Corrigendum

"Search, Liquidity, and the Dynamics of House Prices and Construction",
Allen Head, Huw Lloyd-Ellis, and Hongfei Sun
104(4) (April 2014) 1172-1210.

In the course of our ongoing research, we have discovered two substantive errors in this article. These mistakes do not affect significantly either the qualitative nature of the results or any of the principal conclusions stated in the paper. They do, however, affect the specific quantitative results. One error affects the empirical findings presented primarily in Section I and the online appendix. The other involves the calculation of sales of existing houses in artificial data generated by the model. This affects both the calibration (Section IV) and the equilibrium dynamics in Section V, principally with regard to the dynamics of sales.

The article's quantitative results are presented through tables of statistics and figures depicting impulse responses Virtually all of the specific numerical values in these tables and figures are affected; most only trivially, but some significantly. In this *corrigendum*, we describe the two errors and present corrected values and new versions for the statistics and figures most affected (we number the tables and figures as in the original article and the online appendix). Updated, correct programs have replaced those containing the mistakes on the *Review*'s website, and the online appendix contains a complete set of corrected tables and figures.

1 Incorrect implementation of the Helmert transformation in the system GMM estimation

The system GMM estimation requires a Helmert transformation that requires computation of a weighted difference between every observation of each variable and its forward mean. Due to a programming error in STATA, this was computed incorrectly. The incorrect estimates were, however, very similar to those produced by other methods and the error was not recognized. With the error corrected and the Helmert transformation performed correctly, the system GMM procedure is not tenable when applied to the panel VAR with income and price data in levels owing to a weak instruments problem. This is not an uncommon problem, as documented by Bun and Windmeijer (2010) and others.

In Table 2 below we report, as an alternative baseline specification, the moments in response to income shocks based on a Least Squares Dummy Variable (LSDV) estimation of the panel VAR (Cagala and Glogowsky, 2014). As before, the cross-sectional means of each variable were removed at each date. Figure 1 depicts the associated impulse responses. These results are similar to those incorrectly reported in the original paper. Essentially all of the qualitative results are unchanged. Quantitatively, the most economically significant differences are:

- The estimated income process is less persistent.

• Income shocks account for a smaller share of house price fluctuations.
• House prices exhibit a lower variance and higher degree of persistence in response to income shocks than originally reported.

As in our article, here we adopt a specification with incomes and prices in levels as our benchmark because the theoretical model focusses on shocks to the level of city income. As noted in the original paper, we cannot rule out the possibility of non-stationarity for some panels. Table D3 therefore reports the same moments with income and prices in growth rates for both the LSDV and system GMM estimations. The findings regarding these moments are very similar across all three specifications.

2 Incorrect calculation of annual sales

Although the theoretical model was solved correctly, annual sales were computed incorrectly from the artificial, simulated data. This error is significant in that it results in the volatility of our measure of growth in sales of existing homes being overstated. The model is, in fact, unable to generate volatility in this variable as high as reported in Table 2 for any parameterization known to us. While the volatility of sales growth was not reported as a result in the paper, it was used as a calibration target. As such, the original calibration strategy can no longer be followed when annual sales is calculated correctly.

We therefore adopt here a different strategy for the calibration of the elasticity parameters, δ, α and η. For δ, the elasticity of the matching function with respect to the number of buyers, we use the value estimated independently by Genoseve and Han (2012). This value ($\delta = .84$) is very close to our previously calibrated value of $\delta = .86$. With δ set independently, the parameters α and η jointly determine the ratio of the volatilities of population growth and the construction rate, but not their levels. For this reason, we now choose them to match jointly the volatilities of population and price growth.

The resulting quantitative exercise differs from that presented in Section V of our article: There, we targeted the volatilities of growth in sales, construction, and population and then considered the extent to which the model accounted for both the variance and persistence of house price fluctuations. Here, we target the variances of growth in population and prices and consider the extent to which the model accounts for the autocorrelation in the response of house price appreciation to the income shock, as well as the volatilities of construction and sales growth. In both cases, we also compare the implications for other moments.

Figure 3 depicts the responses of prices, population growth, the construction rate and sales growth to the income shock. Tables 7 and 8 contain moments from data (using the corrected estimation), the new baseline calibration of the search model, and a similarly re-calibrated no-search economy. Overall, the results are very similar to those reported previously (except, of course, for the volatility of sales growth) both qualitatively and quantitatively. While it is true that the model generates slightly less autocorrelation in house price growth than before, the income shock driving the house price movements is itself less persistent. Moreover, the trade-off between the magnitude and persistence of house price fluctuations remains as described in the published version. Table 11 reports sensitivity exercises under the revised calibration and illustrates this trade-off.

Table 2: Moments from Panel VAR: Income shocks only

				$\rho(x_t, x_{t-i})$			
Variable (x)	σ_x/σ_y	$\rho(x,y)$	$\rho(x,p)$	$i=1$	$i=2$	$i=3$	$i=4$
Income growth (y)	1.00	1.00	0.47	0.17	0.00	-0.06	-0.08
Price growth (p)	1.08	0.47	1.00	0.83	0.47	0.10	-0.19
Sales growth (s)	1.37	0.76	0.18	0.54	0.05	-0.17	-0.25
Construction (h)	0.12	0.31	0.80	0.85	0.54	0.23	-0.01
Population Growth (n)	0.17	0.77	0.75	0.62	0.29	0.06	-0.09

Table D3: Moments from panel VAR for different specifications – income shocks

					$\rho(x_t, x_{t-i})$			
	Specification	σ_x/σ_y	$\rho(x,y)$	$\rho(x,p)$	$i=1$	$i=2$	$i=3$	$i=4$
Income Growth	LSDV Levels	1.00	1.00	0.47	0.17	0.00	-0.06	-0.08
	LSDV Growth	1.00	1.00	0.47	0.17	0.01	-0.00	-0.02
	2SLS Growth	1.00	1.00	0.51	0.22	0.06	0.02	0.00
Price Growth	LSDV Levels	1.08	0.47	1.00	0.83	0.47	0.10	-0.19
	LSDV Growth	1.15	0.47	1.00	0.82	0.45	0.11	-0.10
	2SLS Growth	1.08	0.51	1.00	0.85	0.54	0.25	0.06
Sales Growth	LSDV Levels	1.37	0.76	0.18	0.54	0.05	-0.17	-0.25
	LSDV Growth	1.55	0.70	0.14	0.53	-0.07	-0.34	-0.35
	2SLS Growth	1.24	0.84	0.35	0.46	-0.11	-0.23	-0.25
Cons. Rate	LSDV Levels	0.12	0.31	0.80	0.85	0.54	0.23	-0.01
	LSDV Growth	0.14	0.41	0.96	0.87	0.59	0.31	0.11
	2SLS Growth	0.14	0.44	0.97	0.90	0.66	0.42	0.24
Pop. Growth	LSDV Levels	0.17	0.77	0.75	0.62	0.29	0.06	-0.09
	LSDV Growth	0.18	0.83	0.85	0.65	0.28	0.03	-0.08
	2SLS Growth	0.18	0.85	0.88	0.67	0.39	0.18	0.06

Notes: "LSDV Levels" refers to the LSDV estimation with price and income in log levels
"LSDV Growth" refers to the LSDV estimation with price and income in growth rates
"2SLS Growth" refers to the system GMM estimation with price and income in growth rates

Table 7: Volatilities and co-movements

Moment	US Cities	Baseline	No search
σ_p/σ_y	1.08	1.08*	1.28
σ_s/σ_y	1.37	0.31	0.14
σ_h/σ_y	0.12	0.118	0.17
σ_n/σ_y	0.17	0.17*	0.17*
$\rho(p, y)$	0.47	0.98	0.99
$\rho(s, y)$	0.76	0.38	0.12
$\rho(h, y)$	0.31	0.28	0.33
$\rho(n, y)$	0.77	0.48	0.33
$\rho(s, p)$	0.18	0.22	0.03

Note: A * indicates a calibrated target.

Table 8: Autocorrelations (106 cities, 1981-2008)

$\rho(x_t, x_{t-i})$	$i=1$	$i=2$	$i=3$	$i=4$	$i=1$	$i=2$	$i=3$	$i=4$
	Price Growth				Sales Growth			
US Cities	0.83	0.47	0.10	-0.19	0.54	0.05	-0.17	-0.25
Baseline	0.32	0.04	-0.01	-0.04	0.35	0.02	-0.01	-0.03
	Population Growth				Construction Rate			
US Cities	0.62	0.29	0.06	-0.09	0.85	0.54	0.23	-0.01
Baseline	0.83	0.64	0.51	0.42	0.95	0.87	0.78	0.69

Table 11: Volatility and Persistence of Price growth: Sensitivity Results

Moment	Baseline Calibration		New housing supply elasticity (η)		Entry elasticity (α)		Matching Elasticity (δ)	
			.10	10	4.5	18	.50	.95
σ_p/σ_y	1.08		2.11	0.33	0.46	1.68	1.25	1.03
ρ_1^p	0.32		0.26	0.49	0.60	0.25	0.28	0.33
	Land supply elasticity (ξ)		Vacancy Rate (v)		Homeowner exit probability (π_n)		Housing Utility (z^H)	
	0.01	100	.01	.03	.004	.012	.01	.04
σ_p/σ_y	1.87	1.06	1.30	0.92	1.09	1.04	0.73	1.15
ρ_1^p	0.27	0.33	0.27	0.36	0.27	0.38	0.56	0.29

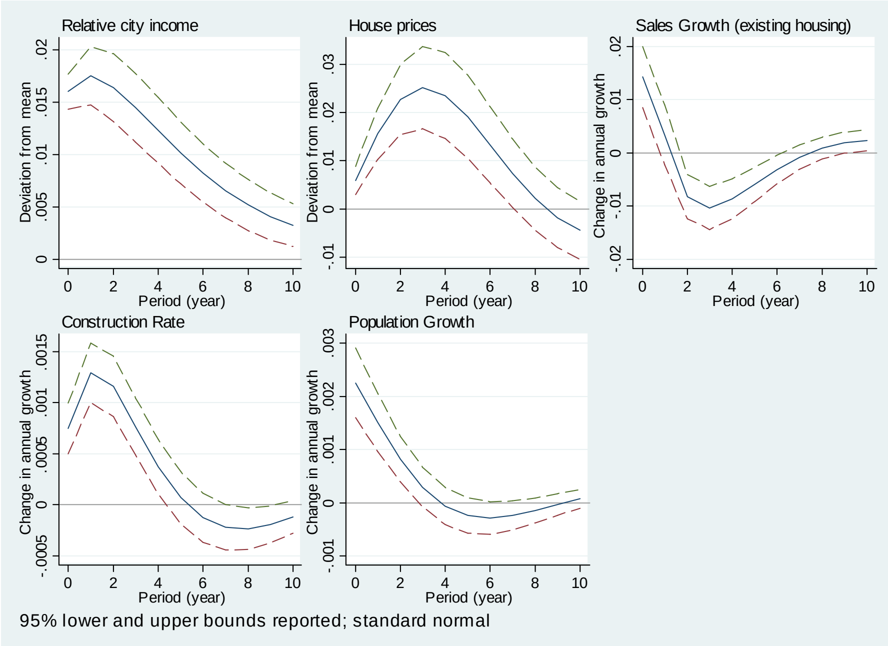


Figure 1: Estimated impulse responses to an income shock

Figure 3: Responses to an income shock with and without search

References

Bun, Maurice J. G. and Frank Windmeijer (2010), "The weak instrument problem of the system GMM estimator in dynamic panel data models," *The Econometrics Journal,* Volume 13, Issue 1, pages 95–126, February 2010.

Cagala, T. and Glogowsky, U. (2014). Panel Vector Autoregressions for Stata (xtvar).

Genoseve and Han, L. (2012), "Search and Matching in the Market for Existing Homes," *Journal of Urban Economics*; Issue: 72; 2012; Pages: 31-45.

Search, Liquidity and the Dynamics of House Prices and Construction

By Allen Head, Huw Lloyd-Ellis and Hongfei Sun

Revised Online Appendix

This online appendix has been revised to reflect the corrections described in the *Corrigendum*. Appendix C contains a complete set of revised tables and figures corresponding to those in the main text of the original article. Appendix D contains revised tables of additional empirical results referenced in Section I of the original article. Appendix E offers an interpretation of $G(\cdot)$ based on a multi-city environment, as referenced in footnote 14 of the originial article. Appendix F provides additional details regarding the robustness exercises discussed in section VI.

Appendix C: Revised Tables and Figures

This appendix contains a complete set of revised tables and figures, numbered so that they correspond to those in the original article

I: Empirical Properties of Housing Markets in U.S. M.S.A.'s

Table 1 — Moments from Panel VAR: all shocks

				$\rho(x_t, x_{t-i})$			
Variable (x)	σ_x/σ_y	$\rho(x, y)$	$\rho(x, p)$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
Income growth (y)	1.00	1.00	0.26	0.17	-0.00	-0.06	-0.08
Price growth (p)	2.74	0.26	1.00	0.67	0.32	0.03	-0.18
Sales growth (s)	6.15	0.13	-0.06	0.12	-0.03	-0.03	-0.04
Construction (h)	0.34	0.14	0.42	0.75	0.48	0.26	0.10
Population Growth (n)	0.62	0.22	0.20	0.26	0.15	0.08	0.02

Note: In all tables, σ_x denotes the standard deviation of x, and $\rho(x, y)$ the correlation of x with y.

Table 2 — Moments from Panel VAR: Income shocks only

				$\rho(x_t, x_{t-i})$			
Variable (x)	σ_x/σ_y	$\rho(x, y)$	$\rho(x, p)$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
Income growth (y)	1.00	1.00	0.47	0.17	0.00	-0.06	-0.08
Price growth (p)	1.08	0.47	1.00	0.83	0.47	0.10	-0.19
Sales growth (s)	1.37	0.76	0.18	0.54	0.05	-0.17	-0.25
Construction (h)	0.12	0.31	0.80	0.85	0.54	0.23	-0.01
Population Growth (n)	0.17	0.77	0.75	0.62	0.29	0.06	-0.09

Table 3 — Prices of claim to local income: Income shock

			$\rho(x_t, x_{t-i})$			
Variable (x)	σ_x/σ_y	$\rho(x, p)$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
Houses in U.S. MSA's	1.08	0.47	0.83	0.47	0.10	-0.19
Claims to MSA Incomes	0.24	0.97	-0.07	-0.07	-0.06	-0.05

IV. Calibration

Table 4 — Baseline Calibration Parameters: Steady State

Parameter	Value	Target
β	0.99	Annual real interest rate = 4 percent
μ	0.003	Annual population growth rate = 1.2 percent
π_f	0.030	Annual mobility of renters = 12 percent
π_n	0.008	Annual mobility of owners = 3.2 percent
θ	0.012	Fraction of moving owners that stay local = 0.6
ϕ	0.25	Quarterly permits/construction employment (1000s of hrs)
$\bar{q}$	3.84	Average land price-income ratio
ξ	1.75	Median price-elasticity of land supply
ψ	0.43	Fraction of households that rent = 32 percent
m	0.0125	Average rent to average income ratio, $r^* = 0.137$
z^H	0.028	Zero net-of-maintenance depreciation
κ	0.76	Vacancy rate = 2 percent
χ	0.0916	Months to sell = months to buy
ζ	0.0028	$P^* = 12.8$

Table 5 — Implied moments for annual income growth process[1]

		$\rho(y_t, y_{t-i})$			
Data	σ_y	$i = 1$	$i = 2$	$i = 3$	$i = 4$
Actual	0.0169	0.17	0.00	-0.06	-0.08
Artificial	0.0169	0.18	-0.05	-0.05	-0.05

Table 6 — Baseline Calibration Parameters: Non Steady-State

Parameter	Value	Target
δ	0.84	Genoseve and Han (2012)
α	9.05	Relative volatility of population growth = 0.17
η	1.85	Relative volatility of house price growth = 1.08

[1] The underlying quarterly income process is $\ln y_t = a \ln y_{t-1} + \varepsilon_t$, where , $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. The parameters are $\sigma_\varepsilon = 0.0104$ and $a = 0.98$ to match σ_y and the sum of the first four autocorrelation coefficients.

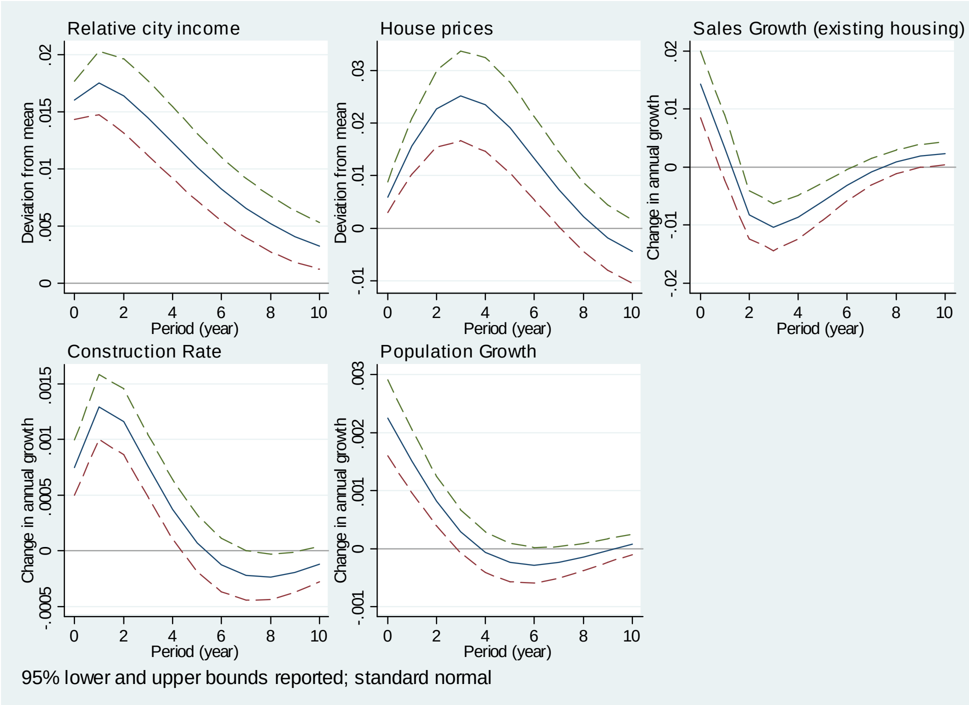


Figure 1: Estimated impulse responses to an income shock

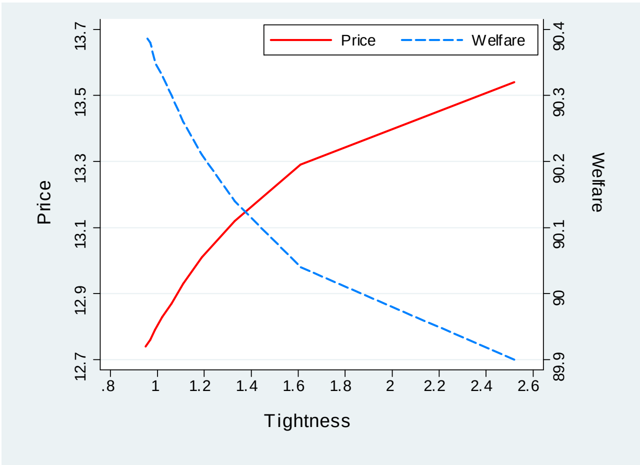


Figure 2: Varying the Productivity of Matching in the Steady-State

V. Equilibrium Dynamics

Table 7 — Volatilities and co-movements[2]

Moment	US Cities	Baseline	No search
σ_p/σ_y	1.08	1.08*	1.28
σ_s/σ_y	1.37	0.31	0.14
σ_h/σ_y	0.12	0.118	0.17
σ_n/σ_y	0.17	0.17*	0.17*
$\rho(p, y)$	0.47	0.98	0.99
$\rho(s, y)$	0.76	0.38	0.12
$\rho(h, y)$	0.31	0.28	0.33
$\rho(n, y)$	0.77	0.48	0.33
$\rho(s, p)$	0.18	0.22	0.03

Note: A * indicates a calibrated target.

Table 8 — Autocorrelations (106 cities, 1981-2008)

$\rho(x_t, x_{t-i})$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 1$	$i = 2$	$i = 3$	$i = 4$
	Price Growth				Sales Growth			
US Cities	0.83	0.47	0.10	-0.19	0.54	0.05	-0.17	-0.25
Baseline	0.32	0.04	-0.01	-0.04	0.35	0.01	-0.02	-0.03
	Population Growth				Construction Rate			
US Cities	0.62	0.29	0.06	-0.09	0.85	0.54	0.23	-0.01
Baseline	0.83	0.64	0.51	0.42	0.95	0.87	0.78	0.69

[2] Sales of existing houses are total sales less a proxy for sales of newly constructed housing.

Figure 3: Responses to an income shock with and without search

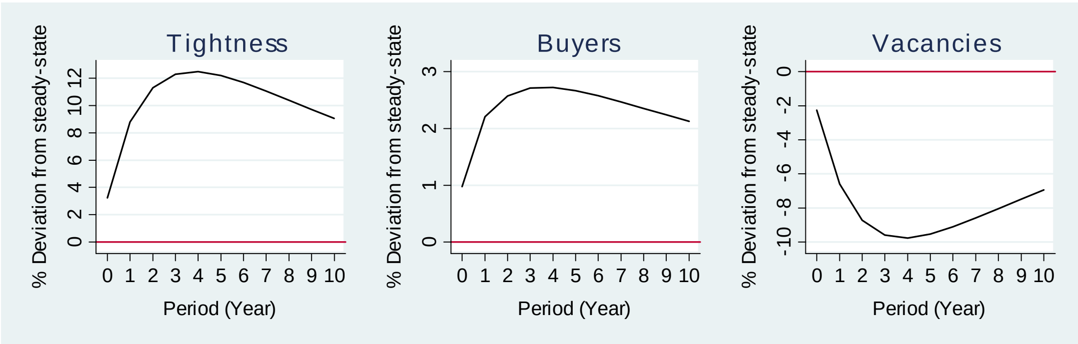


Figure 4: Responses to an income shock: Matching

Table 9 – Construction wages and employment (98 cities, 1981-2008)

	Construction Wage					
			$\rho(w_t, w_{t-i})$			
Moment	σ_w/σ_y	$\rho(w,y)$	$i=1$	$i=2$	$i=3$	$i=4$
US Cities	0.61	0.95	0.43	0.19	-0.03	-0.15
Baseline	0.39	0.99	0.27	0.00	-0.03	-0.05
	Construction Employment					
			$\rho(l_t, l_{t-i})$			
Moment	σ_l/σ_y	$\rho(l,y)$	$i=1$	$i=2$	$i=3$	$i=4$
US Cities	1.24	0.81	0.66	0.27	-0.05	-0.25
Baseline	0.79	0.97	0.33	0.06	0.00	-0.03

Table 10: Growth in Fair Market Rents (106 cities, 1985-2008)

			$\rho(r_t, r_{t-i})$			
Moment	σ_r/σ_y	$\rho(r,y)$	$i=1$	$i=2$	$i=3$	$i=4$
US Cities	0.36	0.25	0.90	0.65	0.34	0.05
Baseline	5.58	0.79	0.62	0.27	0.09	0.00

Table 11—Volatility and Persistence of Price growth: Sensitivity Results

Moment	Baseline Calibration		New housing supply elasticity (η)		Entry elasticity (α)		Matching Elasticity (δ)	
			.10	10	4.5	18	.50	.95
σ_p/σ_y	1.08		2.11	0.33	0.46	1.68	1.25	1.03
ρ_1^p	0.32		0.26	0.49	0.60	0.25	0.28	0.33
	Land supply elasticity (ξ)		Vacancy Rate (v)		Homeowner exit probability (π_n)		Housing Utility (z^H)	
	0.01	100	.01	.03	.004	.012	.01	.04
σ_p/σ_y	1.87	1.06	1.30	0.92	1.09	1.04	0.73	1.15
ρ_1^p	0.27	0.33	0.27	0.36	0.27	0.38	0.56	0.29

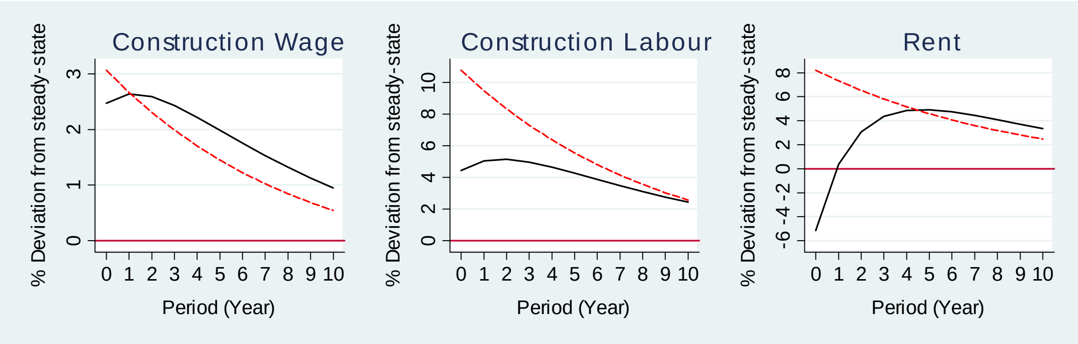
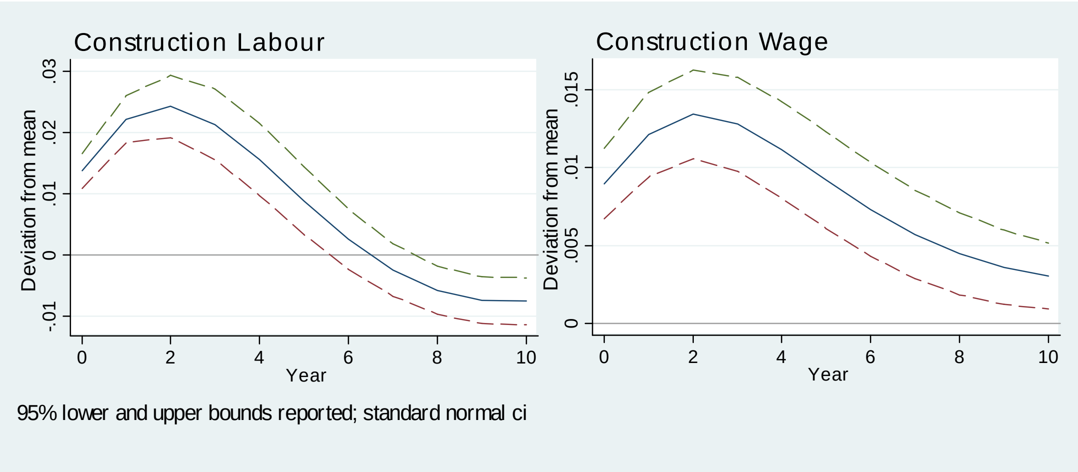


Figure 5: Construction employment, wages and rent with and without search



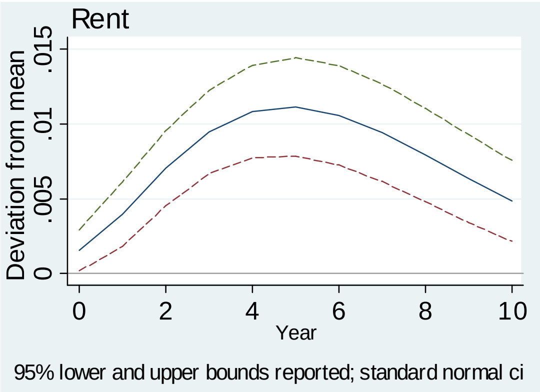


Figure 6: Impulse responses for construction employment, wages and rent

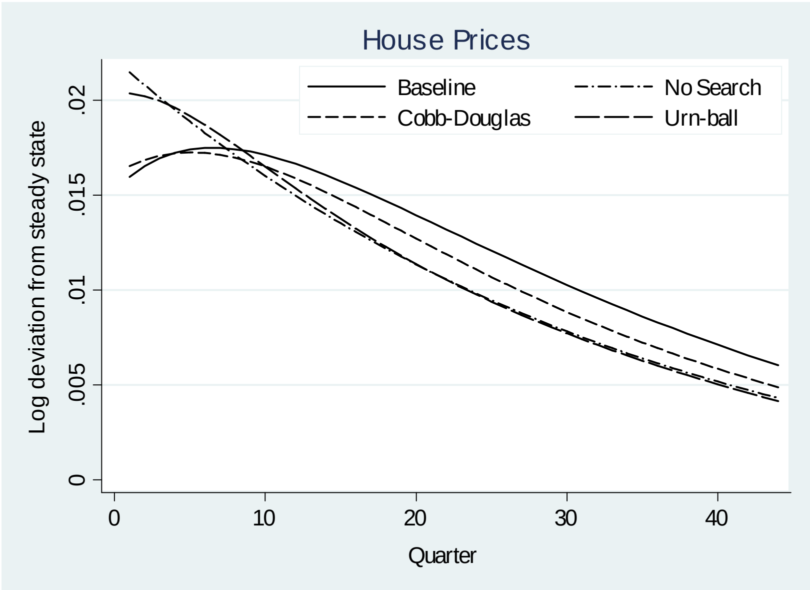


Figure 7: Price growth under competitive search with alternative matching functions

Appendix D: Additional Empirical Results

Table D1 reports the test statistics and P-values for four different panel unit root tests for each of the variables used in the panel VAR (other standard tests produce similar results). Recall that cross-sectional means at each date have been removed. As noted in the main text, the null hypothesis in each case is rejected at the 95% confidence level, with one exception (the Hadri test for sales growth, g^S).

Table D1: Panel Unit Root Test Statistics

	Y		P		g^S		g^H		g^N	
Breitung	−1.82	[0.03]	−12.45	[0.00]	−12.16	[0.00]	−12.52	[0.00]	−6.84	[0.00]
IPS	−4.68	[0.00]	−12.97	[0.00]	−32.88	[0.00]	−11.49	[0.00]	−13.07	[0.00]
- ave. lag	1.07		1.92		0.87		1.19		0.94	
Fisher - P	338.4	[0.00]	785.9	[0.00]	1196.7	[0.00]	445.8	[0.00]	573.21	[0.00]
- Z	−4.34	[0.00]	−16.56	[0.00]	−26.03	[0.00]	−10.30	[0.00]	−12.37	[0.00]
Hadri	80.75	[0.00]	72.95	[0.00]	−1.26	[0.90]	36.12	[0.00]	45.21	[0.00]

P-values in square brackets.

The Breitung (2000) test assumes that all panels have a common autoregressive parameter. The null hypothesis is that all series contain a unit root. The Im-Pesaran-Shin (IPS) (2003) test relaxes the assumption of a common autoregressive parameter and instead allows each panel to have its own. The null hypothesis is that all panels have a unit root. We allow the number of lags to be chosen optimally and the average is reported. The Fisher-type tests conduct unit-root tests for each panel individually, and then combine the p-values from these tests to produce an overall test. We have reported two versions of the test (other versions yield similar results). In both cases augmented Dickey-Fuller unit-root tests are used. The Hadri (2000) LM test differs from the other three in that it has as the null hypothesis that all the panels are stationary.

Table D2 documents the parameter estimates for the baseline estimation of the LSDV specification described in the *Corrigendum*. It replaces the corresponding table referenced in Section I of the original article.

Table D2: LSDV estimates

	Y		P		g^S		g^H		g^N	
$Y(-1)$	1.11	(0.02)	0.29	(0.04)	0.39	(0.13)	0.04	(0.00)	0.07	(0.01)
$P(-1)$	0.04	(0.01)	1.57	(0.02)	-0.70	(0.05)	-0.00	(0.00)	-0.03	(0.00)
$g^S(-1)$	0.01	(0.00)	0.04	(0.01)	-0.04	(0.02)	0.00	(0.00)	0.00	(0.00)
$g^H(-1)$	0.57	(0.08)	0.87	(0.16)	0.61	(0.52)	0.75	(0.02)	0.52	(0.05)
$g^N(-1)$	-0.45	(0.04)	0.20	(0.08)	0.65	(0.24)	0.07	(0.01)	0.03	(0.02)
$Y(-2)$	-0.21	(0.02)	-0.26	(0.04)	-0.35	(0.13)	-0.03	(0.00)	-0.04	(0.01)
$P(-2)$	-0.05	(0.01)	-0.67	(0.02)	0.47	(0.05)	-0.00	(0.00)	0.01	(0.00)
$g^S(-2)$	0.00	(0.00)	-0.01	(0.01)	0.04	(0.02)	-0.00	(0.00)	0.00	(0.00)
$g^H(-2)$	-0.34	(0.08)	-0.92	(0.15)	-2.35	(0.49)	-0.16	(0.02)	-0.10	(0.04)
$g^N(-2)$	0.04	(0.04)	0.27	(0.08)	-0.19	(0.26)	0.04	(0.01)	0.00	(0.02)

Standard errors in parenthesis. No. of observations = 2544

Table D3 compares moments for the baseline specification with three alternatives. "LSDV Levels" refers to the baseline LSDV estimation with price and income in log levels (as in Table 2). "LSDV Growth" refers to the LSDV estimation with price and income in growth rates. "2SLS Growth" refers to the system GMM estimation with price and income in growth rates. "AR(2) income" is the same as "LSDV Levels" but with a univariate income process.

Table D3: Moments from panel VAR for different specifications – income shocks

	Specification	Relative Std. Dev.	Corr. with Income	Corr. with Price	Autocorrelation year 1	year 2	year 3	year 4
Income Growth	LSDV Levels	1.00	1.00	0.47	0.17	0.00	-0.06	-0.08
	LSDV Growth	1.00	1.00	0.47	0.17	0.01	-0.00	-0.02
	2SLS Growth	1.00	1.00	0.51	0.22	0.06	0.02	0.00
	AR(2) income	1.00	1.00	0.43	0.17	-0.04	-0.08	-0.08
Price Growth	LSDV Levels	1.08	0.47	1.00	0.83	0.47	0.10	-0.19
	LSDV Growth	1.15	0.47	1.00	0.82	0.45	0.11	-0.10
	2SLS Growth	1.08	0.51	1.00	0.85	0.54	0.25	0.06
	AR(2) income	1.05	0.43	1.00	0.83	0.45	0.07	-0.21
Sales Growth	LSDV Levels	1.37	0.76	0.18	0.54	0.05	-0.17	-0.25
	LSDV Growth	1.55	0.70	0.14	0.53	-0.07	-0.34	-0.35
	2SLS Growth	1.24	0.84	0.35	0.46	-0.11	-0.23	-0.25
	AR(2) income	1.33	0.77	0.18	0.55	0.04	-0.19	-0.26
Cons. Rate	LSDV Levels	0.12	0.31	0.80	0.85	0.54	0.23	-0.01
	LSDV Growth	0.14	0.41	0.96	0.87	0.59	0.31	0.11
	2SLS Growth	0.14	0.44	0.97	0.90	0.66	0.42	0.24
	AR(2) income	0.13	0.28	0.80	0.85	0.53	0.21	-0.02
Pop. Growth	LSDV Levels	0.17	0.77	0.75	0.62	0.29	0.06	-0.09
	LSDV Growth	0.18	0.83	0.85	0.65	0.28	0.03	-0.08
	2SLS Growth	0.18	0.85	0.88	0.67	0.39	0.18	0.06
	AR(2) income	0.17	0.75	0.75	0.63	0.29	0.04	-0.10

Appendix E: A Multi-City Environment

There are M symmetric cities, indexed by $i = 1, \ldots, M$, where M is finite, but *large* in the sense that no individual city has a significant effect on aggregate quantities. The cities can be of identical or different sizes; what is important is that they all be small in this sense. We will focus on City 1, which will correspond to the *representative* or *average* city that was considered in the text.

Each city can be described as in Section II, except that here city-specific quantities are indexed by i. In particular, at each point in time, income in City i is denoted $\widetilde{y}_i$. Define average income across all cities by $\overline{y} = \sum_{i=1}^{M} \widetilde{y}_i$, and let $y_i \equiv \frac{\widetilde{y}_i}{\overline{y}}$. We will assume that in the steady-state, $y_i = 1$ for all i. We will think of the deviation of City 1 income from the average, y_1, as following a stochastic process just as in the text. This is straightforward under the assumption that City 1 is small so that fluctuations in $\widetilde{y}_1$ have no effect on $\overline{y}$. Alternatively, we can dispense with y_i and consider fluctuations in the *level* of City i income, $\widetilde{y}_i$. What is important in what follows is that the shocks considered be truly *city-specific.* That is, that fluctuations in either y_1 or $\widetilde{y}_1$, have negligible effects on income and/or housing market conditions in all other cities.

As in the text, the population of the economy is given by Q_t, and grows at gross rate $1+\mu$. Every period, each new household that enters the economy draws M potential amenity values, $a_i \in [0, \overline{a}]$ (one for each city), from distributions $F_i(a)$, for $i = 1, \ldots M$. Here for simplicity we will assume $F_i(\cdot) = F(\cdot)$ for all i., and that $\overline{a}$ is sufficiently large that a positive measure of households chooses to enter all cities in each time period. Amenity values are in utils, and like both consumption and housing services enter households' utility linearly. Utility from amenities, also like that from income, is realized only when the household chooses a particular city in which to live, and locates there.

For each new household j, let $\overline{W}_{ij}$ denote the value of being a new entrant to City i, defined just as in (9). Since new entrants to any city are identical, variation in $\overline{W}_{ij}$ across households is induced solely by variation in the amenity value, a_{ij}; in particular, $F_i(\overline{W}(a)) = F_i(a) = F(a)$ where a is the amenity value that generates $\overline{W}(a)$ given all other city attributes (income, house prices, the housing market tightness, etc.). Let $\varepsilon_j \equiv \max[\overline{W}_{2j}, \ldots, \overline{W}_{Mj}]$. That is, ε_j denotes the highest alternative value to entering City 1 for each new household j. Since M is finite, ε exists for all new households and identifies a single best alternative with probability 1. Similarly, the probability that $\varepsilon_j = \overline{W}_{1j}$, that is that a household is indifferent between entering City 1 and some other city, approaches zero as M becomes large.

Let $G(\varepsilon)$ denote the distribution of the highest alternative value, ε, across households. In a situation in which all cities other than City 1 are identical, $\varepsilon_j$is the value of entering that city for which household j has the highest amenity value. Thus, $G(\cdot)$ satisfies:

$$G(\varepsilon) = [F(a^*)]^{M-1} \tag{A28}$$

where $a^* \in [0, \overline{a}]$ is the amenity value which generates the maximum value ε. Note, however, that for $G(\varepsilon)$ to be well-defined, it is not required for all cities other than City 1 to be identical. Finally, note that the entry cutoff, ε_t^c, in this case satisfies $\varepsilon_t^c = \overline{W}_{1t}$, just as in (8). That is, any household with a maximum alternative value below $\overline{W}_{1t}$ enters City 1.

When a household leaves their city of residence due to the realization of a relocation shock (which happens with probability π for both home-owners and permanent renters), we assume that they are effectively in the same situation as a new household who has just arrived in the economy. That is, they re-draw, *in the current period*, from the amenity distribution for each city, and choose the city which yields the highest value. The expected continuation value following a relocation shock for any household currently resident in *any* city is thus given by

$$Z = \overline{\varepsilon} \equiv \int G(\varepsilon) d\varepsilon. \tag{A29}$$

From (A29) it is clear that Z depends only on the distribution of amenity values, $F(\cdot)$. Also, note that since City 1 is small, the probability that a household which exits it due to a relocation shock returns immediately is negligible.

Let POP_t denote the population of City 1 in period t. The population evolves via

$$POP_{t+1} = POP_t + \underbrace{\mu G(\overline{W}_{1t})Q_t + \pi \mathbb{Z}_t G(\overline{W}_{1t})}_{\text{new entrants}} - \underbrace{\pi[N_t + F_t]}_{\text{exiters}}, \tag{A30}$$

where $\mathbb{Z}_t$ denotes the measure of agents that exit *all other* cities in period t, and is assumed to be unaffected by conditions in City 1. On the balanced growth path, we assume first that all cities are symmetric, so that $G(\overline{W}_{1t}) = 1/M$ for all t. Similarly, $\mathbb{Z}_t = M(N_t + F_t)$. Thus, from (A30) $POP_{t+1} = (1 + \mu)POP_t$.

Finally, note that it is not important that we model the shock to City 1's income as being relative to the average. Any stable distribution of income across cities will give rise to a well-defined distribution of alternative values for City 1 (although (A28) will no longer apply). A direct increase in City 1 income, $\widetilde{y}_1$, will thus lead to entry for the same reasons as before. Again, the magnitude of the response will be determined by the properties of $G(\cdot)$ in a neighborhold of $\varepsilon = \widetilde{y}_1$ along the balanced growth path.

Suppose now that the economy is subject to aggregate income shocks which affect all cities symmetrically. Because utility is linear, adding a common component, y_{ct}, to city-level income of the form,

$$\widetilde{Y}_{it} = \widetilde{y}_{it} + y_{ct} \qquad i = 1, \dots, N \tag{A31}$$

will have no effect whatsoever, as it affects neither the ranking of cities by new entrants or relocaters nor the demand for housing. Common shocks to construction costs and/or population growth will affect housing markets within each city, but will have no effect on mobility as they will not change the ranking of cities across prospective entrants as this is determined only by the amenity distribution, $F(\cdot)$.

Appendix F: Details of Robustness Exercises

Exiting Buyers

We assume that unsuccessful buyers exit at the end of the period with the same probability, π_n, as homeowners. This implies that the value of being a searching buyer is now

$$W_t = u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1 - \lambda_t)\, \beta \left[\pi_n Z + (1 - \pi_n) E_t W_{t+1}\right]. \tag{A32}$$

and the stock of buyers at date t is given by:

$$B_t = \theta(1 - \pi_n) N_{t-1} + \psi G(\varepsilon_t^c) \mu Q_{t-1} + (1 - \lambda_{t-1})(1 - \pi_n) B_{t-1}. \tag{A33}$$

The rest of the model remains unchanged.

Note: An implication of the corrections described in the *Corrigendum* is that we fix the value of δ, thereby reducing the extent to which the parameters can be recalibrated to match specific targets. As a result, assuming unsuccessful buyers exit the city at the same rate as homeowners has more of an effect on the results, once the model is re-calibrated. When we re-calibrate the elasticities, α and η, to match our targets (the volatilities of population and price growth), the first-order autocorrelation falls from .32 to 0.28 (in addition, the relative volatility of the construction rate rises and that of sales growth falls). All other assertions concerning robustness in the original article remain correct.

Mismatched Owners remain in their houses

Proposition 2. *In equilibrium, mismatched owners are indifferent between the following two arrangements:*

(1) putting up their house for sale or rent immediately and renting while searching;

(2) remaining in their current house while searching, then putting their vacant house up for sale once they are matched with a new one.

Proof: The value of being a mis-matched owner who remains in their house while they search for a new one is given by

$$\tilde{J}_t = y_t + x_t - m + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) + (1 - \lambda_t) \beta E_t \tilde{J}_{t+1}. \tag{A34}$$

The value of becoming a renter immediately and putting the vacant house up for sale is given by

$$\begin{aligned} W_t + V_t &= u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1 - \lambda_t) \beta E_t W_{t+1} + \gamma_t P_t + (1 - \gamma_t) \beta E_t V_{t+1} \\ &= u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1 - \lambda_t) \beta E_t W_{t+1} + \gamma_t (1 - \chi) \beta E_t \left(J_{t+1} - W_{t+1}\right) \\ &\quad + \gamma_t \chi \beta E_t V_{t+1} + (1 - \gamma_t) \beta E_t V_{t+1} \\ &= y_t + x_t - r_t + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) \\ &\quad + \gamma_t (1 - \chi)) \beta E_t \left(J_{t+1} - W_{t+1} - V_{t+1}\right) + (1 - \lambda_t) \beta \left(E_t W_{t+1} + E_t V_{t+1}\right). \end{aligned} \tag{A35}$$

Given (23), the above implies that

$$
\begin{aligned}
W_t + V_t &= y_t + x_t - m + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) \\
&\quad + (1-\lambda_t)\beta E_t \left[W_{t+1} + V_{t+1}\right]. \qquad \text{(A36)}
\end{aligned}
$$

Since $\lim_{T\to\infty} \beta^T E_t \tilde{J}_{t+T} = \lim_{T\to\infty} \beta^T E_t \left[W_{T+1} + V_{T+1}\right] = 0$, solving forwards implies that

$$
\tilde{J}_t = W_t + V_t. \qquad \text{(A37)}
$$

QED

Let $\tilde{n}_t$ denote mismatched owners who remain in their (owned) homes. Then the flows of households between states is now described by (30) and

$$
\begin{aligned}
(1+\mu)\tilde{n}_t &= \theta(1-\pi_n)n_{t-1} + \left[1-\lambda_{t-1}\right]\tilde{n}_{t-1} \qquad \text{(A38)} \\
(1+\mu)b_t &= \psi\mu G\left(\bar{W}_t\right) + \left[1-\lambda_{t-1}\right]b_{t-1} \qquad \text{(A39)} \\
(1+\mu)n_t &= (1-\theta)(1-\pi_n)n_{t-1} + \lambda_{t-1}\left(b_{t-1} + \tilde{n}_{t-1}\right) \qquad \text{(A40)}
\end{aligned}
$$

Market tightness is given by

$$
\omega_t = \frac{b_t + \tilde{n}_t}{h_t - b_t - \tilde{n}_t - f_t - n_t} \qquad (1)
$$

and the housing stock evolves according to

$$
(1+\mu)h_{t+1} = h_t + \phi^\eta \left(n_t + \tilde{n}_t + b_t + f_t\right)\left(\beta E_t V_{t+1} - \bar{q}\right)^\eta. \qquad \text{(A42)}
$$

Endogenous Exit

The implied value of being a homeowner is now given by

$$
\begin{aligned}
J_t &= u_t^H + \beta\pi_n E_t\left(Z_{t+1}^* + e_{t+1}V_{t+1}\right) + \theta\beta E_t(1-\pi_n e_{t+1})\left(W_{t+1} + V_{t+1}\right) \\
&\quad + \beta(1-\theta)E_t(1-\pi_n e_{t+1})J_{t+1}, \qquad \text{(A43)}
\end{aligned}
$$

where $Z_t^* = \left(\bar{Z}^2 - J_t^2\right)/2\bar{Z}$ and $e_t = 1 - J_t/\bar{Z}$ is the exit probability conditional on having an opportunity.

Allocation of Land Developers' Profits

The profits of land developers is given by

$$
\Pi_t = A_t \int_0^{q_t} (q_t - c)\Lambda(dc).
$$

Given the functional form for the cost distribution given in (38), this can be expressed as

$$\Pi_t = \frac{q_t \Lambda(q_t) A_t}{1+\tau}.$$

We therefore add $\Pi_t/(N_t + B_t + F_t)$ to the income of each household. Note that, since preferences are linear, it does not actually matter how these profits are distributed across households.

Competitive Search

By entering sub-market (ω_t, P_t), a seller sells a house at P_t with probability $\gamma(\omega_t)$. The seller chooses to enter a sub-market that maximizes his/her expected return. It follows that the value of a vacant house for sale satisfies

$$V_t = \max_{(\omega_t, P_t)} \left\{ \gamma(\omega_t) P_t + [1 - \gamma(\omega_t)] \beta E_t \tilde{V}_{t+1} \right\}. \tag{A44}$$

Free entry of sellers implies that all active sub-markets (*i.e.* sub-markets with $\lambda, \gamma \in (0,1)$) in equilibrium must offer the sellers the same payoff V_t, although (ω_t, P_t) varies across sub-markets. It follows that the relationship between the listed price and market tightness that must be satisfied by all active sub-markets:

$$\gamma(\omega_t(P_t)) = \frac{V_t - \beta E_t \tilde{V}_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}}. \tag{A45}$$

Thus, it is sufficient to index sub-markets by the posted price P_t alone.

Buyers also decide in each period which sub-market to enter. The value of being a buyer W_t is therefore given by

$$W_t = u_t^R + \max_{P_t} \left\{ \lambda(\omega_t(P_t)) (\beta E_t J_{t+1} - P_t) + [1 - \lambda(\omega_t(P_t))] \beta E_t W_{t+1} \right\}. \tag{A46}$$

In equilibrium, the set of active sub-markets is *complete* in the sense that there is no other sub-market that could improve the welfare of any buyer or seller.

Let $\epsilon(\omega_t)$ denote the elasticity of the measure of matches with respect to the measure of buyers. We then have the following proposition:

Proposition 3. *In a competitive search equilibrium, there is only one active sub-market. In this market, the share of the surplus from house transactions that accrues to the buyer is equal to the elasticity of the measure of matches with respect to the measure of buyers:*[3]

$$\chi(\omega_t) = \epsilon(\omega_t). \tag{A47}$$

[3] This result is a special case of that derived in Moen (1997).

Proof: The first-order condition to the optimization problem in (A46) yields

$$\lambda'(\omega_t)\omega_t'(P_t)\left(\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}\right) - \lambda\left(\omega_t(P_t)\right) = 0, \tag{A48}$$

where $\omega_t(P_t)$ and $\omega_t'(P_t)$ are implicitly determined by (A45). This implies

$$\frac{\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}} = -\frac{\lambda(\omega_t(P_t))/\lambda'(\omega_t(P_t))}{\gamma\left(\omega_t(P_t)\right)/\gamma'(\omega_t(P_t))}, \tag{A49}$$

which can be used together with (A45) to solve for P_t, given the values $E_t J_{t+1}$, $E_t W_{t+1}$ and $E_t \tilde{V}_{t+1}$. Then one can solve for ω_t from (A45). Note that (A45) implies that $\omega_t'(P_t) < 0$ given $\gamma'(\omega) > 0$ from Assumption 1.

The trade surplus in the housing market is strictly positive. Given the boundary condition $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$, it is clear that the household's equilibrium values are bounded, which implies that the trade surplus is also bounded. Thus $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} \in (0, \infty)$, where we have incorporated that $V = \tilde{V}$ in the equilibrium. Recall condition (ix) of the equilibrium definition that $\gamma\left(\omega_t\right),\ \lambda\left(\omega_t\right) \in (0,1)$ for all active sub-markets. Also recall from part (ii) of Assumption 1 that $\lambda'(\omega) < 0,\ \gamma'(\omega) > 0$. These conditions imply that $\epsilon\left(\omega\right) \in (0,1)$ where

$$\epsilon\left(\omega_t\right) = \frac{B_t}{M} \cdot \frac{\partial M}{\partial B_t} = \frac{1}{1 - \frac{\gamma(\omega_t)/\gamma'(\omega_t)}{\lambda(\omega_t)/\lambda'(\omega_t)}}. \tag{A50}$$

Define $LHS\left(P_t\right)$ as the left-hand side of (A49) and $RHS\left(P_t\right)$ the right-hand side. Given (A50), it is clear that

$$RHS\left(P_t\right) = \frac{\epsilon(\omega_t(P_t))}{1 - \epsilon(\omega_t(P_t))}. \tag{A51}$$

Because $\epsilon\left(\omega\right) \in (0,1)$, we have $RHS\left(P_t\right) \in (0,\infty)$ for all P_t. Moreover, recall $\omega_t'(P_t) < 0$ from (A45) and $\epsilon'\left(\omega\right) \leq 0$ from Assumption 1. Thus $RHS'\left(P_t\right) \geq 0$.

For any given V_t, J_t, W_t, one can verify that $LHS'\left(P_t\right) < 0$ because $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} > 0$. Recall from (20) and (21) that the price in an active sub-market satisfies

$$\beta E_t V_{t+1} \leq P_t \leq \beta E_t J_{t+1} - \beta E_t W_{t+1}. \tag{A52}$$

It follows that

$$\begin{aligned} LHS(P_t = \beta E_t V_{t+1}) &= \infty > RHS(P_t = \beta E_t V_{t+1}) && \text{(A53)} \\ LHS\left(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}\right) &= 0 < RHS\left(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}\right), && \text{(A54)} \end{aligned}$$

where the two inequalities are because $RHS\left(P_t\right) \in (0,\infty)$ for all P_t. The above results imply a unique $P_t^* \in (\beta E_t V_{t+1},\ \beta E_t J_{t+1} - \beta E_t W_{t+1})$ that satisfies

$$\frac{\beta E_t J_{t+1} - P_t^* - \beta E_t W_{t+1}}{P_t^* - \beta E_t V_{t+1}} = -\frac{\lambda(\omega_t^*(P_t^*))/\lambda'(\omega_t^*(P_t^*))}{\gamma\left(\omega_t^*(P_t^*)\right)/\gamma'(\omega_t^*(P_t^*))}, \tag{A55}$$

and a unique $\omega_t^*(P_t^*)$ that satisfies

$$\omega_t^*(P_t^*) = \gamma^{-1}\left(\frac{V_t - \beta E_t V_{t+1}}{P_t^* - \beta E_t V_{t+1}}\right). \tag{A56}$$

Thus, there is a single active sub-market in the directed search equilibrium.

Equation (A55) may be written as

$$\frac{\chi(\omega)}{1-\chi(\omega)} = \frac{\epsilon(\omega)}{1-\epsilon(\omega)}, \tag{A57}$$

where $\chi(\omega)$ denotes the buyer's share of the surplus in a sub-market with tightness ω. The right-hand side of the above is the ratio of the elasticities of the number of matches with respect to the numbers of buyers and sellers. It follows that $\chi(\omega) = \epsilon(\omega)$. **QED**

The share of the surplus accruing to the buyer for the urn ball matching function is given by

$$\chi(\omega) = \epsilon(\omega) = \frac{\varrho\omega}{e^{\varrho\omega} - 1}, \tag{A58}$$

which is decreasing in market tightness ω.

Search, Liquidity and the Dynamics of House Prices and Construction

By Allen Head, Huw Lloyd-Ellis and Honfei Sun

Online Appendix

Appendix C: Empirical Results

Panel Unit Root Tests

Table C1 reports the test statistics and P-values for four different panel unit root tests for each of the variables used in the panel VAR (other standard tests produce similar results). Recall that cross-sectional means at each date have been removed. As noted in the main text, the null hypothesis in each case is rejected at the 95% confidence level, with one exception (the Hadri test for sales growth, g^S).

Table C1: Panel Unit Root Test Statistics

	Y		P		g^S		g^H		g^N	
Breitung	-1.82	[0.03]	-12.45	[0.00]	-12.16	[0.00]	-12.52	[0.00]	-6.84	[0.00]
IPS	-4.68	[0.00]	-12.97	[0.00]	-32.88	[0.00]	-11.49	[0.00]	-13.07	[0.00]
- ave. lag	1.07		1.92		0.87		1.19		0.94	
Fisher - P	338.4	[0.00]	785.9	[0.00]	1196.7	[0.00]	445.8	[0.00]	573.21	[0.00]
- Z	-4.34	[0.00]	-16.56	[0.00]	-26.03	[0.00]	-10.30	[0.00]	-12.37	[0.00]
Hadri	80.75	[0.00]	72.95	[0.00]	-1.26	[0.90]	36.12	[0.00]	45.21	[0.00]

P-values in square brackets.

The Breitung (2000) test assumes that all panels have a common autoregressive parameter. The null hypothesis is that all series contain a unit root. The alternative hypothesis is that the series are stationary. Breitung's (2000) Monte Carlo simulations suggest that his test is substantially more powerful than other panel unit-root tests for the modest-size dataset he considered (N=20, T=30).

The Im-Pesaran-Shin (IPS) (2003) test relaxes the assumption of a common autoregressive parameter and instead allows each panel to have its own. The null hypothesis is that all panels have a unit root. The alternative hypothesis is that the fraction of panels that are stationary is nonzero. Specifically, if we let N_0 denote the number of stationary panels, then the fraction N_0/N tends to a nonzero fraction as N tends to infinity. This allows some (but not all) of the panels to possess unit roots under the alternative hypothesis. We allow the number of lags to be chosen optimally and the average is reported.

The Fisher-type tests conduct unit-root tests for each panel individually, and then combine the p-values from these tests to produce an overall test. We have reported two versions of the test (other versions yield similar results). In both cases augmented Dickey-Fuller unit-root tests are used. The P-test combines p-values using the inverse chi-squared transformation and the Z test uses the normal transformation.

The Hadri (2000) LM test differs from the other three in that it has as the null hypothesis that all the panels are stationary. The alternative hypothesis is that at least some of the panels contain a unit root. Hadri (2000) states that his tests are appropriate for panel datasets in which T is large and N is moderate, such as the Penn World Tables frequently used for cross-country comparisons.

Full Panel VAR Results

Table C2 documents the parameter estimates for the baseline estimation of the Panel VAR discussed in Section 2. Estimating a panel VAR raises a number of econometric issues. A basic problem in dynamic panel data models with fixed effects is that the lagged dependent variables are, by construction, correlated with the individual effects. This renders the least squares estimator biased and inconsistent. Consistent estimation requires some transformation to eliminate fixed effects. A within transformation wipes out the individual effects by taking deviations from sample means, but the resulting within-group estimator is inconsistent when the number of panels becomes large for a given time-dimension (Nickell, 1981).

Table C2: System GMM (2SLS) estimates

	Y		P		g^S		g^H		g^N	
$Y(-1)$	1.23	(0.05)	0.45	(0.09)	0.38	(0.23)	0.01	(0.01)	0.06	(0.02)
$P(-1)$	-0.01	(0.01)	1.37	(0.04)	-0.53	(0.08)	0.01	(0.00)	-0.02	(0.00)
$g^S(-1)$	0.00	(0.00)	-0.06	(0.01)	0.07	(0.03)	0.00	(0.00)	0.01	(0.00)
$g^H(-1)$	0.74	(0.12)	1.28	(0.26)	0.06	(0.83)	0.74	(0.04)	0.39	(0.09)
$g^N(-1)$	-0.23	(0.17)	0.15	(0.28)	2.23	(1.23)	0.08	(0.04)	0.24	(0.19)
$Y(-2)$	-0.33	(0.05)	-0.61	(0.08)	-0.47	(0.22)	-0.02	(0.01)	-0.06	(0.03)
$P(-2)$	0.02	(0.01)	-0.46	(0.05)	0.63	(0.08)	-0.01	(0.00)	0.02	(0.00)
$g^S(-2)$	-0.00	(0.00)	-0.01	(0.01)	0.04	(0.03)	-0.00	(0.00)	0.00	(0.00)
$g^H(-2)$	-0.67	(0.09)	-1.30	(0.19)	-2.77	(0.60)	-0.14	(0.03)	-0.21	(0.04)
$g^N(-2)$	0.16	(0.06)	0.53	(0.12)	1.02	(0.38)	0.03	(0.01)	0.16	(0.05)

Standard errors in parenthesis. No. of observations = 2438

Given this inconsistency, the literature focuses mainly on a first-difference transformation to eliminate the individual effect while handling the remaining correlation with the (transformed) error term using instrumental variables and GMM estimators (e.g. Arellano and Bond, 1991). However, the Arellano-Bond estimator is known to suffer from a weak instruments problem when the relevant time series are highly persistent, as they are in our case. As Blundell and Bond (1998) demonstrate this can result in large finite-sample biases. In our baseline estimation we use the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998). This estimator is consistent when the number of panels becomes large for a given time-dimension and is less likely to suffer from the weak instruments problem. Another reason for focussing on this estimator is that its properties are fairly well understood and it has been studied in the context of panel VARs by Binder, Hsiao and Pesaren (2005).

Analysis of Regional Sub-samples

Table C3: Moments from system GMM estimation for regional sub-samples – income shock

	Region	Relative Std. Dev.	Corr. with Inc. Growth	Corr. with Price App.	Autocorrelation year 1	year 2	year 3	year 4
Income Growth	Coastal	1.00	1.00	0.76	0.27	0.01	-0.07	-0.06
	Interior	1.00	1.00	0.68	0.14	-0.05	-0.10	-0.10
	Sunbelt	1.00	1.00	0.51	0.34	0.12	0.02	-0.04
Price Appreciation	Coastal	1.71	0.76	1.00	0.69	0.20	-0.12	-0.23
	Interior	1.10	0.68	1.00	0.69	0.31	-0.02	-0.25
	Sunbelt	0.89	0.51	1.00	0.89	0.66	0.42	0.20
Sales Growth (existing)	Coastal	3.11	0.19	-0.33	0.90	0.83	0.74	0.65
	Interior	1.11	0.30	-0.01	0.69	0.54	0.47	0.42
	Sunbelt	2.21	0.66	-0.09	0.81	0.58	0.45	0.38
Construction Rate	Coastal	0.05	0.10	0.24	0.92	0.79	0.68	0.61
	Interior	0.15	0.52	0.66	0.88	0.66	0.45	0.27
	Sunbelt	0.15	0.63	0.65	0.86	0.57	0.27	0.02
Population Growth	Coastal	0.15	0.04	-0.30	0.93	0.88	0.83	0.77
	Interior	0.11	0.70	0.39	0.60	0.32	0.16	0.13
	Sunbelt	0.35	0.79	0.26	0.73	0.48	0.26	0.11

We now consider the results of estimating the panel VAR model on various sub-samples of both cities and time periods. Table C2 provides key moments for local earnings, house prices, construction rates, and ratios of housing stocks to city population based on shocks to local income in the panel VAR for each of the three sub-samples. Several, key observations are apparent. The standard deviation of house prices is roughly equal to that of local earnings in the full sample. Both construction rates and housing stock-population ratios are much less volatile than local earnings. House prices, construction rates and housing stock-population ratios are all strongly positively correlated with local earnings, although for inland cities these correlations are somewhat weaker. The higher and more persistent autocorrelation in both house price growth and population growth relative to earnings growth can also be observed in all the sub-samples.

Certain features of these moments and impulse response functions in Figure conform to *a priori* expectations regarding population and price movements. In particular, coastal cities typically have more inelastic land supply than sunbelt cities. Accordingly, in response to demand shocks, price volatility tends to be higher and population and construction volatility tend to be lower in the coastal cities.

Alternative estimators

There are several potential problems with using the system GMM estimator for a sample with the dimensions considered here. While it is usually thought to be suitable for typical microeconometric panels, with only a few waves but a large number of individuals, here we have moderately large number of cities and a moderately long time series. Moreover, GMM estimators tend to have a larger standard error compared to the within-group estimator and may suffer from a finite sample bias due to weak instruments. Here we address these issues by comparing our estimates with those of two alternative estimators: OLS with no fixed effects and a standard within-groups estimator (WGE). Although the WGE is inconsistent as the number of panels becomes large, this should be less of a problem given the dimensions of our sample.

For the sake of brevity we do not report here all of the estimation results for each estimator. Instead Table C4 reports only the sum of the coefficients on the lagged dependent variables for each equation under each estimator, as suggested by Blundell and Bond (1998). As may be seem, the OLS estimates yields the most persistent processes for each variable. This reflects the upward bias due to the fact any fixed effect is attributed to persistent effects of the shocks. The WGE etimates yield the least persistent processes, which reflects the downward bias. The system GMM (2SLS) estimator implies persistence that lie between these two extremes.

Table C4: Implied persistence: sum of coefficients on lagged dependent variable

Equation	WGE	$2SLS$	OLS
Y	0.90	0.90	0.99
P	0.87	0.92	0.98
g^S	-0.12	-0.11	-0.03
g^H	0.60	0.60	0.72
g^N	0.06	0.40	0.45

Table C5 documents the same set of moments as we have previously considered, for each of the estimators. While there are clearly some differences across estimators, the same broad pattern emerges as that depicted in Table 2. The biggest outliers come from those based on OLS estimation. This is because the omission of city level fixed effects forces any permament differences to show up as high persistence. The system GMM (2SLS) estimator implies a price growth response that is the most volatile and the least persistent.

Table C5: Moments from estimation using alternative estimators – income shocks

	Estimator	Relative Std. Dev.	Corr. with Inc. Growth	Corr. with Price App.	Autocorrelation year 1	year 2	year 3	year 4
Income Growth	WGE	1.00	1.00	0.67	0.24	0.04	-0.04	-0.09
	2SLS	1.00	1.00	0.76	0.28	0.03	-0.06	-0.09
	OLS	1.00	1.00	0.47	0.21	0.08	0.02	-0.00
Price Growth	WGE	1.90	0.67	1.00	0.81	0.48	0.12	-0.19
	2SLS	1.60	0.76	1.00	0.75	0.36	0.05	-0.15
	OLS	1.22	0.47	1.00	0.88	0.60	0.31	0.09
Sales Growth	WGE	1.75	0.60	0.02	0.57	0.11	-0.15	0.27
	2SLS	1.32	0.56	0.01	0.59	0.39	0.35	0.35
	OLS	1.98	0.59	0.12	0.71	0.37	0.25	0.25
Cons. Rate	WGE	0.12	0.41	0.90	0.84	0.47	0.09	-0.20
	2SLS	0.11	0.55	0.80	0.88	0.61	0.33	0.11
	OLS	0.40	0.19	0.44	0.98	0.95	0.91	0.88
Pop. Growth	WGE	0.16	0.89	0.84	0.57	0.20	-0.07	-0.25
	2SLS	0.17	0.75	0.60	0.67	0.40	0.19	0.09
	OLS	0.38	0.47	0.55	0.91	0.84	0.77	0.73

Alternative Specifications

Table C6 documents the relevant moments due to income shocks from the panel VAR for two alternative specifications.[1] The first specification, labelled "AR(2) Income", restricts the equation for income so that income depends only on its own lagged values. The specification labelled "All growth" uses growth rates of per capita incomes and prices in the VAR rather than levels. As may be seen by comparing to Table 2, restricting the income process to be univariate has negligible effects. This suggest thats lagged feedback effects of prices and population on per capita income are of second order importance. Specifying the VAR so that incomes and prices are in growth rates rather than in log levels has somewhat larger effects on our results, but does not change the broad conclusions. Note that, by construction, the level of relative income under this specification is permanently high following a shock. However, this has little impact on the moments that we consider here.

Table C6: Moments from system GMM estimation for alternative specifications – income shocks

	Specification	Relative Std. Dev.	Corr. with Inc. Growth	Corr. with Price Growth	Autocorrelation year 1	year 2	year 3	year 4
Income Growth	Baseline	1.00	1.00	0.76	0.28	0.03	-0.06	-0.09
	AR(2) Income	1.00	1.00	0.75	0.26	0.02	-0.05	-0.07
	All growth	1.00	1.00	0.47	0.23	0.07	0.03	0.01
Price Growth	Baseline	1.60	0.76	1.00	0.75	0.36	0.05	-0.15
	AR(2) Income	1.62	0.74	1.00	0.76	0.38	0.07	-0.12
	All growth	1.10	0.47	1.00	0.87	0.58	0.30	0.10
Sales Growth	Baseline	1.32	0.56	0.01	0.59	0.39	0.35	0.35
	AR(2) Income	1.40	0.58	-0.02	0.62	0.44	0.40	0.41
	All growth	1.34	0.83	0.16	0.44	-0.08	-0.23	-0.26
Cons. Rate	Baseline	0.11	0.55	0.80	0.88	0.61	0.33	0.11
	AR(2) Income	0.13	0.52	0.74	0.90	0.68	0.45	0.26
	All growth	0.13	0.44	0.98	0.90	0.67	0.42	0.23
Pop. Growth	Baseline	0.17	0.75	0.60	0.67	0.40	0.19	0.09
	AR(2) Income	0.17	0.77	0.57	0.68	0.42	0.22	0.12
	All growth	0.18	0.85	0.85	0.69	0.40	0.20	0.07

[1] We have considered others including alternative definitions of the construction rate and other definitions of income. Similar patterns emerge in all cases.

Appendix D: A Multi-City Environment

There are M symmetric cities, indexed by $i = 1, \ldots, M$, where M is finite, but *large* in the sense that no individual city has a significant effect on aggregate quantities. The cities can be of identical or different sizes; what is important is that they all be small in this sense. We will focus on City 1, which will correspond to the *representative* or *average* city that was considered in the text.

Each city can be described as in Section II, except that here city-specific quantities are indexed by i. In particular, at each point in time, income in City i is denoted $\widetilde{y}_i$. Define average income across all cities by $\overline{y} = \sum_{i=1}^{M} \widetilde{y}_i$, and let $y_i \equiv \frac{\widetilde{y}_i}{\overline{y}}$. We will assume that in the steady-state, $y_i = 1$ for all i. We will think of the deviation of City 1 income from the average, y_1, as following a stochastic process just as in the text. This is straightforward under the assumption that City 1 is small so that fluctuations in $\widetilde{y}_1$ have no effect on $\overline{y}$. Alternatively, we can dispense with y_i and consider fluctuations in the *level* of City i income, $\widetilde{y}_i$. What is important in what follows is that the shocks considered be truly *city-specific*. That is, that fluctuations in either y_1 or $\widetilde{y}_1$, have negligible effects on income and/or housing market conditions in all other cities.

As in the text, the population of the economy is given by Q_t, and grows at gross rate $1+\mu$. Every period, each new household that enters the economy draws M potential amenity values, $a_i \in [0, \overline{a}]$ (one for each city), from distributions $F_i(a)$, for $i = 1, \ldots M$. Here for simplicity we will assume $F_i(\cdot) = F(\cdot)$ for all i., and that $\overline{a}$ is sufficiently large that a positive measure of households chooses to enter all cities in each time period. Amenity values are in utils, and like both consumption and housing services enter households' utility linearly. Utility from amenities, also like that from income, is realized only when the household chooses a particular city in which to live, and locates there.

For each new household j, let $\overline{W}_{ij}$ denote the value of being a new entrant to City i, defined just as in (9). Since new entrants to any city are identical, variation in $\overline{W}_{ij}$ across households is induced solely by variation in the amenity value, a_{ij}; in particular, $F_i(\overline{W}(a)) = F_i(a) = F(a)$ where a is the amenity value that generates $\overline{W}(a)$ given all other city attributes (income, house prices, the housing market tightness, etc.). Let $\varepsilon_j \equiv \max[\overline{W}_{2j}, \ldots, \overline{W}_{Mj}]$. That is, ε_j denotes the highest alternative value to entering City 1 for each new household j. Since M is finite, ε exists for all new households and identifies a single best alternative with probability 1. Similarly, the probability that $\varepsilon_j = \overline{W}_{1j}$, that is that a household is indifferent between entering City 1 and some other city, approaches zero as M becomes large.

Let $G(\varepsilon)$ denote the distribution of the highest alternative value, ε, across households. In a situation in which all cities other than City 1 are identical, $\varepsilon_j$is the value of entering that city for which household j has the highest amenity value. Thus, $G(\cdot)$ satisfies:

$$G(\varepsilon) = [F(a^*)]^{M-1} \tag{A28}$$

where $a^* \in [0, \overline{a}]$ is the amenity value which generates the maximum value ε. Note, however, that for $G(\varepsilon)$ to be well-defined, it is not required for all cities other than City 1 to be identical. Finally, note that the entry cutoff, ε_t^c, in this case satisfies $\varepsilon_t^c = \overline{W}_{1t}$, just as in (8). That is, any household with a maximum alternative value below $\overline{W}_{1t}$ enters City 1.

When a household leaves their city of residence due to the realization of a relocation shock (which happens with probability π for both home-owners and permanent renters), we assume that they are effectively in the same situation as a new household who has just arrived in the economy. That is, they re-draw, *in the current period*, from the amenity distribution for each city, and choose the city which yields the highest value. The expected continuation value following a relocation shock for any household currently resident in *any* city is thus given by

$$Z = \overline{\varepsilon} \equiv \int G(\varepsilon) d\varepsilon. \tag{A29}$$

From (A29) it is clear that Z depends only on the distribution of amenity values, $F(\cdot)$. Also, note that since City 1 is small, the probability that a household which exits it due to a relocation shock returns immediately is negligible.

Let POP_t denote the population of City 1 in period t. The population evolves via

$$POP_{t+1} = POP_t + \underbrace{\mu G(\overline{W}_{1t}) Q_t + \pi \mathbb{Z}_t G(\overline{W}_{1t})}_{\text{new entrants}} - \underbrace{\pi [N_t + F_t]}_{\text{exiters}}, \tag{A30}$$

where $\mathbb{Z}_t$ denotes the measure of agents that exit *all other* cities in period t, and is assumed to be unaffected by conditions in City 1. On the balanced growth path, we assume first that all cities are symmetric, so that $G(\overline{W}_{1t}) = 1/M$ for all t. Similarly, $\mathbb{Z}_t = M(N_t + F_t)$. Thus, from (A30) $POP_{t+1} = (1 + \mu) POP_t$.

Finally, note that it is not important that we model the shock to City 1's income as being relative to the average. Any stable distribution of income across cities will give rise to a well-defined distribution of alternative values for City 1 (although (A28) will no longer apply). A direct increase in City 1 income, $\widetilde{y}_1$, will thus lead to entry for the same reasons as before. Again, the magnitude of the response will be determined by the properties of $G(\cdot)$ in a neighborhold of $\varepsilon = \widetilde{y}_1$ along the balanced growth path.

Suppose now that the economy is subject to aggregate income shocks which affect all cities symmetrically. Because utility is linear, adding a common component, y_{ct}, to city-level income of the form,

$$\widetilde{Y}_{it} = \widetilde{y}_{it} + y_{ct} \qquad i = 1, \ldots, N \tag{A31}$$

will have no effect whatsoever, as it affects neither the ranking of cities by new entrants or relocaters nor the demand for housing. Common shocks to construction costs and/or population growth will affect housing markets within each city, but will have no effect on mobility as they will not change the ranking of cities across prospective entrants as this is determined only by the amenity distribution, $F(\cdot)$.

Appendix E: Details of Robustness Exercises

Exiting Buyers

We assume that unsuccessful buyers exit at the end of the period with the same probability, π_n, as homeowners. This implies that the value of being a searching buyer is now

$$W_t = u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1-\lambda_t)\, \beta \left[\pi_n Z + (1-\pi_n) E_t W_{t+1}\right]. \tag{A32}$$

and the stock of buyers at date t is given by:

$$B_t = \theta(1-\pi_n) N_{t-1} + \psi G(\varepsilon_t^c) \mu Q_{t-1} + (1-\lambda_{t-1})(1-\pi_n) B_{t-1}. \tag{A33}$$

The rest of the model remains unchanged.

Mismatched Owners remain in their houses

Proposition 2. *In equilibrium, mismatched owners are indifferent between the following two arrangements:*

(1) putting up their house for sale or rent immediately and renting while searching;

(2) remaining in their current house while searching, then putting their vacant house up for sale once they are matched with a new one.

Proof: The value of being a mis-matched owner who remains in their house while they search for a new one is given by

$$\tilde{J}_t = y_t + x_t - m + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) + (1-\lambda_t) \beta E_t \tilde{J}_{t+1}. \tag{A34}$$

The value of becoming a renter immediately and putting the vacant house up for sale is given by

$$\begin{aligned} W_t + V_t &= u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1-\lambda_t)\beta E_t W_{t+1} + \gamma_t P_t + (1-\gamma_t)\beta E_t V_{t+1} \\ &= u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1-\lambda_t)\beta E_t W_{t+1} + \gamma_t (1-\chi) \beta E_t \left(J_{t+1} - W_{t+1}\right) \\ &\quad + \gamma_t \chi \beta E_t V_{t+1} + (1-\gamma_t)\beta E_t V_{t+1} \\ &= y_t + x_t - r_t + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) \\ &\quad + \gamma_t (1-\chi))\beta E_t \left(J_{t+1} - W_{t+1} - V_{t+1}\right) + (1-\lambda_t)\beta \left(E_t W_{t+1} + E_t V_{t+1}\right). \end{aligned} \tag{A35}$$

Given (23), the above implies that

$$\begin{aligned} W_t + V_t &= y_t + x_t - m + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) \\ &\quad + (1-\lambda_t)\beta E_t \left[W_{t+1} + V_{t+1}\right]. \end{aligned} \tag{A36}$$

Since $\lim_{T\to\infty} \beta^T E_t \tilde{J}_{t+T} = \lim_{T\to\infty} \beta^T E_t \left[W_{T+1} + V_{T+1}\right] = 0$, solving forwards implies that

$$\tilde{J}_t = W_t + V_t. \tag{A37}$$

QED

Let $\tilde{n}_t$ denote mismatched owners who remain in their (owned) homes. Then the flows of households between states is now described by (30) and

$$\begin{aligned}(1+\mu)\tilde{n}_t &= \theta(1-\pi_n)n_{t-1} + [1-\lambda_{t-1}]\,\tilde{n}_{t-1} &\text{(A38)}\\ (1+\mu)b_t &= \psi\mu G\left(\bar{W}_t\right) + [1-\lambda_{t-1}]\,b_{t-1} &\text{(A39)}\\ (1+\mu)n_t &= (1-\theta)(1-\pi_n)n_{t-1} + \lambda_{t-1}\left(b_{t-1}+\tilde{n}_{t-1}\right) &\text{(A40)}\end{aligned}$$

Market tightness is given by

$$\omega_t = \frac{b_t + \tilde{n}_t}{h_t - b_t - \tilde{n}_t - f_t - n_t} \tag{1}$$

and the housing stock evolves according to

$$(1+\mu)h_{t+1} = h_t + \phi^\eta\left(n_t + \tilde{n}_t + b_t + f_t\right)\left(\beta E_t V_{t+1} - \bar{q}\right)^\eta . \tag{A42}$$

Endogenous Exit

The implied value of being a homeowner is now given by

$$\begin{aligned}J_t &= u_t^H + \beta\pi_n E_t\left(Z_{t+1}^* + e_{t+1}V_{t+1}\right) + \theta\beta E_t(1-\pi_n e_{t+1})\left(W_{t+1}+V_{t+1}\right)\\ &\quad +\beta(1-\theta)E_t(1-\pi_n e_{t+1})J_{t+1}, &\text{(A43)}\end{aligned}$$

where $Z_t^* = \left(\bar{Z}^2 - J_t^2\right)/2\bar{Z}$ and $e_t = 1 - J_t/\bar{Z}$ is the exit probability conditional on having an opportunity.

Allocation of Land Developers' Profits

The profits of land developers is given by

$$\Pi_t = A_t\int_0^{q_t}(q_t - c)\Lambda(dc).$$

Given the functional form for the cost distribution given in (38), this can be expressed as

$$\Pi_t = \frac{q_t\Lambda(q_t)A_t}{1+\tau}.$$

We therefore add $\Pi_t/(N_t + B_t + F_t)$ to the income of each household. Note that, since preferences are linear, it does not actually matter how these profits are distributed across households.

Competitive Search

By entering sub-market (ω_t, P_t), a seller sells a house at P_t with probability $\gamma(\omega_t)$. The seller chooses to enter a sub-market that maximizes his/her expected return. It follows that the value of a vacant house for sale satisfies

$$V_t = \max_{(\omega_t, P_t)} \left\{ \gamma(\omega_t) P_t + [1 - \gamma(\omega_t)] \beta E_t \tilde{V}_{t+1} \right\}. \tag{A44}$$

Free entry of sellers implies that all active sub-markets (*i.e.* sub-markets with $\lambda, \gamma \in (0,1)$) in equilibrium must offer the sellers the same payoff V_t, although (ω_t, P_t) varies across sub-markets. It follows that the relationship between the listed price and market tightness that must be satisfied by all active sub-markets:

$$\gamma(\omega_t(P_t)) = \frac{V_t - \beta E_t \tilde{V}_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}}. \tag{A45}$$

Thus, it is sufficient to index sub-markets by the posted price P_t alone.

Buyers also decide in each period which sub-market to enter. The value of being a buyer W_t is therefore given by

$$W_t = u_t^R + \max_{P_t} \left\{ \lambda(\omega_t(P_t)) (\beta E_t J_{t+1} - P_t) + [1 - \lambda(\omega_t(P_t))] \beta E_t W_{t+1} \right\}. \tag{A46}$$

In equilibrium, the set of active sub-markets is *complete* in the sense that there is no other sub-market that could improve the welfare of any buyer or seller.

Let $\epsilon(\omega_t)$ denote the elasticity of the measure of matches with respect to the measure of buyers. We then have the following proposition:

Proposition 3. *In a competitive search equilibrium, there is only one active sub-market. In this market, the share of the surplus from house transactions that accrues to the buyer is equal to the elasticity of the measure of matches with respect to the measure of buyers:*[2]

$$\chi(\omega_t) = \epsilon(\omega_t). \tag{A47}$$

Proof: The first-order condition to the optimization problem in (A46) yields

$$\lambda'(\omega_t)\omega_t'(P_t)(\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}) - \lambda(\omega_t(P_t)) = 0, \tag{A48}$$

where $\omega_t(P_t)$ and $\omega_t'(P_t)$ are implicitly determined by (A45). This implies

$$\frac{\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}} = -\frac{\lambda(\omega_t(P_t))/\lambda'(\omega_t(P_t))}{\gamma(\omega_t(P_t))/\gamma'(\omega_t(P_t))}, \tag{A49}$$

[2]This result is a special case of that derived in Moen (1997).

which can be used together with (A45) to solve for P_t, given the values $E_t J_{t+1}$, $E_t W_{t+1}$ and $E_t \tilde{V}_{t+1}$. Then one can solve for ω_t from (A45). Note that (A45) implies that $\omega_t'(P_t) < 0$ given $\gamma'(\omega) > 0$ from Assumption 1.

The trade surplus in the housing market is strictly positive. Given the boundary condition $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$, it is clear that the household's equilibrium values are bounded, which implies that the trade surplus is also bounded. Thus $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} \in (0, \infty)$, where we have incorporated that $V = \tilde{V}$ in the equilibrium. Recall condition (ix) of the equilibrium definition that $\gamma(\omega_t),\ \lambda(\omega_t) \in (0,1)$ for all active sub-markets. Also recall from part (ii) of Assumption 1 that $\lambda'(\omega) < 0,\ \gamma'(\omega) > 0$. These conditions imply that $\epsilon(\omega) \in (0,1)$ where

$$\epsilon(\omega_t) = \frac{B_t}{M} \cdot \frac{\partial M}{\partial B_t} = \frac{1}{1 - \frac{\gamma(\omega_t)/\gamma'(\omega_t)}{\lambda(\omega_t)/\lambda'(\omega_t)}}. \tag{A50}$$

Define $LHS(P_t)$ as the left-hand side of (A49) and $RHS(P_t)$ the right-hand side. Given (A50), it is clear that

$$RHS(P_t) = \frac{\epsilon(\omega_t(P_t))}{1 - \epsilon(\omega_t(P_t))}. \tag{A51}$$

Because $\epsilon(\omega) \in (0,1)$, we have $RHS(P_t) \in (0,\infty)$ for all P_t. Moreover, recall $\omega_t'(P_t) < 0$ from (A45) and $\epsilon'(\omega) \le 0$ from Assumption 1. Thus $RHS'(P_t) \ge 0$.

For any given V_t, J_t, W_t, one can verify that $LHS'(P_t) < 0$ because $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} > 0$. Recall from (20) and (21) that the price in an active sub-market satisfies

$$\beta E_t V_{t+1} \le P_t \le \beta E_t J_{t+1} - \beta E_t W_{t+1}. \tag{A52}$$

It follows that

$$LHS(P_t = \beta E_t V_{t+1}) = \infty > RHS(P_t = \beta E_t V_{t+1}) \tag{A53}$$

$$LHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}) = 0 < RHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}), \tag{A54}$$

where the two inequalities are because $RHS(P_t) \in (0,\infty)$ for all P_t. The above results imply a unique $P_t^* \in (\beta E_t V_{t+1},\ \beta E_t J_{t+1} - \beta E_t W_{t+1})$ that satisfies

$$\frac{\beta E_t J_{t+1} - P_t^* - \beta E_t W_{t+1}}{P_t^* - \beta E_t V_{t+1}} = -\frac{\lambda(\omega_t^*(P_t^*))/\lambda'(\omega_t^*(P_t^*))}{\gamma(\omega_t^*(P_t^*))/\gamma'(\omega_t^*(P_t^*))}, \tag{A55}$$

and a unique $\omega_t^*(P_t^*)$ that satisfies

$$\omega_t^*(P_t^*) = \gamma^{-1}\left(\frac{V_t - \beta E_t V_{t+1}}{P_t^* - \beta E_t V_{t+1}}\right). \tag{A56}$$

Thus, there is a single active sub-market in the directed search equilibrium.

Equation (A55) may be written as

$$\frac{\chi(\omega)}{1 - \chi(\omega)} = \frac{\epsilon(\omega)}{1 - \epsilon(\omega)}, \tag{A57}$$

where $\chi(\omega)$ denotes the buyer's share of the surplus in a sub-market with tightness ω. The right-hand side of the above is the ratio of the elasticities of the number of matches with respect to the numbers of buyers and sellers. It follows that $\chi(\omega) = \epsilon(\omega)$. **QED**

The share of the surplus accruing to the buyer for the urn ball matching function is given by

$$\chi(\omega) = \epsilon(\omega) = \frac{\varrho\omega}{e^{\varrho\omega} - 1}, \tag{A58}$$

which is decreasing in market tightness ω.