The economics of predation: What drives pricing when there is learning-by-doing?
—Online Appendix—

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A1 Omitted expressions

Below we provide several expressions that are omitted from the main text as well as further details on the concept of inclusive price.

A1.1 Firms' decisions: Conditional expectation of scrap value and setup cost

Exit decision of incumbent firm. Given the assumed distribution for scrap values, the probability of incumbent firm 1 exiting the industry in state $\mathbf{e}'$ is

$$\begin{aligned}
\phi_1(\mathbf{e}') &= E_X\left[\phi_1(\mathbf{e}', X_1)\right] \\
&= \int \phi_1(\mathbf{e}', X_1) dF_X(X_1) = 1 - F_X(\widehat{X}_1(\mathbf{e}')) \\
&= \begin{cases} 1 & \text{if} \quad \widehat{X}_1(\mathbf{e}') < \overline{X} - \Delta_X, \\ \frac{1}{2} - \frac{\left[\widehat{X}_1(\mathbf{e}') - \overline{X}\right]}{2\Delta_X} & \text{if} \quad \widehat{X}_1(\mathbf{e}') \in [\overline{X} - \Delta_X, \overline{X} + \Delta_X], \\ 0 & \text{if} \quad \widehat{X}_1(\mathbf{e}') > \overline{X} + \Delta_X \end{cases}
\end{aligned}$$

and the expectation of the scrap value conditional on exiting the industry is

$$\begin{aligned}
E_X\left[X_1 | X_1 \geq \widehat{X}_1(\mathbf{e}')\right] &= \frac{\int_{F_X^{-1}(1-\phi_1(\mathbf{e}'))}^{\overline{X}+\Delta_X} X_1 dF_X(X_1)}{\phi_1(\mathbf{e}')} \\
&= \frac{1}{\phi_1(\mathbf{e}')}\left[Z_X(0) - Z_X\left(1 - \phi_1(\mathbf{e}')\right)\right],
\end{aligned}$$

where

$$Z_X(1-\phi) = \frac{1}{\Delta_X^2}\begin{cases} -\frac{1}{6}\left(\overline{X} - \Delta_X\right)^3 & \text{if} \quad 1-\phi \leq 0, \\ \frac{1}{2}\left(\Delta_X - \overline{X}\right)\left(F_X^{-1}(1-\phi)\right)^2 + \frac{1}{3}\left(F_X^{-1}(1-\phi)\right)^3 & \text{if} \quad 1-\phi \in \left[0, \frac{1}{2}\right], \\ \frac{1}{2}\left(\Delta_X + \overline{X}\right)\left(F_X^{-1}(1-\phi)\right)^2 - \frac{1}{3}\left(F_X^{-1}(1-\phi)\right)^3 - \frac{1}{3}\overline{X}^3 & \text{if} \quad 1-\phi \in \left[\frac{1}{2}, 1\right], \\ \frac{1}{6}\left(\overline{X} + \Delta_X\right)^3 - \frac{1}{3}\overline{X}^3 & \text{if} \quad 1-\phi \geq 1 \end{cases}$$

and

$$F_X^{-1}(1-\phi) = \overline{X} + \Delta_X\begin{cases} -1 & \text{if} \quad 1-\phi \leq 0, \\ -1 + \sqrt{2(1-\phi)} & \text{if} \quad 1-\phi \in \left[0, \frac{1}{2}\right], \\ 1 - \sqrt{2\phi} & \text{if} \quad 1-\phi \in \left[\frac{1}{2}, 1\right], \\ 1 & \text{if} \quad 1-\phi \geq 1. \end{cases}$$

Entry decision of potential entrant. Given the assumed distribution for setup costs, the probability of potential entrant 1 not entering the industry in state $\mathbf{e}'$ is

$$\begin{aligned}
\phi_1(\mathbf{e}') &= E_S\left[\phi_1(\mathbf{e}', S_1)\right] \\
&= \int \phi_1(\mathbf{e}', S_1) dF_S(S_1) = 1 - F_S(\widehat{S}_1(\mathbf{e}')) \\
&= \begin{cases} 1 & \text{if} \quad \widehat{S}_1(\mathbf{e}') < \overline{S} - \Delta_S, \\ \frac{1}{2} - \frac{[\widehat{S}_1(\mathbf{e}') - \overline{S}]}{2\Delta_S} & \text{if} \quad \widehat{S}_1(\mathbf{e}') \in [\overline{S} - \Delta_S, \overline{S} + \Delta_S], \\ 0 & \text{if} \quad \widehat{S}_1(\mathbf{e}') > \overline{S} + \Delta_S \end{cases}
\end{aligned}$$

and the expectation of the setup cost conditional on entering the industry is

$$\begin{aligned}
E_S\left[S_1 | S_1 \leq \widehat{S}_1(\mathbf{e}')\right] &= \frac{\int_{\overline{S}-\Delta_S}^{F_S^{-1}(1-\phi_1(\mathbf{e}'))} S_1 dF_S(S_1)}{(1 - \phi_1(\mathbf{e}'))} \\
&= \frac{1}{\phi_1(\mathbf{e}')} \left[Z_S\left(1 - \phi_1(\mathbf{e}')\right) - Z_S\left(1\right)\right],
\end{aligned}$$

where

$$Z_S\left(1-\phi\right) = \frac{1}{\Delta_S^2} \begin{cases} -\frac{1}{6}\left(\overline{S} - \Delta_S\right)^3 & \text{if} \quad 1-\phi \leq 0, \\ \frac{1}{2}\left(\Delta_S - \overline{S}\right)\left(F_S^{-1}(1-\phi)\right)^2 + \frac{1}{3}\left(F_S^{-1}(1-\phi)\right)^3 & \text{if} \quad 1-\phi \in \left[0, \frac{1}{2}\right], \\ \frac{1}{2}\left(\Delta_S + \overline{S}\right)\left(F_S^{-1}(1-\phi)\right)^2 - \frac{1}{3}\left(F_S^{-1}(1-\phi)\right)^3 - \frac{1}{3}\overline{S}^3 & \text{if} \quad 1-\phi \in \left[\frac{1}{2}, 1\right], \\ \frac{1}{6}\left(\overline{S} + \Delta_S\right)^3 - \frac{1}{3}\overline{S}^3 & \text{if} \quad 1-\phi \geq 1 \end{cases}$$

and

$$F_S^{-1}(1-\phi) = \overline{S} + \Delta_S \begin{cases} -1 & \text{if} \quad 1-\phi \leq 0, \\ -1 + \sqrt{2\left(1-\phi\right)} & \text{if} \quad 1-\phi \in \left[0, \frac{1}{2}\right], \\ 1 - \sqrt{2\phi} & \text{if} \quad 1-\phi \in \left[\frac{1}{2}, 1\right], \\ 1 & \text{if} \quad 1-\phi \geq 1. \end{cases}$$

A1.2 Learning-by-doing: Marginal revenue and inclusive price

$mr_1(p_1, p_2(\mathbf{e})) = p_1 - \frac{\sigma}{1 - D_1(p_1, p_2(\mathbf{e}))}$ is the marginal revenue of incumbent firm 1 with respect to quantity and therefore analogous to the traditional textbook concept. To see this, let $q_1 = D_1(p_1, p_2(\mathbf{e}))$ be demand and $p_1 = P_1(q_1, p_2(\mathbf{e}))$ inverse demand as implicitly defined by $q_1 = D_1(P_1(q_1, p_2(\mathbf{e})), p_2(\mathbf{e}))$. The marginal revenue of incumbent firm 1 is

$$MR_1(q_1, p_2(\mathbf{e})) = \frac{\partial\left[q_1 P_1(q_1, p_2(\mathbf{e}))\right]}{\partial q_1} = q_1 \frac{\partial P_1(q_1, p_2(\mathbf{e}))}{\partial q_1} + P_1(q_1, p_2(\mathbf{e})). \tag{A1}$$

Define $mr_1(p_1, p_2(\mathbf{e})) = MR_1(D_1(p_1, p_2(\mathbf{e})), p_2(\mathbf{e}))$ to be the marginal revenue of incumbent firm 1 evaluated at the quantity $q_1 = D_1(p_1, p_2(\mathbf{e}))$ corresponding to prices p_1 and $p_2(\mathbf{e})$. Then we have

$$\frac{\partial P_1(D_1(p_1, p_2(\mathbf{e})), p_2(\mathbf{e}))}{\partial q_1} = \left[\frac{\partial D_1(p_1, p_2(\mathbf{e}))}{\partial p_1}\right]^{-1} = -\frac{\sigma}{\left[1 - D_1(p_1, p_2(\mathbf{e}))\right] D_1(p_1, p_2(\mathbf{e}))}. \tag{A2}$$

Substituting equation (A2) into equation (A1), it follows that $mr_1(p_1, p_2(\mathbf{e})) = p_1 - \frac{\sigma}{1 - D_1(p_1, p_2(\mathbf{e}))}$.

A1.3 Industry structure, conduct, and performance: Consumer and producer surplus

Consumer surplus in state $\mathbf{e}$ is

$$\begin{aligned} CS(\mathbf{e}) &= \sigma \log\left\{\exp\left(\frac{v - p_0}{\sigma}\right) + \sum\nolimits_{n=1}^{2} \exp\left(\frac{v - p_n(\mathbf{e})}{\sigma}\right)\right\} \\ &= v + \sigma \log\left\{\exp\left(\frac{-p_0}{\sigma}\right) + \sum\nolimits_{n=1}^{2} \exp\left(\frac{-p_n(\mathbf{e})}{\sigma}\right)\right\}. \end{aligned}$$

The producer surplus of firm 1 in state $\mathbf{e}$ is

$$\begin{aligned} PS_1(\mathbf{e}) = 1\left[e_1 > 0\right] \Big\{ & D_0(p_1(\mathbf{e}), p_2(\mathbf{e}))\phi_1(\mathbf{e}) E_X\left[X_1 | X_1 \geq \widehat{X}_1(\mathbf{e})\right] \\ & + D_1(p_1(\mathbf{e}), p_2(\mathbf{e}))\left\{p_1(\mathbf{e}) - c(e_1) + \phi_1(e_1+1, e_2) E_X\left[X_1 | X_1 \geq \widehat{X}_1(e_1+1, e_2)\right]\right\} \\ & + D_2(p_1(\mathbf{e}), p_2(\mathbf{e}))\phi_1(e_1, e_2+1) E_X\left[X_1 | X_1 \geq \widehat{X}_1(e_1, e_2+1)\right] \Big\} \\ -1\left[e_1 = 0\right] \Big\{ & D_0(p_1(\mathbf{e}), p_2(\mathbf{e}))\left(1 - \phi_1(\mathbf{e})\right) E_S\left[S_1 | S_1 \leq \widehat{S}_1(\mathbf{e})\right] \\ & + D_1(p_1(\mathbf{e}), p_2(\mathbf{e}))\left(1 - \phi_1(e_1+1, e_2)\right) E_S\left[S_1 | S_1 \leq \widehat{S}_1(e_1+1, e_2)\right] \\ & + D_2(p_1(\mathbf{e}), p_2(\mathbf{e}))\left(1 - \phi_1(e_1, e_2+1)\right) E_S\left[S_1 | S_1 \leq \widehat{S}_1(e_1, e_2+1)\right] \Big\}. \end{aligned}$$

The first set of terms represents the contingency that firm 1 is an incumbent that participates in the product market and receives a scrap value upon exit; the second set the contingency that firm 1 is an entrant that incurs a setup cost upon entry.

A2 Additional figures and tables

Below we provide additional figures and tables to supplement those in the main text.

A2.1 Equilibrium correspondence: Metrics of industry conduct and performance

Figures A1–A5 complement Figure 3 in the main text. They illustrate the equilibrium correspondence by plotting $\overline{p}^{\infty}$, CS^{∞}, TS^{∞}, CS^{NPV}, and TS^{NPV} against ρ, σ, and $\overline{X}$, respectively.

A2.2 Equilibrium correspondence: Aggressive equilibria with little learning-by-doing

Figure A6 supplements footnote 21 in the main text. It illustrates that aggressive equilibria can arise for $\rho = 0.99$ and $\sigma = 0.10$ and $\rho = 0.98$ and $\sigma = 0.30$ where there is practically no learning-by-doing.

A2.3 Equilibrium correspondence: Multiple equilibria

Figure A7 shows the number of equilibria that we have identified for combinations of ρ and σ, ρ and $\overline{X}$, and σ and $\overline{X}$, respectively. Darker shades indicate more equilibria.

We have found 152 equilibria for $\rho = 0.45$ and $\sigma = 0.9$. In Figures A8–A10 we present some of them to further illustrate the differences between equilibria alluded to in footnote 25 in the main text. Figure A8 presents equilibria #35 and #36 that are fairly similar to each other;[1] the differences between values and policies in any state are less than 9%.

Figure A9 presents equilibria #35 and #5 that are much less similar to each other; the differences between values and policies in some state are 114%. These equilibria differ in the location of the trench: The pricing decision has a single trench along the e_1-axis at $e_2 = 5$ in equilibrium #35 and at $e_2 = 2$ in equilibrium #5. Due to the delayed onset of predation-like behavior, the industry is much more likely to evolve into a mature duopoly in equilibrium #35 than in equilibrium #5.

Figure A10 presents equilibria #14 and #15. In these equilibria the pricing decision has double trenches along the e_1-axis at $e_2 = 2$ and $e_2 = 5$. While the location of the trenches is the same, these equilibria differ in the depth of the trenches.

A2.4 Counterfactual and equilibrium correspondences: Multiple counterfactuals

Figures A11, A12, and A13 show the number of counterfactuals for Definitions 1, 2, and 3 that we have identified for combinations of ρ and σ, ρ and $\overline{X}$, and σ and $\overline{X}$, respectively. Darker shades indicate more counterfactuals. As can be seen from comparing Figures A11,

[1] Equilibria are numbered arbitrarily.

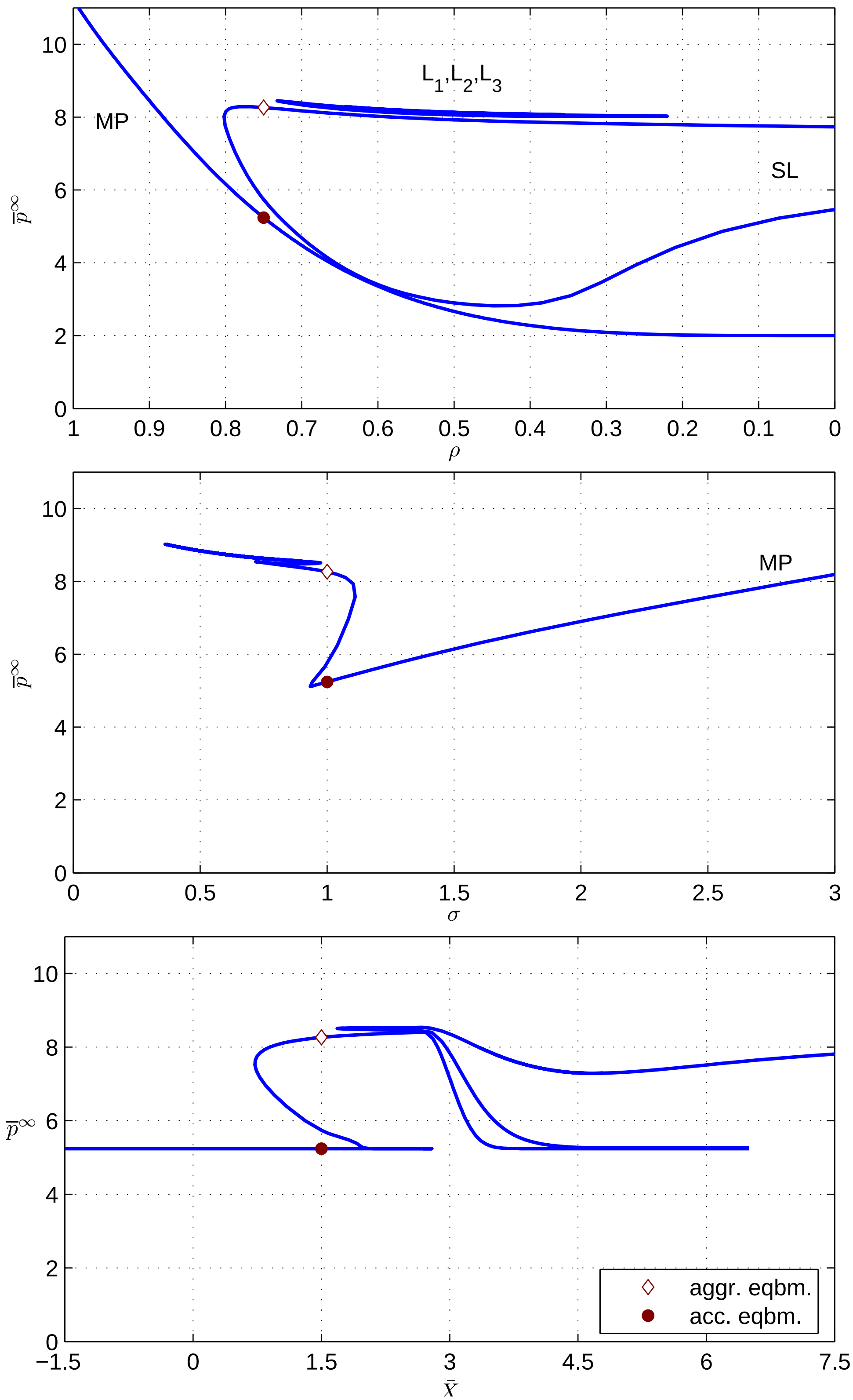

Figure A1: Expected long-run average price. Equilibrium correspondence: slice along $\rho \in [0,1]$ (upper panel), $\sigma \in [0,3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

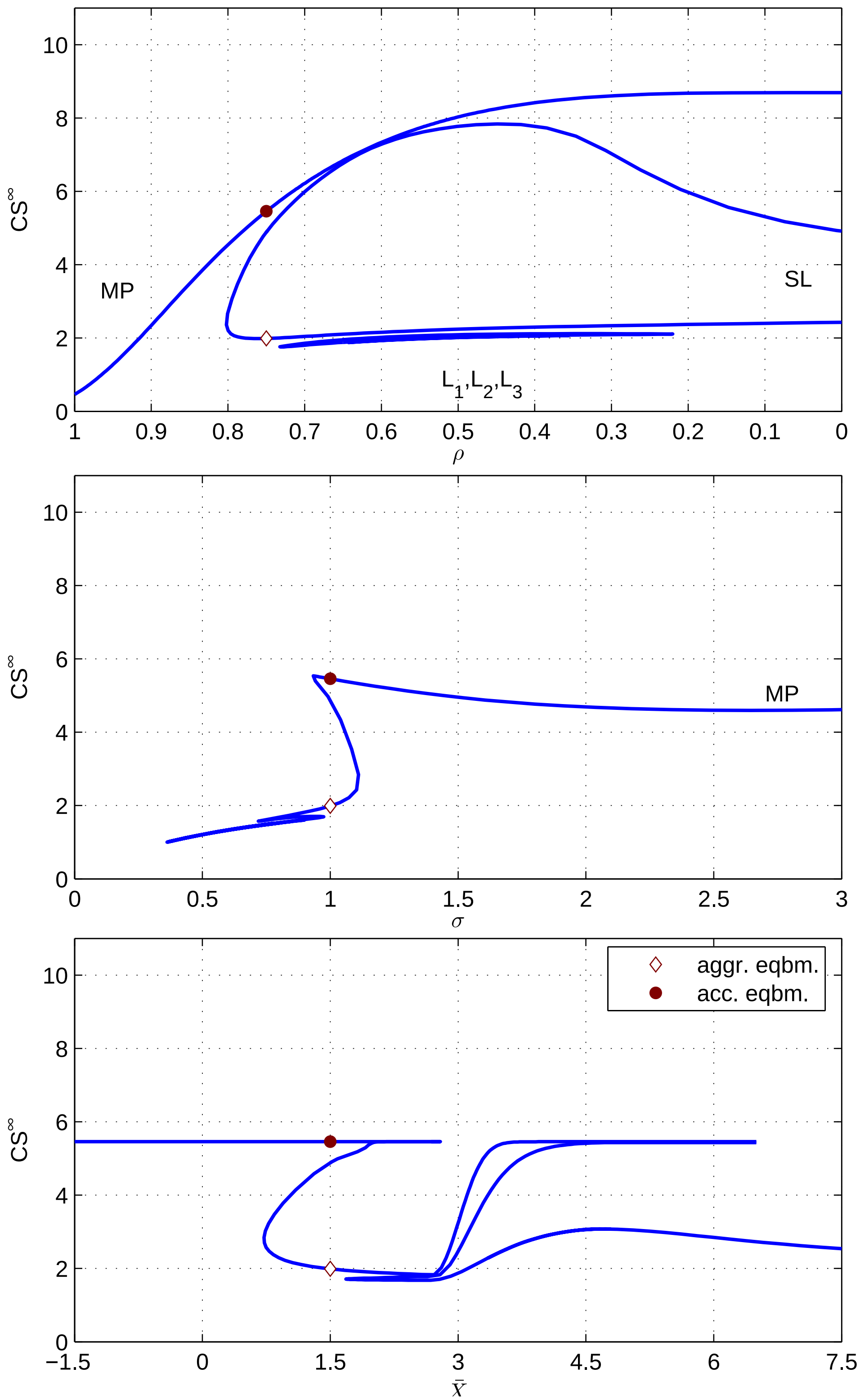

Figure A2: Expected long-run consumer surplus. Equilibrium correspondence: slice along $\rho \in [0, 1]$ (upper panel), $\sigma \in [0, 3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

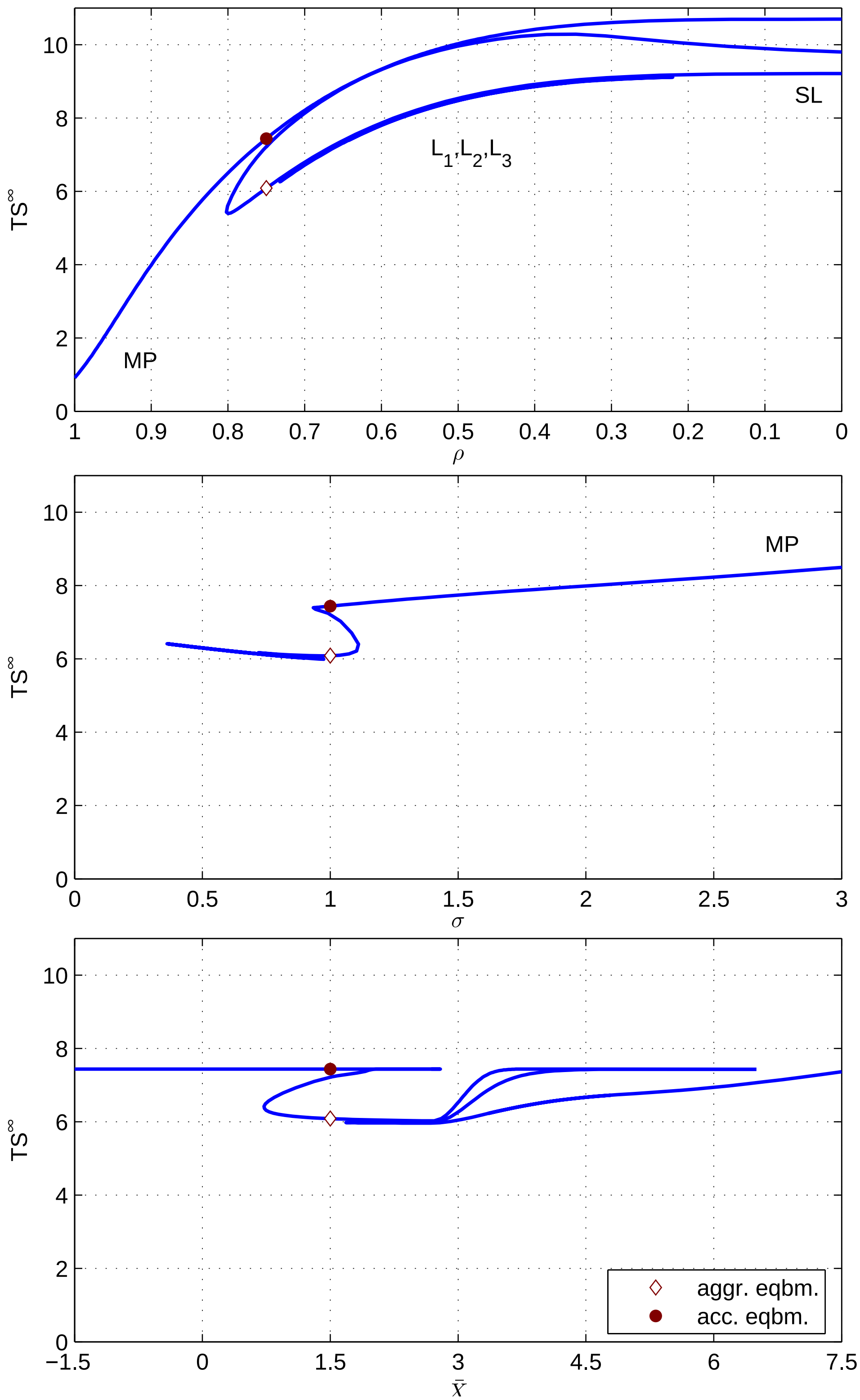

Figure A3: Expected long-run total surplus. Equilibrium correspondence: slice along $\rho \in [0, 1]$ (upper panel), $\sigma \in [0, 3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

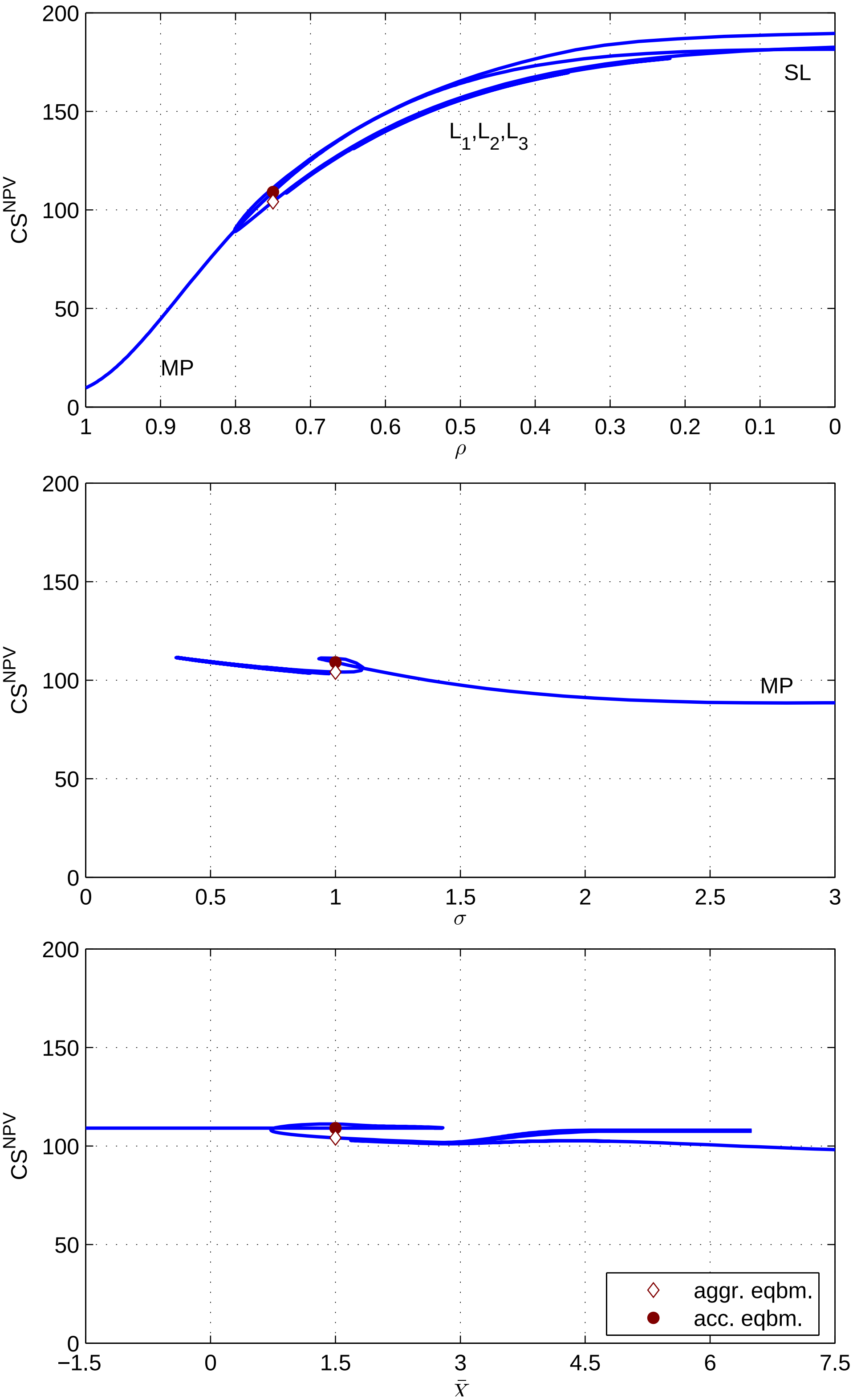

Figure A4: Expected discounted consumer surplus. Equilibrium correspondence: slice along $\rho \in [0,1]$ (upper panel), $\sigma \in [0,3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

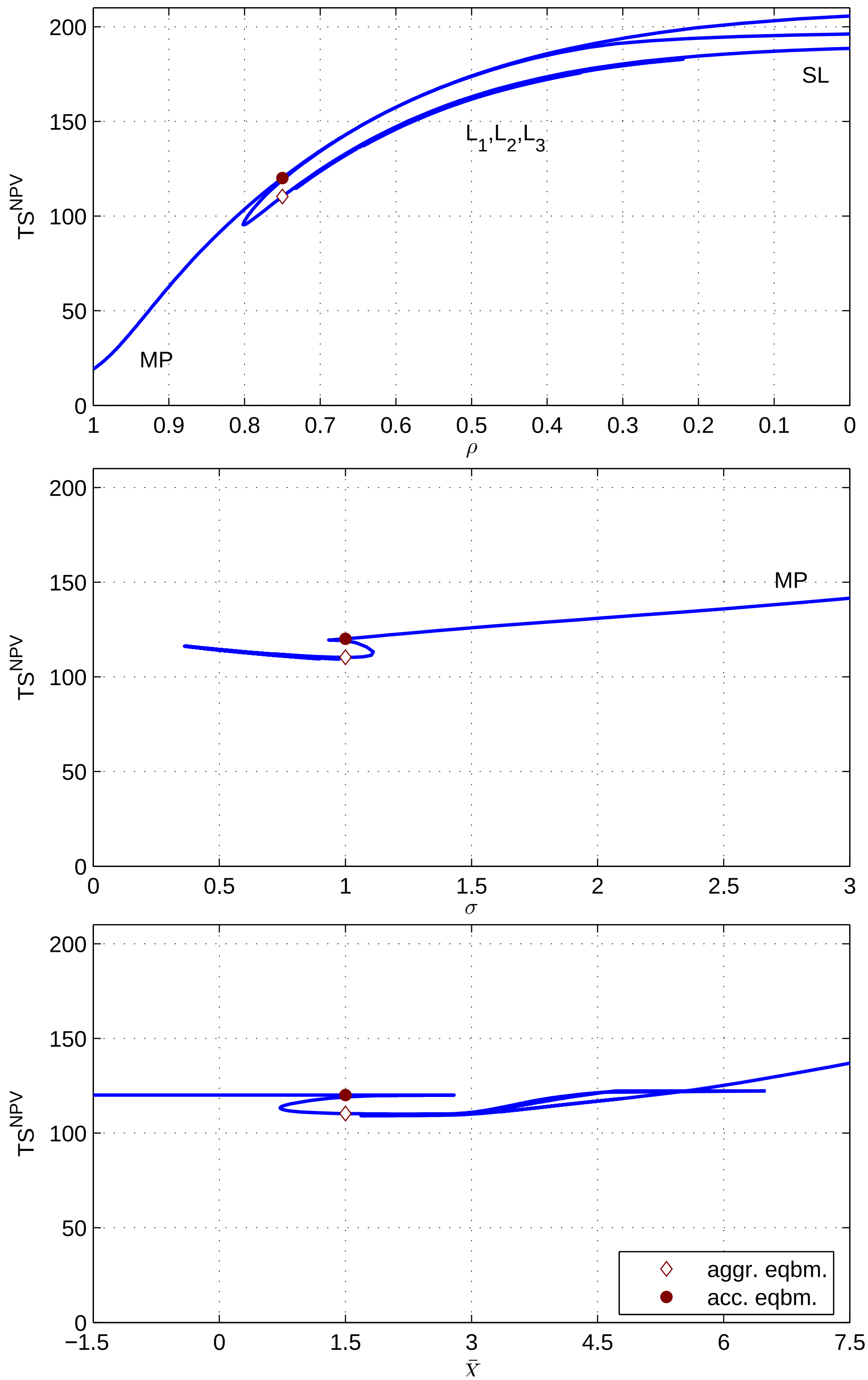

Figure A5: Expected discounted total surplus. Equilibrium correspondence: slice along $\rho \in [0, 1]$ (upper panel), $\sigma \in [0, 3]$ (middle panel), and $\overline{X} \in [-1.5, 7.5]$ (lower panel).

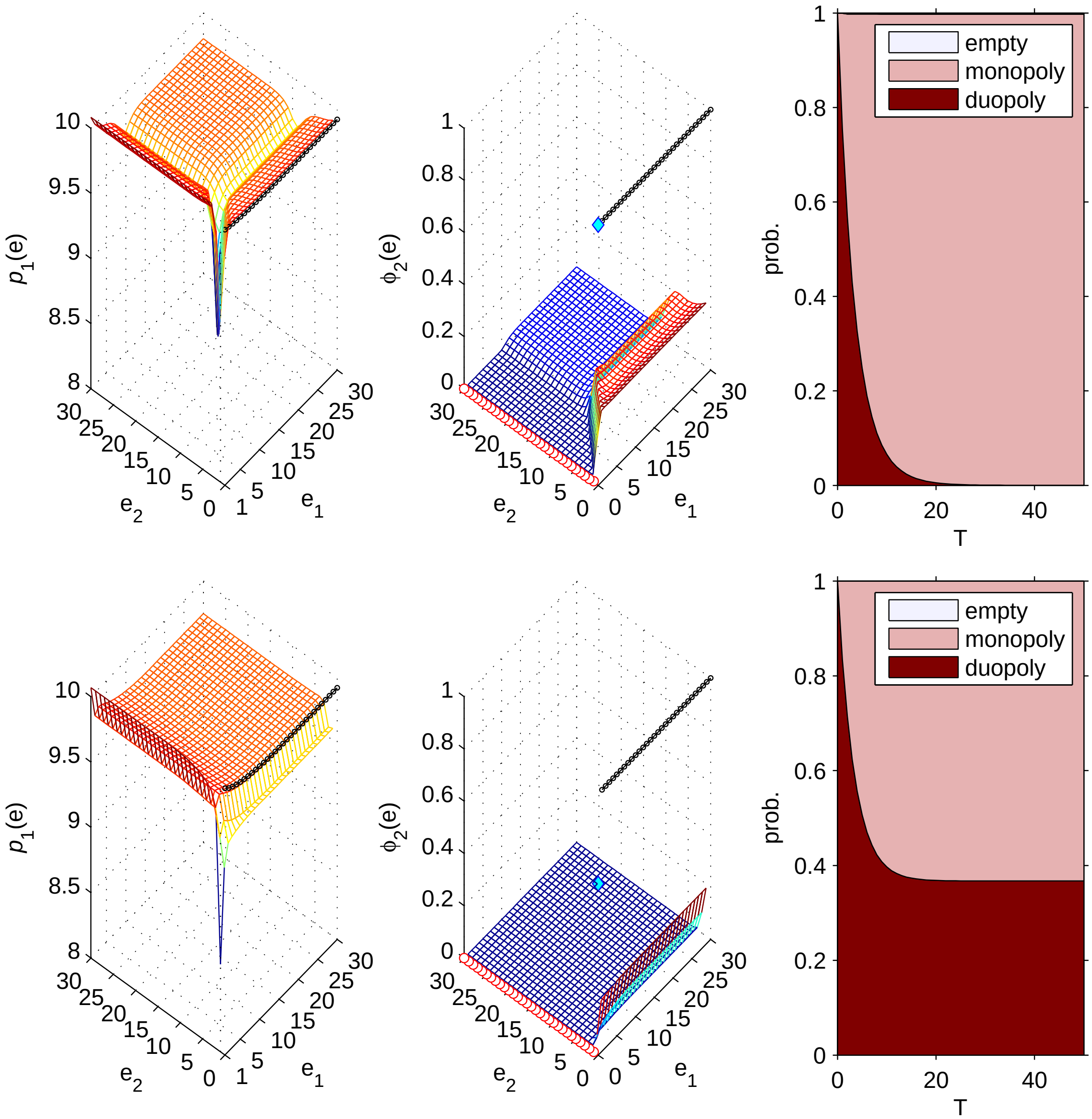

Figure A6: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $\mathbf{e} = (1, 1)$ at $T = 0$ (right panels). Aggressive equilibrium for $\rho = 0.99$ and $\sigma = 0.10$ (upper panels) and $\rho = 0.98$ and $\sigma = 0.30$ (lower panels).

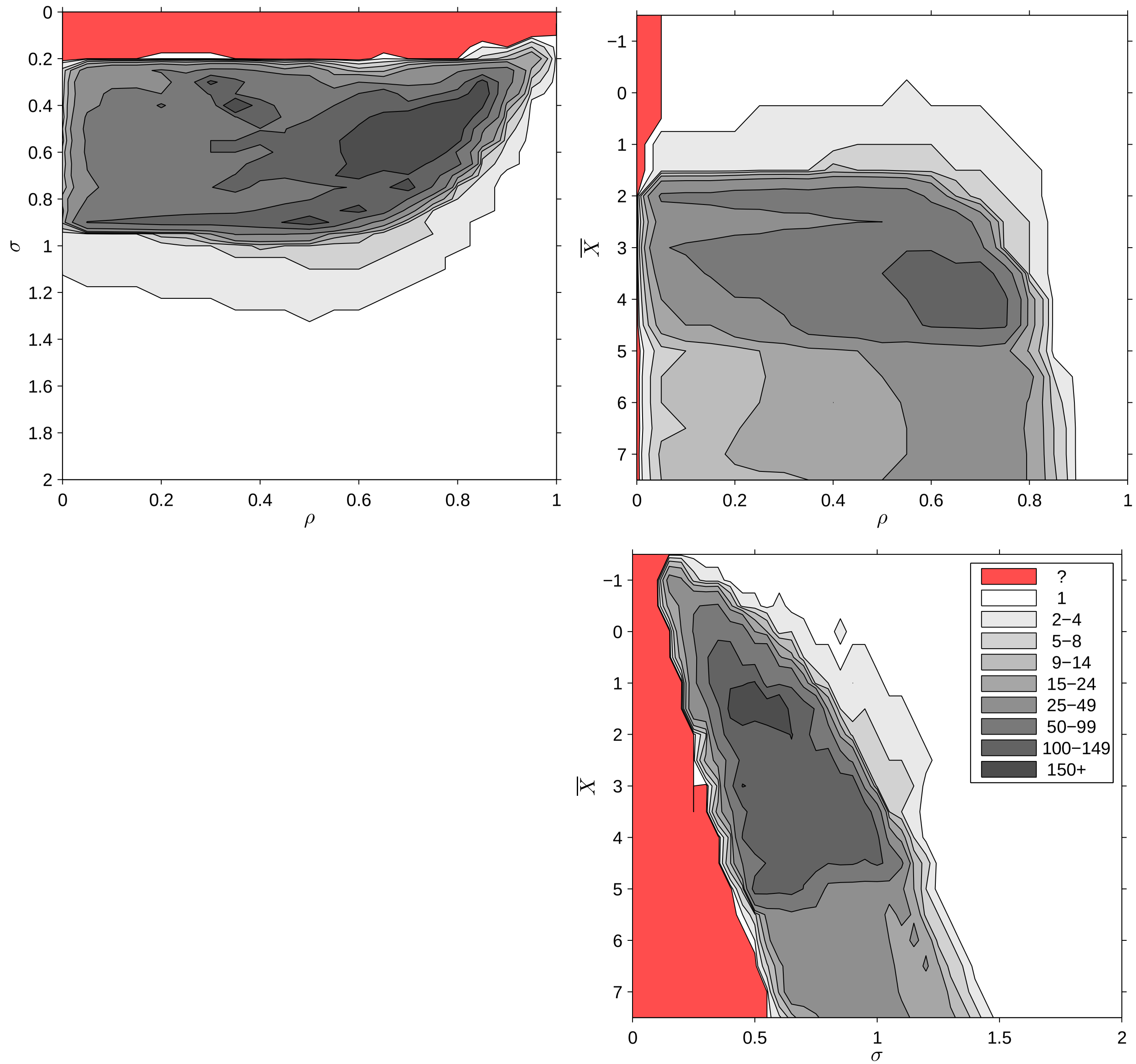

Figure A7: Number of equilibria. Equilibrium correspondence: slice along $(\rho, \sigma) \in [0, 1] \times [0, 3]$ (truncated at $\sigma = 2$, upper left panel), $(\rho, \overline{X}) \in [0, 1] \times [-1.5, 7.5]$ (upper right panel), and $(\sigma, \overline{X}) \in [0, 3] \times [-1.5, 7.5]$ (truncated at $\sigma = 2$, lower right panel). ? indicates that the homotopy algorithm crashed.

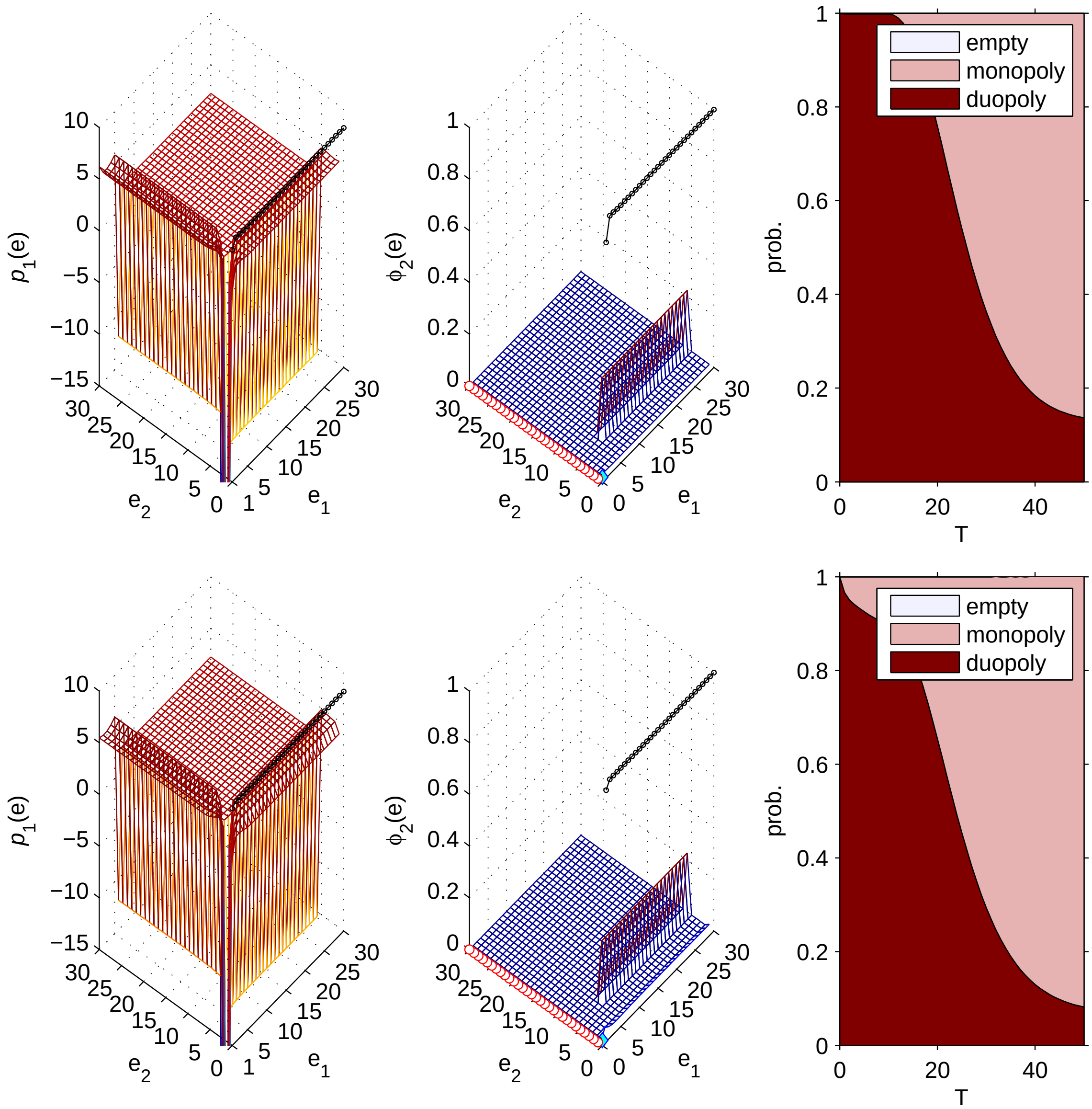

Figure A8: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $\mathbf{e} = (1, 1)$ at $T = 0$ (right panels). Equilibrium #35 (upper panels) and equilibrium #36 (lower panels) for $\rho = 0.45$ and $\sigma = 0.90$.

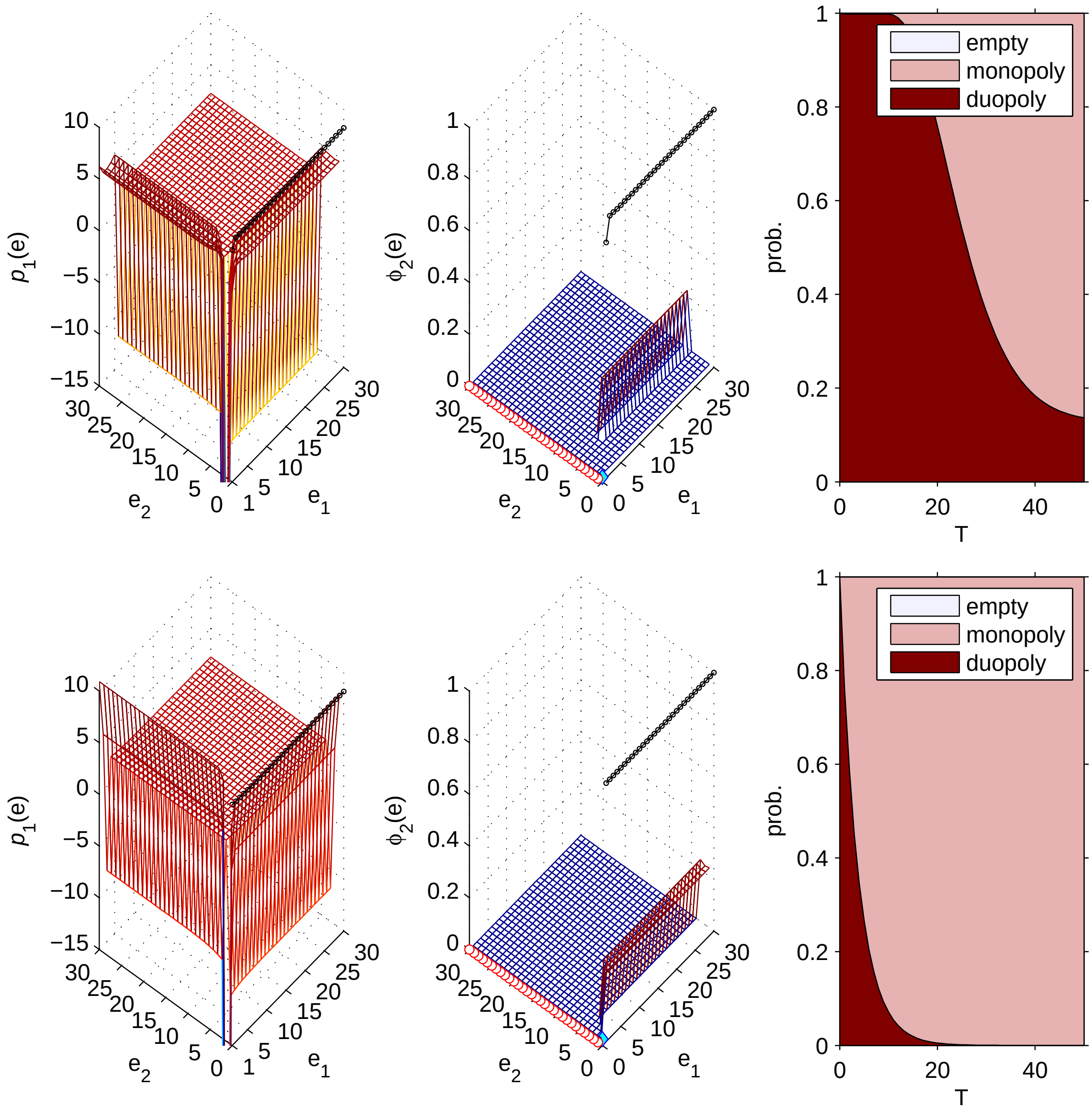

Figure A9: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $\mathbf{e} = (1, 1)$ at $T = 0$ (right panels). Equilibrium #35 (upper panels) and equilibrium #5 (lower panels) for $\rho = 0.45$ and $\sigma = 0.90$.

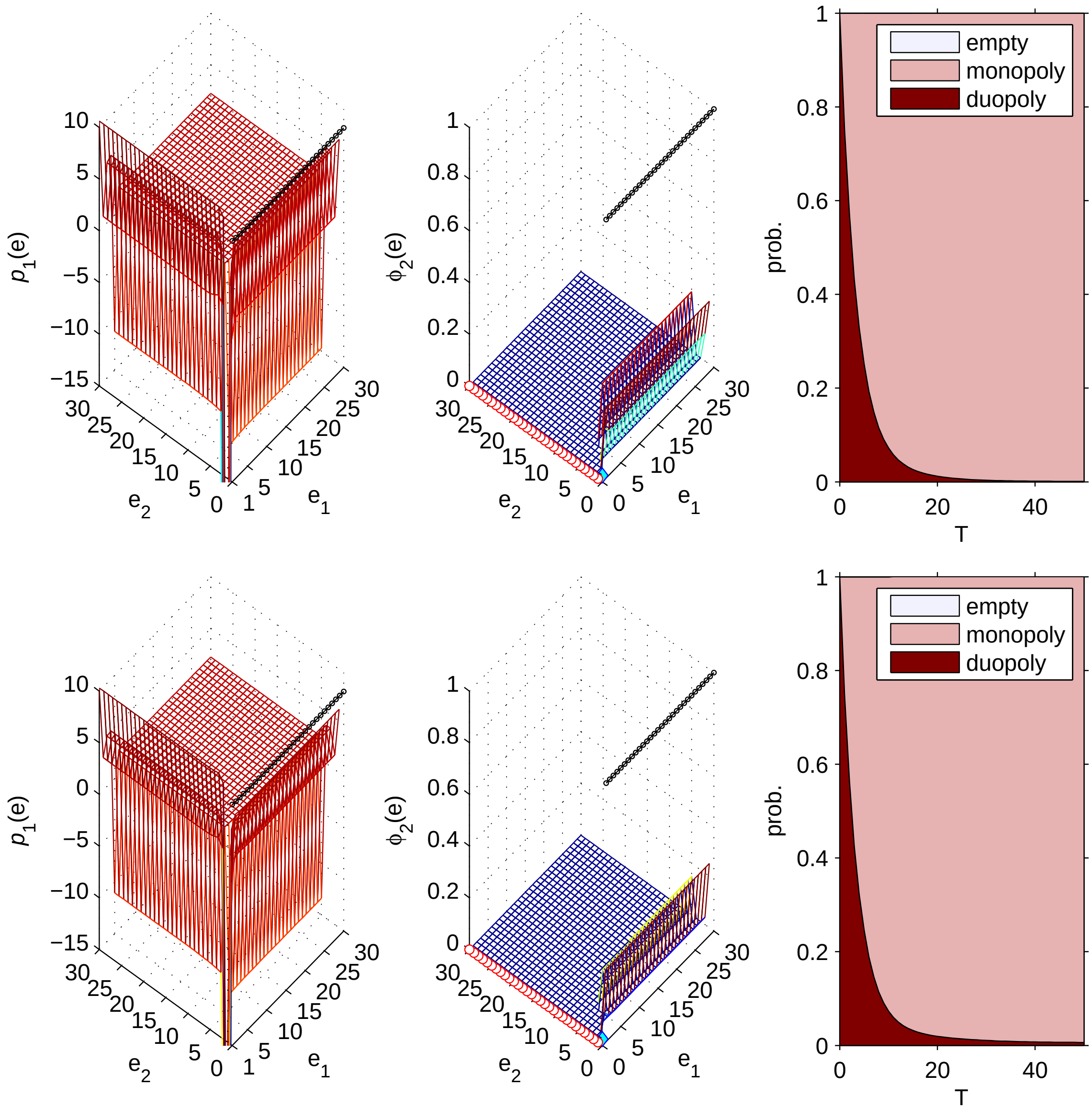

Figure A10: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $\mathbf{e} = (1, 1)$ at $T = 0$ (right panels). Equilibrium #14 (upper panels) and equilibrium #15 (lower panels) for $\rho = 0.45$ and $\sigma = 0.90$.

A12, and A13 to Figure A7, there tend to be less counterfactuals than equilibria for a given parameterization.

A2.5 Counterfactual and equilibrium correspondences and eliminated and surviving equilibria: Product differentiation and scrap value

Figures A14 and A15 complement Figure 4 in the main text. They illustrate the counterfactual correspondence for Definitions 1–3 by plotting HHI^{∞} against σ and $\overline{X}$, respectively. They superimpose the equilibrium correspondences $\mathbf{H}^{-1}(\sigma)$ and $\mathbf{H}^{-1}(\overline{X})$ from Figure 3 and distinguish between surviving and eliminated equilibria.

As in the main text, the counterfactual correspondences for Definition 3 resemble the equilibrium correspondence much more closely than those for Definitions 1 and 2. Furthermore, the stronger Definitions 1 and 2 eliminate many more equilibria that are associated with high expected long-run Herfindahl indices than the weaker Definition 3.

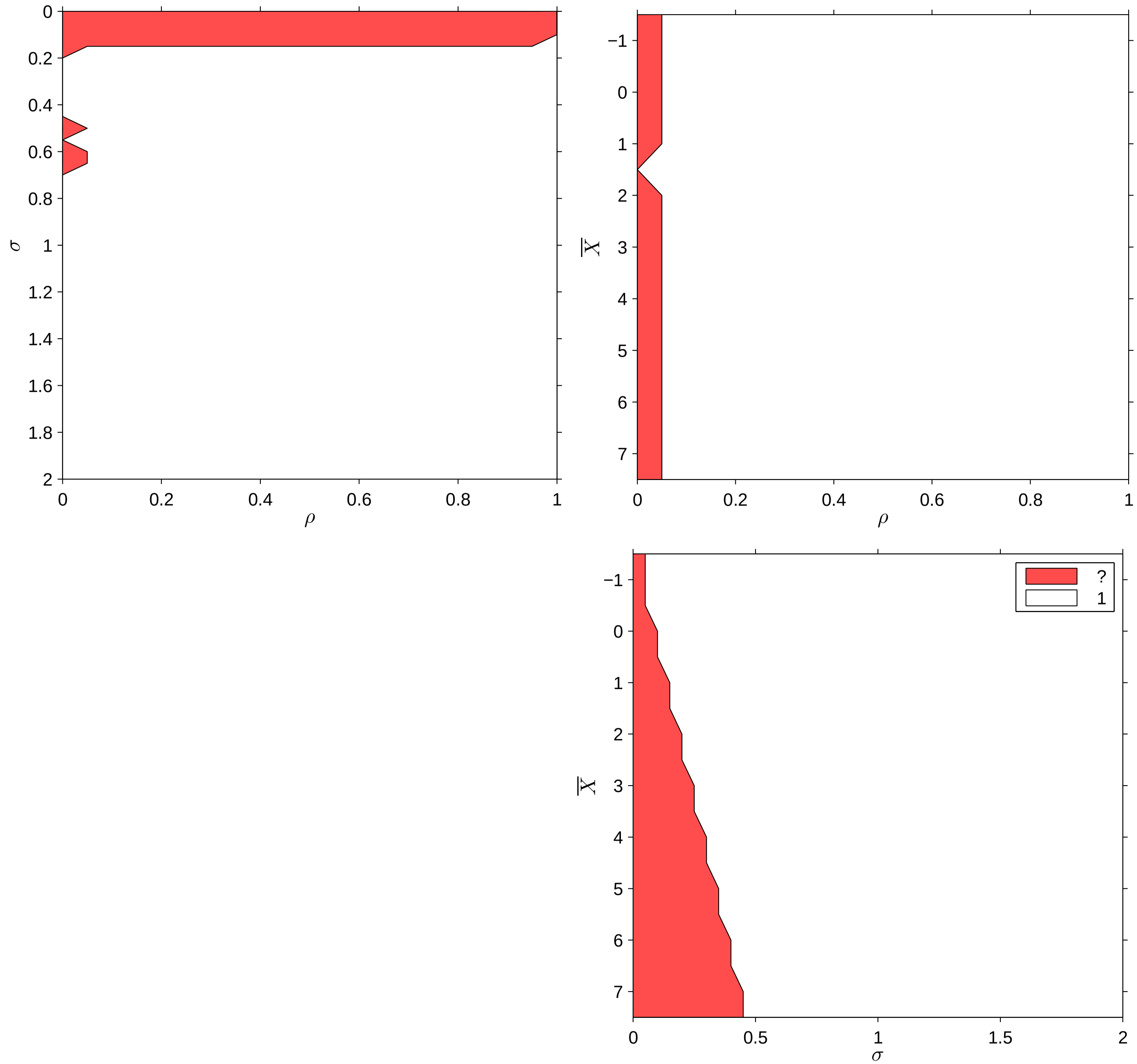

Figure A11: Number of counterfactuals for Definition 1. Counterfactual correspondence: slice along $(\rho, \sigma) \in [0, 1] \times [0, 3]$ (truncated at $\sigma = 2$, upper left panel), $(\rho, \overline{X}) \in [0, 1] \times [-1.5, 7.5]$ (upper right panel), and $(\sigma, \overline{X}) \in [0, 3] \times [-1.5, 7.5]$ (truncated at $\sigma = 2$, lower right panel). ? indicates that the homotopy algorithm crashed.

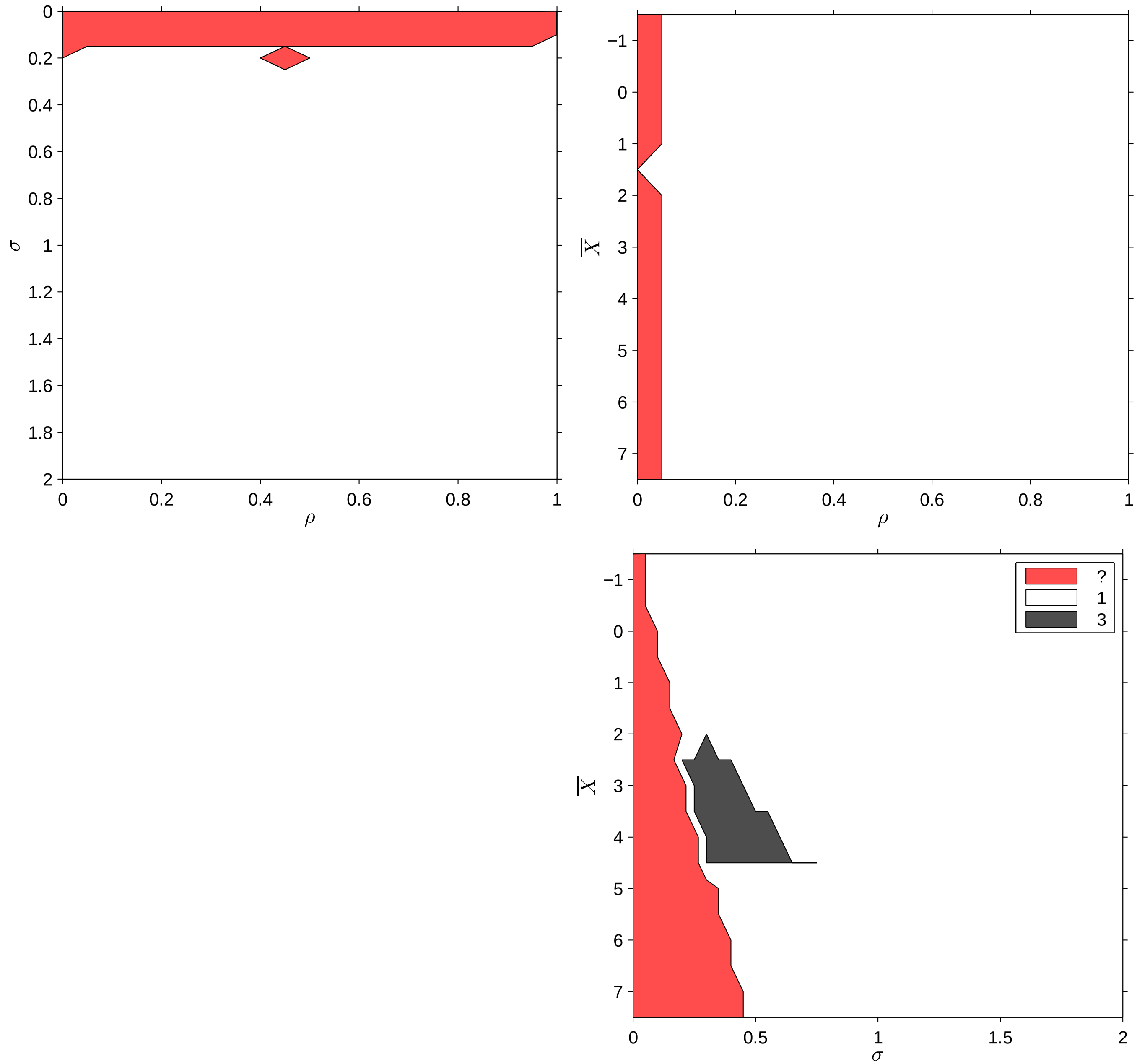

Figure A12: Number of counterfactuals for Definition 2. Counterfactual correspondence: slice along $(\rho, \sigma) \in [0, 1] \times [0, 3]$ (truncated at $\sigma = 2$, upper left panel), $(\rho, \overline{X}) \in [0, 1] \times [-1.5, 7.5]$ (upper right panel), and $(\sigma, \overline{X}) \in [0, 3] \times [-1.5, 7.5]$ (truncated at $\sigma = 2$, lower right panel). ? indicates that the homotopy algorithm crashed.

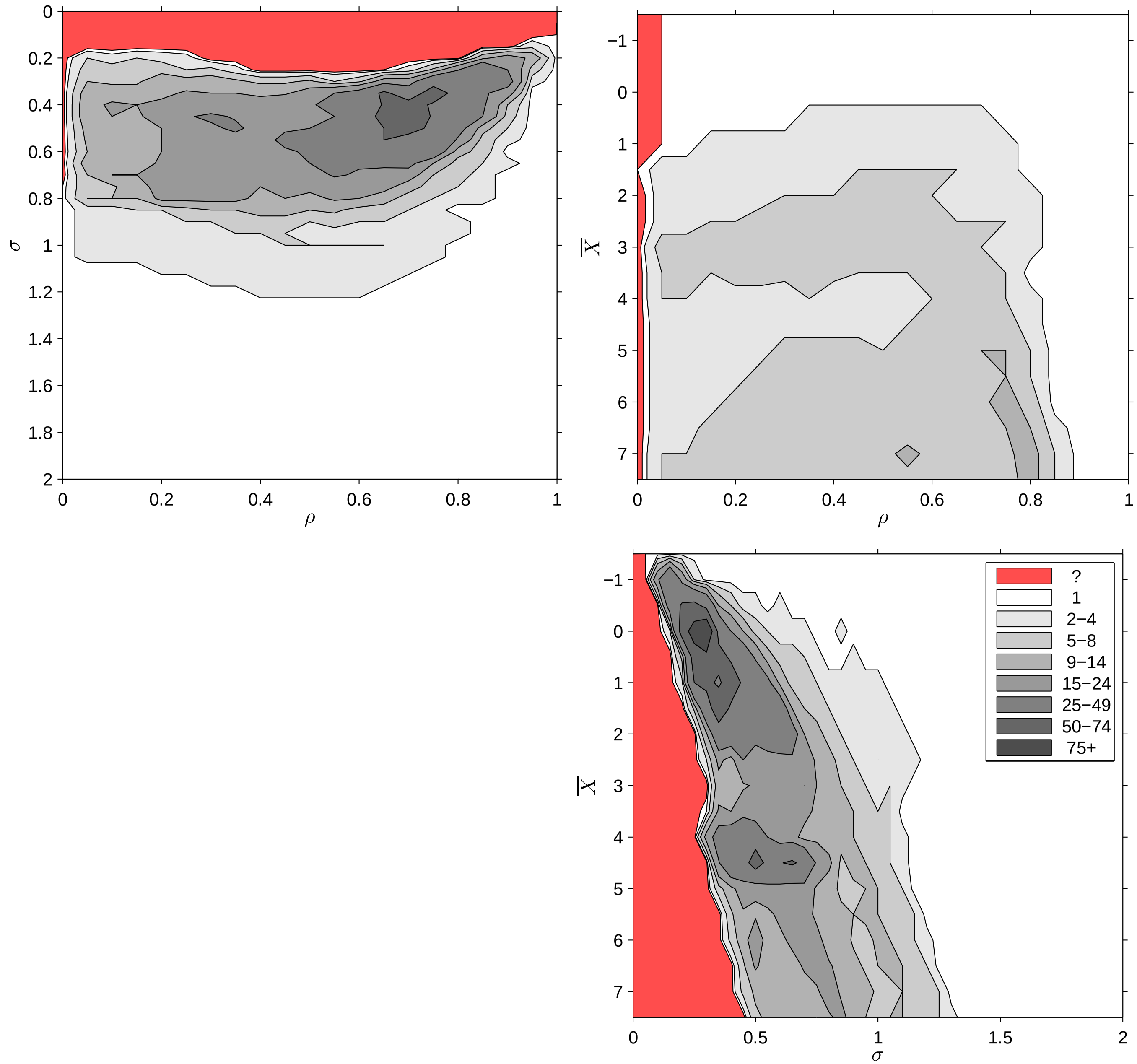

Figure A13: Number of counterfactuals for Definition 3. Counterfactual correspondence: slice along $(\rho, \sigma) \in [0, 1] \times [0, 3]$ (truncated at $\sigma = 2$, upper left panel), $(\rho, \overline{X}) \in [0, 1] \times [-1.5, 7.5]$ (upper right panel), and $(\sigma, \overline{X}) \in [0, 3] \times [-1.5, 7.5]$ (truncated at $\sigma = 2$, lower right panel). ? indicates that the homotopy algorithm crashed.

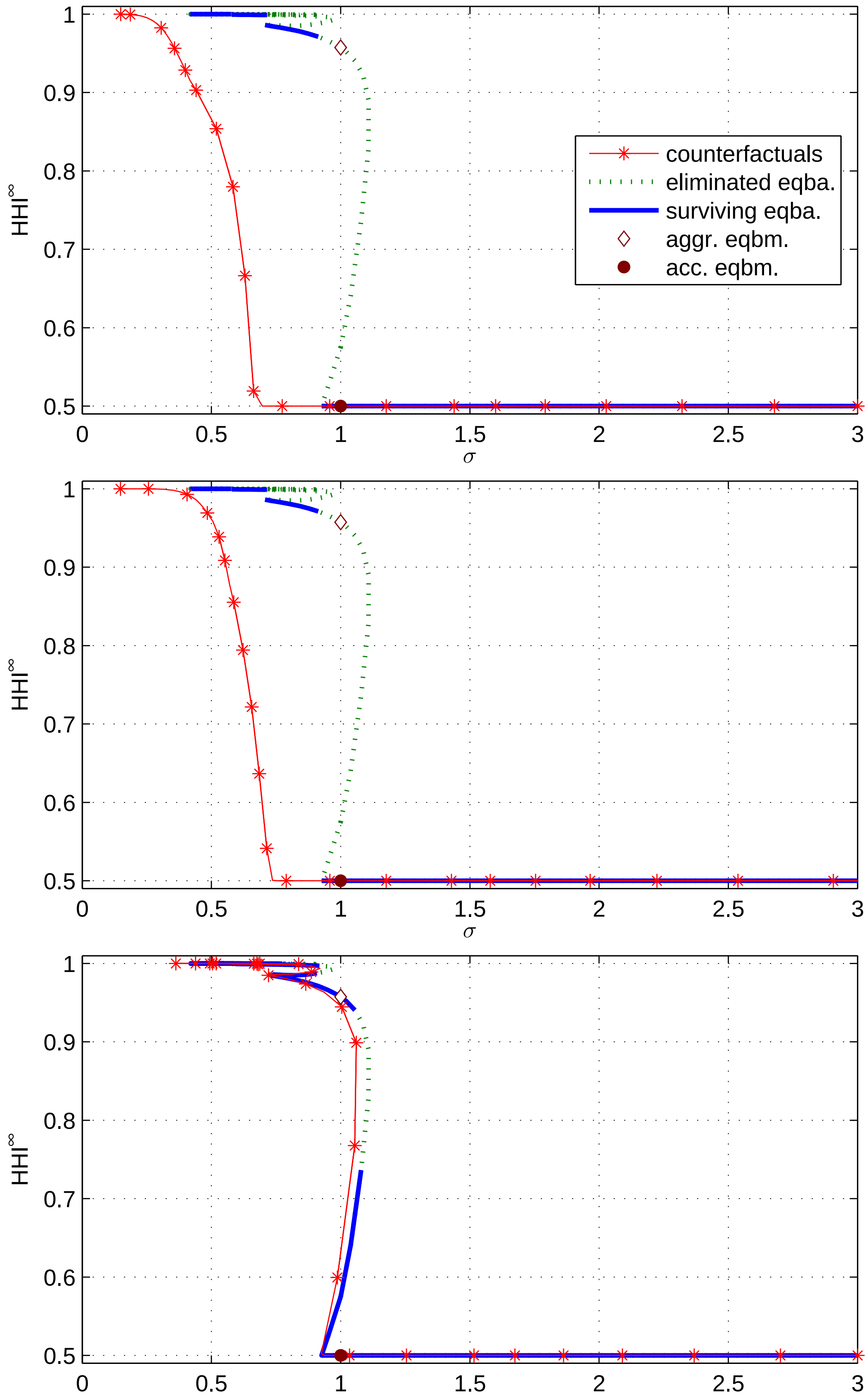

Figure A14: Expected long-run Herfindahl index. Counterfactual (solid red line) and equilibrium correspondences for Definitions 1–3 (upper, middle, and lower panels) along with eliminated (dashed green line) and surviving (solid blue line) equilibria. Slice along $\sigma \in [0, 3]$.

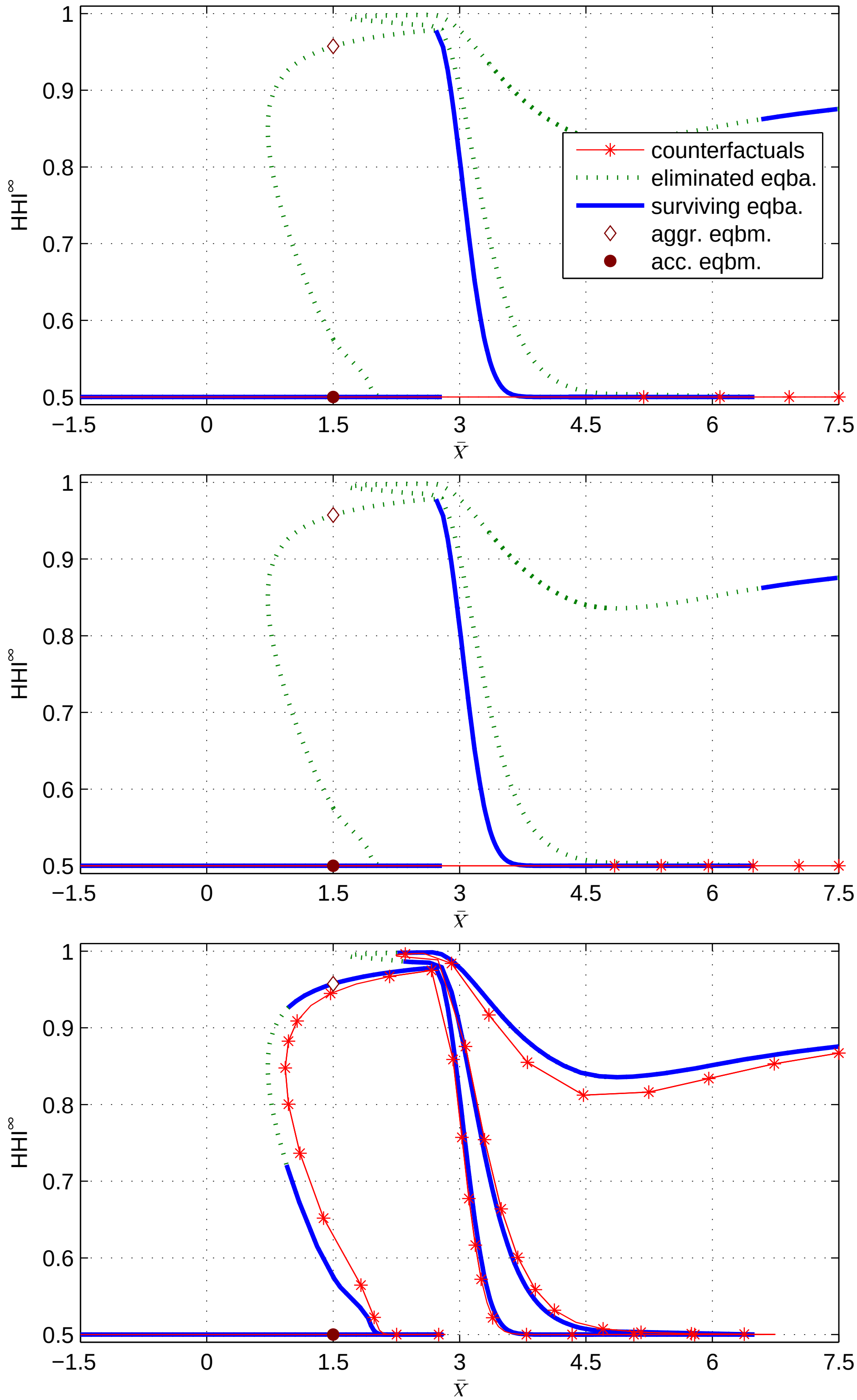

Figure A15: Expected long-run Herfindahl index. Counterfactual (solid red line) and equilibrium correspondences for Definitions 1–3 (upper, middle, and lower panels) along with eliminated (dashed green line) and surviving (solid blue line) equilibria. Slice along $\overline{X} \in [-1.5, 7.5]$.