Appendix: "Treatment Effects and Informative Missingness with an Application to Bank Recapitalization Programs"

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This appendix contains detailed instructions for the Markov chain Monte Carlo (MCMC) algorithm employed in the paper. The appendix first describes the steps for the algorithm followed by definitions of the vectors and matrices involved. Secondly, the appendix offers details on the location of the data.

MCMC Estimation Algorithm for Censored Outcomes

1. Sample $\boldsymbol{\beta}$ from the distribution $\boldsymbol{\beta}\,|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\beta}$. [1]
2. Sample $\boldsymbol{\Omega}$ from the distribution $\boldsymbol{\Omega}|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\Omega}$ in a one block, multi-step procedure.
3. For $i \in N_1,$ sample $\mathbf{y}_1^*$ from the distribution $\mathbf{y}_1^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_1^*$.
4. For $i \in N_2,$ sample $\mathbf{y}_2^*$ from the distribution $\mathbf{y}_2^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_2^*$.
5. For $i \in N_{2o},$ sample $\mathbf{y}_3^*$ from the distribution $\mathbf{y}_3^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_3^*$.
6. For $i \in N_{3o},$ sample $\mathbf{y}_4^*$ from the distribution $\mathbf{y}_4^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_4^*$.
7. For $i \in N_{1o},$ sample $\mathbf{y}_5^*$ from the distribution $\mathbf{y}_5^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_5^*$.

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[1] The notation "\" represents "except", e.g., $\mathbf{y}^* \setminus \mathbf{y}_1^*$ says all elements in $\mathbf{y}^*$ except $\mathbf{y}_1^*$.

Step 1: Sampling $\boldsymbol{\beta}$
Sample $\boldsymbol{\beta}\,|\mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\beta} \sim \mathcal{N}(\mathbf{b}, \mathbf{B})$, where

$$\begin{aligned}
\mathbf{b} &= \mathbf{B}(\mathbf{B}_0^{-1}\mathbf{b}_0 + \sum_{i\in N_1} \mathbf{J}_C'\mathbf{X}_{iC}'\boldsymbol{\Omega}_C^{-1}\mathbf{y}_{iC}^* + \\
&\quad \sum_{i\in N_2} \mathbf{J}_D'\mathbf{X}_{iD}'\boldsymbol{\Omega}_D^{-1}\mathbf{y}_{iD}^* + \sum_{i\in N_3} \mathbf{J}_A'\mathbf{X}_{iA}'\boldsymbol{\Omega}_A^{-1}\mathbf{y}_{iA}^*), \\
\mathbf{B} &= (\mathbf{B}_0^{-1} + \sum_{i\in N_1} \mathbf{J}_C'\mathbf{X}_{iC}'\boldsymbol{\Omega}_C^{-1}\mathbf{X}_{iC}\mathbf{J}_C + \\
&\quad \sum_{i\in N_2} \mathbf{J}_D'\mathbf{X}_{iD}'\boldsymbol{\Omega}_D^{-1}\mathbf{X}_{iD}\mathbf{J}_D + \sum_{i\in N_3} \mathbf{J}_A'\mathbf{X}_{iA}'\boldsymbol{\Omega}_A^{-1}\mathbf{X}_{iA}\mathbf{J}_A)^{-1}.
\end{aligned}$$

Step 2: Sampling $\boldsymbol{\Omega}$
Sample $\Omega|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\Omega}$ in a one block, nine-step procedure by first drawing Ω_{11} , $\Omega_{tt\cdot l} = \Omega_{tt} - \Omega_{tl}\Omega_{ll}^{-1}\Omega_{lt}$, and $B_{lt} = \Omega_{ll}^{-1}\Omega_{lt}$, and then reconstructing $\boldsymbol{\Omega}$ from these quantities

2. (a) $\Omega_{11}|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\Omega} \sim \mathcal{IW}(\nu - 1 + n,\ \mathbf{Q}_{11} + \sum_{N_1,N_2,N_3} \boldsymbol{\eta}_{i1}^*\boldsymbol{\eta}_{i1}^{*'})$
   - i. $\boldsymbol{\eta}_{i1}^* = y_{i1}^* - \mathbf{x}_{i1}\mathbf{J}_1\boldsymbol{\beta}$**,** where $\mathbf{J}_1 = \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \end{array}\right]_{1\times K}$

   (b) $\Omega_{22\cdot 1}|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\Omega} \sim \mathcal{IW}(\nu + n_2 + n_3, R_{22\cdot 1})$
   (c) $B_{12}|\mathbf{y}, \mathbf{y}^*, \Omega_{22\cdot 1} \sim \mathcal{MN}(R_{11}^{-1}R_{21}, \Omega_{22\cdot 1} \otimes R_{11}^{-1})$
   (d) Define $\boldsymbol{\Omega}_u = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}$
   (e) $\Omega_{55\cdot 1}|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\Omega} \sim \mathcal{IW}(\nu + n_1, R_{55\cdot 1})$
   (f) $B_{15}|\mathbf{y}, \mathbf{y}^*, \Omega_{55\cdot 1} \sim \mathcal{MN}(R_{11}^{-1}R_{51}, \Omega_{55\cdot 1} \otimes R_{11}^{-1})$
   (g) $\boldsymbol{\Omega}_{33\cdot u}|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\Omega} \sim \mathcal{IW}(\nu + n_2, \mathbf{R}_{33\cdot u})$
   (h) $\mathbf{B}_{u3}|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\Omega}_{33\cdot u} \sim \mathcal{MN}(\mathbf{R}_u^{-1}\mathbf{R}_{3u}, \boldsymbol{\Omega}_{33\cdot u} \otimes \mathbf{R}_u^{-1})$
   (i) $\boldsymbol{\Omega}_{44\cdot u}|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\theta} \setminus \boldsymbol{\Omega} \sim \mathcal{IW}(\nu + n_3, \mathbf{R}_{44\cdot u})$
   (j) $\mathbf{B}_{u4}|\mathbf{y}, \mathbf{y}^*, \boldsymbol{\Omega}_{44\cdot u} \sim \mathcal{MN}(\mathbf{R}_u^{-1}\mathbf{R}_{4u}, \boldsymbol{\Omega}_{44\cdot u} \otimes \mathbf{R}_u^{-1})$

where $\mathbf{R} = \mathbf{Q} + \sum \boldsymbol{\eta}_i^*\boldsymbol{\eta}_i^{*\prime}$, and the following subsections are obtained by partitioning $\mathbf{R}$ to conform to $\mathbf{Q}$, and $\mathbf{R}_{ll\cdot t} = \mathbf{R}_{ll} - \mathbf{R}_{lt}\mathbf{R}_{tt}^{-1}\mathbf{R}_{tl}$. From these sampling densities, $\boldsymbol{\Omega}$ can be recovered.

Steps 3-7: Sampling $\mathbf{y}^*$

$$\begin{aligned}
\mathbf{y}_1^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_1^* &\sim \mathcal{TN}_{(-\infty,0)}(\mathbf{x}_{i1}'\boldsymbol{\beta}_1 + E(\varepsilon_{i1}|\varepsilon_{i\setminus 1}), var(\varepsilon_{i1}|\varepsilon_{i\setminus 1})), \quad i \in N_1, \\
\mathbf{y}_2^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_2^* &\sim \mathcal{TN}_{(-\infty,0)}(\mathbf{x}_{i2}'\boldsymbol{\beta}_2 + E(\varepsilon_{i2}|\varepsilon_{i\setminus 2}), var(\varepsilon_{i2}|\varepsilon_{i\setminus 2})), \quad i \in N_2, \\
\mathbf{y}_3^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_3^* &\sim \mathcal{TN}_{(-\infty,0)}((\mathbf{x}_{i3}'\ y_{i1})\boldsymbol{\beta}_3 + E(\varepsilon_{i3}|\varepsilon_{i\setminus 3}), var(\varepsilon_{i3}|\varepsilon_{i\setminus 3})), \quad i \in N_{2o}, \\
\mathbf{y}_4^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_4^* &\sim \mathcal{TN}_{(-\infty,0)}((\mathbf{x}_{i4}'\ y_{i1}\ y_{i2})\boldsymbol{\beta}_4 + E(\varepsilon_{i4}|\varepsilon_{i\setminus 4}), var(\varepsilon_{i4}|\varepsilon_{i\setminus 4})), \quad i \in N_{3o}, \\
\mathbf{y}_5^*|\mathbf{y}, \boldsymbol{\theta}, \mathbf{y}^* \setminus \mathbf{y}_5^* &\sim \mathcal{TN}_{(-\infty,0)}(\mathbf{x}_{i5}'\boldsymbol{\beta}_5 + E(\varepsilon_{i5}|\varepsilon_{i\setminus 5}), var(\varepsilon_{i5}|\varepsilon_{i\setminus 5})), \quad i \in N_{1o}.
\end{aligned}$$

Definitions

Priors: It is assumed that $\boldsymbol{\beta}$ has a joint normal distribution with mean $\boldsymbol{\beta}_0$ and variance $\mathbf{B}_0$, and (independently) that the covariance matrix $\boldsymbol{\Omega}$ has an inverted Wishart distribution with parameters υ and $\mathbf{Q}$,

$$\pi(\boldsymbol{\beta}, \boldsymbol{\Omega}) = \mathcal{N}(\boldsymbol{\beta} \,|\boldsymbol{\beta}_0, \mathbf{B}_0)\mathcal{IW}(\boldsymbol{\Omega}|\upsilon, \mathbf{Q}).$$

Data: For the i-th observation, define the following vectors and matrices,

$$\mathbf{y}^*_{iC} = (y^*_{i1}, y^*_{i5})', \quad \mathbf{y}^*_{iD} = (y^*_{i1}, y^*_{i2}, y^*_{i3})', \quad \mathbf{y}^*_{iA} = (y^*_{i1}, y^*_{i2}, y^*_{i4})',$$

$$\mathbf{X}_{iC} = \begin{pmatrix} \mathbf{x}'_{i1} & 0 \\ 0 & \mathbf{x}'_{i5} \end{pmatrix}, \quad \mathbf{X}_{iD} = \begin{pmatrix} \mathbf{x}'_{i1} & 0 & 0 \\ 0 & \mathbf{x}'_{i2} & 0 \\ 0 & 0 & (\mathbf{x}'_{i3}\; y_{i1}) \end{pmatrix}, \quad \mathbf{X}_{iA} = \begin{pmatrix} \mathbf{x}'_{i1} & 0 & 0 \\ 0 & \mathbf{x}'_{i2} & 0 \\ 0 & 0 & (\mathbf{x}'_{i4}\; y_{i1}\; y_{i2}) \end{pmatrix}.$$

Let $N_1 = \{i : y_{i1} = 0\}$ be the n_1 observations in the non-selected sample and $N_2 = \{i : y_{i1} > 0 \; and \; y_{i2} = 0\}$ be the n_2 observations in the selected untreated sample. Set $N_3 = \{i : y_{i1} > 0 \; and \; y_{i2} > 0\}$ to be the n_3 observations in the selected treated sample.

In order to isolate the vectors and matrices that correspond to the 3 different subsets of the sample, define

$$\mathbf{J}_C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}_{2\times K}, \; \mathbf{J}_D = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}_{3\times K} \text{ and } \mathbf{J}_A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}_{3\times K}$$

where $K = k_1 + k_2 + k_3 + k_4 + k_5$, which represents the number of covariates in each equation.

For $i \in N_1$ (non-selected sample),

$$\boldsymbol{\eta}^*_{iC} \;=\; \mathbf{y}^*_{iC} - \mathbf{X}_{iC}\mathbf{J}_C\boldsymbol{\beta},$$

for $i \in N_2$ (selected untreated sample),

$$\boldsymbol{\eta}^*_{iD} \;=\; \mathbf{y}^*_{iD} - \mathbf{X}_{iD}\mathbf{J}_D\boldsymbol{\beta},$$

and for $i \in N_3$ (selected treated sample),

$$\boldsymbol{\eta}^*_{iA} \;=\; \mathbf{y}^*_{iA} - \mathbf{X}_{iA}\mathbf{J}_A\boldsymbol{\beta},$$

Finally, N_{2o} is defined as the truncated portion of the N_2 sample in $\mathbf{y}_3$, N_{3o} is defined as the truncated region of the N_3 sample in $\mathbf{y}_4$, and N_{1o} is defined as the discrete part of the N_1 sample in $\mathbf{y}_5$ since all the equations are Tobit equations with censored dependent variables.

Data

- RFC Card Index to Loans Made to Banks and Railroads, 1934-57
  - National Archives, College Park, MD
  - Record Group 234 / Reconstruction Finance Corp.
- Declined and Cancelled Loans, 1933-1941
  - National Archives, College Park, MD
  - Record Group 234 / Reconstruction Finance Corp.
  - Location: 570
- Paid Loans, 1933-1941
  - National Archives, College Park, MD
  - Record Group 234 / Reconstruction Finance Corp.
  - Location: 570
- Rand McNally Banker's Directory
  - Years 1932 - 1935

Programs

The MCMC algorithm employed in this paper follows from the steps listed above. This algorithm extends the sampling techniques developed in the below reference:

Chib, Siddhartha, Edward Greenberg, and Ivan Jeliazkov. 2009. "Estimation of Semiparametric Models in the Presence of Endogeneity and Sample Selection." *Journal of Computational and Graphical Statistics* 18(2): 321-348.

The GAUSS programs are available in the supplementary materials associated with Chib, Greenberg, and Jeliazkov (2009) found at the below link.

http://amstat.tandfonline.com/doi/suppl/10.1198/jcgs.2009.07070