LOSSES WITH CRRA UTILITY

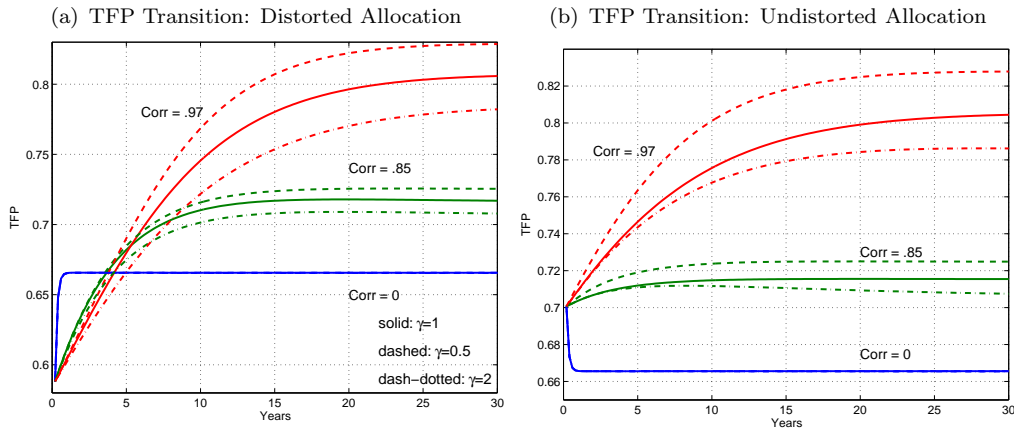


FIGURE F2. TFP TRANSITIONS WITH CRRA UTILITY

F3. Proofs

Proof of Proposition D.1 Consider next the law of motion for aggregate capital

$$(F4) \quad \begin{aligned} \dot{K}(t) &= \int_0^\infty s(z, t)\omega(z, t)dzK(t) \\ &= \int_0^\infty [\lambda \max\{z\pi(t) - r(t) - \delta, 0\} + r(t) - \tilde{c}(z, t)]\omega(z, t)dzK(t). \end{aligned}$$

Using that the shares $\omega(z)$ integrate to one, we have that

$$\frac{\dot{K}}{K} = \lambda\pi \int_{\underline{z}}^\infty z\omega(z)dz - \lambda(r + \delta) \int_{\underline{z}}^\infty \omega(z)dz + r - \int_0^\infty \tilde{c}(z)\omega(z)dz.$$

Using capital market clearing (14),

$$(F5) \quad \frac{\dot{K}}{K} = \lambda\pi X - (\tilde{\rho} + \delta), \quad X \equiv \int_{\underline{z}}^\infty z\omega(z)dz, \quad \tilde{\rho} = \int_0^\infty \tilde{c}(z)\omega(z)dz.$$

Substituting (A4) into (F5) and rearranging, we get

$$\dot{K} = \alpha Z K^\alpha L^{1-\alpha} - (\tilde{\rho} + \delta)K, \quad Z = (\lambda X)^\alpha.$$

After substituting for λ from (14), this is equation (12) in Proposition 1. Substituting the definition of π from Lemma 1 into (A4) and rearranging yields the expression for w . Substituting (A4) into the cutoff condition $\underline{z}\pi = R$ and rearranging yields the expression for R . \square

Proof of Lemma D.1 From Lemma 1, we know that $\dot{a} = A(z, t)a - c$ where $A(z, t) = \lambda \max\{z\pi(t) - r(t) - \delta, 0\} + r(t)$. The Bellman equation is then (see Ch.2 in Stokey, 2009),

$$\rho V(a, z, t) = \max_c \frac{c^{1-\gamma}}{1-\gamma} + V_a(a, z, t)(A(z, t)a - c) + V_z(a, z, t)\mu(z) + \frac{1}{2}V_{zz}(a, z, t)\sigma^2(z) + V_t(a, z, t).$$

The first order condition is $c^{-\gamma} = V_a(a, z, t)$ and hence

$$\rho V(a, z, t) = -\frac{V_a(a, z, t)^{1-1/\gamma}}{1-1/\gamma} + V_a(a, z, t)A(z, t)a + V_z(a, z, t)\mu(z) + \frac{1}{2}V_{zz}(a, z, t)\sigma^2(z) + V_t(a, z, t).$$

The proof proceeds with a guess and verify strategy. Guess that the value function takes the form

$$V(a, z, t) = v(z, t)a^{1-\gamma}.$$

With this guess $V_a(a, z, t) = (1 - \gamma)v(z, t)a^{-\gamma}$. Using this guess in the Bellman

equation and canceling the terms $a^{1-\gamma}$ yields

$$(F6) \quad \begin{aligned} \rho v(z, t) = & \gamma v(z, t)^{1-1/\gamma} + (1 - \gamma)v(z, t)[\lambda \max\{z\pi(t) - r(t) - \delta(t), 0\} + r(t)] \\ & + v_z(z, t)\mu(z) + \frac{1}{2}v_{zz}(z, t)\sigma^2(z) + v_t(z, t) \end{aligned}$$

From the first-order condition $c^{-\gamma} = v(z, t)a^{-\gamma}$, consumption can be written as $c(a, z, t) = \tilde{c}(z, t)a$ where $\tilde{c}(z, t) = v(z, t)^{-1/\gamma}$ and therefore savings are given by (B3). Finally substitute $v = \tilde{c}^{-\gamma}$, $v_z = -\gamma\tilde{c}^{-\gamma-1}\tilde{c}_z$, $v_{zz} = \gamma(1 + \gamma)\tilde{c}^{-\gamma-2}\tilde{c}_z - \gamma\tilde{c}^{-\gamma-1}\tilde{c}_{zz}$ into (F6), and multiply by $\tilde{c}^{1+\gamma}$ to get a PDE for $\tilde{c}(z, t)$:

$$(F7) \quad \begin{aligned} \rho\tilde{c}(z, t) = & \gamma\tilde{c}(z, t)^2 + (1 - \gamma)\tilde{c}(z, t)[\lambda \max\{z\pi(t) - r(t) - \delta(t), 0\} + r(t)] \\ & - \gamma\tilde{c}_z(z, t)\mu(z) + \frac{1}{2} \left(\gamma(1 + \gamma)\frac{\tilde{c}_z(z, t)}{\tilde{c}(z, t)} - \gamma\tilde{c}_{zz}(z, t) \right) \sigma^2(z) - \gamma\frac{\partial\tilde{c}(z, t)}{\partial t}. \end{aligned}$$

F4. Numerical Solution of Model with CRRA Utility

This section describes how I extend the computational algorithm in J to handle the case of CRRA utility. An additional challenge relative to the logarithmic case, is that the optimal consumption policy function depends on prices w and r (because these enter the HJB equation). I therefore employ an iterative procedure. As an initial guess for the case $\gamma \neq 1$, I use the prices from the steady state equilibrium with logarithmic utility ($\gamma = 1$) which is computed as described in Appendix J. I denote these by (w_0, r_0) , and then for $j = 1, 2, \dots$ I follow

- 1) Given (w_j, r_j) , solve the HJB equation (F6) using the algorithm described in the end of this section.
- 2) Given $\tilde{c}_j(z) = v_j(z)^{-1/\gamma}$, compute $\tilde{s}_j(z)$ defined in (F1) and $\tilde{\rho}_j$ defined in (F3).
- 3) Given $\tilde{s}_j(z)$ and $\tilde{\rho}_j$, solve the differential equation for wealth shares (21) as described in Appendix J, simply replacing $s(z)$ and ρ by $\tilde{s}_j(z)$ and $\tilde{\rho}_j$ everywhere.
- 4) Compute the steady state prices (w_{j+1}, r_{j+1}) .

When (w_{j+1}, r_{j+1}) is close enough to (w_j, r_j) , I call this a steady state equilibrium with CRRA utility.

Numerical Solution of Value Function with CRRA Utility The stationary value function corresponding to (F6) is

$$(F8) \quad \rho v(z) = \gamma v(z)^{1-1/\gamma} + (1 - \gamma)v(z)[\lambda \max\{z\pi - r - \delta, 0\} + r] + v'(z)\mu(z) + \frac{1}{2}v''(z)\sigma^2(z)$$

and since the z -process is reflected at \bar{z} we have the corresponding boundary condition:

$$v'(\bar{z}) = 0.$$

As I will show momentarily, a boundary condition at $z = 0$ is not needed because “takes care of itself.” I solve (F8) with a finite difference method. The discretization of (F8) is:

$$\frac{v_i^{n+1} - v_i^n}{\Delta} + \rho v_i^{n+1} = \gamma(v_i^n)^{1-1/\gamma} + (1-\gamma)v_i^{n+1}\Pi_i + \frac{v_{i+1}^{n+1} - v_i^{n+1}}{\Delta z} \mu_i + \frac{1}{2} \frac{v_{i+1}^{n+1} - 2v_i^{n+1} + v_{i-1}^{n+1}}{(\Delta z)^2} \sigma_i^2$$

which can be written as

$$\begin{aligned} v_{i+1}^{n+1} \left(-\frac{\mu_i}{\Delta z} - \frac{1}{2} \frac{\sigma_i^2}{(\Delta z)^2} \right) + v_i^{n+1} \left(\frac{1}{\Delta} + \rho - (1-\gamma)\Pi_i + \frac{\mu_i}{\Delta z} + \frac{\sigma_i^2}{(\Delta z)^2} \right) + v_{i-1}^{n+1} \left(-\frac{1}{2} \frac{\sigma_i^2}{(\Delta z)^2} \right) \\ = \gamma(v_i^n)^{1-1/\gamma} + \frac{v_i^n}{\Delta} \end{aligned}$$

At the upper boundary and $v'(z_I) = 0$ so that $v_{I+1} = v_I$ and therefore.

$$v_I^{n+1} \left(\frac{1}{\Delta} + \rho - (1-\gamma)\Pi_I + \frac{1}{2} \frac{\sigma_I^2}{(\Delta z)^2} \right) + v_{I-1}^{n+1} \left(-\frac{1}{2} \frac{\sigma_I^2}{(\Delta z)^2} \right) = \gamma(v_I^n)^{1-1/\gamma} + \frac{v_I^n}{\Delta}$$

At the lower boundary, $z = 0$, we have that $\mu(0) = \sigma^2(0) = 0$ and therefore

$$v_1^{n+1} \left(\frac{1}{\Delta} + \rho - (1-\gamma)\Pi_1 + \frac{\sigma_1^2}{(\Delta z)^2} \right) = \gamma(v_1^n)^{1-1/\gamma} + \frac{v_1^n}{\Delta}$$

so v_0 is never used and therefore no boundary condition is needed. I solve this using an iterative procedure. As an initial guess I use $v_i^0 = \rho^{-\gamma}$.

DECREASING RETURNS TO SCALE

The purpose of this Appendix is to show that many formulas on the production side of the economy are continuous in returns to scale. To this end, assume that production technologies are given by $y = ((zk)^\alpha \ell^{1-\alpha})^\nu$, $\nu < 1$. Consider first the first-best. It is easy to show that the optimal resource allocation and GDP are given by

$$\begin{aligned} \frac{k(z; \nu)}{K} = \frac{\ell(z; \nu)}{L} = \frac{z^{\frac{\alpha\nu}{1-\nu}}}{\int_0^{\bar{z}} z^{\frac{\alpha\nu}{1-\nu}} \psi(z) dz} \\ Y^{FB} = Z^{FB}(\nu)(K^\alpha L^{1-\alpha})^\nu, \quad Z(\nu) = \left(\int_0^{\bar{z}} z^{\frac{\alpha\nu}{1-\nu}} \psi(z) dz \right)^{1-\nu}. \end{aligned}$$

These expressions are continuous in ν and it is easy show that my formulas for the constant returns case obtain in the limit as $\nu \rightarrow 1$:

$$\lim_{\nu \rightarrow 1} \frac{k(z; \nu)}{K} = \lim_{\nu \rightarrow 1} \frac{\ell(z; \nu)}{L} = \begin{cases} 1, & z = \bar{z} \\ 0, & z < \bar{z} \end{cases}$$

$$\lim_{\nu \rightarrow 1} Z^{FB}(\nu) = \bar{z}^\alpha.$$

Next consider, the economy with collateral constraints. Entrepreneurs still solve (5) but with the decreasing returns technology. The solution is

$$k(a, z) = \min\{k^u(z), \lambda a\}$$

$$k^u(z) = z^{\frac{\alpha\nu}{1-\nu}} \left(\frac{\alpha\nu}{r + \delta} \right)^{\frac{1-(1-\alpha)\nu}{1-\nu}} \left(\frac{(1-\alpha)\nu}{w} \right)^{\frac{(1-\alpha)\nu}{1-\nu}}$$

$$\ell(a, z) = \left(\frac{(1-\alpha)\nu(zk(a, z))^{\alpha\nu}}{w} \right)^{\frac{1}{1-(1-\alpha)\nu}}$$

Again, it can be seen that the expressions in Lemma 1 obtain in the limit as $\nu \rightarrow 1$. In particular, note that in this limit $k^u(z) \rightarrow \infty$ for all z so that all firms are constrained. Substituting these expressions into the capital and labor market clearing conditions (7) and (8), we have expressions defining the equilibrium r and w (for a given distribution $G(a, z)$). Substituting into $Y = \int (zk(a, z))^{\alpha\nu} \ell(a, z)^{(1-\alpha)\nu} dG(a, z)$, we have an expression for GDP. Again taking the limit as $\nu \rightarrow 1$, one obtains my expressions for GDP and TFP in the constant returns case, (11) and (13).

IMPLICATIONS OF RESULTS IN SECTION II.B FOR LITERATURE ON PRODUCTIVITY LOSSES FROM FINANCIAL FRICTIONS

As shown in Figure 2, TFP is a very “steep” function of autocorrelation for *high* values of the latter. Third, the same is not true for *low* values of autocorrelation for which TFP is relatively “flat” (that is, TFP is convex as a function of autocorrelation). Put differently, in terms of TFP losses an autocorrelation of, say, .95 is relatively far apart from one of .99. For instance, in the extreme case of no capital markets, $\lambda = 1$, the TFP losses with $Corr = .95$ are 26 percent whereas with $Corr = .99$ they are only 16.6 percent.

This “steepness” of TFP with respect to persistence potentially allows for a reconciliation of some of the very different quantitative results in the literature. For instance, Buera, Kaboski and Shin (2011) find that financial frictions can explain TFP losses of up to 40% and that roughly half of this due to intensive margin capital misallocation (see their Figure 4). In contrast, Midrigan and Xu (forthcoming) find that intensive margin capital misallocation only leads to relatively small TFP losses of 5 to 10%. One main difference between the two papers lies

in the form and calibration of the productivity process faced by entrepreneurs. Buera, Kaboski and Shin (2011) assume that every period entrepreneurs get a new productivity draw from an exogenous distribution with some probability. In contrast, Midrigan and Xu (forthcoming) work with a stochastic process that features both a permanent and a transitory component and in their calibration the permanent component accounts for two thirds of the cross-sectional variance of productivity. While neither of these two productivity processes is directly comparable to the one in the present paper, my analysis captures the logic that leads them to obtain such different numbers, namely that Midrigan and Xu’s stochastic process features higher persistence (broadly defined) than the one of Buera, Kaboski and Shin.⁶⁰ When Midrigan and Xu instead calibrate a model without a permanent component and a transitory component with an autocorrelation of .92, the productivity losses they report increase to 18.1% (see their Table 5).⁶¹ When they additionally lower the autocorrelation to 0.8, TFP losses increase further to 29.5%. Therefore their framework seems to display the same “steepness” as in my model so that similar values of persistence may be quite far apart from each other in terms of TFP losses.

CLOSED FORM EXAMPLES FOR STEADY STATES WITH $\lambda = 1$

The main purpose of this Appendix is to illustrate the role of the *autocorrelation* of productivity shocks for capital misallocation and implied TFP losses. To do so, I specialize to the extreme case of no capital markets, $\lambda = 1$. The case $\lambda = 1$ is restrictive but carries all intuition for the more general case $\lambda \geq 1$. The latter is analyzed numerically in the main text. By specializing the stochastic process (20), I can solve the ODE for stationary wealth shares $\omega(z)$, (22), in closed form. All aggregate variables in the model can then be obtained in closed form as well. I present two examples corresponding to two particular forms of the stochastic process (20).

11. Example 1: Feller Square Root Process

The following stochastic process which is known as a Feller square root process is convenient:⁶²

$$(II) \quad dz = (1/\theta)(1 - z)dt + \sigma\sqrt{z/\theta}dW,$$

⁶⁰Note in particular that for the stochastic process (26) the limit as $Corr = \exp(-1/\theta) \rightarrow 1$ means that individual productivities are fixed. This is therefore also the limiting case of Midrigan and Xu with only a permanent component, or the one in Buera, Kaboski and Shin in which the probability of getting a new draw is zero. Similarly, my limit as $Corr = \exp(-1/\theta) \rightarrow 0$ means that productivity shocks are iid over time. This corresponds to Midrigan and Xu’s process with only a transitory component which is also iid, and Buera, Kaboski and Shin’s process when new draws arrive with probability one.

⁶¹These are productivity losses for an economy with the external-finance-to-GDP ratio of Colombia equal to .3. Unfortunately their table 5 does not report the TFP losses for an economy with no debt $\lambda = 1$. However, using a similar calculation as in Table 1, the Colombian external-finance-to-GDP ratio implies that $\lambda = 1.1$ so their TFP losses corresponding to $\lambda = 1$ should be only slightly larger.

⁶²See Cox, Ingersoll and Ross (1985) for an application in finance.

where θ and σ are positive. This is just the special case of (20) with a drift term $\mu(z) = (1/\theta)(1 - z)$ and a diffusion term $\sigma(z) = \sigma\sqrt{z/\theta}$. Importantly, this process is mean-reverting and therefore allows for a stationary distribution. The speed of mean reversion is determined by the parameter θ . The stationary distribution is given by

$$(I2) \quad \psi(z) \propto e^{-\gamma z} z^{\gamma-1}, \quad \gamma = \frac{2}{\sigma^2}.$$

This is the formula for a Gamma distribution with both parameters equal to γ . The mean and variance are

$$(I3) \quad \mathbb{E}[z] = 1, \quad Var[z] = \frac{\sigma^2}{2} = \frac{1}{\gamma}.$$

I impose the parameter restriction $\sigma^2 < 2$. As can be seen from (I2), this assumption ensures that the stationary distribution has zero density at $z = 0$.

This section is chiefly concerned with the persistence of productivity shocks. Wong (1964) shows that – similarly to the Ornstein-Uhlenbeck process (26) in the main text – the autocorrelation of z between two dates t and $t + s, s \geq 0$ is given by

$$(I4) \quad Corr[z(t), z(t + s)] = e^{-(1/\theta)s} \in (0, 1].$$

Under the specific functional form (II), one can solve the ODE for wealth shares (22) using a guess-and-verify strategy.

PROPOSITION 5: *Consider an economy with no credit markets $\lambda = 1$, and where productivity follows the stochastic process (II). Then the stationary wealth shares are given by*

$$(I5) \quad \omega(z) \propto e^{-\beta z} z^{\gamma-1}, \quad \beta = \gamma - \theta(\rho + \delta), \quad \gamma = \frac{2}{\sigma^2}.$$

(The proof is at the end of this section.) The behavior of the wealth shares is similar to those for the Ornstein-Uhlenbeck process in Figure 1 in the main text. In particular, as we increase the autocorrelation of productivity shocks above zero, self-financing becomes more and more feasible and wealth becomes more and more concentrated among high productivity types. Under the specific functional form for the productivity process (II), one can also obtain an expression for aggregate TFP. Using that the wealth shares are Gamma and that therefore TFP is $Z = \mathbb{E}_\omega(z)^\alpha = (\gamma/\beta)^\alpha$, we obtain the expression

$$(I6) \quad Z = \left(\frac{1}{1 - \theta(\rho + \delta)/\gamma} \right)^\alpha.$$

Figure I1 shows how TFP changes with autocorrelation $Corr(z(t), z(t+1)) = \exp(-1/\theta)$. As expected, TFP is higher the more correlated are productivity

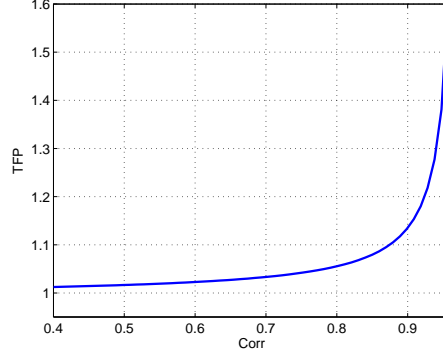


FIGURE I1. TFP AND AUTOCORRELATION WITH FELLER SQUARE ROOT PROCESS FOR PRODUCTIVITY.

Note: TFP as in (16) plotted against $Corr(z(t), z(t+1)) = \exp(-1/\theta)$. Other parameters are $\alpha = 1/3, \gamma = 3, \rho = \delta = 0.05$.

shocks. This follows immediately from the fact that wealth is more concentrated among high productivity types so that there is less capital misallocation. Two limiting cases are also of interest: first

$$Z \rightarrow \mathbb{E}[z]^\alpha = 1 \quad \text{as} \quad \theta \rightarrow 0 \quad (\text{so that} \quad Corr \rightarrow 0).$$

As already discussed, this limit corresponds to the case where productivity shocks are iid over time, also implying that $\omega(z) = \psi(z)$. TFP is then given by the (unweighted) average productivity which here equals unity. Second,

$$Z \rightarrow \max\{z\}^\alpha = \infty \quad \text{as} \quad \theta \rightarrow \bar{\theta} = \frac{\rho + \delta}{\gamma}, \quad (\text{so that} \quad Corr \rightarrow \overline{Corr} = e^{-(1/\bar{\theta})}).$$

That is, if autocorrelation is sufficiently high, all wealth is held by the highest productivity type (here $z = \infty$) so that TFP is first-best. This is true even though capital markets are completely shut down, $\lambda = 1$. As in the main text, TFP is also convex as a function of autocorrelation.

Proof of Proposition 5: With $\lambda = 1$, $s(z) = z\pi - \rho - \delta$. Using the drift and diffusion in the stochastic process (11), the ODE (22) becomes

$$(I7) \quad 0 = \theta[z\pi - \rho - \delta]\omega(z) - \frac{d}{dz} [(1-z)\omega(z)] + \frac{1}{2} \frac{d^2}{dz^2} [\sigma^2 z\omega(z)].$$

There is an additional restriction ensuring that aggregate capital is constant

$$(I8) \quad 0 = \int_0^\infty s(z)\omega(z) = \pi \int_0^\infty z\omega(z)dz - \rho - \delta.$$

This will be crucial below. Guess a functional form $\omega(z) = e^{-\beta z} z^{\gamma-1}$ (provided that the solution integrates, it can always be scaled so as to integrate to one). Substitute the guess into the ODE and proceed by equating coefficients on three terms:

$$(I9) \quad e^{-\beta z} z^\gamma, \quad e^{-\beta z} z^{\gamma-1}, \quad e^{-\beta z} z^{\gamma-2}.$$

Consider first the coefficients on the third term. The level term does not contribute to this, the drift term contributes $-(\gamma-1)$ and the diffusion term $\sigma^2\gamma(\gamma-1)/2$. Thus $0 = -(\gamma-1) + \sigma^2\gamma(\gamma-1)/2$, or $\gamma = 2/\sigma^2$. Consider next the second term of (I9). The level term contributes $-\theta(\rho+\delta)$, the drift term $\beta+\gamma$ and the diffusion term $-\beta\gamma\sigma^2$ so that $0 = -\theta(\rho+\delta) + \beta + \gamma - \beta\gamma\sigma^2$. Rearrange and use the expression for γ ,

$$(I10) \quad \beta = \gamma - \theta(\rho + \delta).$$

For the first term in (I9), the level term contributes $\theta\pi$, the drift term $-\beta$ and the diffusion term $\sigma^2\beta^2/2$ so that $0 = \theta\pi - \beta + \sigma^2\beta^2/2$. Using (I10), this can be written as

$$(I11) \quad \pi = \frac{1}{\theta} \left(\beta - \frac{\sigma^2}{2}\beta^2 \right) = \frac{\beta}{\theta\gamma} (\gamma - \beta) = \frac{\beta}{\gamma} (\rho + \delta)$$

Consider next condition (I8). For a Gamma distribution, $\mathbb{E}_\omega(z) = \gamma/\beta$ so that, $\pi\mathbb{E}_\omega(z) = \pi(\gamma/\beta) = \rho + \delta$. This coincides with (I11) so that the guess indeed solves (I7).

I2. Example 2: Geometric Mean Reverting Process

The following stochastic process also yields closed form solutions:

$$(I12) \quad \frac{dz}{z} = (1/\theta) \left[(1-z) + \frac{\sigma^2}{2} \right] dt + \sigma\sqrt{1/\theta}dW,$$

where θ and σ are positive.⁶³ The parameter θ governs the speed of mean-reversion. The stationary distribution is given by $\psi(z) \propto e^{-\beta z} z^{\beta-1}$, $\beta = 2/\sigma^2$, which is again a Gamma distribution. The mean and variance are $\mathbb{E}(z) = 1$,

⁶³ The term $\sigma^2/2$ in the drift makes algebra easier below. It is a correction for the fact that (I12) is a geometric process, i.e. it has *relative* increments dz/z . Applying Itô's Lemma, $\hat{z} \equiv \log z$ evolves according to $d\hat{z} = (1/\theta)(1-\exp(\hat{z})) + \sigma\sqrt{1/\theta}dW$ which does not involve the term $\sigma^2/2$. See the discussion in Dixit (1993) of what he terms the "Jensen-Itô effect".

$Var(z) = \sigma^2/2 = 1/\beta$. Again, the assumption $\sigma^2 < 2$ is imposed. As in Proposition 5, the wealth shares can be found through a guess-and-verify strategy and are

$$\omega(z) \propto e^{-\beta z} z^{\gamma-1}, \quad \beta = \frac{2}{\sigma^2}, \quad \gamma = \frac{\beta}{2} \left(1 + \sqrt{1 + \frac{4}{\beta} \theta (\rho + \delta)} \right).$$

The reader can verify that these behave qualitatively exactly as in Figure 1 for the wealth shares resulting from the geometric mean reverting process in the main text. Similarly, TFP is given by

$$(I13) \quad Z = \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 + \frac{4}{\beta} \theta (\rho + \delta)} \right)^\alpha.$$

TFP also behaves qualitatively as (I6) in Figure 4 (i.e. it is also convex in $Corr = \exp(-(1/\theta))$).

COMPUTATION OF EQUILIBRIUM

In this section I describe the algorithm I use to calculate equilibria, that is time paths for functions $(\omega(z, t), K(t), \underline{z}(t), Z(t), Y(t), w(t), r(t))$ satisfying (11)-(15) and (21). I approximate these functions at N discrete points in the time dimension, $t^n, n = 1, \dots, N$, and the wealth shares at I discrete points in the space dimension, $z_i, i = 1, \dots, I$. I use equispaced grids, denote by Δz and Δt the distance between grid points, and use the short-hand notation $\omega_i^n \equiv \omega(z_i, t^n)$, $K^n \equiv K(t^n)$ and so on. The partial differential equation (21) is solved using an implicit finite difference method. A MATLAB version of the code is available at <http://www.princeton.edu/~moll/transition.m>

I set $(K^0, \omega_i^0) = (K(0), \omega(z_i, 0)), i = 1, \dots, I$, the exogenously given initial state. Then for $n = 0, 1, 2, \dots, N$, I follow

Step 1 (solve equilibrium at given point in time t^n): Given $K^n, \omega_i^n, i = 1, \dots, I$, compute \underline{z}^n from (14), Z^n from (13), Y^n from (11), and r^n, w^n from (15).

Step 2 (compute wealth shares and capital stock at next point in time t^{n+1}): Use a forward-difference approximation for the aggregate capital stock

$$\dot{K}(t^n) \approx \frac{K^{n+1} - K^n}{\Delta t}$$

and use (12) to solve for $K^{n+1} = \Delta t[\alpha Y^n - (\rho + \delta)K^n] + K^n$. Compute $s_i^n = s(z_i, t^n)$ from (6). Next solve for ω^{n+1} from the PDE (21) as follows. Write (21) as

$$(J1) \quad \frac{\partial \omega(z, t)}{\partial t} = a(z, t)\omega(z, t) + b(z)\frac{\partial \omega(z, t)}{\partial z} + c(z)\frac{\partial^2 \omega(z, t)}{\partial z^2}$$

where⁶⁴

$$a(z, t) = s(z, t) - \frac{\dot{K}(t)}{K(t)} - \mu'(z) + \frac{1}{2}(\sigma^2)''(z), \quad b(z) = -\mu(z) + (\sigma^2)'(z), \quad c(z) = \frac{1}{2}\sigma^2(z)$$

Use a forward-difference approximation in the time dimension and central-difference approximations in the space dimension

$$\frac{\partial \omega(z_i, t^n)}{\partial t} \approx \frac{\omega_i^{n+1} - \omega_i^n}{\Delta t}, \quad \frac{\partial \omega(z_i, t^n)}{\partial z} \approx \frac{\omega_{i+1}^n - \omega_{i-1}^n}{2\Delta z}, \quad \frac{\partial^2 \omega(z_i, t^n)}{\partial z^2} \approx \frac{\omega_{i+1}^n - 2\omega_i^n + \omega_{i-1}^n}{(\Delta z)^2}$$

to write (J1) as

$$\frac{\omega_i^{n+1} - \omega_i^n}{\Delta t} = a_i^n \omega_i^{n+1} + b_i \frac{\omega_{i+1}^{n+1} - \omega_{i-1}^{n+1}}{2\Delta z} + c_i \frac{\omega_{i+1}^{n+1} - 2\omega_i^{n+1} + \omega_{i-1}^{n+1}}{(\Delta z)^2}$$

Note that the spatial derivatives on the right hand side are evaluated at the $n+1$ time level so this is an implicit scheme.⁶⁵ Rearrange to obtain

$$(J2) \quad \omega_{i-1}^{n+1} x_i + \omega_i^{n+1} y_i + \omega_{i+1}^{n+1} z_i = \omega_i^n$$

where

$$x_i = b_i \frac{\Delta t}{2\Delta z} - c_i \frac{\Delta t}{(\Delta z)^2}, \quad y_i = 1 - a_i \Delta t + c_i \frac{2\Delta t}{(\Delta z)^2}, \quad z_i = -b_i \frac{\Delta t}{2\Delta z} - c_i \frac{\Delta t}{(\Delta z)^2}$$

Impose the boundary conditions $\omega_1^{n+1} = 0$ and

$$(J3) \quad \Delta z \sum_{j=1}^I \omega_j^{n+1} = 1.$$

For a given n , (J2) and (J3) is then simply a system of $I-1$ equations in $I-1$

⁶⁴For example, with the Ornstein-Uhlenbeck process (26), $\mu(z) = (1/\theta)(-\log z + \sigma^2/2)z$, $\sigma^2(z) = \sigma^2 z^2/\theta$:

$$a(z, t) = s(z, t) - \frac{\dot{K}(t)}{K(t)} + \frac{1}{\theta} \left(\log z + 1 + \frac{\sigma^2}{2} \right), \quad b(z) = (1/\theta)(\log z - \sigma^2/2)z + 2(\sigma^2/\theta)z, \quad c(z) = \frac{\sigma^2}{2\theta} z^2.$$

⁶⁵Explicit finite difference schemes are well-known to be computationally slow because the time step is limited by the so-called Courant-Friedrichs-Lewy condition. In contrast, implicit schemes often (but not always) allow for arbitrarily large time steps.

unknowns $(\omega_2^{n+1}, \dots, \omega_I^{n+1})$. In matrix form it can be written as

$$\begin{bmatrix} y_2 & z_2 & 0 & 0 & \cdots & 0 \\ x_3 & y_3 & z_3 & 0 & \cdots & 0 \\ 0 & x_4 & y_4 & z_4 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \cdots & \cdots & \cdots & x_{I-1} & y_{I-1} & z_{I-1} \\ \Delta z & \Delta z & \cdots & \cdots & \Delta z & \Delta z \end{bmatrix} \begin{bmatrix} \omega_2^{n+1} \\ \omega_3^{n+1} \\ \omega_4^{n+1} \\ \vdots \\ \omega_{I-2}^{n+1} \\ \omega_{I-1}^{n+1} \\ \omega_I^{n+1} \end{bmatrix} = \begin{bmatrix} \omega_2^n \\ \omega_3^n \\ \omega_4^n \\ \vdots \\ \omega_{I-2}^n \\ \omega_{I-1}^n \\ 1 \end{bmatrix}$$

This system is then solved for ω^{n+1} by inverting the matrix on the left-hand-side.

DETAILS ON TRANSITION EXPERIMENTS

Transition After Removing Distortionary Taxes (Experiment 1). Following Buera and Shin (2013), I start transitions from a stationary equilibrium under financial frictions and idiosyncratic taxes or wedges, τ_i . Entrepreneur i solves

$$\Pi(a_i, z_i) = \max_{k_i, \ell_i} (1 - \tau_i) f(z_i, k_i, \ell_i) - w\ell_i - (r + \delta)k_i \quad \text{s.t.} \quad k_i \leq \lambda a_i.$$

I also assume that τ_i is a random variable with two possible outcomes $\tau_+ \geq 0$ and $\tau_- \leq 0$ and with probability $\Pr(\tau_i = \tau_+ | z) = 1 - e^{-qz}$ and adopt their values $\tau_+ = 0.5, \tau_- = -0.15, q = 1.55$. The analytical tractability of my model remains unchanged. In particular, one can simply replace productivity z by $\tilde{z}(z) = ((1 - \tau_-)e^{-qz} + (1 - \tau_+)(1 - e^{-qz}))^{1/\alpha} z$ in firms' profits and savings rates, and by $z^\alpha \tilde{z}(z)^{1-\alpha}$ in firms' and aggregate output and productivity. For example, aggregate TFP becomes $Z = \mathbb{E}_\omega [z^\alpha \tilde{z}(z)^{1-\alpha} | z \geq \underline{z}]^\alpha$.

Figure K1 plots the transition dynamics of TFP, the capital stock, GDP and the interest rate. TFP in panel (a), for example, jumps up when the reform is implemented and then converges gradually if shocks are persistent and remains constant if shocks are iid.

Transition from Relatively Undistorted Initial Wealth Shares (Experiment 3). Figure K2 plots transition dynamics from exogenously given initial wealth shares (29). In contrast to Figure 4 these are relatively undistorted with $m = 0.25$. It is instructive to contrast the time paths for TFP in panel (a) with those in Figure 4. With iid productivity shocks, TFP still jumps immediately to its steady state level where wealth and productivity are independent. However, since wealth and ability are positively correlated in the initial distribution, steady state TFP is now lower than initial TFP. In contrast, with more persistent shocks, there is still some scope for self-financing to undo capital misallocation. Consequently, steady state TFP is higher than initial TFP.

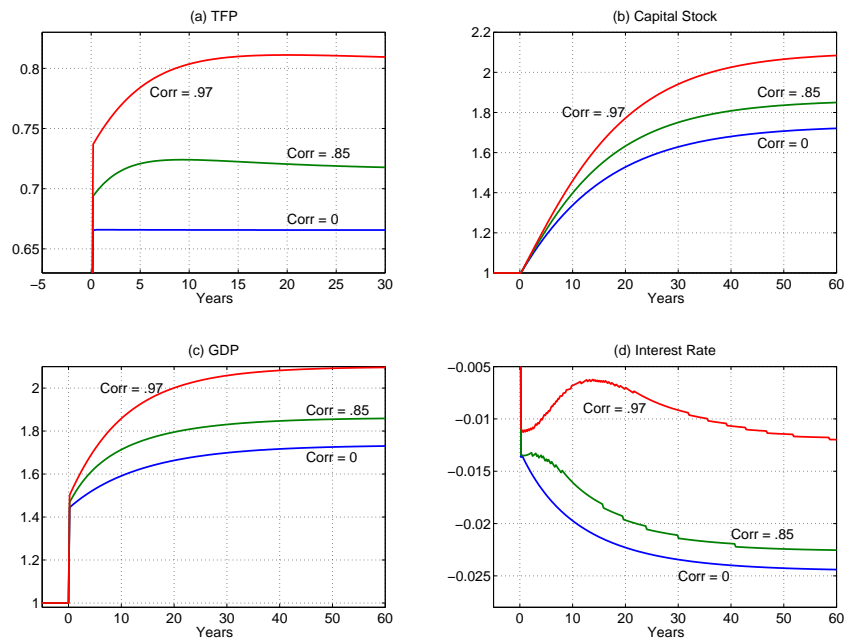


FIGURE K1. TRANSITION DYNAMICS AFTER REMOVING DISTORTIONARY TAXES

Note: Parameter values are $\alpha = 1/3$, $\rho = \delta = 0.05$ and $\lambda = 1.2$, consistent with the external-finance-to-GDP ratio for India (see Table E1). For the benchmark exercise (red), I use $Corr = \exp(-(1/\theta)) = 0.85$ and $\sigma\sqrt{1/\theta} = 0.56$. The lines for $Corr = 0$ and $Corr = 0.97$ vary θ while holding constant $Var(\log z) = \sigma^2/2$. The pre-reform steady state is computed as described in the text.

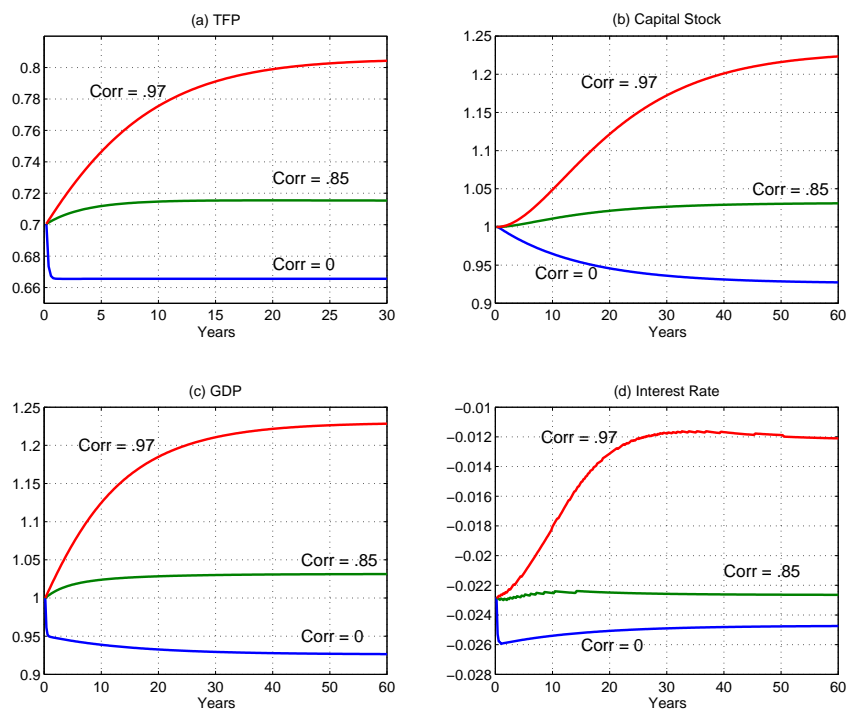


FIGURE K2. TRANSITION DYNAMICS FROM RELATIVELY UNDISTORTED INITIAL WEALTH SHARES

Note: Parameter values are $\alpha = 1/3$, $\rho = \delta = 0.05$ and $\lambda = 1.2$, consistent with the external-finance-to-GDP ratio for India (see Table E1). For the benchmark exercise (red), I use $Corr = \exp(-(1/\theta)) = 0.85$ and $\sigma\sqrt{1/\theta} = 0.56$. The lines for $Corr = 0$ and $Corr = 0.97$ vary θ . Initial wealth shares are given by (29) with $m = 0.25$.