Supplementary Appendix

Debt versus Pensions

It is well known that a pension system and government debt are basically equivalent policy instruments, at least in deterministic models or models with complete markets. Here we derive conditions under which equivalence holds in a stochastic incomplete markets model with distortive taxation. This means that all taxes and transfers that a household has to pay are linearly related to the household decisions on labor supply and saving. There are no lump sum taxes or transfers. For reasons of space, we confine ourselves to the case of two-period lives, where households work in the first period and retire in the second. Similar results hold with many cohorts, if the government has age-dependent taxes available. In order to establish these equivalence results, we stay within an incomplete-markets framework with only one asset and therefore assume that government bonds yield the same stochastic return as physical capital, so that government debt and capital are perfect substitutes for the household. We also introduce capital income taxes.

In a stochastic setup, the pension system serves two purposes. First, it shifts resources from future to current generations, which we assume reflects the preferences of the policy maker. Second, it helps to spread the effects of shocks efficiently between different generations. To highlight both aspects, we distinguish between three different fiscal policy regimes:

1) Debt policy (**DP**). The instruments are government debt, a labor income tax at rate τ_t^d and the capital tax τ_t^c. The capital tax is state dependent, i.e., the tax rate that applies in period t is only determined in t. Government debt follows

(31)
$$(1+r_t)d_{t-1,0}N_{t-1,0}-d_{t,0}N_{t,0} = \tau_t^d w_{t,0}L_{t,0}N_{t,0}+\tau_t^c r_t(k_{t-1,0}+d_{t-1,0})N_{t-1,0} \quad \text{for all } t.$$

Here, $k_{t,0}$ and $d_{t,0}$ denote asset holdings in the form of capital and debt, respectively, such that $k_{t,0} + d_{t,0} = A_{t,0}$. As only the young are savers in our simplified two generations economy, we have that $K_t = k_{t,0}N_{t,0}$ and $D_t = d_{t,0}N_{t,0}$. We assume a no-Ponzi condition on the government.

The household budget constraints are

(32a)
$$k_{t,0} + d_{t,0} + C_{t,0} = w_{t,0}(1 - \tau_t^d - \tau^l)L_{t,0} + T_{t,0}$$

(32b)
$$C_{t+1,1} = \frac{1 + r_{t+1}(1 - \tau_{t+1}^c)}{\varsigma_{t,0}}(k_{t,0} + d_{t,0})$$

and the household first order conditions are

(33a)
$$U_C\left(C_{t,0}, L_{t,0}\right) = \beta\varsigma_{t,0}\mathbb{E}_t\left[\frac{1 + r_{t+1}(1 - \tau_{t+1}^c)}{\varsigma_{t,0}}U_C\left(C_{t+1,1}, 0\right)\right]$$

(33b)
$$-U_L\left(C_{t,0}, L_{t,0}\right) = w_{t,0}(1 - \tau_t^d - \tau^l)U_C\left(C_{t,0}, L_{t,0}\right)$$

2) Pension policy with a predetermined pension factor (**PP1**). The instruments are pension contributions τ_t^p, the capital tax τ_t^c, and a pension benefit factor $\tilde{b}_t$. The capital tax is specified as in **DP**. Pension income of the old in period $t+1$ is given by

$$p_{t+1,1} = \tau_{t,0}^p w_{t,0} L_{t,0} \frac{1 + r_{t+1}(1 - \tau_{t+1}^c)}{\varsigma_{t,0}} \tilde{b}_t. \tag{34}$$

It is the product of three components: the past contributions to the pension system, $\tau_{t,0}^p w_{t,0} L_{t,0}$; the adjusted interest on contributions, $\frac{1+r_{t+1}(1-\tau_{t+1}^c)}{\varsigma_{t,0}}$; the pension factor, $\tilde{b}_t$, which is already fixed in period t (policy instruments carry a tilde if they are predetermined, which means that the rate applied in $t+1$ is already determined in t). The government budget is balanced in every period:

$$\tilde{b}_{t-1} \frac{1 + r_t(1 - \tau_t^c)}{\varsigma_{t-1,0}} \tau_{t-1,0}^p w_{t-1,0} L_{t-1,0} N_{t,1} = \tau_{t,0}^p w_{t,0} L_{t,0} N_{t,0} + \tau_t^c r_t k_{t-1,0} N_{t-1,0} \quad \text{for all } t. \tag{35}$$

The household budget constraints are

$$k_{t,0} + C_{t,0} = w_{t,0}(1 - \tau_t^p - \tau^l) L_{t,0} + T_{t,0} \tag{36a}$$

$$C_{t+1,1} = \frac{1 + r_{t+1}(1 - \tau_{t+1}^c)}{\varsigma_{t,0}} \left[k_{t,0} + \tilde{b}_t \tau_t^p w_{t,0} L_{t,0} \right] \tag{36b}$$

and the household first order conditions are

$$U_C\left(C_{t,0}, L_{t,0}\right) = \beta \varsigma_{t,0} \mathbb{E}_t \left[\frac{1 + r_{t+1}(1 - \tau_{t+1}^c)}{\varsigma_{t,0}} U_C\left(C_{t+1,1}, 0\right) \right] \tag{37a}$$

$$\begin{aligned} -U_L\left(C_{t,0}, L_{t,0}\right) = w_{t,0} \Big\{ & (1 - \tau_t^p - \tau^l) U_C\left(C_{t,0}, L_{t,0}\right) \\ & + \beta \varsigma_{t,0} \tau_t^p \tilde{b}_t \mathbb{E}_t \left[\frac{1 + r_{t+1}(1 - \tau_{t+1}^c)}{\varsigma_{t,0}} U_C\left(C_{t+1,1}, 0\right) \right] \Big\} \end{aligned} \tag{37b}$$

3) Pension policy with state-dependent pension factor (**PP2**). In this case, the capital tax to be paid in $t+1$ is alread fixed in t, and denoted by $\tilde{\tau}_t^c$. Now the pension factor b_{t+1} is state-dependent, it is only determined in $t+1$. Formulas are the same as in (**PP1**), except for the timing of $\tilde{\tau}_t^c$ and b_{t+1}, and are given in Appendix V.

Proposition 8. *The three fiscal policy regimes* **DP**, **PP1** *and* **PP2** *are equivalent in the sense that every real allocation that can be implemented in one policy regime can also be implemented in any of the other two regimes.*

The equivalence between **DP** *and* **PP1** *and between* **DP** *and* **PP2** *holds true except for a set of allocations that has measure zero, namely when* $\tau_t^d = -d_{t,0}/(w_{t,0} L_{t,0})$ *and* $d_{t,0} \neq 0$. *It is certainly true if* $\tau_t^d \geq 0$ *and* $d_{t,0} \geq 0$ *for all* t *and all realizations of shocks.*

PROOF:

In **PP2**, *the government budget constraint is*

$$b_t \frac{1 + r_t(1 - \tilde{\tau}^c_{t-1})}{\varsigma_{t-1,0}} \tau^p_{t-1,0} w_{t-1,0} L_{t-1,0} N_{t,1} = \tau^p_{t,0} w_{t,0} L_{t,0} N_{t,0} + \tilde{\tau}^c_{t-1} r_t k_{t-1,0} N_{t-1,0} \quad \textit{for all } t. \tag{38}$$

The household budget constraints are

$$k_{t,0} + C_{t,0} = w_{t,0}(1 - \tau^p_t - \tau^l) L_{t,0} + T_{t,0} \tag{39a}$$

$$C_{t+1,1} = \frac{1 + r_{t+1}(1 - \tilde{\tau}^c_t)}{\varsigma_{t,0}} \left[k_{t,0} + b_{t+1} \tau^p_t w_{t,0} L_{t,0}. \right] \tag{39b}$$

and the household first order conditions are

$$U_C(C_{t,0}, L_{t,0}) = \beta \varsigma_{t,0} \mathbb{E}_t \left[\frac{1 + r_{t+1}(1 - \tilde{\tau}^c_t)}{\varsigma_{t,0}} U_C(C_{t+1,1}, 0) \right] \tag{40a}$$

$$\begin{aligned} -U_L(C_{t,0}, L_{t,0}) = w_{t,0} \Big\{ &(1 - \tau^p_t - \tau^l) U_C(C_{t,0}, L_{t,0}) \\ &+ \beta \varsigma_{t,0} \tau^p_t \mathbb{E}_t \left[b_{t+1} \frac{1 + r_{t+1}(1 - \tilde{\tau}^c_t)}{\varsigma_{t,0}} U_C(C_{t+1,1}, 0) \right] \Big\} \end{aligned} \tag{40b}$$

Equivalence between DP and PP1: *first set the state-dependent capital taxes equal in both regimes. Then with the choices in (44), simple algebra shows that both the household budget constraints and first order conditions are satisfied in both regimes, for the same capital holdings. It can be easily checked that, under (44), the government budget constraints (31) and (35) are equivalent as well. Notice that this already follows from aggregate feasibility and the fact that HH budget constraints are satisfied in each period.*

Equivalence between PP1 and PP2 *requires that the contribution rate to the pension system, τ^p_t, is the same in both regimes and that the following conditions hold:*

$$\mathbb{E}_t \left[\frac{1 + r_{t+1}(1 - \tau^c_{t+1})}{\varsigma_{t,0}} U_C(C_{t+1,1}, 0) \right] = \mathbb{E}_t \left[\frac{1 + r_{t+1}(1 - \tilde{\tau}^c_t)}{\varsigma_{t,0}} U_C(C_{t+1,1}, 0) \right] \tag{41}$$

$$\frac{1 + r_{t+1}(1 - \tau^c_{t+1})}{\varsigma_{t,0}} \left[k_{t,0} + b_t \tau^p_t w_{t,0} L_{t,0} \right] = \frac{1 + r_{t+1}(1 - \tilde{\tau}^c_t)}{\varsigma_{t,0}} \left[k_{t,0} + b_{t+1} \tau^p_t w_{t,0} L_{t,0} \right] \tag{42}$$

$$\mathbb{E}_t \left[b_t \frac{1 + r_{t+1}(1 - \tau^c_{t+1})}{\varsigma_{t,0}} U_C(C_{t+1,1}, 0) \right] = \mathbb{E}_t \left[b_{t+1} \frac{1 + r_{t+1}(1 - \tilde{\tau}^c_t)}{\varsigma_{t,0}} U_C(C_{t+1,1}, 0) \right] \tag{43}$$

First choose $\tilde{\tau}^c_t$ such that (41) is satisfied. Given that, choose b_{t+1} for each state of the world in $t+1$ such that (42) is satisfied. Then (41) and (42) together imply (43). To see this, multiply (42) on both sides by $U_C(C_{t+1,1}, 0)$ and form conditional expectations. From (41) we see that the term involving $k_{t,0}$ cancels on both sides of (42). The remainder implies (43).

To show that the two government budget constraints (35) and (38) are equivalent (again, this already follows from feasibility and the individual budget constraints), just *subtract (38) from (35), add* $\frac{1+r_t}{\varsigma_{t-1,0}} k_{t-1,0}$ *on both sides of the resulting equation and use* $N_{t-1,0} = N_{t,1}/\varsigma_{t,0}$*. Then (42) implies that this difference is zero.*

The equivalence between **DP** *and the two pension regimes can break down for certain combinations of* τ_t^d *and* $d_{t,0}$*. From (44) we get* $\tau_t^p = \tau_t^d + d_{t,0}/(w_{t,0} L_{t,0})$*. Thus it is possible to get* $\tau_t^p = 0$ *although* $d_{t,0} \neq 0$*. Then (44b) is not satisfied.*

The equivalence between **DP** and **PP1** is quite intuitive; we just set

$$\tau_t^d = \tau_t^p (1 - \tilde{b}_t) \tag{44a}$$

$$d_{t,0} = \tau_t^p \tilde{b}_t w_{t,0} L_{t,0}. \tag{44b}$$

The fraction $\tilde{b}_t$ of the pension contribution is the fraction that the household will receive with interest during retirement. It is therefore like a credit from the household to the government, which is expressed in (44b). The fraction $1 - \tilde{b}_t$ is like a tax on labor income, which explains (44a).

Efficiency requires that the effect of a shock be shared also by old households. In **DP** and **PP1**, this is achieved by a state-dependent capital tax. As the equivalence between **PP1** and **PP2** shows, a state-dependent pension factor b_{t+1} can take over the role of a shock absorber instead of the capital income tax. Therefore, the capital income tax of period $t+1$ can already be determined in period t. To have both the pension factor and the capital tax determined only in $t+1$ would be redundant.

As also shown, the equivalence between **DP** and the two pension regimes may break down. Although this is only the case under knife-edge conditions, namely if $\tau_t^d = -d_{t,0}/(w_{t,0} L_{t,0})$ and $d_{t,0} \neq 0$, it points to the fact that a pension system is not a natural policy regime for all kinds of intergenerational redistributions. It is natural if there is a systematic redistribution from future to current generations: if $\tau_t^d \geq 0$ and $d_{t,0} \geq 0$ always, then the above problem cannot occur.

The results of this section motivate why we abstract from government debt and capital income taxes in our analysis. Although debt and a pension system are not completely equivalent in a model with many cohorts and age-independent taxes, they are still almost equivalent. If we were allowing debt and a pension system simultaneously in the model, the result would be an optimal second-best policy that consists of taking very large offsetting positions in the two instruments. Such a policy is both unrealistic and hard to interpret. It is therefore better to shut down one of the instruments.

Capital income taxes are ignored for similar reasons. In fact, with our calibrations, the optimal tax on capital income would be positive in the steady state. We find that it is close to 10 percent. Nevertheless, in the numerical examples below, we don't allow capital taxes for various reasons. We have shown above that capital taxes and the pension factor b_t can both play the role of a shock absorber, in an equivalent way. The purpose of the present paper is to show how the pension system, not the capital tax, optimally reacts to demographic shocks. This is what seems relevant for practical policy purposes. The tax system in many countries is designed to react to demographic developments, while the capital tax appears to be governed by different considerations. Having both instruments active would make it hard to interpret the results. Moreover, if we allow for capital taxes, it is not clear where the tax revenues should go. If they enter the general budget from which pensions are paid, this may create effects in the model that are unrealistic, since pensions in reality are mostly paid by payroll taxes.

Third, we focus exclusively on a Bismarck pension system, where benefits are linearly linked to contributions, and ignore systems of the Beveridge type, where pension payments are indepedent of past contributions. In the latter case, the full pension contribution rate τ_t^p acts as a distortive tax, such that the first order condition for labor supply equals $U_L\left(C_{t,0}, L_{t,0}\right) = w_{t,0} U_C\left(C_{t,0}, L_{t,0}\right)\left(1 - \tau_t^p - \tau^l\right)$. It is obvious that such a pension system is not equivalent to **DP**, **PP1** or **PP2**, because it introduces an additional labor market distortion. A Beveridge pension system may have a role to play if intra-generational redistribution is important. But in a model such as ours where people of the same cohort are all alike, a Beveridge pension system is pointless.