

*A1. Algebraic Description of the CGE Model*

The applied general equilibrium model is formulated as a system of nonlinear inequalities. The inequalities correspond to the two classes of conditions associated with a general equilibrium: (i) exhaustion of product (zero profit) conditions for constant-returns-to-scale producers; and (ii) market clearance for all goods and factors. The former class determines activity levels and the latter determines price levels. In equilibrium, each of these variables is linked to one inequality condition: an activity level to an exhaustion of product constraint and a commodity price to a market clearance condition.

In our algebraic exposition, the notation is used to denote the unit profit function (calculated as the difference between unit revenue and unit cost) for constant-returns-to-scale production of sector  $i$  in region  $r$  where  $z$  is the name assigned to the associated production activity. Differentiating the unit profit function with respect to input and output prices provides compensated demand and supply coefficients (Hotelling’s Lemma), which appear subsequently in the market clearance conditions. We use  $g$  as an index comprising all sectors/commodities  $i$  ( $g = i$ ), the final consumption composite ( $g = C$ ), the public good composite ( $g = G$ ), and aggregate investment ( $g = I$ ). The index  $r$  (aliased with  $s$ ) denotes regions. The index  $EG$  represents the subset of all energy goods (here: coal, oil, gas, electricity) and the label  $FF$  denotes the subset of fossil fuels (here: coal, oil, gas). Tables A1 - A6 explain the notation for variables and parameters employed within our algebraic exposition. Figures A1 - A3 provide a graphical exposition of the production structure. Numerically, the model is implemented in GAMS and solved using PATH.

**Zero-profit conditions:**

- Production of goods except fossil fuels ( $g \notin FF$ ):

$$\Pi_{gr}^Y = p_{gr} - \left[ \theta_{gr}^M p_{gr}^{M(1-\sigma_{gr}^{KLEM})} + (1 - \theta_{gr}^M) p_{gr}^{KLE(1-\sigma_{gr}^{KLEM})} \right]^{1/(1-\sigma_{gr}^{KLEM})} \leq 0$$

- Sector-specific energy-value-added aggregate:

$$\Pi_{gr}^{KLE} = p_{gr}^{KLE} - \left[ \theta_{gr}^E p_{gr}^{E(1-\sigma_{gr}^{KLE})} + (1 - \theta_{gr}^E) p_{gr}^{KL(1-\sigma_{gr}^{KLE})} \right]^{1/(1-\sigma_{gr}^{KLE})} \leq 0$$

- Sector-specific material aggregate:

$$\Pi_{gr}^M = p_{gr}^M - \left[ \sum_{i \notin EG} \theta_{igr}^{MN} p_{igr}^{A(1-\sigma_{gr}^M)} \right]^{1/(1-\sigma_{gr}^M)} \leq 0$$

- Sector-specific energy aggregate:

$$\Pi_{gr}^E = p_{gr}^E - \left[ \sum_{i \in EG} \theta_{igr}^{EN} (p_{igr}^A + p_r^{CO_2} a_{igr}^{CO_2})^{(1-\sigma_{gr}^E)} \right]^{1/(1-\sigma_{gr}^E)} \leq 0$$

- Sector-specific value-added aggregate:

$$\Pi_{gr}^{KL} = p_{gr}^{KL} - \left[ \theta_{gr}^K v_{gr}^{(1-\sigma_{gr}^{KL})} + (1 - \theta_{gr}^K) w_r^{(1-\sigma_{gr}^{KL})} \right]^{1/(1-\sigma_{gr}^{KL})} \leq 0$$

- Production of fossil fuels ( $g \in FF$ ):

$$\Pi_{gr}^Y = p_{gr} - \left[ \theta_{gr}^Q q_{gr}^{(1-\sigma_{gr}^Q)} + (1 - \theta_{gr}^Q) \left( \theta_{gr}^L w_r + \theta_{gr}^K v_{gr} + \sum_{i \notin FF} \theta_{igr}^{FF} p_{igr}^A \right)^{(1-\sigma_{gr}^Q)} \right]^{1/(1-\sigma_{gr}^Q)} \leq 0$$

- Armington aggregate:

$$\Pi_{igr}^A = p_{igr}^A - \left( \theta_{igr}^A p_{ir}^{(1-\sigma_{ir}^A)} + (1 - \theta_{igr}^A) p_{ir}^{IM(1-\sigma_{ir}^A)} \right)^{1/(1-\sigma_{ir}^A)} \leq 0$$

- Aggregate imports across import regions:

$$\Pi_{ir}^{IM} = p_{ir}^{IM} - \left[ \sum_s \theta_{isr}^{IM} (1 + \tau_{isr}) p_{is}^{(1-\sigma_{ir}^{IM})} \right]^{1/(1-\sigma_{ir}^{IM})} \leq 0$$

### Market-clearance conditions:

- Labor:

$$\bar{L}_r \geq \sum_g Y_{gr}^{KL} \frac{\partial \Pi_{gr}^{KL}}{\partial w_r}$$

- Capital:

$$\bar{K}_{gr} \geq Y_{gr}^{KL} \frac{\partial \Pi_{gr}^{KL}}{\partial v_{gr}}$$

- Fossil fuel resources ( $g \in FF$ ):

$$\bar{Q}_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial q_{gr}}$$

- Material composite:

$$M_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial p_{gr}^M}$$

- Energy composite:

$$E_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial p_{gr}^E}$$

- Value-added composite:

$$KL_{gr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial p_{gr}^{KL}}$$

- Import composite:

$$IM_{ir} \geq \sum_g A_{igr} \frac{\partial \Pi_{igr}^A}{\partial p_{ir}^{IM}}$$

- Armington aggregate:

$$A_{igr} \geq Y_{gr} \frac{\partial \Pi_{gr}^Y}{\partial p_{igr}^A}$$

- Commodities ( $g = i$ ):

$$Y_{ir} \geq \sum_g A_{igr} \frac{\partial \Pi_{igr}^A}{\partial p_{ir}} + \sum_{s \neq r} IM_{is} \frac{\partial \Pi_{is}^{IM}}{\partial p_{ir}}$$

- Private consumption ( $g = C$ ):

$$Y_{Cr} p_{Cr} \geq w_r \bar{L}_r + \sum_g v_{gr} \bar{K}_{gr} + \sum_{i \in FF} q_{ir} \bar{Q}_{ir} + p_r^{CO_2} C \bar{O}_{2r} + \bar{B}_r$$

- Public consumption ( $g = G$ ):

$$Y_{Gr} \geq \bar{G}_r$$

- Investment ( $g = I$ ):

$$Y_{Ir} \geq \bar{I}_r$$

- Carbon emissions:

$$C \bar{O}_{2r} \geq \sum_g \sum_{i \in FF} E_{gr} \frac{\partial \Pi_{gr}^E}{\partial (p_{igr}^A + p_r^{CO_2} a_{igr}^{CO_2})} a_{igr}^{CO_2}$$

TABLE A1—INDICES &amp; SETS

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$i, j$	Sectors and goods
$g$	The union of produced goods $i$ , private consumption $C$ , public demand $G$ and investment $I$
$r, s, t$	Regions
$EG$	Energy goods; coal, crude oil, refined oil, natural gas and electricity
$FF$	Fossil fuels; coal, crude oil and natural gas.

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TABLE A2—ACTIVITY LEVELS

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$Y_{gr}$	Production of item $g$ in region $r$
$E_{gr}$	Energy composite for item $g$ in region $r$
$KL_{gr}$	Value-added composite for item $g$ in region $r$
$KLE_{gr}$	Energy-value-added composite for item $g$ in region $r$
$A_{igr}$	Armington aggregate for commodity $i$ for demand category (item) $g$ in region $r$
$IM_{ir}$	Aggregate imports of commodity $i$ in region $r$

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TABLE A3—PRICES

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$p_{gr}$	Price of item $g$ in region $r$
$p_{gr}^M$	Price of material composite for item $g$ in region $r$
$p_{gr}^E$	Price of energy composite for item $g$ in region $r$
$p_{gr}^{KL}$	Price of value-added composite for item $g$ in region $r$
$p_{gr}^{KLE}$	Price of energy-value-added composite for item $g$ in region $r$
$p_{igr}^A$	Price of Armington good $i$ for demand category $g$ in region $r$
$p_{ir}^{IM}$	Price of import composite for good $i$ in region $r$
$\tau_{isr}$	Tariff rate good $i$ imported from region $s$ to region $r$
$w_r$	Wage rate in region $r$
$v_{ir}$	Capital rental rate in sector $i$ in region $r$
$q_{ir}$	Rent to fossil fuel resources in region $r$ ( $i \in FF$ )
$p_r^{CO_2}$	Implicit price of carbon in region $r$

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TABLE A4—ENDOWMENTS AND CARBON EMISSIONS SPECIFICATION

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$\bar{L}_r$	Aggregate labor endowment for region $r$
$\bar{K}_{ir}$	Capital endowment for sector $i$ in region $r$
$\bar{Q}_{ir}$	Endowment of fossil energy resource $i$ in region $r$ ( $i \in FF$ )
$\bar{B}_r$	Initial balance for payment deficit or surplus in region $r$ (note: $\sum_r \bar{B}_r = 0$ )
$\bar{CO}_{2r}$	Aggregate carbon emission cap in region $r$
$a_{igr}^{CO_2}$	Carbon emission coefficient for fossil fuel $i$ in demand category $g$ in region $r$ ( $i \in FF$ )

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TABLE A5—COST SHARE PARAMETERS

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$\theta_{gr}^M$	Cost share of material composite in production of item $g$ in region $r$
$\theta_{gr}^E$	Cost share of energy composite in the aggregate of energy and value added of item $g$ in region $r$
$\theta_{igr}^{MN}$	Cost share of material input $i$ in the material composite of item $g$ in region $r$
$\theta_{igr}^{EN}$	Cost share of energy input in the energy composite of item $g$ in region $r$
$\theta_{gr}^K$	Cost share of capital within the value-added composite of item $g$ in region $r$
$\theta_{gr}^Q$	Cost share of fossil fuel resource in fossil fuel production ( $g \in FF$ ) in region $r$
$\theta_{gr}^L$	Cost share of labor in non-resource inputs to fossil fuel production ( $g \in FF$ ) in region $r$
$\theta_{gr}^K$	Cost share of capital in non-resource inputs to fossil fuel production ( $g \in FF$ ) in region $r$
$\theta_{igr}^{FF}$	Cost share of good $i$ in non-resource inputs to fossil fuel production ( $g \in FF$ ) in region $r$
$\theta_{igr}^A$	Cost share of domestic output $i$ within the Armington item $g$ in region $r$
$\theta_{isr}^M$	Cost share of exports of good $i$ from region $s$ in the import composite of good $i$ in region $r$

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TABLE A6—ELASTICITY PARAMETERS

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$\sigma_{gr}^{KLEM}$	Substitution between the material composite and the energy-value-added aggregate in the production of item $g$ in region $r^*$
$\sigma_{gr}^{KLE}$	Substitution between energy and the value-added composite in the production of item $g$ in region $r^*$
$\sigma_{gr}^M$	Substitution between material inputs within the energy composite in the production of item $g$ in region $r^*$
$\sigma_{gr}^{KL}$	Substitution between capital and labor within the value-added composite in the production of item $g$ in region $r^*$
$\sigma_{gr}^E$	Substitution between energy inputs within the energy composite in the production of item $g$ in region $r$ (by default = 0.5)
$\sigma_{gr}^Q$	Substitution between natural resource input and the composite of other inputs in the fossil fuel production ( $g \in FF$ ) of region $r^{***}$
$\sigma_{ir}^A$	Substitution between domestic variety and the composite of imported varieties from different regions for good $i$ in region $r^{**}$
$\sigma_{ir}^{IM}$	Substitution between imports from different regions within the import composite for good $i$ in region $r^{**}$

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\* — Calibrated based on estimates from Okagawa and Ban (2008).

\*\* — Calibrated based on estimates from Narayanan, Aguiar and McDougall (2012) with the exception for elasticities in the market for crude oil which are assumed equal to  $+\infty$ .

\*\*\* — Calibrated based on estimates from Graham, Thorpe and Hogan (1999) and Krichene (2002).

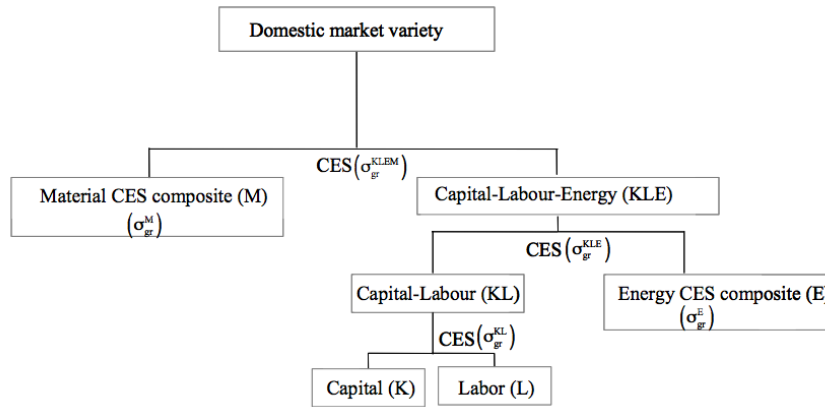


FIGURE A1. NESTING IN NON-FOSSIL-FUEL PRODUCTION

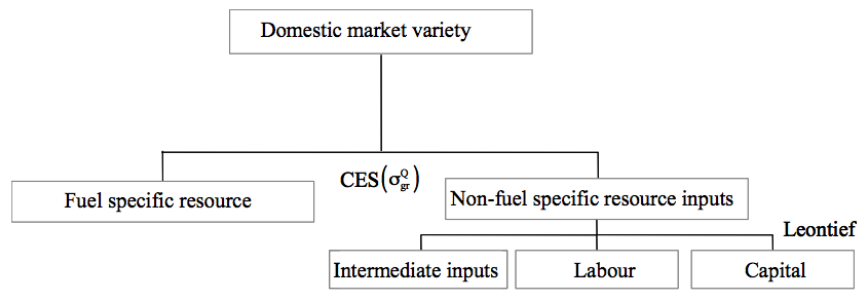


FIGURE A2. NESTING IN FOSSIL-FUEL PRODUCTION

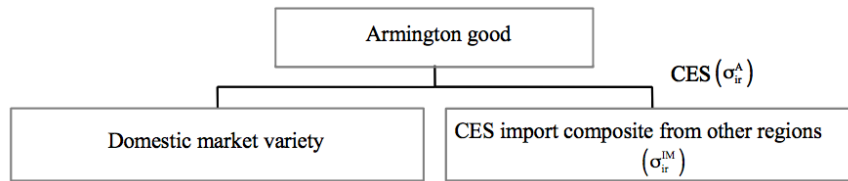


FIGURE A3. NESTING IN ARMINGTON COMPOSITE PRODUCTION

A2. *Region and Sector Mappings*

TABLE A7—MAPPING OF REGIONS FROM THE GTAP 8 DATASET

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<i>United States</i>	United States
<i>EU-27 plus European Free Trade Area</i>	France, Germany, Italy, United Kingdom, Austria, Belgium, Denmark, Finland, Greece, Ireland, Luxembourg, Netherlands, Portugal, Spain, Sweden, Czech Republic, Hungary, Malta, Poland, Romania, Slovakia, Slovenia, Estonia, Latvia, Lithuania, Bulgaria, Cyprus, Switzerland, Norway, Rest of EFTA
<i>Other Annex I minus Russia</i>	Canada, Japan, Belarus, Ukraine, Australia, New Zealand, Turkey
<i>China and Hong Kong</i>	China, Hong Kong
<i>India</i>	India
<i>Russian Federation</i>	Russian Federation
<i>Other Energy-Exporting Countries</i>	Indonesia, Rest of North Africa, Nigeria, Rest of South Central Africa, Ecuador, Venezuela, Islamic Republic of Iran, Rest of Western Asia, Egypt, Bolivia, Malaysia
<i>Other Middle-Income Countries</i>	Albania, Armenia, Argentina, Azerbaijan, Bulgaria, Brazil, Botswana, Chile, Columbia, Costa Rica, Georgia, Guatemala, Kazakhstan, Sri Lanka, Morocco, Mauritius, Mexico, Panama, Peru, Philippines, Paraguay, Thailand, Tunisia, Uruguay, South Africa, Rest of Oceania, Rest of South America, Caribbean, Rest of North Africa, Rest of South African Customs Union
<i>Other Low-Income Countries</i>	Bangladesh, Ethiopia, Kyrgyzstan, Cambodia, Rest of East Asia, Lao People's Democratic Republic, Madagascar, Myanmar, Malawi, Mozambique, Nicaragua, Pakistan, Senegal, Tanzania, Uganda, Vietnam, Zambia, Zimbabwe, Rest of South Asia, Rest of Southeast Asia, Rest of Eastern Europe, Rest of Former Soviet Union, Rest of Western Africa, West of Central Africa, Rest of South Central Africa, Rest of Eastern Africa

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TABLE A8—MAPPING OF SECTORS FROM GTAP 8 DATASET

<i>Coal</i>	Coal
<i>Crude Oil</i>	Crude Oil
<i>Natural Gas</i>	Natural Gas
<i>Refined Petroleum and Coal</i>	Refined Petroleum and Coal
<i>Electricity</i>	Electricity
<i>Chemical, Rubber, Plastic Products</i>	Chemical, Rubber, Plastic Products
<i>Iron and Steel</i>	Iron and steel
<i>Non-Ferrous Metals</i>	Non-Ferrous Metal
<i>Non-Metallic Minerals</i>	Non-Metallic Mineral, Other Minerals
<i>Water Transport</i>	Water Transport
<i>Air Transport</i>	Air Transport
<i>Other Transport</i>	Other Transport
<i>All Other Goods</i>	All Other Goods

TABLE A9—MAPPING OF FACTORS FROM GTAP 8 DATASET

<i>Physical Capital</i>	Physical Capital
<i>Labor</i>	Unskilled Labor, Skilled Labor
<i>Natural Resources</i>	Natural Resources