General Equilibrium Impacts
Of a Federal Clean Energy Standard

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Appendix B (Online)

Definitions of the Marginal Cost of Funds for Capital and Labor

The MCPF for the capital and labor tax are given by

(B.1)
$$\eta_K \equiv \frac{rK}{rK + \tau_K \frac{\partial(rK)}{\partial \tau_K} + \tau_L \frac{\partial(wL)}{\partial \tau_K}}$$

and

(B.2)
$$\eta_L \equiv \frac{wL}{wL + \tau_K \frac{\partial(rK)}{\partial \tau_L} + \tau_L \frac{\partial(wL)}{\partial \tau_L}}$$

respectively. In each of these expressions, the numerator is the cost to the representative agent of a marginal increase in the tax, while the denominator is the marginal revenue from that tax increase. Thus the ratio is the MCPF.[1]

Derivation of Equation (11)

Taking a total derivative of utility (1) with respect to τ_Z, substituting in the consumer first-order conditions (3), subtracting total derivatives of the production functions (4) and (5) with respect to τ_Z, substituting in the firm first-order conditions (6) and (7), substituting in total derivatives of the factor-market clearing conditions (8) and (9) and the government budget

[1] Each expression is a "nonenvironmental" MCPF, one that ignores any welfare effects stemming directly from policy-induced changes in emissions or environmental quality.

constraint (10) with respect to τ_Z, and rearranging yield the following equation for the marginal change in welfare for a marginal change in either the CES or C&T:

(B.3) $$\frac{1}{\lambda}\frac{dU}{d\tau_Z}=\tau_Z\frac{dZ}{d\tau_Z}+\tau_Y\frac{dY}{d\tau_Z}+\tau_K\frac{d(rk)}{d\tau_Z}+\tau_L\frac{d(wL)}{d\tau_Z}$$

Expanding the total derivatives allows this to be rewritten as

(B.4) $$\frac{1}{\lambda}\frac{dU}{d\tau_Z}=\tau_Z\frac{dZ}{d\tau_Z}+\tau_K\frac{\partial(rk)}{\partial\tau_Z}+\tau_L\frac{\partial(wL)}{\partial\tau_Z}+\frac{d\tau_K}{d\tau_Z}\left[\tau_K\frac{\partial(rK)}{\partial\tau_K}+\tau_L\frac{\partial(wL)}{\partial\tau_K}\right]+\frac{d\tau_L}{d\tau_Z}\left[\tau_K\frac{\partial(rK)}{\partial\tau_L}+\tau_L\frac{\partial(wL)}{\partial\tau_L}\right]$$

The next step is to derive expressions for $d\tau_K/d\tau_Z$ and $d\tau_L/d\tau_Z$. Taking a total derivative of the government budget constraint (10) and substituting in $dG=0$ and $\tau_Y=0$ yields

(B.5) $$\left(Z+\tau_Z\frac{dZ}{d\tau_Z}+\tau_L\frac{\partial(wL)}{\partial\tau_Z}+\tau_K\frac{\partial(rK)}{\partial\tau_Z}\right)d\tau_Z+\left(rK+\tau_L\frac{\partial(wL)}{\partial\tau_K}+\tau_K\frac{\partial(rK)}{\partial\tau_K}\right)d\tau_K+\left(wL+\tau_L\frac{\partial(wL)}{\partial\tau_L}+\tau_K\frac{\partial(rK)}{\partial\tau_L}\right)d\tau_L=0$$

The share α_K of marginal revenue from the carbon tax (the first term in (B.5) is used to reduce the capital tax and the share α_L is used to reduce the labor tax. Together with (B.5), these imply

(B.6) $$\frac{d\tau_K}{d\tau_Z}=-\alpha_K\frac{Z+\tau_Z\frac{dZ}{d\tau_Z}+\tau_L\frac{\partial(wL)}{\partial\tau_Z}+\tau_K\frac{\partial(rK)}{\partial\tau_Z}}{rK+\tau_L\frac{\partial(wL)}{\partial\tau_K}+\tau_K\frac{\partial(rK)}{\partial\tau_K}}$$

and

(B.7) $$\frac{d\tau_L}{d\tau_Z} = -\alpha_L \frac{Z + \tau_Z \frac{dZ}{d\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} + \tau_K \frac{\partial(rK)}{\partial\tau_Z}}{wL + \tau_L \frac{\partial(wL)}{\partial\tau_L} + \tau_K \frac{\partial(rK)}{\partial\tau_L}}$$

Substituting (B.6) and (B.7) into (B.5) and rearranging, using (12), (B.1), and (B.2), yields

(B.8) $$\frac{1}{\lambda}\frac{dU}{d\tau_Z} = \tau_Z \frac{dZ}{d\tau_Z} + \eta_R \left[\tau_K \frac{\partial(rk)}{\partial\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} \right] + (\eta_R - 1)\left(Z + \tau_Z \frac{dZ}{d\tau_Z} \right)$$ (

The last remaining step is to expand the tax-interaction term (the second term on the right-hand side of (B.8)). To do this, recognize that the marginal burden of τ_Z is *Z*, and the marginal burden of τ_K is *rK*. So having γ_{ZK} share of the burden fall on capital implies that portion of the tax-interaction effect is equivalent to an increase in τ_K of $\gamma_{ZK}Z/rK$. Following the analogous step for τ_L lets us express the tax-interaction term as

(B.9) $$\eta_R \left[\tau_K \frac{\partial(rk)}{\partial\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} \right] = \eta_R Z \left[\frac{\gamma_{ZK}}{rK}\left(\tau_K \frac{\partial(rk)}{\partial\tau_K} + \tau_L \frac{\partial(wL)}{\partial\tau_K} \right) + \frac{\gamma_{ZL}}{wL}\left(\tau_K \frac{\partial(rk)}{\partial\tau_L} + \tau_L \frac{\partial(wL)}{\partial\tau_L} \right) \right]$$

Rearranging (B.9), using (B.1) and (B.2), yields

(B.10) $$\eta_R \left[\tau_K \frac{\partial(rk)}{\partial\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} \right] = \eta_R Z \left[\gamma_{ZK} \frac{1-\eta_K}{\eta_K} + \gamma_{ZL} \frac{1-\eta_K}{\eta_K} \right]$$

Substituting (B.10) and (13) into (B.8) and dividing through by $dZ/d\tau_Z$ yields (11).

Derivation of Equation (14)

Expanding the total derivatives from (B.3) for the case of the CES yields

$$\text{(B.11)} \qquad \begin{aligned} &\frac{1}{\lambda}\frac{dU}{d\tau_Z} = \tau_Z \frac{dZ}{d\tau_Z} + \tau_Y \frac{dY}{d\tau_Z} + \tau_K \frac{\partial(rk)}{\partial\tau_Z} + \tau_L \frac{\partial(wL)}{\partial\tau_Z} \\ &+\frac{d\tau_Y}{d\tau_Z}\left[\tau_K \frac{\partial(rk)}{\partial\tau_Y} + \tau_L \frac{\partial(wL)}{\partial\tau_Y}\right] + \frac{d\tau_K}{d\tau_Z}\left[\tau_K \frac{\partial(rK)}{\partial\tau_K} + \tau_L \frac{\partial(wL)}{\partial\tau_K}\right] \\ &+\frac{d\tau_L}{d\tau_Z}\left[\tau_K \frac{\partial(rK)}{\partial\tau_L} + \tau_L \frac{\partial(wL)}{\partial\tau_L}\right] \end{aligned}$$

The CES is equivalent to a revenue-neutral combination of τ_Y and τ_Z. This implies that

$$\text{(B.12)} \qquad \frac{d\tau_Y}{d\tau_Z} = -\frac{Z + \tau_Z \dfrac{dZ}{d\tau_Z}}{Y + \tau_Y \dfrac{dY}{d\tau_Z}}$$

Note however, that because the CES still affects the revenue from other taxes, it still implies a change in τ_K and τ_L. Following a similar set of steps to those that gave (B.6) and (B.7) yields

$$\text{(B.13)} \qquad \frac{d\tau_K}{d\tau_Z} = -\alpha_K \frac{\tau_L \dfrac{\partial(wL)}{\partial\tau_Z} + \tau_K \dfrac{\partial(rK)}{\partial\tau_Z} + \dfrac{d\tau_Y}{d\tau_Z}\left[\tau_L \dfrac{\partial(wL)}{\partial\tau_Y} + \tau_K \dfrac{\partial(rK)}{\partial\tau_Y}\right]}{rK + \tau_L \dfrac{\partial(wL)}{\partial\tau_K} + \tau_K \dfrac{\partial(rK)}{\partial\tau_K}}$$

and

$$\text{(B.14)} \qquad \frac{d\tau_L}{d\tau_Z} = -\alpha_L \frac{\tau_L \dfrac{\partial(wL)}{\partial\tau_Z} + \tau_K \dfrac{\partial(rK)}{\partial\tau_Z} + \dfrac{d\tau_Y}{d\tau_Z}\left[\tau_L \dfrac{\partial(wL)}{\partial\tau_Y} + \tau_K \dfrac{\partial(rK)}{\partial\tau_Y}\right]}{wL + \tau_L \dfrac{\partial(wL)}{\partial\tau_L} + \tau_K \dfrac{\partial(rK)}{\partial\tau_L}}$$

Substituting (B.13) and (B.14) into (B.11) and rearranging, using (12), (B.1), and (B.2), yields

(B.15)

$$\frac{1}{\lambda}\frac{dU}{d\tau_Z}=\tau_Z\frac{dZ}{d\tau_Z}+\tau_Y\frac{dY}{d\tau_Z}$$
$$+\eta_R\left\{\tau_K\frac{\partial(rk)}{\partial\tau_Z}+\tau_L\frac{\partial(wL)}{\partial\tau_Z}+\frac{d\tau_Y}{d\tau_Z}\left[\tau_K\frac{\partial(rk)}{\partial\tau_Y}+\tau_L\frac{\partial(wL)}{\partial\tau_Y}\right]\right\}$$

which is analogous to (B.8). Then to get from (B.15) to (14), follow the same steps that led from (B.8) to (11).