Online Appendix B of "Cycles of Conflict: An Economic Model" by Daron Acemoglu and Alexander Wolitzky

Additional Extensions

Two-Sided Errors

The analysis of the baseline model was simplified by the assumption that only the good action can generate the good signal. This section shows that our main conclusions still apply when either action can generate either signal.

In particular, assume now that the signal $\tilde{y}_{t-1}$ is distributed as follows:

$$\Pr(\tilde{y}_{t-1} = 1 | y_{t-1} = 1) = 1 - \pi$$
$$\Pr(\tilde{y}_{t-1} = 1 | y_{t-1} = 0) = \pi',$$

where $\pi, \pi' \in (0,1)$ and $\pi + \pi' < 1$. The assumption that $\pi + \pi' < 1$ means that the good action is more likely to generate the good signal than is the bad action, and is thus essentially a normalization.

As in the baseline model, Assumption 1 guarantees that normal player t plays $x_t = 1$ if and only if $\tilde{y}_{t-1} = 1$. It is straightforward to see that the appropriate analog of Assumption 2, which guarantees that normal player t plays $y_t = 1$ if and only if her assessment of the probability that the other group is bad after observing $\tilde{y}_{t-1}$ is below a threshold $\mu^*_{2-SIDED}$, is the following.

Assumption 2'

$$\mu_0 < \mu^*_{2-SIDED} \equiv 1 - \frac{u(0,0) - u(1,0)}{(1-\pi)(u(1,1) - u(1,0)) + \pi'(u(0,0) - u(0,1))}.$$

The analog of Assumption 3 is then:

Assumption 3' $\mu_t \neq \mu^*_{2-SIDED}$ for all $t \in \mathbb{N}$.

Denote normal player t's assessment of the probability that the other group is bad after observing $\tilde{y}_{t-1} = 0$ by μ_t (as usual), and denote her assessment of this probability after observing $\tilde{y}_{t-1} = 1$ (which equals 0 in the baseline model, due to one-sided errors) by μ'_t. To compute these probabilities, let

$$M = \begin{pmatrix} 1-\pi & \pi' \\ \pi & 1-\pi' \end{pmatrix}$$

be the Markov transition matrix governing the evolution of $\tilde{y}_t$ in the event that both groups are normal, under the hypothesis that normal players play $y_t = 1$ if and only if $\tilde{y}_{t-1} = 1$. That is, if both groups are normal and $\tilde{y}_t = 1$, then $\tilde{y}_{t+1} = 1$ with probability $1 - \pi$; if, on the other hand, $\tilde{y}_t = 0$, then $\tilde{y}_{t+1} = 1$ with probability π'. Then, by Bayes rule,

$$\mu_t = \frac{\mu_0 (1 - \pi')}{\mu_0 (1 - \pi') + (1 - \mu_0) \left(1 - M^t_{(1,1)}\right)},$$

where $M^t_{(1,1)}$ is the $(1,1)$ coordinate of the t^{th} power of M. This is simply because the probability of observing $\tilde{y}_{t-1} = 0$ conditional on the other group being bad equals $1 - \pi'$, while the probability of observing $\tilde{y}_{t-1} = 0$ conditional on the other group being good equals $1 - M^t_{(1,1)}$. Similarly,

$$\mu'_t = \frac{\mu_0 \pi'}{\mu_0 \pi' + (1 - \mu_0) M^t_{(1,1)}}.$$

In the baseline model, it was the case that $\mu_t \to \mu_0$ as $t \to \infty$, so Assumption 2 guaranteed the existence of a time T such that $\mu_T < \mu^*_{2-SIDED}$. With two-sided errors, $M^t_{(1,1)} \to \frac{\pi'}{\pi + \pi'}$ as $t \to \infty$, so $\mu_t \to \mu_\infty$ as $t \to \infty$, where

$$\mu_\infty = \frac{\mu_0 (1 - \pi')}{\mu_0 (1 - \pi') + (1 - \mu_0) \frac{\pi}{\pi + \pi'}}.$$

If $\mu_\infty < \mu^*_{2-SIDED}$, then Assumption 2 guarantees the existence of a smallest time $T_{2-SIDED}$ such that $\mu_{T_{2-SIDED}} < \mu^*_{2-SIDED}$, and there is a deterministic cycle with period $T_{2-SIDED}$, as in the baseline model. If on the other hand $\mu_\infty \geq \mu^*_{2-SIDED}$, then there is no deterministic cycle, and in particular a bad signal always leads to a spiral of bad actions that lasts until the next accidental good signal.

Summarizing, we have the following result.

Proposition 6 *Under Assumptions 1, 2', and 3', the model with two-sided errors has a unique sequential equilibrium. If $\mu_\infty < \mu^*_{2-SIDED}$, then the equilibrium has the following properties:*

1. *At every time $t \neq 0 \bmod T_{2-SIDED}$, normal player t plays good actions $(x_t = 1, y_t = 1)$ if she gets the good signal $\tilde{y}_{t-1} = 1$, and plays bad actions $(x_t = 0, y_t = 0)$ if she gets the bad signal $\tilde{y}_{t-1} = 0$.*

2. *At every time $t = 0 \bmod T_{2-SIDED}$, normal player t plays the good action $x_t = 1$ toward player $t - 1$ if and only if she gets the good signal $\tilde{y}_{t-1} = 1$, but plays the good action $y_t = 1$ toward player $t + 1$ regardless of her signal.*

3. Bad players always play bad actions ($x_t = 0, y_t = 0$).

If instead $\mu_\infty \geq \mu^*_{2-SIDED}$, then the equilibrium has the following properties:

1. *At every time $t > 0$, normal player t plays good actions ($x_t = 1, y_t = 1$) if she gets the good signal $\tilde{y}_{t-1} = 1$, and plays bad actions ($x_t = 0, y_t = 0$) if she gets the bad signal $\tilde{y}_{t-1} = 0$.*

2. *Normal player 0 plays the good action $y_0 = 1$ toward player 1.*

3. *Bad players always play bad actions ($x_t = 0, y_t = 0$).*

Proof. Since player $t+1$ plays $x_{t+1} = 1$ if and only if he is normal and $\tilde{y}_t = 1$, it follows that (normal) player t plays $y_t = 1$ if and only if his belief that the other group is bad is below the cutoff $\mu^*_{2-SIDED}$. Now one can compute that $M^t_{(1,1)} = \frac{\pi' + \pi(1-\pi-\pi')^t}{\pi+\pi'}$. In particular, $M^t_{(1,1)} > \pi'$ for all t, and hence $\mu'_t < \mu_0$ for all t. Therefore, Assumption 2' implies that player t always plays $y_t = 1$ after seeing signal $\tilde{y}_{t-1} = 1$. Finally, $\mu_t > \mu^*_{2-SIDED}$ for all $t < T_{2-SIDED}$ (with the convention that $T_{2-SIDED} = \infty$ if $\mu_\infty \geq \mu^*_{2-SIDED}$), by definition of $T_{2-SIDED}$, so player t plays $y_t = 0$ after seeing $\tilde{y}_{t-1} = 0$, for all $t < T_{2-SIDED}$. The remainder of the argument is as in the baseline model. ■

Forward-Looking Behavior

Another stark assumption in the baseline model is that players do not care at all about future periods. We now relax this by retaining the assumption that agents are short-lived but assuming that they care about their group's future utility.[23] Even though, not surprisingly, forward-looking behavior can introduce multiple equilibria, we can obtain a clean characterization of the subset of equilibria that have the same cyclic structure as the unique equilibrium in the baseline model. In particular, we show that in every such equilibrium cooperation restarts at least as frequently as in the baseline model. Hence, the average duration of conflict is reduced.

Formally, modify the baseline model by supposing that normal player t's payoff is now

$$\sum_{\tau=0}^{\infty} \delta^{2\tau} u_{t+2\tau}$$

for some $\delta \in (0, 1)$, where u_τ is player τ's payoff in the baseline model. Everything else is exactly as in the baseline model. Our result is the following.

[23] An alternative interpretation is that each group consists of a single long-lived agent that can only remember the most recent signal.

Proposition 7 *Let μ_t be defined as in the baseline model. For odd integers k let*

$$\mu^*_{T'-k} = \frac{\frac{1-\delta^k(1-\pi)^k}{1-\delta^2(1-\pi)^2}\left[(2-\pi)\,u(1,1)+\pi u(1,0)-2u(0,0)\right]-\left(u(1,1)-u(0,0)\right)}{\frac{1-\delta^k(1-\pi)^k}{1-\delta^2(1-\pi)^2}\left[(2-\pi)\,u(1,1)+\pi u(1,0)-2u(0,0)\right]+2u(0,0)-u(1,1)-u(1,0)},$$

and for even integers k let

$$\mu^*_{T'-k} = \frac{\frac{1-\delta^{k-1}(1-\pi)^{k-1}}{1-\delta^2(1-\pi)^2}\left[(2-\pi)\,u(1,1)+\pi u(1,0)-2u(0,0)\right]-\left(1-\delta^k\right)\left(u(1,1)-u(0,0)\right)}{\frac{1-\delta^{k-1}(1-\pi)^{k-1}}{1-\delta^2(1-\pi)^2}\left[(2-\pi)\,u(1,1)+\pi u(1,0)-2u(0,0)\right]+\left(2-\delta^k\right)u(0,0)-\left(1-\delta^k\right)u(1,1)-u(1,0)}.$$

Then the model with forward-looking behavior has a sequential equilibrium that coincides with the sequential equilibrium of the baseline model, but with restart time T' rather than T, if and only if

1. *$\mu^*_{T'-k} \leq \mu_{T'-k}$ for all $k \in \{1,\ldots,T'-1\}$, and*

2. *$\mu^*_0 \geq \mu_{T'}$.*

In particular, in every such equilibrium the restart time T' is no greater than T.

The intuition is the following: Consider a candidate equilibrium with restart time T'. Since player T' restarts cooperation whatever player $T'-1$ does, the incentives of player $T'-1$ are exactly in the baseline model, so she will play just as in the baseline model. But player $T'-2$ now has an additional reason to take a good action toward player $T'-1$ after a bad signal: provided that the other group is normal, this will help player T' obtain payoff $u(1,1)$ rather than $u(0,0)$ against player $T'-1$. Similarly, player $T'-4$ has yet stronger incentives to restart cooperation because of the additional payoffs that this might generate for $T'-2$ against $T'-3$ and $T'-1$. One can now compute the cutoff belief for player $T'-k$ to restart cooperation as $\mu^*_{T'-k}$, which then implies that no player will restart cooperation prior to time T' if and only if $\mu^*_{T'-k} \leq \mu_{T'-k}$ for all $k \in \{1,\ldots,T'-1\}$ (the first condition in Proposition 7).[24] If, on the other hand, player T' does not restart cooperation, then in this candidate equilibrium cooperation will not restart until time $2T'$. Hence, cooperation restarts at time T' if and only if $\mu_{T'} \leq \mu^*_{T'-T'}$, i.e., if and only if $\mu^*_0 \geq \mu_{T'}$ (the second condition in Proposition 7). Finally, the restart time T' cannot exceed T, as player T's incentive to restart cooperation in the model with forward-looking behavior is never less than her incentive to restart cooperation in the

[24] The need to distinguish between odd and even k comes because the only "extra incentive" from helping player T' comes only from her interaction with player $T'-1$ (as she always takes the good action toward player $T'+1$), while for earlier players the extra incentive comes from their interactions with both their predecessors and their successors.

baseline model, and her posterior would be the same as in the baseline model if (counterfactually) the restart time did exceed T.[25]

Proof. For the first part of the proposition, it is clear that the only potentially profitable deviations are deviations by player $T'-k$ to $y=1$ after the bad signal for $k \in \{1,\ldots,T'-1\}$ and deviations by player T' to $y=0$ after the bad signal.

Consider first deviations by player $T'-k$. Since player T' always restarts cooperation, player $T'-k$'s action is inconsequential for the expected payoff of players $t \geq T'+1$, so player $T'-k$ needs only take into account the effect of her action of the payoff on players $T'-k+2, T'-k+4, \ldots, T'-1$ (for k odd) or $T'-k+2, T'-k+4, \ldots, T'$ (for k even). Now if the opposing group is bad, then player $T'-k+2\tau$ gets payoff $u(0,0)$ against each of her opponents, regardless of player $T'-k$'s action, for $\tau \in \{1,\ldots,\lceil (k-1)/2 \rceil\}$. If instead the opposing group is normal, then by taking action $y=1$ rather than $y=0$, player $T'-k$ increases player $T'-k+2\tau$'s probability of getting $u(1,1)$ rather than $u(0,0)$ against his predecessor and getting $(1-\pi)u(1,1)+\pi u(1,0)$ rather than $u(0,0)$ against his successor from 0 to $(1-\pi)^{2\tau}$. This increases player $T'-k+2\tau$'s expected payoff by a total of

$$(1-\pi)^{2\tau}\left[(2-\pi)u(1,1)+\pi u(1,0)-2u(0,0)\right].$$

Thus, for k odd, playing $y=1$ is optimal for player $T'-k$ with belief μ if and only if

$$\begin{aligned}&-\mu\left(u(0,0)-u(1,0)\right)-(1-\mu)\left(u(1,1)-u(0,0)\right)\\&\qquad+(1-\mu)\left[(2-\pi)u(1,1)+\pi u(1,0)-2u(0,0)\right]\left[\begin{array}{c}1+\delta^2(1-\pi)^2+\delta^4(1-\pi)^4\\+\ldots+\delta^{k-1}(1-\pi)^{k-1}\end{array}\right]\geq 0,\end{aligned}$$

or

$$\mu \leq \mu^*_{T'-k}.$$

The expression for k even is similar, except that since player T' always plays $y_{T'}=1$ his benefit from player $T'-k$'s taking action $y=1$ rather than $y=0$ when the opposing group is normal is only

$$(1-\pi)^k\left(u(1,1)-u(0,0)\right).$$

So, for k even, playing $y=1$ is optimal for player $T'-k$ with belief μ if and only if

$$\begin{aligned}&-\mu\left(u(0,0)-u(1,0)\right)-(1-\mu)\left(1-\delta^k\right)\left(u(1,1)-u(0,0)\right)\\&\qquad+(1-\mu)\left[(2-\pi)u(1,1)+\pi u(1,0)-2u(0,0)\right]\left[\begin{array}{c}1+\delta^2(1-\pi)^2+\delta^4(1-\pi)^4\\+\ldots+\delta^{k-2}(1-\pi)^{k-2}\end{array}\right]\geq 0,\end{aligned}$$

[25] Note that Proposition 7 allows for multiple equilibria, because the expectation that player T' will restart cooperation reduces earlier players' incentive to restart cooperation (as they can count on player T' to restart) and increases player T''s own incentive to restart cooperation (as she knows that if she does not restart then no one will restart until time $2T'$).

or

$$\mu \leq \mu^*_{T'-k}.$$

Next, consider deviations by player T'. The argument here is nearly identical, noting that if player T' does not restart cooperation then the next restart occurs in T' periods.

Finally, to show that the cycle length T' in any such equilibrium is at most T, consider the strategy profile with cycle length $T' > T$. Then player T will deviate by restarting cooperation after the bad signal, as his posterior is μ_T and his benefit from playing $y = 1$ rather than $y = 0$ is at least as large as in the baseline model. ■

Recurrent Conflict Versus Escalation

As noted in footnote 16, the cycles of conflict captured in our baseline model resemble recurrent episodes of conflicts alternating with episodes of peace, rather than escalation of the intensity of conflict within a given conflict episode. In this subsection, we show that our mechanism can also generate this type of "escalation spiral".[26]

Consider the unobserved calendar time model of Section II, modified to have three possible actions, 0, $\frac{1}{2}$, and 1, and three possible signals, also called 0, $\frac{1}{2}$, and 1 (there are still two possible types, and action 0 is still dominant for bad types). Here, 0 and 1 are the bad and good actions/signals as usual, while $\frac{1}{2}$ is a new, intermediate action/signal, corresponding to "limited conflict," so that the switch from $\frac{1}{2}$ to 0 is an escalation of conflict. We assume that the game remains a coordination game (in particular, $\left(\frac{1}{2}, \frac{1}{2}\right)$ is a Nash equilibrium), that "more cooperative" equilibria are Pareto-preferred, that a group's payoff has increasing differences in its own action and the other group's action, and that a group is always better off when the other group is more cooperative. Formally, the following conditions are sufficient to ensure this:

1. $1 \in \arg\max_{x \in \left\{0, \frac{1}{2}, 1\right\}} u(x, 1)$, $\frac{1}{2} \in \arg\max_{x \in \left\{0, \frac{1}{2}, 1\right\}} u\left(x, \frac{1}{2}\right)$, $0 \in \arg\max_{x \in \left\{0, \frac{1}{2}, 1\right\}} u(x, 0)$.

2. $u(x, y)$ has increasing differences in (x, y): if $x \geq x'$ and $y \geq y'$, then $u(x, y) - u(x', y) \geq u(x, y') - u(x', y')$.

3. $u(x, y)$ is non-decreasing in y.

4. $u(1, 1) > u\left(\frac{1}{2}, \frac{1}{2}\right) > u(0, 0)$.

[26] See, for example, Jervis (1976). We thank a referee for drawing our attention to this issue.

We also assume that the conditional distribution of signals is given by

$$\begin{array}{ccc} \Pr(\tilde{y}_t = 1 | y_t = 1) = 1 - \pi & \Pr\left(\tilde{y}_t = \frac{1}{2} | y_t = 1\right) = \pi & \Pr(\tilde{y}_t = 0 | y_t = 1) = 0 \\ \Pr\left(\tilde{y}_t = 1 | y_t = \frac{1}{2}\right) = \rho & \Pr\left(\tilde{y}_t = \frac{1}{2} | y_t = \frac{1}{2}\right) = 1 - \rho - \rho' & \Pr\left(\tilde{y}_t = 0 | y_t = \frac{1}{2}\right) = \rho' \\ \Pr(\tilde{y}_t = 1 | y_t = 0) = 0 & \Pr\left(\tilde{y}_t = \frac{1}{2} | y_t = 0\right) = \pi' & \Pr(\tilde{y}_t = 0 | y_t = 0) = 1 - \pi' \end{array}$$

Thus, a good action can generate a good signal or an intermediate signal; a bad action can generate a bad signal or an intermediate signal; and an intermediate action can generate any signal. Finally, assume that $\pi + \rho < 1$ and $\pi' + \rho' < 1$, so that a good action is more likely to generate a good signal than is an intermediate action, and a bad action is more likely to generate a bad signal than is an intermediate action.

Below, we derive conditions under which a sequential equilibrium of the following form exists.[27]

1. Normal player 0 plays $y_0 = 1$.
2. At every time $t > 0$, normal player t plays good actions $(x_t = 1, y_t = 1)$ if she gets the good signal $\tilde{y}_{t-1} = 1$, and plays intermediate actions $\left(x_t = \frac{1}{2}, y_t = \frac{1}{2}\right)$ if she gets the intermediate signal $\tilde{y}_{t-1} = \frac{1}{2}$. If she gets the bad signal $\tilde{y}_{t-1} = 0$, she plays the bad action $x_t = 0$ toward player $t-1$, and mixes between playing the bad action $y_t = 0$ and the intermediate action $y_t = \frac{1}{2}$ toward player $t+1$.
3. Bad players always play bad actions $(x_t = 0, y_t = 0)$.

Note that such an equilibrium displays recurrent conflict exactly as in our baseline model or our model with unobserved calendar time but also displays escalation within each conflict spiral: each conflict spiral starts with the misperception of a good action as an intermediate action (which leads to genuine intermediate actions), and then may involve the misperception of an intermediate action as a bad action (which leads to genuine bad actions). Thus, our framework can fairly naturally accommodate escalation of conflict as well as periodic conflict.

To understand when an equilibrium of the conjectured form exists, first observe that in any equilibrium players take more aggressive actions when they believe the opposing group is more likely to be bad.

Lemma 1 *In any sequential equilibrium, normal player t's optimal action toward her successor y_t is non-increasing in her belief.*

[27] Unlike in the baseline model, we do not claim that equilibrium is unique here.

Proof. In any sequential equilibrium, normal player $t+1$ plays $x_{t+1} = \tilde{y}_t$ and bad player $t+1$ plays $x_{t+1} = 0$. Hence, letting $U(y_t)$ be normal player t's expected payoff from taking action y_t against player $t+1$ given belief μ, we have

$$\begin{aligned}
U(0) &= (1-\pi')\,u(0,0) + \pi'\left((1-\mu)\,u\left(0,\frac{1}{2}\right) + \mu u(0,0)\right) \\
U\left(\frac{1}{2}\right) &= (1-\rho-\rho')\left[(1-\mu)\,u\left(\frac{1}{2},\frac{1}{2}\right) + \mu u\left(\frac{1}{2},0\right)\right] + \rho\left[(1-\mu)\,u\left(\frac{1}{2},1\right) + \mu u\left(\frac{1}{2},0\right)\right] + \rho' u\left(\frac{1}{2},0\right) \\
U(1) &= (1-\pi)\left[(1-\mu)\,u(1,1) + \mu u(1,0)\right] + \pi\left[(1-\mu)\,u\left(1,\frac{1}{2}\right) + \mu u(1,0)\right].
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{\partial U(0)}{\partial \mu} &= -\pi'\left[u\left(0,\frac{1}{2}\right) - u(0,0)\right] \\
\frac{\partial U\left(\frac{1}{2}\right)}{\partial \mu} &= -(1-\rho-\rho')\left[u\left(\frac{1}{2},\frac{1}{2}\right) - u\left(\frac{1}{2},0\right)\right] - \rho\left[u\left(\frac{1}{2},1\right) - u\left(\frac{1}{2},0\right)\right] \\
\frac{\partial U(1)}{\partial \mu} &= -(1-\pi)\left[u(1,1) - u(1,0)\right] - \pi\left[u\left(1,\frac{1}{2}\right) - u(1,0)\right].
\end{aligned}$$

Proof. Increasing differences implies that $u\left(\frac{1}{2},\frac{1}{2}\right) - u\left(\frac{1}{2},0\right) \geq u\left(0,\frac{1}{2}\right) - u(0,0)$, and $u(x,y)$ non-decreasing in y then implies that $u\left(\frac{1}{2},1\right) - u\left(\frac{1}{2},0\right) \geq u\left(0,\frac{1}{2}\right) - u(0,0)$. The assumption that $\pi' + \rho' < 1$ now implies that $\frac{\partial U(0)}{\partial \mu} \geq \frac{\partial U\left(\frac{1}{2}\right)}{\partial \mu}$. Increasing differences also implies that $u(1,1) - u(1,0) \geq u\left(\frac{1}{2},1\right) - u\left(\frac{1}{2},0\right)$ and $u\left(1,\frac{1}{2}\right) - u(1,0) \geq u\left(\frac{1}{2},\frac{1}{2}\right) - u\left(\frac{1}{2},0\right)$, and therefore $\frac{\partial U\left(\frac{1}{2}\right)}{\partial \mu} \geq \frac{\partial U(1)}{\partial \mu}$. It follows that normal player t's optimal action y_t is non-increasing in her belief. ■ ■

Now let $q_{\tilde{y}}$ be the long-run fraction of periods t in which $\tilde{y}_t = \tilde{y}$ when both groups are normal. In an equilibrium of the form conjectured above, letting p be the probability that normal player t plays the bad action $y_t = 0$ after the bad signal $\tilde{y}_{t-1} = 0$, we have

$$\begin{aligned}
q_0 &= q_{\frac{1}{2}}\rho' + q_0\left[(1-p)\,\rho' + p\,(1-\pi')\right] \\
q_{\frac{1}{2}} &= q_1\pi + q_{\frac{1}{2}}(1-\rho-\rho') + q_0\left[(1-p)(1-\rho-\rho') + p\pi'\right] \\
q_1 &= q_1(1-\pi) + q_{\frac{1}{2}}\rho + q_0(1-p)\,\rho
\end{aligned}$$

This system of equations may easily be solved for q_0, $q_{\frac{1}{2}}$, and q_1; we omit the details.

Finally, let $\mu^*_{\frac{1}{2}}$ be the cutoff belief that makes normal player t indifferent between actions $y_t = 0$ and $y_t = \frac{1}{2}$, and let μ^*_1 be the cutoff belief that makes her indifferent between actions $y_t = \frac{1}{2}$ and $y_t = 1$, which may be easily computed from the above formulas for $U(0)$, $U\left(\frac{1}{2}\right)$, and $U(1)$. Letting

$\mu^{\tilde{y}_{t-1}}$ be normal player t's posterior belief after observing signal $\tilde{y}_{t-1}$, we have

$$\begin{aligned}\mu^0 &= \frac{\mu_0\left(1-\pi'\right)}{\mu_0\left(1-\pi'\right)+\left(1-\mu_0\right)q_0} \\ \mu^{\frac{1}{2}} &= \frac{\mu_0\pi'}{\mu_0\pi'+\left(1-\mu_0\right)q_{\frac{1}{2}}} \\ \mu^1 &= 0.\end{aligned}$$

So an equilibrium of the desired form exists only if $\mu^0 = \mu^*_{\frac{1}{2}}$, or equivalently if

$$q_0 = \left(1-\pi'\right)\frac{\mu_0}{1-\mu_0}\frac{1-\mu^*_{\frac{1}{2}}}{\mu^*_{\frac{1}{2}}}.$$

This equation implicitly fixes the mixing probability p at some $p^* \in [0,1]$. Finally, by Lemma 1, an equilibrium of the desired form exists if and only if Assumption 2 holds and it is optimal for normal player t to play action $y_t = \frac{1}{2}$ after observing signal $\tilde{y}_{t-1} = \frac{1}{2}$ when $p = p^*$; that is, if and only if Assumption 2 holds and $\mu^{\frac{1}{2}} \in \left[\mu^*_1, \mu^*_{\frac{1}{2}}\right]$ when $p = p^*$.[28]

While this characterization is not very explicit, it does show that an equilibrium of the conjectured form should exist for a wide range of parameters.

[28] Assumption 2 implies that it is optimal for normal player 0 to play $y_0 = 1$. It also implies that it is optimal for normal player t to play action $y_t = 1$ after observing signal $\tilde{y}_{t-1} = 1$, as $\mu^1 = 0 < \mu_0$.