Finance and Misallocation: Evidence from Plant-Level Data:

Online Appendix: Not for Publication

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Contents

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A Additional Model Details

In this section we discuss some of the details of the model and derivations that we have not included in the main text.

A.1 Benchmark Model

A.1.1 Decision Rules

The labor and capital decision rules of producers in the modern sector are:

$$l^m(a_{it}, z_i, e_{it}) = \left(\frac{\alpha\eta}{W}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{\tilde{r}_{it}+\delta}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \exp(z_i + e_{it} + \phi),$$

and

$$k^m(a_{it}, z_i, e_{it}) = \left(\frac{\alpha\eta}{W}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{\tilde{r}_{it}+\delta}\right)^{\frac{1-\alpha\eta}{1-\eta}} \exp(z_i + e_{it} + \phi),$$

where $\tilde{r}_{it}$ is the producer's shadow cost of funds and is equal to

$$\tilde{r}_{it} = \begin{cases} r \text{ if } \left(\frac{\alpha\eta}{W}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{r+\delta}\right)^{\frac{1-\alpha\eta}{1-\eta}} \exp(z_i + e_{it} + \phi) < \frac{1}{1-\theta}(a_{it} + \theta\kappa\exp(z_i)), \\ \left(\frac{\eta\alpha}{W}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} ((1-\alpha)\eta) \left(\frac{\exp(z_i+e_{it}+\phi)}{\frac{1}{1-\theta}(a_{it}+\theta\kappa\exp(z_i))}\right)^{\frac{1-\eta}{1-\alpha\eta}} - \delta \text{ otherwise.} \end{cases}$$

The output of a producer in the modern sector is therefore:

$$y_{it} = \eta^{\frac{\eta}{1-\eta}} \left(\frac{\alpha\eta}{W}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\frac{\eta(1-\alpha)}{(\tilde{r}_{it}+\delta)}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \exp(z_i + e_{it} + \phi).$$

Similarly, the decision rules in the traditional sector are:

$$l_{it}^{\tau} = l^{\tau}(z_i, e_{it}) = \left(\frac{\eta}{w}\right)^{\frac{1}{1-\eta}} \exp(z_i + e_{it})$$

and

$$y_{it}^{\tau} = y^{\tau}(z_i, e_{it}) = \left(\frac{\eta}{w}\right)^{\frac{\eta}{1-\eta}} \exp(z_i + e_{it}).$$

A.1.2 The distribution of the permanent productivity component

All decision rules in the model are independent of z, the permanent productivity component. Hence, the only role of z is to determine moments of the cross-section distribution of output across producers. The four moments we used to calibrate the model are the standard deviation of log output and the autocorrelation of output at various horizons. All these moments depend only on the variance of z across producers and hence we only need to choose the variance of the permanent productivity component to match the four moments. The equations below describe how these moments depend on the variance of the z given data on the rescaled output $\hat{Y}_{it} = Y_{it}/\exp(z_i)$:

$$\text{s.d.}(\log Y_{it}) = var\left(\log \hat{Y}_{it} + z_i\right)^{\frac{1}{2}} = \left[var\left(\log \hat{Y}_{it}\right) + var(z_i)\right]^{\frac{1}{2}}$$

$$corr(\log Y_{it}, Y_{it-j}) = \frac{cov\left(\log \hat{Y}_{it}, \hat{Y}_{it-j}\right) + var(z_i)}{var\left(\log \hat{Y}_{it}\right) + var(z_i)}$$

A.1.3 First-Best TFP and Losses from Misallocation

The planner solves:

$$\max_{K_i, L_i} \int_{i \in M} \exp(e_i + z_i + \phi)^{1-\eta} \left(L_i^{\alpha} K_i^{1-\alpha}\right)^{\eta} di$$

s.t.

$$K = \int_{i \in M} K_i di, \quad L = \int_{i \in M} L_i di$$

Clearly, the planner sets

$$\alpha\eta \exp(e_i + z_i + \phi)^{1-\eta} \left(L_i^{\alpha} K_i^{1-\alpha}\right)^{\eta} = \gamma L_i,$$

$$(1-\alpha)\eta \exp(e_i + z_i + \phi)^{1-\eta} \left(L_i^{\alpha} K_i^{1-\alpha}\right)^{\eta} = \xi K_i,$$

where γ and ξ are multipliers on the planner's resource constraints and thus satisfy:

$$\gamma = \alpha\eta\frac{Y}{L}$$

$$\xi = (1-\alpha)\,\eta\frac{Y}{K}$$

The FOCs of the planner also imply:

$$\frac{L_i}{L} = \frac{K_i}{K} = \frac{Y_i}{Y}$$

We further have:

$$L_i = \left(\frac{L}{Y}\right)^{\frac{1}{1-\eta}} \left(\frac{L}{K}\right)^{\frac{(\alpha-1)\eta}{1-\eta}} \exp\left(e_i + z_i + \phi\right)$$

$$K_i = \left(\frac{K}{Y}\right)^{\frac{1}{1-\eta}} \left(\frac{L}{K}\right)^{\frac{\alpha\eta}{1-\eta}} \exp\left(e_i + z_i + \phi\right)$$

Integrating across all producers, we have

$$L = \left(\frac{L}{Y}\right)^{\frac{1}{1-\eta}} \left(\frac{L}{K}\right)^{\frac{(\alpha-1)\eta}{1-\eta}} \int_{i\in M} \exp\left(e_i + z_i + \phi\right) di$$

$$K = \left(\frac{K}{Y}\right)^{\frac{1}{1-\eta}} \left(\frac{L}{K}\right)^{\frac{\alpha\eta}{1-\eta}} \int_{i\in M} \exp\left(e_i + z_i + \phi\right) di$$

which implies

$$Y = \left(\int_{i\in M} \exp\left(e_i + z_i + \phi\right)\right)^{1-\eta} \left(L^{\alpha} K^{1-\alpha}\right)^{\eta}$$

so the first-best TFP in this economy is

$$TFP^{best} = \left(\int_{i\in M} \exp\left(e_i + z_i + \phi\right)\right)^{1-\eta}$$

Also notice that the original allocations satisfy:

$$L_i = \frac{\exp\left(z_i + e_i + \phi\right)\left(r + \delta + \mu_i\right)^{-\frac{(1-\alpha)\eta}{1-\eta}}}{\int_{i\in M} \exp\left(z_i + e_i + \phi\right)\left(r + \delta + \mu_i\right)^{-\frac{(1-\alpha)\eta}{1-\eta}}} L$$

and

$$K_i = \frac{\exp\left(z_i + e_i + \phi\right)\left(r + \delta + \mu_i\right)^{\frac{\alpha\eta-1}{1-\eta}}}{\int_{i\in M} \exp\left(z_i + e_i + \phi\right)\left(r + \delta + \mu_i\right)^{\frac{a\eta-1}{1-\eta}}} K$$

where μ_i is the multiplier on the borrowing constraint of producer i.

Integrating across producers gives

$$Y = \frac{\left(\int_{i\in M} \exp\left(e_i + z_i + \phi\right)\left(r_i + \delta\right)^{-\frac{(1-\alpha)\eta}{1-\eta}}\right)^{1-\alpha\eta}}{\left(\int_{i\in M} \exp\left(e_i + z_i + \phi\right)\left(r_i + \delta\right)^{\frac{\alpha\eta-1}{1-\eta}}\right)^{(1-\alpha)\eta}} \left(L^{\alpha}K^{1-\alpha}\right)^{\eta}$$

so that TFP in the economy with financial frictions is:

$$TFP = \frac{\left(\int_{i\in M} \exp\left(e_i + z_i + \phi\right)\left(r + \delta + \mu_i\right)^{-\frac{(1-\alpha)\eta}{1-\eta}}\right)^{1-\alpha\eta}}{\left(\int_{i\in M} \exp\left(e_i + z_i + \phi\right)\left(r + \delta + \mu_i\right)^{\frac{\alpha\eta-1}{1-\eta}}\right)^{(1-\alpha)\eta}}$$

To arrive at the expressions in text, notice that μ_i and the decision to enter the modern sector is independent of z_i. Moreover, we have normalized the mean permanent productivity component, $\int \exp z_i$, to unity, so that the z_i terms drop out from the TFP expression.

A.1.4 Efficient Allocations

Given a stock of efficiency units of labor $L_t = \gamma^t$ and a measure of producers $N_t = \gamma^t$, the planner chooses how to allocate labor and capital across different producers, as well as across sectors, to maximize agents' welfare. We assume here that the planner can transfer resources across the three types of agents (workers and the entrepreneurs in the two sectors) freely and can thus equate the marginal utility of consumption across these agents.

Each period the planner must choose the capital stock K_{t+1} with which to operate next period, as well as the measure of producers of each productivity, $n_{t+1}^{\tau}(e)$ and $n_{t+1}^{m}(e)$, that will operate in the two sectors. The two measures must satisfy the technological constraints that we assume in the original economy: entering the modern sector entails a one-time sunk cost κ and the planner choose who to send to the modern sector prior to observing the producer's productivity in the following period. In effect, the planner chooses a cutoff productivity level $\bar{e}$ such that all producers with productivity $e_{it} > \bar{e}$ are sent to the modern sector, and all other producers remain in the traditional sector.

In the rescaled (by γ^t and z_i) space these two measures therefore satisfy:

$$n_j^{\tau} = \frac{1}{\gamma} \int \sum_i f_{i,j} \mathrm{I}_{\{e_i<\bar{e}\}} n_i^{\tau} + \left(1 - \frac{1}{\gamma}\right) \bar{f}_j, \tag{1}$$

$$n_j^{m} = \frac{1}{\gamma} \left(\sum_i f_{i,j} n_i^{m} + \sum_i f_{i,j} \mathrm{I}_{\{e_i \geqslant \bar{e}\}} n_i^{\tau} \right). \tag{2}$$

Given these measures, the output produced by the two sectors is:

$$Y^{\tau} = \left(\sum_i \exp\left(e_i\right) n_i^{\tau} \right)^{1-\eta} \left(L^{\tau}\right)^{\eta}$$

and

$$Y^m = \left(\sum_i \exp\left(e_i + \phi\right) n_i^m \right)^{1-\eta} \left(\left(L^m\right)^\alpha \left(K\right)^{1-\alpha} \right)^\eta$$

where K is the (rescaled by γ^t) amount of capital with which the planner enters the period and L^m and L^τ are the amount of efficiency units of labor (rescaled by γ^t) in each sector. These must satisfy

$$L^m + L^\tau = 1$$

Since investment in the sunk cost and the physical capital occurs one period in advance of production, and since the planner faces a discount factor equal to $\beta U'\left(C_{t+1}\right)/U'\left(C_t\right) = \frac{\beta}{\gamma}$, the planner's problem is to

$$\max \frac{\beta}{\gamma}\left(Y^\tau + Y^m\left(K\right) + \left(1-\delta\right)K\right) - K - \left(1 - \frac{1}{\gamma}\right)\kappa \sum_i n_i^m.$$

A.2 Heterogeneity in Entering Producer's Wealth

Our Benchmark model assumes that all entering producers start with zero initial wealth. We now modify the model to assume that at entry all producers receive an endowment of a_i units of the good. Importantly, producers differ in the amount of endowment they receive. We assume that $x_i = a_i/z_i$ is a Pareto random variable with a lower bound equal to 0 and a cdf given by

$$F(x) = 1 - \left(1 + x\right)^{-\sigma_a}.$$

Intuitively, greater dispersion in initial wealth (relative to productivity) may generate greater dispersion in the marginal product of capital of young producers that enter the modern sector and thus larger losses from misallocation.

Table A1 reports the TFP losses from misallocation in our Benchmark model under various parameterizations of the amount of dispersion in initial net worth, σ_a. We see that the losses from misallocation are greater when σ_a is lower so that there is more dispersion in net worth. Intuitively, since a is bounded from below by 0, increasing the variance of a_i also raises the average wealth of entering producers and decreases the losses from misallocation by making young producers less constrained than old producers. When $\sigma_a = 1$ (high dispersion in net worth), differences in age across producers generate smaller TFP losses (2% compared to 3.7% in the Benchmark model). Misallocation among young producers alone is indeed fairly high (6.3% TFP losses), but this effect is offset by the decrease in the average product of capital of young producers. Recall that in the data the measured TFP losses from misallocation are fairly similar across young and old producers, so it is difficult to argue that dispersion in initial net worth is a quantitatively important source of misallocation.

A.3 Economy with Technology Adoption

Let V^p be the value of a producer in the modern sector that has adopted the productive technology, V^u the value of a producer in a modern sector that has not adopted the productive technology, and V^τ be the value of a producer in the traditional sector.

Let π^p, π^u and π^τ denote the profits of each type of producer. The value of a producer that has adopted the productive technology is:

$$V^p(a,i) = \max_c \log(c) + \beta \sum_j f_{i,j} V^p(a', j)$$

subject to

$$a' = (1-\theta\chi)\pi^p(a,e) + (1+r)a - c.$$

The value of a modern-sector producer that has not yet adopted reflect the option value of doing so in future periods:

$$V^u(a,i) = \max_c \log(c) + \beta \max\left\{ \sum_j f_{i,j} V^u(a', j),\ \left(\sum_j f_{i,j} V^p(a' - \kappa_p, j)\right) \mathrm{I}_{a' > \kappa_p + \underline{a}_p} \right\}$$

subject to:

$$a' = (1-\theta\chi)\pi^u(a,e) + (1+r)a - c$$

Finally, the value of a producer in the traditional sector is:

$$V^\tau(a,i) = \max_c \log(c) + \beta \max\left\{ \sum_j f_{i,j} V^\tau(a', j),\ \left(\sum_j f_{i,j} V^u(a' - \kappa_u +, j)\right) \mathrm{I}_{a' > \underline{a}_u} \right\}$$

subject to:

$$x = \pi^\tau(a,e) + (1+r)a - c$$

where x is the amount the entrepreneur has saved at the end of the period. Its next period net worth is:

$$\begin{aligned} a' &= x \text{ if stay} \\ a' &: a' = x - \kappa_u + \theta\chi p^u(a', e) \text{ if switch} \end{aligned}$$

We also note that the borrowing constraint of a producer that has adopted the efficient technology is given by

$$k \leq \frac{1}{1-\theta} a + \frac{\theta}{1-\theta}(\kappa_u + \kappa_p),$$

reflecting our assumption that the sunk cost of intangible investment is pledgeable and can be used as collateral.

A.4 Economy without Producer Entry and Growth

For simplicity, we work in the rescaled (by z_i) space here.

A.4.1 Predetermined Capital

The producer solves:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t.

$$c_t + k_{t+1} = \exp(e_t)^{1-\eta} \left(l_t^{\alpha} k_t^{1-\alpha}\right)^{\eta} - w l_t + (1-\delta) k_t - (1+r) d_t + d_{t+1}$$

and we now assume that k_{t+1} is not measurable with respect to e_{t+1}.

Consider first the static problem of choosing labor after learning productivity. The choice of l is:

$$\max_l \exp(e)^{1-\eta} \left(l^{\alpha} k^{1-\alpha}\right)^{\eta} - wl$$

or

$$l = \left(\frac{\alpha\eta}{w}\right)^{\frac{1}{1-\alpha\eta}} \exp(e)^{\frac{1-\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}}$$

Output net of labor spending is:

$$y - wl = (1-\alpha\eta) \left(\frac{\alpha\eta}{w}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} \exp(e)^{\frac{1-\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}}$$

The firm chooses the capital stock to maximize:

$$\max_k (1-\alpha\eta) \left(\frac{\alpha\eta}{w}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} \sum_j f_{i,j} \exp(e_j)^{\frac{1-\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}} - (r+\delta) k$$

or

$$\max_k (1-\alpha\eta) \left(\frac{\alpha\eta}{w}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}} \sum_j f_{i,j} \exp(e_j)^{\frac{1-\eta}{1-\alpha\eta}} - (r+\delta) k$$

subject to the borrowing constraint

$$k \leq \frac{1}{1-\theta} a$$

The firm thus sets (let $\tilde{r}$ be again the shadow cost of funds)

$$k = \left(\frac{(1-\alpha)\eta}{\tilde{r}+\delta}\right)^{\frac{1-\alpha\eta}{1-\eta}} \left(\frac{\alpha\eta}{w}\right)^{\frac{\alpha\eta}{1-\eta}} \left(\sum_j f_{i,j} \exp(e_j)^{\frac{1-\eta}{1-\alpha\eta}}\right)^{\frac{1-\alpha\eta}{1-\eta}}$$

Its choice of labor is then:

$$l = \left(\frac{\alpha\eta}{w}\right)^{\frac{1-(1-\alpha)\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{\tilde{r}+\delta}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \left(\sum_j f_{i,j} \exp(e_j)^{\frac{1-\eta}{1-\alpha\eta}}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} \exp(e_i)^{\frac{1-\eta}{1-\alpha\eta}}$$

and its output is:

$$y=\left(\frac{\alpha\eta}{w}\right)^{\frac{\alpha\eta}{1-\eta}}\left(\frac{(1-\alpha)\,\eta}{\tilde{r}+\delta}\right)^{\frac{(1-\alpha)\eta}{1-\eta}}\left(\sum_j f_{i,j}\exp\left(e_j\right)^{\frac{1-\eta}{1-\alpha\eta}}\right)^{\frac{(1-\alpha)\eta}{1-\eta}}\exp\left(e_i\right)^{\frac{1-\eta}{1-\alpha\eta}}$$

while its dividends are:

$$\pi\left(a,e,e_{-1}\right)=\left(1-\alpha\eta\right)y-\left(r+\delta\right)k$$

The value function is now a function of the capital stock with which the producer enters the period:

$$V\left(a,e_i,k\right)=\max_{c,k'}\log\left(c\right)+\beta\sum_j f_{i,j}V\left(a',e_j,k'\right)$$

subject to

$$c+a'=\pi\left(a,e_i,k\right)+\left(1+r\right)a$$

A.4.2 Economy with Constant Markups

Before we describe our economy in which markups positively comove with productivity, consider first an economy with constant markups. We now assume that final goods (used for consumption and investment) are produced using

$$Q=\left(\int q_i^{\frac{\gamma-1}{\gamma}}di\right)^{\frac{\gamma}{\gamma-1}},$$

where q_i is the quantity of intermediate good i used in production and γ is the elasticity of substitution across different intermediate goods. Demand for each intermediate input is thus given by

$$p_i=P\left(\frac{q_i}{Q}\right)^{-\frac{1}{\gamma}}, \tag{3}$$

where

$$PQ=\int p_iq_i$$

We assume now that producers of intermediate inputs operate using a constant returns technology:

$$q_i=\exp\left(\nu_i\right)l_i^{\alpha}k_i^{1-\alpha}$$

and solve:

$$\max_{p_i,l_i,k_i}p_iq_i-Wl_i-\left(r+\delta\right)Pk_i$$

s.t.

$$k_i\leq\frac{1}{1-\theta}a_i$$

and the demand function (3). Substituting the demand function, we have

$$\pi_i = q_i^{1-\frac{1}{\gamma}} P Q^{\frac{1}{\gamma}} - Wl_i - (r+\delta) P k_i.$$

Finally, assume that the final good is the numeraire, and let $\eta = 1 - \frac{1}{\gamma}$ and

$$A_i = \left[Q^{\frac{1}{\gamma}} \left(\exp\left(\nu_i\right)\right)^{1-\frac{1}{\gamma}} \right]^{\frac{1}{1-\eta}}.$$

We then have

$$\pi_i = A_i^{1-\eta} \left(\exp\left(\nu_i\right) l_i^{\alpha} k_i^{1-\alpha}\right)^{1-\frac{1}{\gamma}} - Wl_i - (r+\delta) k_i.$$

The problem of an individual producer is thus identical to that in our original setup, except for the fact that now the decreasing returns arise from imperfect substitutability across goods.

To compute TFP in this economy, note that the decision rules for capital and labor are

$$l_i = \frac{\exp\left(\nu_i\right)^{\gamma-1} \left(r+\delta+\mu_i\right)^{-(1-\alpha)(\gamma-1)}}{\int \exp\left(\nu_i\right)^{\gamma-1} \left(r+\delta+\mu_i\right)^{-(1-\alpha)(\gamma-1)} di} L$$

and

$$k_i = \frac{\exp\left(\nu_i\right)^{\gamma-1} \left(r+\delta+\mu_i\right)^{\alpha+\gamma(1-\alpha)}}{\int \exp\left(\nu_i\right)^{\gamma-1} \left(r+\delta+\mu_i\right)^{\alpha+\gamma(1-\alpha)} di} K$$

Consider next the expression for aggregate output. Given that $Q^{\frac{\gamma-1}{\gamma}} = \int q_i^{\frac{\gamma-1}{\gamma}}$, we have:

$$Q^{1-\frac{1}{\gamma}} = \left(L^{\alpha} K^{1-\alpha}\right)^{1-\frac{1}{\gamma}} \frac{\left(\int \exp\left(\nu_i\right)^{\gamma-1} \left(r+\delta+\mu_i\right)^{-(1-\alpha)(\gamma-1)}\right)^{1-\alpha\left(1-\frac{1}{\gamma}\right)}}{\left(\int \exp\left(\nu_i\right)^{\gamma-1} \left(r+\delta+\mu_i\right)^{-(\alpha+\gamma(1-\alpha))} di\right)^{(1-\alpha)\left(1-\frac{1}{\gamma}\right)}}$$

and therefore

$$Q = L^{\alpha} K^{1-\alpha} \frac{\left(\int \exp\left(\nu_i\right)^{\gamma-1} \left(r+\delta+\mu_i\right)^{-(1-\alpha)(\gamma-1)}\right)^{\left(\frac{\gamma}{\gamma-1}-\alpha\right)}}{\left(\exp\left(\nu_i\right)^{\gamma-1} \int \left(r+\delta+\mu_i\right)^{-(\alpha+\gamma(1-\alpha))} di\right)^{(1-\alpha)}}$$

The first-best level of output is equal to:

$$Q^{best} = L^{\alpha} K^{1-\alpha} \left(\int \exp\left(\nu_i\right)^{\gamma-1}\right)^{\frac{1}{\gamma-1}}$$

Notice that setting $\eta = 1 - \frac{1}{\gamma}$, $Y = Q^{1-\frac{1}{\gamma}}$ and $\exp\left(e_i + z_i\right) = \exp\left(\nu_i\right)^{\gamma-1}$, our original setup with decreasing returns to scale and perfectly substitutable inputs is equivalent to that arising in the model with constant markups and differentiated inputs. The only difference is that in the original model we have reported results about the productivity with which the economy produces Y, rather than Q. To recast our original numbers in this alternative setup we simply have to magnify all the TFP numbers we have previously reported by a factor of $\frac{\gamma}{\gamma-1} = \frac{1}{\eta} = 1/0.85$.

A.4.3 Economy with Variable Markups

To allow markups to vary with productivity we next assume that the producer's price, $p_{i,t}$, is not measurable with respect to the producer's transitory productivity, $e_{i,t}$, in that period. Since quantities are demand-determined:

$$q_{i,t} = Qp_{i,t}^{-\gamma},$$

this assumption implies that the producer's revenue,

$$y_{i,t} = p_{i,t}q_{i,t}$$

is also not measurable with respect to $e_{i,t}$.

To solve this problem, we use the equivalence results in the previous subsection and note that in our original notation the fact that the producer's output (equivalently revenue) is predetermined at some level $\bar{y}$, implies an optimal use of labor and capital given by

$$l_{i,t} = \left(\frac{\alpha}{w}\right)^{1-\alpha} \left(\frac{(1-\alpha)}{r+\delta+\mu_{i,t}}\right)^{-(1-\alpha)} e_{i,t}^{\frac{\eta-1}{\eta}} \bar{y}_{i,t}^{\frac{1}{\eta}}$$

and

$$k_{i,t} = \left(\frac{\alpha}{w}\right)^{-\alpha} \left(\frac{(1-\alpha)}{r+\delta+\mu_{i,t}}\right)^{\alpha} e_{i,t}^{\frac{\eta-1}{\eta}} \bar{y}_{i,t}^{\frac{1}{\eta}}$$

where $\mu_{i,t} = 0$ if the borrowing constraint does not bind so that

$$\left(\frac{\alpha}{w}\right)^{-\alpha} \left(\frac{(1-\alpha)}{r+\delta+\mu_{i,t}}\right)^{\alpha} e_{i,t}^{\frac{\eta-1}{\eta}} \bar{y}_{i,t}^{\frac{1}{\eta}} < \frac{1}{1-\theta} a_{i,t}$$

and

$$\mu_{i,t} = W\left(1-\theta\right)^{\frac{1}{\alpha}} a^{-\frac{1}{\alpha}} \frac{1-\alpha}{\alpha} e_{i,t}^{\frac{1}{\alpha}\frac{\eta-1}{\eta}} \bar{y}_{i,t}^{\frac{1}{\alpha}\frac{1}{\eta}} - r - \delta$$

otherwise.

Given these choices, the cost of production is

$$c_{i,t} = Wl_{i,t} + (r+\delta)\,k_{i,t} = \alpha^{-\alpha}\left(1-\alpha\right)^{-(1-\alpha)} \left(\alpha + \frac{r+\delta}{r+\delta+\mu_{i,t}}\left(1-\alpha\right)\right) W^{\alpha}\left(r+\delta+\mu_{i,t}\right)^{1-\alpha} e_{i,t}^{\frac{\eta-1}{\eta}} \bar{y}_{i,t}^{\frac{1}{\eta}}$$

To solve for the level of output (and thus prices and quantities), we solve

$$\max_{\bar{y}_{i,t}} \bar{y}_{i,t} - E_{i,t-1}c_{i,t}$$

where $E_{i,t-1}$ is the expectation operator conditional on the information set of producer i in period t. Notice that the cost of production, $c_{i,t}$, is itself a function of $y_{i,t}$ both because of the direct effect arising from the downward-sloping demand function, as well as because of the effect a commitment to a certain price has on the multiplier on the borrowing constraint.

As in the economy with predetermined capital, the value function has one additional argument, the amount of output (equivalently the price) that the producer has committed to sell prior to observing its productivity:

$$V\left(a, e_i, y\right) = \max_{c, y'} \log\left(c\right) + \beta \sum_j f_{i,j} V\left(a', e_j, y'\right)$$

subject to

$$c + a' = \pi\left(a, e_i, y\right) + \left(1 + r\right) a$$

A.4.4 Low Elasticity of Substitution Between Capital and Labor

The firms' static, profit-maximization program now reduces to:

$$\max_{k_i, l_i} \exp\left(e_i\right)^{1-\eta} \left[\alpha\left(l_i\right)^{\frac{\gamma-1}{\gamma}} + \left(1-\alpha\right)\left(k_i\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}\eta} - wl_i - \left(r+\delta\right)k_i,$$

subject to

$$k_i \leq \frac{1}{1-\theta} a_i.$$

The solution is

$$k_i = \eta^{\frac{1}{1-\eta}} \left(\frac{r+\delta+\mu_i}{1-\alpha}\right)^{-\gamma} \left[\alpha^\gamma W^{1-\gamma} + \left(1-\alpha\right)^\gamma \left(r+\delta+\mu_i\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}\left(\gamma - \frac{1}{1-\eta}\right)} e_i$$

$$l_i = \eta^{\frac{1}{1-\eta}} \left(\frac{W}{\alpha}\right)^{-\gamma} \left[\alpha^\gamma W^{1-\gamma} + \left(1-\alpha\right)^\gamma \left(r+\delta+\mu_i\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}\left(\gamma - \frac{1}{1-\eta}\right)} e_i$$

and

$$y_i = \eta^{\frac{\eta}{1-\eta}} \left[\alpha^\gamma W^{1-\gamma} + \left(1-\alpha\right)^\gamma \left(r+\delta+\mu\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}\frac{\eta}{\eta-1}} e_i$$

Note that here the ratio of payments to labor to that of payments to capital is equal to

$$\frac{Wl}{\left(r+\delta\right)k} = \frac{\alpha}{1-\alpha}\left(\frac{W}{r+\delta}\right)^{1-\gamma}$$

for an unconstrained producer. Since we chose $\alpha = 2/3$ earlier, this ratio was equal to 2 under the original Cobb-Douglas specification with $\gamma = 1$. We set $\gamma = 0.25$ in this economy and adjust α accordingly to maintain a ratio of payments to labor and capital equal to 2 as we decrease the elasticity of substitution between the two factors:

$$\alpha = \frac{2\left(\frac{W}{r+\delta}\right)^{\gamma-1}}{1 + 2\left(\frac{W}{r+\delta}\right)^{\gamma-1}}.$$

A.4.5 Capital-Specific Productivity Shocks

We modify the technology to

$$y_i = \left[\alpha\left(l_i\right)^{\frac{\gamma-1}{\gamma}} + (1-\alpha)\left(e_i k_i\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}\eta}$$

The decision rules are:

$$k_i = \eta^{\frac{1}{1-\eta}} \left(\frac{r+\delta+\mu}{(1-\alpha)\, e_i^{\frac{\gamma-1}{\gamma}}}\right)^{-\gamma} \left[\alpha^\gamma W^{1-\gamma} + ((1-\alpha))^\gamma e_i^{\gamma-1}\left(r+\delta+\mu\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}\left(\gamma-\frac{1}{1-\eta}\right)}$$

$$l_i = \eta^{\frac{1}{1-\eta}} \left(\frac{w}{\alpha}\right)^{-\gamma} \left[\alpha^\gamma W^{1-\gamma} + ((1-\alpha))^\gamma e_i^{\gamma-1}\left(r+\delta+\mu\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}\left(\gamma-\frac{1}{1-\eta}\right)}$$

and the amount of output produced is:

$$y_i = \eta^{\frac{\eta}{1-\eta}} \left[\alpha^\gamma W^{1-\gamma} + ((1-\alpha))^\gamma e_i^{\gamma-1}\left(r+\delta+\mu\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}\frac{\eta}{\eta-1}}.$$

A.4.6 Heterogeneity in Borrowing Rates

Consider the problem of any individual producer that can save at an interest rate r_L and borrows at r_H. Such a producer solves

$$\max \sum \log c_{it}$$

s.t.

$$c_{it} + k_{it+1} - (1-\delta)\, k_{it} = y_{it} - w l_{it} - \left(1 + r\left(d_{it}\right)\right) d_{it} + d_{it+1}.$$

$$y_{it} = \exp\left(e_{it}\right)^{1-\eta} \left(l_{it}^{\alpha} k_{it}^{1-\alpha}\right)^{\eta}$$

where $r(d) = r_L$ if $d < 0$ and $r(d) = r_H$ otherwise.

As earlier, let $a_{it} = k_{it} - d_{it}$ and rewrite the budget constraint as

$$c_{it} + a_{it+1} = y_{it} - \left(r\left(d_{it}\right) + \delta\right) k_{it} - w l_{it} + \left(1 + r\left(d_{it}\right)\right) a_{it}$$

where

$$r(k,a) = \begin{cases} r_L \text{ if } k < a \\ r_H \text{ if } k > a \end{cases}$$

To solve this problem, we first solve for the optimal choice of labor given a certain level of capital:

$$\max_{l} \exp(e)^{1-\eta} \left(l^{\alpha} k^{1-\alpha}\right)^{\eta} - W l,$$

which gives

$$l = \left(\frac{\alpha\eta}{W}\right)^{\frac{1}{1-\alpha\eta}} \exp\left(e\right)^{\frac{1-\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}}$$

Let

$$f\left(k\right) = y - Wl = \left(1-\alpha\eta\right)\left(\frac{\alpha\eta}{w}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} \exp\left(e\right)^{\frac{1-\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}}$$

To solve for the producer's decision of whether to borrow or save, we first evaluate $f'\left(k\right)$ at $k = a$, the point at which the producer neither borrow nor lends. There are three possible cases.

First, if $f'\left(a\right) < r_L + \delta$, then by the concavity of f it follows that there exists an interior solution with $k < a$ that satisfies $f'\left(k\right) = r_L + \delta$. Such a producer finds it optimal to save and faces a shadow cost of funds equal to r_L.

Second, if $f'\left(a\right) > r_H + \delta$, then the concavity of f implies that there is an interior solution $k > a$ that satisfies $f'\left(k\right) = r_H + \delta$ so the agent borrows and faces a shadow cost of funds equal to r_H.

Finally, if $f'\left(a\right) \in \left(r_L + \delta, r_H + \delta\right)$, then the producer finds it optimal to set $k = a$ and thus neither borrows nor lends. Such a producer faces a shadow cost of funds equal to $f'\left(a\right) - \delta$.

The producer's decision rules are thus very similar to those depicted in Figure 1 in the main text (the shadow cost of funds declines with a), except that now $r + \mu$ is bounded from above by r_H.

B Data Description

We have used three separate micro-level datasets in this paper: two plant-level surveys from Colombia and Korea, as well as a firm-level survey from China. We briefly describe each of the datasets we used.

B.1 Korea

The data is derived from the Korean Annual Manufacturing Survey and covers the years 1991 - 1999. The survey covers all manufacturing plants in Korean economy with 5 or more workers. We have information on each plant's age (based on the reported year of birth), revenue, number of workers, total wage bill, total fringe benefit, intermediate inputs (materials), and energy use. The survey also reports the book value, purchase, retirement, and depreciation for the major categories of capital, including land, building, machinery, and transportation equipment. This information allows us to construct a measure of plant-level capital using the perpetual inventory method, using the reported book value of capital to initialize each series and augmenting each year's series to include purchases net of depreciation and retirements.[1] We construct a broad measure of each plant's capital stock by adding up all buildings, machinery, and equipment a plant owns, as well as by adding the amount of capital a plant rents in any given year. We define labor expenditure as wage and fringe benefit payments to workers. The intermediate inputs include raw materials, water, energy and fuel. We define value added as total revenue net of spending on intermediate inputs. All series are real. We use the aggregate CPI to deflate value added and the producer's wage bill, and the price deflator for investment in the manufacturing sector to construct a measure of capital stock.[2]

We drop observations that are clearly an outcome of measurement error: observations with negative values of value-added, expenditure of labor, and constructed capital series. This leaves us with around $700,000$ plant-year observations over a 9 year period from 1991 to 1999.

B.2 Colombia

The second data set we use is the Colombian Industrial Survey from 1985 to 1990. The survey covers all manufacturing plants with more than ten workers. The information contained in Colombian data is very similar to that in the Korean data and includes each plant's age (again based on the reported year of birth), revenue, number of workers, total wage bill, total fringe benefits, intermediate inputs (materials), and energy use. The survey also provides information on the book value, purchase, retirement, and depreciation for detailed capital categories including land, building, machinery and equipment. After excluding observations that are an obvious outcome of measurement error using the same criteria as those applied to the Korean data, we are left with around $40,000$ plant-year observations over 1985 to 1990.

B.3 China

The third data set we use is the Annual Survey of Chinese Manufacturing Firms from 1998 to 2007. This survey reports data for all manufacturing firms that have annual sales of at least 5 million RMB (about 600,000 US dollars). The unit of observation in the Chinese data is a firm instead of a plant. The survey reports information on age, total revenue, wage bill, intermediate inputs, and the book value of a firm's capital stock. In addition, it also includes several financial variables. Each firm reports its debt holding and ownership structure (state, privately-owned or foreign-owned). Unlike the Korean and Colombian data, the data from

[1] See, for example, Caballero et al. (1995).

[2] As a robustness check, we have also used (2-digit) industry-specific deflators for gross output and investment from the OECD STAN Database and found very similar results.

China does not report detailed information on capital purchases and retirement. For this reason we use the reported book value of each firm's capital. We exclude observations that we suspect are measured with error using the same criteria as above. This leaves us with about to 2 million firm-year observations over a ten year period.

Table A2 lists the key moments we used in calibrating the model for all three countries, as well as their bootstrapped standard errors.

B.4 Measures of Value Added

Most of our analysis has measured value added using the *single-deflation index* by subtracting payments to intermediate inputs from total producer sales. This approach correctly identifies value added under the assumption of constant returns and perfect competition. To see this, assume that gross output, O, is produced using value added, Y, and intermediate inputs, M, using a constant-returns technology:

$$O = G(Y, M) \tag{4}$$

Cost minimization and perfect competition implies

$$G_Y = P_y \tag{5}$$

and

$$G_M = P_m, \tag{6}$$

while constant returns implies that:

$$O = P_y Y + p_m M$$

so that value added is indeed given by

$$P_y Y = O - P_m M.$$

Moreover, since we have also assumed that

$$Y = \exp(e + z)^{1-\eta} \left(L^\alpha K^{1-\alpha}\right)^\eta$$

efficiency requires that

$$(1 - \alpha) \frac{P_y Y}{K} = r + \delta,$$

so we can back out wedges in the producer's choice of capital using data on $P_y Y/K$. Hsieh and Klenow (2009) refer to this object as revenue productivity.

An alternative popular way to measure value added is to construct the *Divisia index*:

$$\ln Y = \frac{\ln O - s_m \ln M}{1 - s_m}.$$

As Basu and Fernald (1995) show, the single-deflated index is equal to the Divisia index plus an additional term that involves the cumulative change in the relative price of intermediate inputs, P_m, over time.

Since in our data idiosyncratic variation in value added is much larger than aggregate changes in relative prices, we find that the two measures produce very similar results. For example, the standard deviation of the growth rate of the Divisia index is equal to 0.62 in Korea (0.59 for the single-deflation index), the autocorrelation is equal to 0.92 (0.90 for the single deflation index), etc.

C Measures of Plant Productivity

We next describe how we have constructed plant-level measures of productivity. Let $\nu_{it} = z_i + e_{it}$ denote a producer's productivity in period t. Recall that the technology with which producers in the modern sector operate is:

$$\ln(Y_{it}) = \eta\alpha \ln(L_{it}) + \eta(1 - \alpha)\ln(K_{it}) + (1 - \eta)\nu_{it}.$$

Our first approach follows the tradition of the index number literature by utilizing the first order condition that determines the producer's choice of labor:

$$\alpha = \frac{WL_{it}}{\eta Y_{it}}.$$

We set $\eta = 0.85$, as in our quantitative analysis and thus obtain estimates of $\hat{\alpha}$ for each 2-digit industry by computing the average labor share in value added.

Since value added is measured with error, we next isolate a purified measure of productivity using the insights of Ackerberg et. al. (2006) and De Loecker and Warzynski (2012). Suppose that value added is measured with an error, u_{it}, so that the observed data on value added, Y^D, is given by

$$\ln(Y^D_{it}) = \eta\alpha \ln(L_{it}) + \eta(1 - \alpha)\ln(K_{it}) + (1 - \eta)\nu_{it} + u_{it}$$

Assuming that the pair $(L_{it},\ K_{it})$ is invertible with respect to ν_{it},[3] we project the data on value added, $\ln(Y^D_{it})$, on a non-parametric function of $\ln(L_{it})$ and $\ln(K_{it})$, in order to isolate

[3]This assumption holds in our Benchmark model since data on L and K alone is sufficient to pin down both the producer's productivity as well as its net worth.

the error term u_{it} as the residual in this non-parametric regression. Intuitively, measurement error is the component of productivity to which neither capital nor labor respond. Given an estimate of α and purified value added, $\hat{Y}_{it}$, we compute the producer's productivity using

$$(1-\eta)\hat{\nu}_{it} = \ln(\hat{Y}_{it}) - \eta\hat{\alpha}\ln(L_{it}) - \eta(1-\hat{\alpha})\ln(K_{it})$$

We isolate the permanent productivity component, $\hat{z}_i$, by simply taking the time-series average of each producer's productivity Alternatively, we can characterize the persistence and variability of the transitory component by estimating an AR(1) process with fixed effects for the composite productivity process:

$$\hat{\nu}_{it} = \rho\hat{\nu}_{it-1} + (1-\rho)z_i + \varepsilon_{it}$$

The estimate of ρ and variance of ε_{it} correspond to the persistence and variance of innovation of e_{it}, the transitory productivity component of a producer's productivity.

Our second approach uses the dynamic panel estimator developed by Blundell and Bond (1998) to estimate the labor elasticity α as well as the degree of returns to scale, η. Note that the Blundell and Bond (1998) estimator is applicable here since we have assumed that e_{it} follows a linear first order Markov process.

Recall that we assume that the data on output is measured with error, u_{it}, so that

$$\begin{aligned} \ln(Y_{it}^D) &= \eta\alpha\ln(L_{it}) + \eta(1-\alpha)\ln(K_{it}) + (1-\eta)(z_i + e_{it}) + u_{it} \\ e_{it} &= \rho e_{it-1} + \varepsilon_{it} \end{aligned}$$

We can substitute out e_{it} from this system of two equations to arrive at:

$$\ln(Y_{it}^D) = \pi_1\ln(L_{it}) + \pi_2\ln(L_{it-1}) + \pi_3\ln(K_{it}) + \pi_4\ln(K_{it-1}) + \pi_5\ln(Y_{it-1}^D) + \gamma_i + \omega_{it}$$

where $\gamma_i = (1-\eta)(1-\rho)z_i$, $\omega_{it} = (1-\eta)\varepsilon_{it} + u_{it} - \rho u_{it-1}$, $\pi_1 = \eta\alpha$, $\pi_3 = \eta(1-\alpha)$, $\pi_5 = \rho$, $\pi_2 = -\pi_1\pi_5$ and $\pi_4 = -\pi_3\pi_5$. The Blundell Bond estimator uses two sets of moment restrictions in order to overcome the weak instrument problem in Arellano and Bond (1991). The first restriction is

$$E[x_{t-s}\Delta\omega_t] = 0,$$

where $x_t = [\ln(L_t), \ln(K_t), \ln(Y_t^D)]$ and $s \geq 2$. The second restriction is

$$E[\Delta x_{t-s}(\gamma + \omega_t)] = 0$$

where again $s \geq 2$.

We apply this dynamic panel estimator to each 2-digit industry in Korean manufacturing and compare the estimated labor input elasticity α with that obtained from the index number approach in Table A3. Both methods produce very similar estimates of α, which leads us to use the much simple index number approach throughout the rest of the paper. Table A3 also reports the implied estimates of the persistence of the transitory productivity component obtained using both methodologies. Notice that the persistence is in all cases close to zero and does not vary much across industries.[4]

Table A4 compares the implied estimates of productivity when we apply the two different methods to the Korean data. Notice that these estimates are highly correlated. One discrepancy that arises is that the TFP losses from capital misallocation predicted by the Blundell-Bond method are smaller than those under the index number approach. The reason is that the Blundell-Bond estimate of η is somewhat smaller (0.74) than the 0.85 we used in our index number approach. Since the size of the TFP losses increases with η, our use of a greater span-of-control parameter tends to overstate the misallocation losses. When we impose the restriction that $\eta = 0.85$ on our Blundell-Bond estimates, the TFP losses from misallocation that we obtain are very similar to those based on the index number approach.

Table A5 reports the elasticities of the labor input derived using the index number approach for the Colombian and Chinese data. As documented by Hsieh and Klenow (2009), the Chinese data does not include non-wage compensation to measure the total wage bill. We thus follow these authors and adjust the labor expenditure share by a constant factor in order to match the Chinese aggregate labor share in manufacturing while preserving the dispersion in sectoral labor shares. As in the case of the Korean data, the estimates of α in China and Colombia are very close to the 0.67 used in our quantitative analysis and not very dispersed across industries.

D Additional Evidence on Role of Credit Constraints

In this section, we discuss how the average product of capital and its cross-sectional dispersion are affected by several producer characteristics that arguably reflect the extent to which a producer is financially constrained. These characteristics, which include an index of external finance dependence due to Rajan and Zingales (1998), a producer's debt to capital ratio, as well as its ownership status (state- or privately-owned) in the case of China, have often been used in the existing literature to measure the role of financial frictions. We also discuss how the average product of capital varies across fast, as opposed to slow-growing producers. The former are more constrained in our model since they need external finance to be able to expand.

[4]The persistence is also stable across different time-periods.

D.1 Differences in External Finance Dependence

We follow a large literature in corporate finance to categorize manufacturing industries into industries with low, medium, and high external financial dependence (EFD), using the approach of Rajan and Zingales (1998).[5] The basic idea of Rajan and Zingales (1998) is that firms listed on U.S. stock market are relatively unconstrained and that for such firms data on the amount of investment that is financed externally identifies an industry's "technological demand" for financing. For example, industries like *Apparel* and *Leather* have low EFD, while industries like *Drugs* and *Electronics* have high EFD according to the Rajan and Zingales (1998) classification. As a consequence, producers in high EFD industries are interpreted be more constrained than producers in low EFD industries in environments with imperfect financial markets. We next ask whether this is indeed the case in the datasets we study. We ask: are industries with a greater need for external finance more constrained (as measured by their average product of capital) and do they exhibit more misallocation (as measured by the dispersion in the average product of capital across producers)?.

In Table A6 we report two sets of micro statistics for each category of low, medium, and high EFD industries — the average and standard deviation of $\ln(\frac{Y_i}{K_i})$ — and compare these moments for the three countries in our sample.

We note that the average output-capital ratio is very similar in the three types of industries in all countries we consider. We thus find little evidence to suggest that industries with high needs of external finance are more financially constrained. Moreover, the variance of the average product of capital across producers is also very similar across the three types of industries, suggesting that industries with greater needs for external finance do not exhibit greater capital misallocation.

D.2 Differences in Debt-to-Capital Ratio

We have information on the debt to capital ratio of producers in our Chinese dataset, as well as for two years (1991 and 1992) in the Korean data set. We next ask whether differences in the debt to capital ratio across producers are correlated with measures of how constrained individual producers are, as it is in versions of our model in which borrowing constraints are severe so that most producers are constrained and in which producers differ in their collateral constraint, θ.

Table A7 shows a pronounced difference in the average product of capital, $\ln(\frac{Y}{K})$, across producers in the three different terciles of the debt to capital ratio. Producers with higher levels of debt to capital tend to have substantially lower (as much as 50% lower) average

[5] See Larrain and Stumpner (2012) for a recent application that studies the effect of financial frictions on misallocation.

product of capital, consistent with the interpretation that the higher debt levels reflect looser collateral constraints.

We find small differences, however, in the variance of the average product of capital (except for a non-monotonic relationship in the case of China) across producers in the different terciles. Once again, differences in this particular measure of the extent to which producers are constrained do not translate into visible differences in the amount of measured misallocation in the data. This result is consistent with the predictions of our model in which differences in θ across producers do not greatly increase the degree of misallocation.

D.3 Ownership Differences in China

We finally compare the micro-level moments and the extent of misallocation across producers in our Chinese dataset that differ in their ownership structure. It has been well documented that the Chinese financial system is dominated by large and inefficient state-owned banks. (See, for example, Allen et al (2007)). As a result of the state-owned banks' preferential treatment, state-owned manufacturing firms face much more lenient borrowing constraints compared to privately-owned or foreign-owned producers.

Table A8 summarizes our findings. As Hsieh and Klenow (2009) do, we find a significant difference in the level of $\ln(\frac{Y}{K})$ across ownership groups. Consistent with the largely anecdotal evidence, the shadow cost of borrowing of private- and foreign-owned firms is 12% and 8% higher than that of state-owned firms. (As in all our analysis in this Appendix, we have controlled for 2-digit industry-year effects in making these calculations). Weighing producers by their size, the average product of capital of private-owned producers is even greater, around 25% greater than than of state-owned firms. This dispersion, as in our model with heterogeneity in borrowing rates, does not translate into large TFP losses, however. The TFP losses due to the heterogeneity in the average product of capital across the three groups are only equal to 0.7%.

Differences in the severity of borrowing constraints also do not translate into much greater measures of misallocation within a given category. The variance of the average product of capital is very close for the different type of producers — 0.87 for state-owned enterprises vs. 0.84 for privately-owned producers, suggesting once again that financial frictions do not greatly increase the amount of measured misallocation in the data.

D.4 Differences in Growth Rates

We next exploit the prediction of our model that financial constraints mostly affect the fast-growing firms whose productivity is increasing and who thus need to increase their capital stocks, as opposed to slow-growing firms whose productivity is declining and need to sell capital.

Table A9 reports the mean and variance of the average product of capital for fast-growing producers (producers in the top decile by output, capital, or productivity growth) and compares them to the mean and variance of the average product of capital for slow-growing producers (producers in the lowest decile of growth), in both our Benchmark model and in the data. Our Benchmark model predicts that in the economy without external finance fast-growing producers have an average product of capital that is about 40% greater than that of slow-growing producers. Moreover, the fast-growing producers are characterized by greater dispersion in the average product of capital (about 0.2 greater variance). These differences disappear in an economy with well-developed financial markets, such as the one calibrated to data from Korea, in which few producers are constrained.

Consider next the evidence from the three countries we study. Note first that fast-growing producers (as measured by output or productivity growth) do indeed have much greater average product of capital than slow-growing producers, but that this gaps are largest for Korea and lowest for Colombia, the opposite of what theory predicts. Recall, however, that this counter-factual implication of the model can be remedied by introducing physical capital adjustment frictions which would also prevent fast-growing producers from increasing their capital stocks, thus leading to greater average product of capital.

As for the dispersion in the average product of capital, we do not find large differences in the variance of Y_i/K_i across fast and slow-growing producers, as measured by output or capital growth rates. The variance of Y_i/K_i is, however, greater for producers whose productivity increases faster, and the gap in this variance does increase with a country's level of financial development (from a difference in variance of 0.09 in Korea to 0.21 in Colombia and 0.31 in China). These differences are in the neighborhood of what the model predicts, thus leading us to conclude that our model does not understate much the extent to which faster productivity growth translates into greater dispersion in the average product of capital.

E Additional Microeconomic Implications

In this section we briefly discuss a number of micro-economic implications of the model and compare these to the data.

E.1 Model

Table A10 reports the key microeconomic implications of tighter borrowing constraints in various versions of our models. Consider first the Benchmark model.

A first implication of the Benchmark model is that financial frictions act like an adjustment cost and prevent constrained firms from adjusting their capital stocks in response to changes in productivity. One way to see this is to notice that the standard deviation of the growth rates of output and capital declines substantially as we tighten the borrowing constraint. The standard deviation of changes in both capital and output is equal to about 0.58 in an economy calibrated to Korea's financial statistics, and about twice smaller in an economy with no external finance. Another way to see that financial frictions act like an adjustment cost is to consider how the capital stock, output, and productivity comove from one period to another. In the relatively frictionless version of the model calibrated to Korea, output and capital comove almost perfectly and both respond almost one-for-one to changes in productivity (the elasticities of changes in output and capital to changes in productivity are equal to 0.9 and 0.85, respectively). In contrast, absent external finance, the capital stock changes by only 0.61% for every 1% change in output. Moreover, both capital and output respond much less to changes in productivity: output increases by only 0.44% for every 1% increase in a producer's productivity, while the capital stock increases by only 0.14%.

A second prediction of the Benchmark model is that more severe borrowing constraints tend to disproportionately affect young producers who have not yet managed to accumulate sufficient internal funds. One way to measure the extent to which young producers are constrained is to calculate their average product of capital. We calculate the average product of capital of producers in different age groups and note in Table A10 that in an economy with Korea's level of financial development the average product of capital of young (ages 1-5) producers is only about 0.08 log-points greater than that of old (ages 11 and above) producers. In contrast, when we eliminate the producer's ability to issue equity by setting χ equal to 0 but leaving θ unchanged, the average product of capital of young producers is 0.50 log-points greater than that of old producers. Eliminating external finance altogether by setting θ equal to zero increases the relative average product of capital of the young producers to 0.73 log-points.

Yet another statistic that is often used to measure the extent to which entering producers are constrained is their relative growth rate. If young producers are more constrained, they grow faster than older producers as they accumulate internal funds.[6] Interestingly, our model does not predict a monotone relationship between the relative growth rate of young producers and the size of financial frictions. For example, the annual growth rate of output of the

[6]See, for example, Cooley and Quadrini (2001).

youngest producers is about 38% larger than that of old producers in an economy with no equity issuance and the mild collateral constraint ($\theta = 0.86$) that is consistent with Korea's debt to output ratio, but declines to 0.12 in an economy without external finance. The reason for this negative relationship is a *selection* effect. A decline in θ reduces the fraction of producers that enter the modern sector. Since those producers that do enter are the least constrained ones, the relative growth rate of young producers does not necessarily increase when θ declines.

A final set of micro-statistics that we report are the cross-sectional dispersion in the average product of capital, which in our Benchmark model are directly related to the TFP losses from misallocation within the modern sector. As Table A10 shows, the variance of the average product of capital increases from about 0.01 in the economy with Korea's level of financial development to 0.14 in the economy without external finance.

We argue next that most of this increase in dispersion and thus most of the TFP losses from misallocation are accounted for by differences in age across producers. To see this, consider a regression of the logarithm of the average product of capital on age dummies:

$$\ln(Y_i/K_i) = \sum_a \gamma_a D_{a,i} + \varepsilon_i, \tag{7}$$

where a is the age and $D_{a,i}$ is an age dummy for producer i. As Table A10 shows, the variance of the fitted values from this regression is equal to 0.12 in the Benchmark economy with no external finance, so that differences in age across producers account for about 85% of all variation in the average product of capital.

Table A10 also reports how the variance of the residuals of the regression in (7) varies across young (ages 1 to 5) and old (ages 11 and above) producers. The variance of the residuals is somewhat higher for younger producers (0.03) than old producers (0.02), reflecting both the fact that the younger producers are more constrained and less able to respond to productivity shocks, as well as the fact that the net worth of younger producers is more dispersed.

Importantly, most of these micro-level implications of the Benchmark model are not robust across the different versions of the model we have considered. First, note that in the economy with technology adoption financial frictions strongly reduce the growth rate of young producers. Financial frictions in this environment prevent the adoption of the more efficient technology and hinder growth. This version of the model can thus rationalize the observation of Hsieh and Klenow (2012) that plants in less-developed economies exhibit less growth than plants in the U.S.[7] Moreover, the relative average product of capital of the

[7]See the work of Cole, Greenwood and Sanchez (2012) who explicitly model the frictions that prevent producers in developing countries from adopting the high-growth technologies adopted in the U.S. and can quantitatively account for the pattern of plant growth in Mexico, India and the U.S.

youngest producers *declines* as we tighten the borrowing constraint, in contrast to what the Benchmark model without technology adoption predicts. This feature of the model arises again because of a selection effect. In the economy with relatively more developed financial markets producers pay the fixed cost of adopting the efficient technology early in the life-cycle and are therefore more constrained when young, both because the fixed cost depletes their net worth and because the efficient scale of production increases after adopting the more efficient technology. In the economy with no external finance producers pay the fixed cost at a much later stage in the life-cycle and are thus less constrained early on. Nevertheless, differences in age across producers continue to account for the bulk of the TFP losses from misallocation in this version of the model.

In contrast, age differences across producers account for a much smaller fraction of the overall TFP losses from misallocation in the economy with exit. The reason is the selection effect we discussed earlier. Even though the youngest producers are indeed most constrained, the fact that the least profitable (most constrained) producers exit in any given period reduces the overall amount of dispersion accounted for by age. The relative average product of capital is thus only slightly greater in the economy without external finance (0.19) than it is in the economy with Korea's level of financial development (0.13) and for this reason the TFP losses due to age are only equal to about one-fifth of the 4.0% overall losses in the economy without external finance.

Finally, notice that in the economy with predetermined capital tighter borrowing constraints do not imply that output and capital are less volatile and less responsive to productivity shocks. Indeed, the capital stock comoves little with changes in productivity (the elasticities are close to zero, and in fact negative) in both economies with weak and strong borrowing constraints. Moreover, the variance of the average product of capital is much higher than in versions of the model without physical capital adjustment frictions (0.28) and essentially insensitive to the degree of financial development.

Overall, we conclude that the micro implications of financial frictions are very sensitive to the details of the model and it is therefore difficult to use these facts to argue for or against the importance of borrowing constraints.

E.2 Cross-Country Evidence

Consider next how the set of statistics we reported above varies in the data across the three countries with different levels of financial development. We report these statistics in Table A11.

We first note that the data shows little relationship between the variability of output and capital growth rates and the degree of a country's financial development. Although output and capital growth is indeed less volatile in Colombia (for example, the standard deviation of output growth is equal to 0.46) than in Korea (0.59), both output and capital are more volatile in China (0.89).

Similarly, the data does not show a strong relationship between the extent to which output and capital respond to productivity shocks and the degree of financial development. Indeed, the capital stock responds negatively to changes in productivity, a feature that is consistent only with the version of our model with predetermined capital in which financial frictions do not have much effect on the ability of producers to reallocate capital in response to changes in productivity.

Notice further that the relative growth rates of capital and output across young and old producers are very similar across countries. Younger producers do grow faster than old producers (by about 10%), but the extent to which this is the case does not systematically vary across countries. Once again, this feature of the data is consistent with the version of our model with exit in which a selection effect is mostly responsible for the differences in growth rates across young and old producers. In that model tighter financial constraints simply lead the youngest producers (who would otherwise grow much faster) to exit.

Consider next how the average product of capital varies across age groups in the three countries we study. Interestingly, the youngest producers have the highest average product of capital in Korea and the smallest relative average product of capital in Colombia. Once again, this feature of the data is inconsistent with the predictions of our Benchmark model, but can be rationalized by the model with technology adoption in which producers are more constrained later in their life-cycle after they adopt the more productive technology.

The last rows of Table A8 report how the cross-section variance of the average product of capital varies across producers of various age groups and across countries. We see that this dispersion is similar in Korea and Colombia (the variance of $\ln Y_i/K_i$ is equal to 0.55 and 0.53, respectively), but somewhat greater in China (0.94), perhaps due to the fact that the Chinese data is on firms rather than plants and the measure of capital used there is imperfect given the lack of data on investments. Importantly, the variance of the average product of capital does not systematically vary across age groups in these three countries.

E.3 Evidence from the Korean Financial Crisis

In Figure A1 we report how the key micro-level statistics respond to a credit crunch – a one-time, unanticipated decrease in θ from its steady-state level of 0.86 to a new level $\theta' = 0.59$ chosen so as to match the halving of the debt to equity ratio in Korean Manufacturing from 1997 to 1999. We assume a permanent shock to θ since the debt to equity ratio has been quite stable in subsequent years and has not returned to its pre-crisis level. In conducting this experiment, we assume a small open economy and for simplicity eliminate the producer's ability to issue equity by setting $\chi = 0$ in both the pre- and post-crisis environment. We solve the model's transition dynamics using a shooting algorithm by iterating on the dynamics of the appropriate measures and equilibrium wage rates until convergence.

Figure A1 illustrates how the key micro-level statistics evolve over time in our model. Notice that both the standard deviation of output and capital growth rates sharply increases in the first years after the shock, as does the dispersion of the average product of capital. The reason is that the credit crunch affects relatively poor young producers a lot more than relatively wealthy old producers, thus generating dispersion in the growth rates of different types of producers. The middle panels of Figure A1 illustrate this by showing that the average product of capital increases sharply for young producers (ages 1-5) and barely changes for the much wealthier old producers (ages 11+). Similarly, the output of young producers declines considerably, while that of old producers increases because of the reduction in wages. Finally, the young producer's average product of capital becomes much more sensitive to changes in productivity in the first year of the credit crunch, reflecting these producer's inability to borrow to respond to positive productivity shocks.

Figure A2 reports a similar set of statistics in the Korean data. We note a number of facts.

First, the data shows a much more modest increase in the standard deviation of output and capital growth rates, as well as the standard deviation of the average product of capital compared to the model. Second, in contrast to the model, the data shows little relationship between the growth experience of young and old producers. Although younger producers indeed have a greater average product of capital than old producers do and also grow faster, the financial crisis did not increase this the productivity gap much, nor did it affect the growth rates of output of young producers differentially. As earlier, we conclude that differences in age across producers do not translate into large differences in the extent to which producers are constrained.

Finally, one prediction of the model that is much more in line with the data is that young producers' average product of capital becomes more sensitive to changes in productivity in the aftermath of the crisis. As in our model, therefore, financial frictions distort the reallocation of capital in response to changes in productivity and the extent to which this is the case increases in times of financial crises.

E.4 Distribution of Growth Rates of Young and Old Producers

Here we report a number of statistics that summarize the distribution of output growth rates of young and old producers in the model and in the data. Consider first our Benchmark model.

First, as Table A12 indicates, a tightening of borrowing constraints leads to a decrease in the variability of growth rates of all producers (as measured by the standard deviation or interquartile range), and especially the younger ones. Second, younger producers (who are more constrained) are characterized by less dispersed output growth rates. All these predictions are a reflection of the adjustment channel through which financial constraints prevent capital from responding to productivity shocks.

Consistent with what the model predicts, countries with less developed financial markets (such as Colombia) are indeed characterized by less volatile output growth rates. Contrary to what the model predicts, however, the youngest producers are the ones that have more volatile growth rates, in all countries we consider. This once again suggests that our model does not understate the extent to which young producers are constrained in the data.

E.5 Alternative Measures of the Average Product of Capital

Our claim that the age channel is weak in the data was based on the observation that the average product of capital is similar in the data for young and old producers and that this gap is similar across countries with different levels of financial development. While our use of the average product of capital to proxy for the marginal product is very much consistent with the existing literature (see for example Hsieh and Klenow (2009)), the concern is that the two objects may be quite different in the presence of, say, sunk capital investments or other non-convexities.

To fix ideas, suppose that operating a plant requires a sunk investment of F units of capital. After the initial investment takes place, the plant operates with a Cobb-Douglas technology which transforms *variable* capital, K, and labor, L, into output:

$$Y = A\left(L^{\alpha}K^{1-\alpha}\right)^{\eta}.$$

Under these assumptions data on Y/K alone is enough to pin down a producer's marginal product of capital. If, however, data on capital stocks include both the variable and sunk components, $K+F$, then the average product of capital, $Y/(K+F)$ no longer summarizes how constrained a particular producer is. In particular, even if young producers start very constrained (high Y/K relative to old producers) but grow over time (so that both Y and K increase), then the *measured* average product of capital, $Y/(K+F)$ may indeed change little as the producer ages, leading us to incorrectly conclude that younger producers are no less constrained than old producers.

One way in which we tried address this concern is by disaggregating the data on capital we use into various components in order to try to disentangle the sunk component, F, from the variable one, K. We consider three measures of the capital stock: all the capital a producer uses (buildings, machinery and equipment), as used throughout all of our analysis, only machinery and equipment, as well as using electricity to proxy for the variable component of the capital stock. We can disaggregate capital data for only two countries in our sample, Korea and Colombia. We find, as Table A13 reports, that the gap in the average product of capital between young and old producers is very similar (a maximum gap of 20%) for all measures of capital we consider in both countries. Since machinery and equipment and especially electricity are likely to have a smaller sunk cost component than buildings do, we interpret our results as suggesting that measurement problems are not necessarily the only explanation for our finding of a weak age channel in the data.

An alternative way to address this concern is to explicitly calculate what theory predicts about how adding the sunk component of capital, F, changes the relationship between the average product of capital and age in the data. Consider the following thought experiment. Suppose that the marginal product of capital, Y/K, declines over a producer's life cycle and evolves according to

$$\ln(Y/K)_{age} = -\gamma \ln(age),$$

where γ measures the extent to which the marginal product of capital declines over time. In our Benchmark model, tighter financial frictions disproportionately affect young producers, and so γ is positive.

In Table A14 we use information on the share of producers of various ages in Korea, together with equation (23) in text, to compute the TFP losses associated with the dispersion in marginal product of capital accounted for by age given various values of γ. Clearly, the more steeply the marginal product declines with age, the large the TFP losses are. When $\gamma = 0.25$, so that the marginal product of capital is about 47% greater for young producers relative to old producers, the TFP losses are equal to 1.7%, slightly smaller than what our model predicts. In contrast, when $\gamma = 1.25$, so that the gap in the marginal product of capital between young and old producers is equal to 250%, TFP losses are equal to 21%.

Consider next what this simple thought experiment implies about the average product of capital, $Y/(K + F)$. To answer this question, we need to make an assumption about the ratio of the sunk capital investment to total capital, $F/(K + F)$, for an entering producer, as well an assumption about how fast the typical producer grows over its life-cycle. We use the data for Korea to compute these growth rates (a plant grows about ten-fold over its life cycle in Korea, a number similar to that reported by Hsieh and Klenow (2012) for the U.S.) and report what the model predicts about the average product of capital in Table A14.

The Table shows that adding the sunk component to the measure of capital we use indeed flattens the life-cycle profile of the average product of capital relative to its marginal product.

Intuitively, since producers grow over the life-cycle, the sunk component is relatively large for young producers (who thus appear less productive) and relatively small for old producers (who thus appear more productive). For example, when $\gamma = 0.25$, the average product of capital of young producers is only about 15% greater than that of old producers when the share of sunk capital is equal to 1/2, and about 61% smaller when the share of sunk capital is equal to 0.9. These numbers are lower than the actual difference of 47% in the marginal product. Similarly, when $\gamma = 1.25$ and the share of sunk capital is equal to 0.9, the gap between the average product of capital across young and old producers is cut in half (to 1.27) relative to the gap in the marginal product of capital (2.5).

The relevant question for our analysis, however, is whether the introduction of a sunk cost changes the model's predictions about how the gap between the average product of capital of young and old producers changes with the gap between the marginal product of capital. This is clearly the case in the example we consider here. To see this, notice that moving from an economy with little dispersion in the marginal product ($\gamma = 0.25$) to an economy with a lot of dispersion in the marginal product ($\gamma = 1.25$), raises the gap in the average product as well. When, say, the share of sunk capital is equal to 0.9, the gap in the average product of capital increases by about 1.9 log-points (from -0.61 to 1.27), only slightly less than 2.0 increase in the the marginal product of capital (from 0.5 to 2.5).

We thus conclude that although adding a sunk capital component indeed makes it difficult to interpret the level of the gap between the average product of capital between young and old producers, non-convexities do not change much the model's implications about how the gap in the average product varies with the gap in the marginal product. This suggests that our observation that in the data the gap between the average product of capital across young and old producers does not vary much across countries is also suggestive of the fact that the gap in the marginal product does not vary much as well.

F Additional Robustness Checks

We next report on several additional robustness checks that we have studied. We explore the role of the nature of sunk costs in driving our results, the nature of productivity changes, as well as endogenize the initial wealth distribution of producers in the modern sector. All the analysis here is conducted in the context of a closed economy.

F.1 Role of Pledgeability of Sunk Costs

Our baseline experiments have assumed that producers can pledge both their variable capital stock, as well as their sunk investment as collateral. That is, we have assumed a collateral constraint given by:

$$D_{t+1} \leqslant \theta \left(K_{t+1} + \kappa \exp\left(z\right)\right)$$

We next relax this assumption and assume that only the variable stock can be used as collateral:

$$D_{t+1} \leqslant \theta K_{t+1}$$

The first two columns of Table A15 report the effect this modification has on our Benchmark economy calibrated to Korea with high values of external finance (clearly, this modification has no consequence in an economy with $\theta = 0$ and thus no external finance). We note that the losses from misallocation increase slightly, from 0.3% to 1.9%, but that the TFP in the modern sector drops quite a bit, by about 6%, reflecting the reduction in the measure of producers that enter the modern sector. Intuitively, producers are less likely to enter the modern sector now given that they take longer to accumulate the internal funds necessary to pay for the sunk cost of entering.

F.2 Sunk Costs vs. Fixed Costs

We have argued that the important role played by the extensive margin distortions arises because financial frictions preclude producers from paying the upfront sunk investment costs. To see this, we next study a version of the model in which producers must pay a fixed, rather than sunk cost, in each period after entering the modern sector, as in Buera, Kaboski and Shin (2011). As above, we assume that the fixed cost is proprtional to a producer's permanent productivity component, z. We set the fixed cost equal to

$$F\left(z\right) = \frac{\kappa\left(z\right)}{\left(\beta/\gamma\right)^{-1} - 1},$$

where recall that β/γ is the planner's discount factor. Given this choice of the size of the fixed costs, the allocations under the efficient allocations without credit frictions would be identical in the economy with fixed costs and sunk costs, since the present value of the two streams of payments is identical. With borrowing constraints, however, producers prefer to pay the fixed costs gradually over time and are thus better off in the economy with fixed costs. The last two columns of Table A15 illustrate this: allocations are very close in the two economies when financial markets are well-developed (we use the numbers for θ and χ from our benchmark Korean calibration), but the levels of TFP, consumption and output are significantly reduced in the sunk cost economy compared to the fixed cost economy. For example, in the sunk cost model only 29% of producers reside in the modern sector in an economy without external finance, while in the fixed cost model 63% of producers reside in the modern sector. Consequently TFP, consumption and output are significantly reduced in the economy with sunk costs and no external finance. Also note that the TFP losses from misallocation are greater as well in the economy with sunk costs (9.8% vs. 7.4%), reflecting the fact that young producers are more constrained.

F.3 Role of Transitory Shocks

We have shown above that the process for transitory shocks plays a key role in our analysis: the more volatile such shocks are, the greater the TFP losses from misallocation. Here we eliminate the transitory shocks altogether and study the implications of the resulting model in Table A16.

Notice in the table that the model without transitory shocks clearly cannot reproduce the variability of output in the data. The standard deviation of output changes is equal to only 0.08 in an economy with Korea's level of financial development, much lower than the 0.59 in the data and in our benchmark model with transitory shocks. We find, however, the the two models' implications for the TFP losses from financial frictions are virtually identical: both the extensive margin distortions (the number of producers in the modern sector is equal to 46% in an economy without external finance in both models) and the TFP losses from misallocation (7.2% in the economy with transitory shocks vs. 5.9% in the economy without transitory shocks) are quite similar in the economies with and without transitory shocks. This reflects the minor role played by the adjustment channel in this model.

F.4 Economies with Endogenous Initial Wealth Distributions

We next study a version of the model in which we increase the amount of dispersion in entering producer's wealth (as we have done earlier), but do so endogenously. In particular, we assume that a constant fraction of producers experience a sudden change in their permanent productivity component. In addition, if a producer resides in the modern sector, we assume that it loses its sunk cost investment and must pay the entry cost again in order to return to the modern sector. Intuitively, with such shocks the model can produce a more dispersed distribution of initial wealth of the producers that join the modern sector. Now there are two types of entering producers: those who have never been to the modern sector, and those who have received the *destruction* shock and thus inherit their pre-existing wealth but need to pay the sunk cost again and draw a new level of productivity. This modification nests both our original model, as well as elements from Buera, Kaboski and Shin (2011).

The entrepreneur's dynamic programming problem is given by

$$V^{m}(a,i) = \max_{c} \log(c) + \beta\rho V^{m}(a',i) + \beta(1-\rho)\sum_{j=1}^{K} g_j V^{\tau}(a',j)$$

subject to:

$$a' = \pi_i^m(a) + (1+r)a - c$$

and

$$V^\tau(a,i) = \max_c \log(c) + \beta \max \left\{ \begin{array}{c} \rho V^\tau(a',i) + (1-\rho)\sum g_j V^\tau(a',j), \\ \rho V^m(a' - \kappa \exp(z_i), i) + (1-\rho)\sum g_j V^m(a' - \kappa \exp(z_i), j) \end{array} \right\}$$

subject to:

$$a' = \pi_i^\tau(a) + (1+r)a - c,$$

where $V^m(a,i)$ is the value of a producer that starts in the modern sector with net worth a and productivity i and $V^\tau(a,i)$ is the value of a producer that starts in the traditional sector.

Given that we have shown above that our model's aggregate implications do not depend much on the presence of transitory productivity shocks, we abstract from such shocks in these experiments. The arguments i and j above therefore refer to the producer's permanent productivity component. Notice that the producer either keeps its current level of productivity (with probability ρ) or draws a new productivity (with probability $1-\rho$) from an invariant distribution g_i. In addition, a producer that experiences a productivity change and resides in the modern sector is forced out of that sector and can only return after paying the sunk cost again.

In Table A17 we compare the aggregate implications of economies with perfect credit markets ($\theta = 1$) and economies with no external finance ($\theta = 0$). We report results for the economy with destruction shocks (upper panel) and without such shocks (lower panel). We also consider two variations of the model with and without sunk entering costs.

In terms of the parameterization, we set $\rho = 0.95$ in the economy with destruction shocks and lower the exogenous growth rate γ to ensure a steady-state entry rate that is identical to that in the Benchmark model. We set the size of the sunk cost equal to 1/3 its value in the original calibration since otherwise no producer finds it optimal to enter the modern sector given that such investments depreciate over time at rate $1-\rho$. We assume a Gaussian distribution of the permanent productivity component and choose the standard deviation, σ_z, equal to 2.5 so that the model can reproduce the fact that the largest 10% of producers in the Korean data jointly account for 85% of all output. We leave all other parameters of the model unchanged.

Comparison of the upper and lower panels of Table A17 makes it clear that adding destruction shocks and thus increasing the dispersion in entering producers' wealth does not change our model's results much. The losses from misallocation in an economy without external finance are equal to 10.2% absent such shocks and increase to only 10.7% with such shocks in the economy with sunk costs. Absent sunk costs, the model with destruction shocks generates only somewhat greater misallocation (12.1% vs. 10%).

F.5 Uncorrelated modern/traditional productivity

We next explore the consequence of relaxing the assumption that a producer that switches to a modern sector inherits its existing level of productivity. In particular, we now set $\rho = 1$ in the economy described above but assume that the producer that switches to the modern sector experiences a change in its productivity with a probability of 1/2 and draws from the same invariant distribution g we assumed earlier. Table A17 shows that the losses from misallocation increase now to 14.4% (10.2% with perfectly correlated shocks), reflecting the wedge between the net worth and productivity of entering producers.

One way to motivate our benchmark model's assumption of perfect correlation of producer productivity across sectors is to note that in the data the variance of the average product of capital across producers of a given age does not systematically vary with age in countries with low levels of financial development. (See Table A11). The model with perfectly correlated productivity shocks is consistent with this prediction (the variance of the average product of capital is 0.07 for producers of ages 1 to 5 and 0.09 for producers older than 10 years), while the model with imperfectly correlated shocks predicts that the variance is much greater (by about 0.10 points) for young than for old producers.

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Table A1: Role of Heterogeneity in Initial Net Worth

	Benchmark	$\sigma_a = 1$	$\sigma_a = 2$	$\sigma_a = 5$
TFP losses, %	4.7	3.4	4.3	4.7
Losses due to age, %	3.7	2.0	3.2	3.7
Losses among 1-5, %	1.3	6.3	4.4	1.4
Losses among 11+, %	0.8	0.9	0.8	0.8
var(Y/K)	0.14	0.12	0.14	0.14
var(Y/K) due to age	0.12	0.06	0.10	0.12
var(Y/K) 1-5	0.03	0.14	0.07	0.03
var(Y/K) 11+	0.02	0.02	0.02	0.02

Note: All numbers refer the version of the model with no external finance

Table A2: Moments

		Korea	s.e.	China	s.e.	Colombia	s.e.
A. Size dispersion	std. dev. changes log output	0.59	0.002	0.89	0.001	0.46	0.008
	std. dev. changes log labor	0.49	0.002	0.64	0.001	0.34	0.005
	std. dev. changes log capital	0.57	0.002	0.67	0.002	0.41	0.005
	std. dev. log output	1.31	0.004	1.45	0.002	1.61	0.013
	std. dev. log labor	1.21	0.004	1.34	0.002	1.41	0.013
	std. dev. log capital	1.44	0.005	1.73	0.002	1.78	0.016
B. Persistence	1-year autocorrelation output	0.90	0.001	0.80	0.001	0.96	0.002
	3-year autocorrelation output	0.86	0.002	0.70	0.001	0.93	0.003
	5-year autocorrelation output	0.84	0.003	0.65	0.002	0.92	0.005
	1-year autocorrelation labor	0.92	0.001	0.88	0.001	0.97	0.001
	3-year autocorrelation labor	0.89	0.001	0.80	0.001	0.94	0.002
	5-year autocorrelation labor	0.86	0.002	0.75	0.002	0.92	0.003
	1-year autocorrelation capital	0.92	0.001	0.92	0.000	0.97	0.001
	3-year autocorrelation capital	0.89	0.001	0.85	0.001	0.93	0.002
	5-year autocorrelation capital	0.86	0.003	0.80	0.002	0.90	0.004
4. Fraction of firms by age	1-5	0.51	0.001	0.44	0.001	0.22	0.004
	5-10	0.27	0.001	0.26	0.000	0.21	0.004
	11+	0.23	0.001	0.33	0.001	0.57	0.006
5. Fraction of value-adde by age	1-5	0.20	0.011	0.29	0.005	0.06	0.006
	5-10	0.20	0.014	0.25	0.005	0.08	0.011
	11+	0.60	0.023	0.44	0.008	0.86	0.015
6. Fraction of wage bill by age	1-5	0.21	0.010	0.25	0.005	0.06	0.004
	5-10	0.20	0.010	0.23	0.009	0.08	0.006
	11+	0.58	0.018	0.48	0.009	0.87	0.009
7. Fraction of capital by age	1-5	0.22	0.015	0.27	0.006	0.13	0.038
	5-10	0.21	0.023	0.22	0.005	0.10	0.027
	11+	0.57	0.035	0.50	0.009	0.76	0.062
8. Employment to capital ratio, by age	aggegate, 1-5	-0.81	0.033	-1.97	0.010	-1.44	0.139
	aggregate, 6-10	-0.77	0.033	-1.90	0.014	-0.88	0.096
	aggregate 11+	-0.79	0.030	-1.88	0.015	-0.64	0.049
	mean 1-5	-0.23	0.003	-1.24	0.002	-0.40	0.020
	mean 6-10	-0.32	0.004	-1.23	0.003	-0.25	0.016
	mean 11+	-0.35	0.005	-1.29	0.004	0.01	0.014
9. Relative aggregate growth rates	output, plants aged 1-5 vs. 11+	0.11	0.015	0.11	0.010	0.12	0.036
	output, plants aged 6-10 vs. 11+	0.05	0.038	0.04	0.009	0.00	0.015
	employment, plants aged 1-5 vs. 11+	0.09	0.006	0.10	0.005	0.07	0.010
	employment, plants aged 6-10 vs. 11+	0.02	0.005	0.05	0.005	0.03	0.006
	capital, plants aged 1-5 vs. 11+	0.09	0.023	0.07	0.007	-0.04	0.014
	capital, plants aged 6-10 vs. 11+	-0.03	0.022	0.02	0.007	-0.03	0.018
# plants		165137		559013		10195	
# observations		479719		2001757		41751	

Table A3: Production Function Estimates. Korea

	Index Number (η = 0.85)		Blundell Bond		
	α	ρ	α	ρ	η
Food	0.64	0.11	0.65	0.03	0.83
Tobacco	0.72	NA	NA	NA	NA
Textile	0.72	0.11	0.80	0.16	0.82
Apparel	0.82	0.07	0.66	0.22	0.94
Leather	0.74	0.03	0.73	0.16	0.83
Wood	0.69	0.08	0.68	0.00	0.67
Paper	0.65	0.07	0.71	0.00	0.81
Publishing	0.67	0.04	0.65	0.01	0.70
Petroleum	0.57	0.13	0.56	0.00	0.43
Chemical	0.60	0.12	0.75	0.02	0.59
Rubber/Plastic	0.67	0.11	0.71	0.01	0.68
Non-metallic Mineral	0.65	0.12	0.72	0.06	0.85
Basic Metal	0.64	0.13	0.61	0.02	0.61
Fabricated Metal	0.68	0.05	0.67	0.06	0.78
Machinery and equipment	0.67	0.09	0.73	0.05	0.79
Computer/Office Mach	0.70	0.10	0.67	0.17	0.75
Electrical Mach	0.68	0.08	0.72	0.03	0.68
Electronic Component	0.72	0.14	0.73	0.04	0.69
Medical Precision Inst.	0.71	0.13	0.71	0.03	0.67
Motor Vehicles	0.69	0.13	0.76	0.02	0.81
Other Transport	0.70	0.12	0.63	0.00	0.63
Furniture/Other	0.72	0.06	0.71	0.14	0.73
Mean	0.68	0.10	0.69	0.06	0.73
Std. dev.	0.05	0.03	0.05	0.07	0.11

Table A4: Comparison of Alternative Estimates of Plant Productivity

		Index number	Blundell Bond Estimates	
		η = 0.85	η = 0.74	η = 0.85
Correlation with index number estimate				
	e+z	1	0.93	0.97
	e	1	0.92	0.95
Measured TFP losses, %				
	overall	16.2	9.4	14.5
	due to age	0.2	0.1	0.2
	if K fixed	2.4	1.2	2.0

Table A5: Labor Input Elasticity. Index Number Estimates

I. Colombia

Food	0.54
Beverage	0.45
Tobacco	0.58
Textile	0.65
Apparel	0.78
Leather	0.75
Wood	0.69
Furniture	0.84
Paper	0.55
Printing/publishing	0.68
Chemical	0.54
Petroleum	0.61
Rubber	0.60
Plastic	0.57
Glass	0.78
Non-metallic mineral	0.59
Iron and steel	0.61
Nonferrous metal	0.53
Metal products	0.69
Machinery	0.68
Electronic Machinery	0.60
Transportation	0.68
Professional equipment	0.68

II. China

Food	0.56
Beverage/alcohol	0.44
Tobacco	0.40
Textile	0.75
Apparel	0.95
Leather	0.85
Wood	0.62
Furniture	0.70
Paper	0.64
Printing/publishing	0.80
Stationary/sporting goods	0.87
Petroleum	0.47
Basic chemical	0.52
Drug	0.44
Chemical fiber	0.57
Rubber	0.67
Plastic	0.67
Non-metallic Mineral	0.65
Steel/Iron	0.54
Non-ferrous metal	0.53
Fabricated metal	0.69
Machinery/equipment (general)	0.67
Machinery/equipment (specialized)	0.65
Transportation	0.69
Electrical Machinery	0.63
Electronics	0.74
Precision Instruments	0.70

Table A6: Role of External Finance Dependence

		Korea	China	Colombia
TFP losses, %	all	16.2	22.4	17.7
	due to EFD differences	0.3	0.1	0.2
avg(Y/K)	low EFD	0	0	0
	medium EFD	-0.17	-0.03	0.00
		[0.003]	[0.000]	[0.009]
	high EFD	0.01	0.00	-0.03
		[0.003]	[0.002]	[0.011]
var(Y/K)	low EFD	0.49	0.84	0.50
	medium EFD	0.50	0.87	0.51
	high EFD	0.48	0.86	0.51

Note: standard errors reported in parantheses

Table A7: Role of Differences in Debt-to-Capital Ratio

		Korea	China
Measured TFP losses, %	overall	16.2	22.4
	due to D/K differences	1.2	2.8
avg. ln (Y/K)	medium-low D/K	-0.26	-0.57
		[0.001]	[0.009]
	high-low D/K	-0.04	-0.39
		[0.002]	[0.01]
var(Y/K)	low D/K firms	0.46	0.88
	medium D/K firms	0.46	0.57
	high D/K firms	0.45	0.83

Note: standard errors reported in parantheses

Table A8: Role of Ownership Structure in China

TFP losses, %	all	22.4
	Due to Ownership	0.7
avg. ln (Y/K)	private-SOE	0.12
		[0.002]
	foreign-SOE	0.08
		[0.002]
var(Y/K)	SOE	0.87
	private	0.84
	foreign	0.89

Note: standard errors reported in parantheses

Table A9: Role of Differences in Growth Rates

	Benchmark Model		Korea	China	Colombia
	"Korea"	No External Finance			
mean(ln Y/K) for producers in:					
top vs. bottom 10% by output growth	0.03	0.48	0.56	0.29	0.04
top vs. bottom 10% by capital growth	0.01	0.33	-0.66	-0.68	-0.39
top vs. bottom 10% by productivity growth	0.06	0.37	0.88	0.55	0.36
var(ln Y/K) for producers in:					
top 10% by output growth	0.005	0.19	0.59	1.05	0.53
bottom 10% by output growth	0.000	0.01	0.57	1.09	0.58
top 10% by capital growth	0.002	0.20	0.53	0.87	0.45
bottom 10% by capital growth	0.000	0.00	0.51	1.08	0.60
top 10% by productivity growth	0.013	0.14	0.57	1.17	0.75
bottom 10% by productivity growth	0.000	0.07	0.48	0.86	0.53

Table A10: Micro-Economic Implications. Model

	Benchmark Model			Adoption		Exit		Predet. Capital	
	"Korea"	No equity	No finance	"Korea"	No finance	"Korea"	No finance	"Korea"	No finance
Debt to output	1.2	1.1	-0.6	1.2	-0.2	1.2	-0.6	1.2	0.0
Equity to output	0.3	0.0	0.0	0.3	0.0	0.3	0	0	0
Std. dev. output growth	0.58	0.55	0.32	0.58	0.26	0.59	0.40	0.59	0.61
Std. dev. capital growth	0.57	0.57	0.25	0.53	0.27	0.52	0.31	0.16	0.11
Elasticity ΔK to ΔY	0.98	1.00	0.61	0.85	0.36	0.82	0.54	-0.13	-0.09
Elasticity ΔY to Δe	0.90	0.75	0.44	0.64	0.27	0.60	0.41	0.31	0.32
Elasticity ΔK to Δe	0.85	0.62	0.14	0.44	-0.12	0.39	0.10	-0.05	-0.03
Rel. output growth 1-5 vs. 11+	0.09	0.32	0.12	0.28	0.08	0.10	0.08	-	-
Rel. capital growth 1-5 vs. 11+	0.13	0.46	0.19	0.28	0.15	0.12	0.07	-	-
avg(Y/K) 1-5 vs. 11+	0.08	0.50	0.73	0.27	0.22	0.13	0.19	-	-
var (Y/K)	0.01	0.09	0.14	0.04	0.14	0.04	0.10	0.28	0.27
var(Y/K) due to age	0.00	0.08	0.12	0.03	0.09	0.00	0.01	-	-
var(Y/K) 1-5	0.01	0.04	0.03	0.02	0.03	0.05	0.11	-	-
var(Y/K) 11+	0.00	0.00	0.02	0.00	0.04	0.01	0.05	-	-

Table A11: Micro-Economic Implications. Data

	Korea	China	Colombia
Std. dev. output growth	0.59	0.89	0.46
Std. dev. capital growth	0.57	0.67	0.41
Elasticity DK to DY	0.23	0.07	0.39
Elasticity DY to De	0.27	0.05	0.13
Elasticity DK to De	-0.20	-0.40	-0.14
Rel. output growth 1-5 vs. 11+	0.11	0.11	0.12
Rel. capital growth 1-5 vs. 11+	0.09	0.07	-0.04
avg(Y/K) 1-5 vs. 11+	0.21	0.15	-0.25
var (Y/K)	0.55	0.94	0.53
var(Y/K) due to age	0.06	0.09	0.06
var(Y/K) 1-5	0.56	1.02	0.48
var(Y/K) 11+	0.52	0.85	0.56

Table A12: Distribution of Growth Rates: Young vs. Old

	Benchmark Model		Korea	China	Colombia
	"Korea"	No External Finance			
std. dev. Δ ln y					
young (1-5)	0.46	0.20	0.51	0.45	0.33
old (11+)	0.63	0.40	0.38	0.39	0.23
25th percentile Δ ln y					
young (1-5)	-0.24	0.02	-0.10	-0.06	-0.07
old (11+)	-0.36	-0.15	-0.13	-0.11	-0.08
50th percentile Δ ln y					
young (1-5)	0.15	0.15	0.07	0.11	0.06
old (11+)	0.00	0.05	0.01	0.03	0.02
75th percentile Δ ln y					
young (1-5)	0.36	0.28	0.33	0.33	0.24
old (11+)	0.36	0.29	0.18	0.18	0.13

Table A13: Alternative measures of the Average Product of Capital

		Korea	Colombia
All capital	avg(Y/K) 1-5 vs. 11+	0.21	-0.25
	var (Y/K)	0.55	0.53
	var(Y/K) due to age	0.02	0.03
	var(Y/K) 1-5	0.56	0.48
	var(Y/K) 11+	0.52	0.56
Machinery and Equipment	avg(Y/K) 1-5 vs. 11+	0.18	-0.18
	var (Y/K)	0.86	0.87
	var(Y/K) due to age	0.01	0.01
	var(Y/K) 1-5	0.88	0.81
	var(Y/K) 11+	0.81	0.91
Energy	avg(Y/K) 1-5 vs. 11+	0.09	0.19
	var (Y/K)	0.79	0.69
	var(Y/K) due to age	0.00	0.01
	var(Y/K) 1-5	0.79	0.71
	var(Y/K) 11+	0.79	0.67

Table A14: MPK vs. APK with sunk capital investment

	$\gamma = 0.25$			$\gamma = 0.75$			$\gamma = 1.25$		
share of capital that is sunk, age = 1	0.5	0.7	0.9	0.5	0.7	0.9	0.5	0.7	0.9
avg(MPK) 1-5 vs. 11+		0.47			1.45			2.50	
avg(APK) 1-5 vs. 11+	0.15	-0.10	-0.61	1.11	0.86	0.26	2.13	1.87	1.27
shadow cost of funds, 1-5		0.13			0.63			2.70	
shadow cost of funds, 11+		0.06			0.10			0.17	
TFP losses, %		1.7			10.9			21.0	

Table A15: Additional Robustness Checks. Closed Economies

	Benchmark	Sunk cost model		Fixed cost model	
Is κ pledgeable?	Yes	No	No	No	No
	$\theta = 0.86$, $\chi = 0.10$	$\theta = 0.86$, $\chi = 0.10$	$\theta = 0$	$\theta = 0.86$, $\chi = 0.10$	$\theta = 0$
Debt to output (modern)	1.18	1.05	0	1.16	0
Equity to output (modern)	0.29	0.42	0	0.25	0
Interest rate	0.047	0.038	-0.060	0.039	-0.060
Fraction constrained	0.17	0.36	1.00	0.14	1.00
Capital to output (modern)	2.59	2.72	2.18	2.81	2.09
TFP (modern)	1.000	0.938	0.766	0.959	0.881
Loss misallocation, %	0.3	1.9	9.8	0.2	7.4
Fraction producers modern	0.93	0.67	0.29	0.69	0.63
Fraction output modern	0.99	0.94	0.45	0.95	0.80
Consumption	1.00	0.94	0.71	0.95	0.77
Output	1.68	1.55	1.06	1.63	1.26

Table A16: Role of Transitory Shocks. Closed economies

	Benchmark with transitory shocks		No transitory shocks	
	$\theta = 0.86, \chi = 0.10$	$\theta = 0.50, \chi = 0$	$\theta = 0.86, \chi = 0.10$	$\theta = 0.50, \chi = 0$
s.d. output growth	0.58	0.26	0.08	0.10
s.d. output	1.30	1.54	1.30	1.58
autocorr. output (1 year)	0.90	0.99	0.99	1
autocorr. output (5 years)	0.86	0.96	0.99	1
Debt to output (modern)	1.05	1.51	1.38	1.59
Equity to output (modern)	0.42	0	0.24	0
Interest rate	0.047	0.010	0.058	0.012
Fraction constrained	0.17	0.93	0.21	1.00
Capital to output (modern)	2.59	2.83	2.35	2.71
TFP (modern)	1.000	0.841	0.981	0.836
Loss misallocation, %	0.3	7.2	0.2	5.9
Fraction producers modern	0.93	0.46	0.93	0.46
Fraction output modern	0.99	0.80	0.99	0.79
Consumption	1.00	0.88	0.99	0.87
Output	1.68	1.33	1.16	1.29

Table A17: Economies with Endogenous Initial Wealth Heterogeneity
Closed Economies

A. With destruction shocks	With Sunk Costs		No Sunk Costs	
	Frictionless	No Borrowing	Frictionless	No Borrowing
Interest rate	0.046	-0.060	0.052	-0.060
Fraction constrained	0.00	1.00	0.00	1.00
TFP (modern)	1.00	0.90	1.00	0.88
Loss misallocation, %	0.0	10.7	0.0	12.1
Fraction producers modern	0.36	0.25	0.93	0.86
Fraction output modern	0.77	0.15	1.00	0.98
Consumption	1.94	0.79	1.00	0.81
Output	1.24	0.85	1.29	1.01

B. Without destruction shocks	With Sunk Costs		No Sunk Costs	
	Frictionless	No Borrowing	Frictionless	No Borrowing
Interest rate	0.048	-0.060	0.046	-0.060
Fraction constrained	0.00	1.00	0.00	1.00
TFP (modern)	1.00	0.88	1.00	0.91
Loss misallocation, %	0.0	10.2	0.0	10.0
Fraction producers modern	0.12	0.11	0.93	0.54
Fraction output modern	0.80	0.48	1.00	0.99
Consumption	1	0.88	1.00	0.89
Output	1.45	1.12	1.59	1.27

Table A18: Imperfectly Correlated Productivity Across Sectors
Closed Economies

	Perfectly Correlated		Imperfectly Correlated	
	Frictionless	No Borrowing	Frictionless	No Borrowing
Interest rate	0.048	-0.060	0.040	-0.060
Fraction constrained	0.00	1.00	0.00	1.00
TFP (modern)	1.00	0.88	1.00	0.72
Loss misallocation, %	0.0	10.2	0.0	14.4
Fraction producers modern	0.12	0.11	0.21	0.09
Fraction output modern	0.80	0.48	0.75	0.13
Consumption	1	0.88	1.00	0.94
Output	1.45	1.12	1.44	0.99

Figure A1: Micro-level statistics during a credit crunch

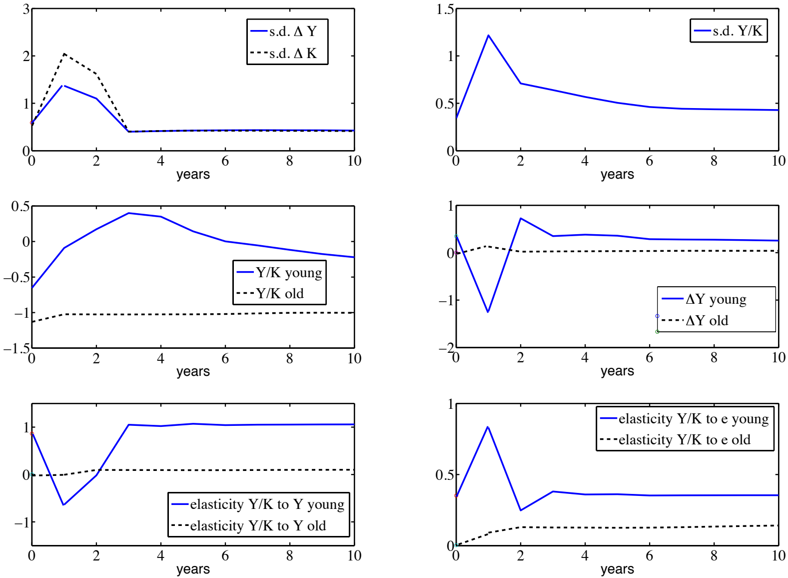

Figure A2: Micro-level statistics. Korean Manufacturing