

Online Appendix for "Auctions in Markets: Common Outside Options and the Continuation Value Effect"

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1 First-price and second-price auctions

In this Section, we provide our revenue comparison for first and second-price auctions. First, we show that our first-price auction has the same equilibrium as a standard common-value auction with value function $v(s, y) = v - (\rho(s, y)U_H(\rho_{lose}(s, y)) + (1 - \rho(s, y))U_L(\rho_{lose}(s, y)))$. Let $H_s(t)$ denote the probability that all other bidders have signal less than t if a bidder has signal s , and $h_s(t)$ the corresponding density of the highest other signal. In our first-price auction, the profit from bidding $b(y)$ when one has signal s is

$$\pi^{1st}(s, y) = (v - b(y))H_s(y) + \int_y^{\bar{s}} (\rho(s, t)U_H(\rho_{lose}(s, y)) + (1 - \rho(s, t))U_L(\rho_{lose}(s, y)))h_s(t)dt.$$

The common-value auction has profit function (see Milgrom and Weber (1982)):

$$\tilde{\pi}^{1st}(s, y) = \int_{\underline{s}}^y v(s, t)h_s(t)dt - b(y)H_s(t).$$

It holds that $\partial\tilde{\pi}^{1st}(s, y)/\partial y = \partial\pi^{1st}(s, y)/\partial y$, which then proves that the two auctions provide the same incentives for the bidders, so equilibrium bids are identical. A similar reasoning presented in the proof of Proposition 1 in the regular Appendix shows that the same holds for our second-price auction, and the common-value second-price auction of Milgrom and

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Weber (1982). But then the linkage principle directly applies as we can compare the revenue of a second-price and a first-price interdependent-value auction with the same value function $v(s, y)$.

2 Comparing revenues from second-price auctions and English auctions

In this Section, we provide the proof of the revenue comparison result for the English- and the second-price auctions considered in an observation of Section 5.2.

In the second-price auction the middle bidder's bid is the revenue, and the bid function can be written as

$$b^{II} = v - [\hat{\rho}_{tie}(s)U_H(\hat{\rho}_{lose}(s)) + (1 - \hat{\rho}_{tie}(s))U_L(\hat{\rho}_{lose}(s))],$$

where $\hat{\rho}_{tie}, \hat{\rho}_{lose}$ are the relevant tying and losing posteriors. These beliefs can be written as $\hat{\rho}_{tie}(s) = \Pr(H \mid s_1 = s_2 = s > s_3) = \frac{s^2 \rho(s, s)}{s^2 \rho(s, s) + (2s - s^2)(1 - \rho(s, s))}$, and $\hat{\rho}_{lose}(s) = \frac{s^2 \rho_{lose}}{s^2 \rho_{lose} + (2s - s^2)(1 - \rho_{lose})}$

Let us now calculate the revenue in the English auction. Let z be lowest of the three types, and s be the medium one. Then the revenue is equal to

$$b^E(s, z) = v - [\hat{\rho}_{tie}(s, z)U_H(\hat{\rho}_{lose}(s, z)) + (1 - \hat{\rho}_{tie}(s, z))U_L(\hat{\rho}_{lose}(s, z))],$$

where $\hat{\rho}_{tie}(s, z) = \Pr(H \mid s_1 = s_2 = s > s_3 = z) = \frac{z \rho(s, s)}{z \rho(s, s) + (1 - z)(1 - \rho(s, s))}$, and $\hat{\rho}_{lose}(s, z) = \Pr(H \mid s_1 > s_2 = s > s_3 = z) = \frac{z \rho_{lose}}{z \rho_{lose} + (1 - z)(1 - \rho_{lose})}$. To calculate the expected revenues, let $g_{mid}(s) = 12s - 42s^2 + 60s^3 - 30s^4$ denote the density of the medium type, and let $h(z \mid s) = \frac{2\rho_{lose}(s)z + 2(1 - \rho_{lose}(s))(1 - z)}{\rho_{lose}(s)s^2 + (1 - \rho_{lose}(s))(2s - s^2)}$ be the density of the low type given the medium types. Then the expected revenues from the two auctions can be written as $R^{II} = \int_{\underline{s}}^{\bar{s}} g_{mid}(s)b^{II}(s)ds$ and $R^E = \int_{\underline{s}}^{\bar{s}} g_{mid}(s) \int_{\underline{s}}^s h(z \mid s)b^E(s, z)dzds$.

After substituting in the relevant functional form assumptions about U_H, U_L , and the parameter values from Example 3, we can use the above bid functions to show that $R^{II} > R^E$ when $d = 1$. To establish the existence of an appropriate \bar{d} , we inspect how the bid functions b^{II}, b^E change as d changes. Straightforward algebra yields that

$$R^E - R^{II} = d \left[\int_{\underline{s}}^{\bar{s}} g_{mid}(s)(1 - \hat{\rho}_{tie}(s))ds - \int_{\underline{s}}^{\bar{s}} g_{mid}(s) \int_{\underline{s}}^s h(z \mid s)(1 - \hat{\rho}_{tie}(s, z))dzds \right] + K,$$

where K represents terms that do not depend on d . Upon substituting from Example 3, it follows that the term in the bracket is positive, which completes our proof.

3 Revenue comparison when signals are precise

In this Section we prove the revenue comparison result of the main text for the case where bidders' signals become arbitrarily precise. Take the model from the main text with two bidders an equal prior for the two states, $\rho_0 = 1/2$, and $\mu_H = \mu_L = 1^1$. To be able to study a monotone equilibrium we assume that V is strictly decreasing, and continuously differentiable, and $V'(x) < 0$ for all $x \in [0, 1]$. Each bidder observes a private signal $s \in [0, 1]$ distributed according to a continuous density function $g_w^\alpha(s)$ in state $w \in \{L, H\}$, where $\alpha \in [0, 1]$ parameterizes the precision of the signal. Similarly we can parameterize the posteriors by α , that is $\tilde{\rho}^\alpha(s) = g_H^\alpha(s) / (g_H^\alpha(s) + g_L^\alpha(s))$ denotes the probability of the high state upon observing signal s . Assume, as in the main text, that ρ^α is strictly increasing in s for all α . In what follows we often suppress the α superscript to simplify notation. To model convergence to full information, assume that the probability of having the correct belief in each state converges to 1 as α converges to 1. Formally, for any $\varepsilon > 0$

$$\lim_{\alpha \rightarrow 1} \Pr(\tilde{\rho}^\alpha(s) \geq \varepsilon \mid L) = 0 \quad (1)$$

and

$$\lim_{\alpha \rightarrow 1} \Pr(\tilde{\rho}^\alpha(s) \leq 1 - \varepsilon \mid H) = 0. \quad (2)$$

To establish our result we need to assume that the posterior upon low signals does not converge much faster than the posterior upon receiving some high signals:

Assumption CR: There exists $T > 0$ and $\hat{\varepsilon} > 0$ such that for all $0 < \varepsilon \leq \hat{\varepsilon}$

$$\lim_{\alpha \rightarrow 1} \frac{\int_{s: \tilde{\rho}^\alpha(s) \geq 1 - \varepsilon} g_{(2)}^\alpha(s) (1 - \tilde{\rho}^\alpha(s))^2 ds}{\int_{s: \tilde{\rho}^\alpha(s) \leq \varepsilon} g_{(2)}^\alpha(s) \tilde{\rho}^\alpha(s) ds} \leq T.$$

To interpret this assumption, imagine that for any fixed α the signals are distributed symmetrically in the sense that $\Pr(\tilde{\rho}^\alpha(s) \leq t \mid L) = \Pr(\tilde{\rho}^\alpha(s) \geq 1 - t \mid H)$ for any $t \in [0, 1]$. In this case the relevant ratio converges to 0, as it can be shown.² Indeed, the only way

¹The case of unequal priors could be handled without any major modification of the treatment. The same is true for the case where the matching probabilities μ_L, μ_H may be less than 1.

²The key idea is that the ratio $\lim_{\alpha \rightarrow 1} \frac{\int_{s: \rho(s) \geq 1 - \varepsilon} g_{(2)}(s) (1 - \rho(s)) ds}{\int_{s: \rho(s) \leq \varepsilon} g_{(2)}(s) \rho(s) ds}$ (that is, omitting the square sign from the numerator) is equal to 1 for any α , as the two states are completely symmetric. On the other hand,

to violate assumption CR is to assume that signals are much more precise in the low state than in the high state in the limit. At the end of this online Appendix we consider such an example, and show that the conclusion of our Proposition below fails.

Proposition *Under Assumption CR there exists $\hat{\alpha} < 1$ such that for all $\alpha \in (\hat{\alpha}, 1)$ the linkage effect is stronger than the continuation value effect, if there are two bidders.*

Proof Step 1: First, we characterize the continuation value (CV) and the linkage (LP) effects. The formula from the main text yields that the linkage effect can be written as³

$$LP = \int_{\underline{s}}^{\bar{s}} g_{(2)}(s)(V(0) - V(1))(\rho_{lose}(s) - \rho(s, s))ds.$$

Similarly, the continuation value effect can be written as

$$CV = \int_{\underline{s}}^{\bar{s}} g_{(2)}(s)[\rho(s, s)(V(1) - W_H(\rho_{lose}(s))) + (1 - \rho(s, s))(V(0) - W_L(\rho_{lose}(s)))]ds.$$

Using that $W_H(x) = V(x) + (1 - x)V'(x)$ and $W_L(x) = V(x) - xV'(x)$ we obtain that

$$\begin{aligned} CV &= \int_{\underline{s}}^{\bar{s}} g_{(2)}[\rho(s, s)(V(1) - V(\rho_{lose})) - (1 - \rho_{lose})V'(\rho_{lose})] + \\ &\quad + (1 - \rho(s, s))(V(0) - V(\rho_{lose}) + \rho_{lose}V'(\rho_{lose}))]ds = \\ &= \int_{\underline{s}}^{\bar{s}} g_{(2)}[\rho(s, s)V(1) + (1 - \rho(s, s))V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose})]ds = \\ &= \int_{\underline{s}}^{\bar{s}} g_{(2)}[\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))(V'(\rho_{lose}) + \\ &\quad + V(0) - V(1))]ds = CV = LP + \\ &\quad + \int_{\underline{s}}^{\bar{s}} g_{(2)}[\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose})]ds. \end{aligned}$$

To show that $LP > CV$ for precise information we perform pointwise comparison to establish that the last sum (that is, $CV - LP$) is negative (when signals are precise). We

$\lim_{\alpha \rightarrow 1} \frac{(1 - \rho(s))^2}{1 - \rho(s)} = 0$ if it is known that $\rho(s) \geq 1 - \varepsilon > 0$, since in this case the high state is very likely and thus the posterior converges to 1 in probability.

³This rewriting follows, because

$$\int_{\underline{s}}^{\bar{s}} g_{(2)}(s)\rho_{lose}(s)ds = \rho_0$$

by Bayes rule when there are two bidders.

use the three-wise grouping we made above, and show non-positivity of $\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose})$ for some s , and show that although this expression can be positive for some other signals s , but it is of higher order and can be neglected.

Step 2:

In this Step we show that there exists an $\varepsilon^* > 0$ such that

$$\frac{\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose})}{\rho_{lose}} < \frac{V'(1)}{2} < 0$$

whenever $\rho \leq \varepsilon^*$. For this we first show that $(\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + \rho_{lose}V'(\rho_{lose}))/\rho_{lose} \leq V'(1) < 0$ for all $\rho_{lose} \in [0, 1]$. Let $H(x) = xV(1) + (1 - x)V(0) - V(x) + xV'(x)$. By the intermediate value theorem for some $0 \leq t \leq k \leq x$

$$H(x) = x(V(1) - V(0) + V'(x) - V'(t)) = x(V(1) - V(0) + V''(k)(x - t)).$$

Also,

$$H'(x) = V(1) - V(0) + V''(x)x.$$

Therefore,

$$\left(\frac{H}{x}\right)' = \frac{xH' - H}{x^2} = \frac{xV''(x) - V''(k)(x - t)}{x} \geq 0,$$

because V is convex and $t \leq k \leq x$. Thus for all $x > 0$ it holds that $H/x \leq H(1)/1 = V'(1) < 0$. Next, let $\tilde{H}(x) = xV(1) + (1 - x)V(0) - V(x) + (x - \tau)V'(x)$ for some $\tau > 0$. Using standard continuity arguments it follows that for all $x \geq 0$, $\tilde{H}(x)/x < V'(1)/2 < 0$ whenever $\tau \leq \tau^*$ for some $\tau^* > 0$.

Now, suppose that $\rho \leq \varepsilon^*$. Then as we noticed $\rho_{lose} \geq \rho$ and $\rho(s, s) = \frac{\rho^2}{\rho^2 + (1 - \rho)^2}$ and thus $\frac{\rho(s, s)}{\rho_{lose}} \leq \frac{\rho}{\rho^2 + (1 - \rho)^2} \leq \frac{\varepsilon^*}{1 - 2\varepsilon^*}$. Write

$$\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose}) =$$

$$\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + \rho_{lose}\left(1 - \frac{\rho(s, s)}{\rho_{lose}}\right)V'(\rho_{lose}) = \tilde{H}(x)$$

with the notation $\rho_{lose} = x$ and $\rho(s, s)/\rho_{lose} = \tau$. Let $\varepsilon^*/(1 - 2\varepsilon^*) \leq \tau^*$ that is $\varepsilon^* \leq \tau^*/(1 + 2\tau^*)$. Then $\frac{\rho(s, s)}{\rho_{lose}} \leq \tau^*$ and thus it follows that if $\rho \leq \varepsilon^*$ then

$$\frac{\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose})}{\rho_{lose}} < V'(1)/2 < 0.$$

Since $\rho_{lose} > \rho$, therefore if $\rho \leq \varepsilon^*$ then

$$\frac{\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose})}{\rho} < V'(1)/2 < 0. \quad (3)$$

Step 3:

First, we establish important results for three carefully constructed signal groups, and then the proof concludes after putting those results together.

Low signals: Upon integration and taking limit of (3), we obtain that

$$\begin{aligned} \lim_{\alpha \rightarrow 1} \int_{s: \rho(s) \leq \varepsilon^*} g_{(2)}[\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose})] ds &\leq \\ &\leq 0.5V'(1) \int_{s: \rho(s) \leq \varepsilon^*} g_{(2)}(s)\rho(s) ds < 0. \end{aligned} \quad (4)$$

Medium signals: Take any signal s such that $0 < \varepsilon^* \leq \rho(s) \leq 1 - \varepsilon^* < 1$. Bayes rule implies that

$$\begin{aligned} \rho_{lose}(s) &= \frac{g_H(s)(1 - G_H(s))}{g_H(s)(1 - G_H(s)) + g_L(s)(1 - G_L(s))} = \\ &= \frac{\rho(s)(1 - G_H(s))}{\rho(s)(1 - G_H(s)) + 1 - G_L(s)}. \end{aligned}$$

By our convergence formulas (1), (2) the fact that signals become precise implies that $\lim_{\alpha \rightarrow 1} 1 - G_H(s) = 1$ and $\lim_{\alpha \rightarrow 1} 1 - G_L(s) = 0$.⁴ Therefore, $\lim_{\alpha \rightarrow 1} \rho_{lose}(s) = 1$ for all such s . Moreover, the convergence is uniform, because ρ_{lose} is strictly increasing in s . To summarize: for all

$\varepsilon > 0$ there exists an $\hat{\alpha}((1 - \varepsilon^*), \varepsilon)$ such that if $\alpha \leq \hat{\alpha}((1 - \varepsilon^*), \varepsilon)$ then for all s such that $\varepsilon^* \leq \rho(s) \leq 1 - \varepsilon^*$ it holds that $1 - \rho_{lose}(s) \leq \varepsilon$. Moreover, since $\rho(s, s) = \frac{\rho^2}{\rho^2 + (1 - \rho)^2} \leq \frac{(1 - \varepsilon^*)^2}{(1 - \varepsilon^*)^2 + \varepsilon^{*2}} < 1$. Therefore, there exists an $\tilde{\varepsilon} < 1$ such that $\rho_{lose} \geq \tilde{\varepsilon}$ implies that $\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose}) \leq 0$. Thus if $\alpha \leq \hat{\alpha}(1 - \varepsilon^*, \tilde{\varepsilon}) = \tilde{\alpha}(1 - \varepsilon^*)$ then for all s such that $\varepsilon^* \leq \rho(s) \leq 1 - \varepsilon^*$ it holds that

$$\rho_{lose}V(1) + (1 - \rho_{lose})V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s))V'(\rho_{lose}) \leq 0. \quad (5)$$

⁴Those formulas imply that $\lim_{\alpha \rightarrow 0} \Pr[\rho \geq \bar{\varepsilon} | H] = 1$ and $\lim_{\alpha \rightarrow 0} \Pr[\rho \leq \varepsilon^* | L] = 1$. Therefore, since ρ is strictly monotone in the signals it indeed follows that $\lim_{\alpha \rightarrow 0} 1 - G_H(s) = 1$ and $\lim_{\alpha \rightarrow 0} 1 - G_L(s) = 0$.

Upon integration and taking limit of (5), we obtain that

$$\lim_{\alpha \rightarrow 1} \int_{s: \rho(s) \in (\varepsilon^*, 1 - \varepsilon^*)} g_{(2)} [\rho_{lose} V(1) + (1 - \rho_{lose}) V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s)) V'(\rho_{lose})] ds \leq 0. \quad (6)$$

High signals: Take any signal such that $\rho(s) \geq 1 - \varepsilon^*$. First, note that for all $x \in [0, 1]$ it holds that

$$\frac{xV(1) + (1-x)V(0) - V(x)}{1-x} \leq V(0) - V(1). \quad (7)$$

Therefore, for all s it holds that

$$\frac{\rho_{lose} V(1) + (1 - \rho_{lose}) V(0) - V(\rho_{lose})}{1 - \rho_{lose}} \leq V(0) - V(1).$$

Noting, that $\rho_{lose} \geq \rho(s, s) = \frac{\rho^2}{\rho^2 + (1 - \rho)^2}$ implies that

$$\frac{1 - \rho_{lose}}{(1 - \rho)^2} \leq \frac{1 - \rho(s, s)}{(1 - \rho)^2} = \frac{1}{\rho^2 + (1 - \rho)^2} \leq 2,$$

and thus $\frac{\rho_{lose} V(1) + (1 - \rho_{lose}) V(0) - V(\rho_{lose})}{(1 - \rho)^2} \leq 2(V(0) - V(1))$. Finally, since $V' < 0$ it follows that

$$\frac{\rho_{lose} V(1) + (1 - \rho_{lose}) V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s)) V'(\rho_{lose})}{(1 - \rho)^2} \leq 2(V(0) - V(1)). \quad (8)$$

Upon integration and taking limit of (8), we obtain that

$$\lim_{\alpha \rightarrow 1} \int_{s: \rho(s) \geq 1 - \varepsilon^*} g_{(2)} [\rho_{lose} V(1) + (1 - \rho_{lose}) V(0) - V(\rho_{lose}) + (\rho_{lose} - \rho(s, s)) V'(\rho_{lose})] ds \leq 2(V(0) - V(1)) \lim_{\alpha \rightarrow 1} \int_{s: \rho(s) \geq 1 - \varepsilon^*} g_{(2)}(s) (1 - \rho(s))^2 ds \quad (9)$$

Inspecting (4), (6) and (9) implies the result under assumption CR after choosing T appropriately. ■

To highlight the importance of Assumption CR, let us consider an example where Assumption CR does not hold, and in this example our main revenue comparison result fails. Assume that there are two signals l, h and that $\Pr(l | L) = 1 - \sqrt[k]{x}$, $\Pr(h | L) = \sqrt[k]{x}$, $\Pr(l | H) = x$, $\Pr(h | L) = 1 - x$ with x being the precision parameter. It is clear that the posteriors $\rho_H = \frac{1-x}{1-x+\sqrt[k]{x}}$ and $\rho_L = \frac{x}{1-\sqrt[k]{x}+x}$ violate Assumption CR when $k > 2$, and the posterior in the high state converges to 1 much slower than the posterior in the low state converges to 0. The value function is $V = \frac{(1+d-x)^n}{n(n-1)}$ for $n = 1.5$ and $d = 0.01$. One can show

that the continuation value effect is greater than the linkage effect when x is close to zero and k is chosen to be 3. As one varies k there are two possibilities: either the continuation value effect is greater than the linkage effect for all signal precision x , or it is greater if and only if signals are close to being fully informative or they are not informative at all (with the linkage effect dominating in the intermediate precision case).

4 Imprecise seller information

4.1 Setup

In this Section we consider the case of imprecise seller's signal, and we show that most of our results are immune to such a modification. Let \tilde{s} be the signal of the seller, and let

$$\tilde{\rho}_{tie}(s, \tilde{s}) = \Pr(H \mid s_i = s_j = s \geq s_k, \tilde{s}) = \frac{\rho(s, s) \Pr(\tilde{s} \mid H)}{\rho(s, s) \Pr(\tilde{s} \mid H) + (1 - \rho(s, s)) \Pr(\tilde{s} \mid L)}$$

denote the posterior upon tying at the top if the seller's signal is \tilde{s} . Similarly, let $\tilde{\rho}_{lose}(s, \tilde{s})$ denote the posterior upon losing if the seller's signal is \tilde{s} . Let $s_{(2)}$ denote the second highest signal among all bidders, and let $\rho_{(2)}(s) = \Pr(H \mid s_{(2)} = s)$.

First, suppose that the seller reveals his signal \tilde{s} before the auction is run. The bid function becomes

$$b(s, \tilde{s}) = v - [\tilde{\rho}_{tie} U_H(\tilde{\rho}_{lose}) + (1 - \tilde{\rho}_{tie}) U_L(\tilde{\rho}_{lose})].$$

The expected revenue from an ex-ante point of view can be calculated as

$$\begin{aligned} ER^{Before} &= v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\bar{s}} g_{(2)} \{ \rho_{(2)} E_{\tilde{s}|H}[\tilde{\rho}_{tie}] + (1 - \rho_{(2)}) E_{\tilde{s}|L}[\tilde{\rho}_{tie}] \} ds + \\ &+ \int_{\underline{s}}^{\bar{s}} g_{(2)} \{ \rho_{(2)} E_{\tilde{s}|H}[\tilde{\rho}_{tie} (V(1) - U_H(\tilde{\rho}_{lose})) + (1 - \tilde{\rho}_{tie}) (V(0) - U_L(\tilde{\rho}_{lose}))] + \\ &+ (1 - \rho_{(2)}) E_{\tilde{s}|L}[\tilde{\rho}_{tie} (V(1) - U_H(\tilde{\rho}_{lose})) + (1 - \tilde{\rho}_{tie}) (V(0) - U_L(\tilde{\rho}_{lose}))] \} ds. \end{aligned}$$

Let $\tilde{\rho}(s) = \rho_{(2)} E_{\tilde{s}|H}[\tilde{\rho}_{tie}(s, \tilde{s})] + (1 - \rho_{(2)}) E_{\tilde{s}|L}[\tilde{\rho}_{tie}(s, \tilde{s})]$ denote the expected posterior upon tying for the bidder with the second highest signal s . Then the revenue with signal revelation (before the auction) can be written as

$$ER^{Before} = v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\bar{s}} g_{(2)} \tilde{\rho}(s) ds +$$

$$+ \int_{\underline{s}}^{\bar{s}} g_{(2)}[\tilde{\rho}(s)(V(1) - U_H(\tilde{\rho}_{lose})) + (1 - \tilde{\rho}(s))(V(0) - U_L(\tilde{\rho}_{lose}))].$$

A similar argument as in the main text implies that if the signal of the seller (\tilde{s}) is revealed after the auction, then the bid becomes

$$b^a(s) = v - \{\rho(s, s)E_{\tilde{s}|H}U_H(\tilde{\rho}_{lose}) + (1 - \rho(s, s))E_{\tilde{s}|L}[U_L(\tilde{\rho}_{lose})]\},$$

and the (ex-ante) expected revenue

$$\begin{aligned} ER^{After} &= v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\bar{s}} g_{(2)}\rho(s, s)ds + \\ &+ \int_{\underline{s}}^{\bar{s}} g_{(2)}\{\rho(s, s)(V(1) - E_{\tilde{s}|H}[U_H(\tilde{\rho}_{lose})]) + (1 - \rho(s, s))(V(0) - E_{\tilde{s}|L}[U_L(\tilde{\rho}_{lose})])\}ds. \end{aligned}$$

The revenue without information revelation does not depend on how precise the seller's signal is, so it is as before:

$$\begin{aligned} ER^{None} &= v - V(0) + (V(0) - V(1)) \int_{\underline{s}}^{\bar{s}} g_{(2)}\rho(s, s)ds + \\ &+ \int_{\underline{s}}^{\bar{s}} g_{(2)}[\rho(s, s)(V(1) - U_H(\rho_{lose}(s))) + (1 - \rho(s, s))(V(0) - U_L(\rho_{lose}(s)))]. \end{aligned}$$

The two effects can be written as

$$\begin{aligned} LP &= ER^{Before} - ER^{After} \\ &= (V(0) - V(1)) \int_{\underline{s}}^{\bar{s}} g_{(2)}\{\tilde{\rho}(s) - \rho(s, s)\}ds + \\ &+ \int_{\underline{s}}^{\bar{s}} g_{(2)}(\tilde{\rho} - \rho(s, s))[(V(1) - E_{\tilde{s}|H}[U_H(\tilde{\rho}_{lose})]) - (V(0) - E_{\tilde{s}|L}[U_L(\tilde{\rho}_{lose})])]ds, \end{aligned}$$

and

$$\begin{aligned} CV &= ER^{None} - ER^{After} = \\ &= \int_{\underline{s}}^{\bar{s}} g_{(2)}[\rho(s, s)(E_{\tilde{s}|H}\{U_H(\tilde{\rho}_{lose})\} - U_H(\rho_{lose})) + (1 - \rho(s, s))(E_{\tilde{s}|L}\{U_L(\tilde{\rho}_{lose})\} - U_L(\rho_{lose}))]ds. \end{aligned}$$

While few general results are available about the behavior of the two effects when signals are imperfectly precise, two observations still hold. First, when bidders have uninformative

signals ($\alpha = 0$), the LP effect is still zero, while $CV > 0$ so revealing information (even not fully precise information) hurts the revenues of the seller. Second, when the bidders are fully informed ($\alpha = 1$), by construction $LP = CV = 0$, and thus the information policy does not matter.

4.2 Numerical example

Now, we conduct a numerical analysis to investigate how imprecise seller signals may influence the optimal information policy. Let us suppose that the seller can receive only two signals $\tilde{s} = s_H, s_L$ with $\Pr[s_H | H] = \Pr[s_L | L] = z \in [0.5, 1]$, that is, z measures the precision of the seller's signal. Then $\tilde{\rho}_{tie}(s, s_H) = \frac{\rho(s,s)z}{\rho(s,s)z + (1-\rho(s,s))(1-z)}$, $\tilde{\rho}_{tie}(s, s_L) = \frac{\rho(s,s)(1-z)}{\rho(s,s)(1-z) + (1-\rho(s,s))z}$ and $E_{\tilde{s}|H}[\tilde{\rho}_{tie}(s, \tilde{s})] = z\tilde{\rho}_{tie}(s, s_H) + (1-z)\tilde{\rho}_{tie}(s, s_L)$, $E_{\tilde{s}|L}[\tilde{\rho}_{tie}(s, \tilde{s})] = (1-z)\tilde{\rho}_{tie}(s, s_H) + z\tilde{\rho}_{tie}(s, s_L)$. Similar calculations can be made for the other expected values. After substituting back from Example 2 in the main text, one can calculate the CV and LP effects numerically for any values of z , and α (the signal precision of the bidders). The main text depicts (as Figure 2) the two effects for a fixed level of z (chosen $z = 0.7$) varying α between 0 and 1. The picture is qualitatively similar to the full precision case (that is, when $z = 1$): the seller reveals information if and only if the bidders have precise private signals.

Our other observation concerns the question whether the seller has an incentive to reveal his signal with a noise. To study this, imagine that the seller fully observes the state, but he can choose to reveal a signal with an arbitrary precision $z \in [0.5, 1]$. Extensive numerical analysis suggests that for any fixed α , the seller maximizes his revenue by revealing the state perfectly (when $\alpha \geq 0.225$) or not revealing it at all (when $\alpha \leq 0.225$), so garbling his information is not profitable for the seller in this example. For illustration, let us study the most interesting case when α is such that the seller is close to being indifferent to revealing his fully informative signal or not revealing it at all.⁵ Depicting the revenue from revealing a signal with precision z (see Figure 1 above) shows a non-monotonic dependence when $\alpha = 0.24$. The graph below provides two interesting observations. First, revealing a fully precise signal is the best policy for the seller. Second, revealing a partially precise signal ($z \in (0.5, 1)$) may be worse than not revealing any signal at all. Future research should shed light on how such a non-monotonic dependence of revenues in the signal precision of the seller may occur when outside options are endogenous.

⁵If α is much larger than this cutoff value (0.225) then the seller's revenue is increasing in z for all values of z , and when it is much smaller then it the revenue is decreasing in z for all values of z .

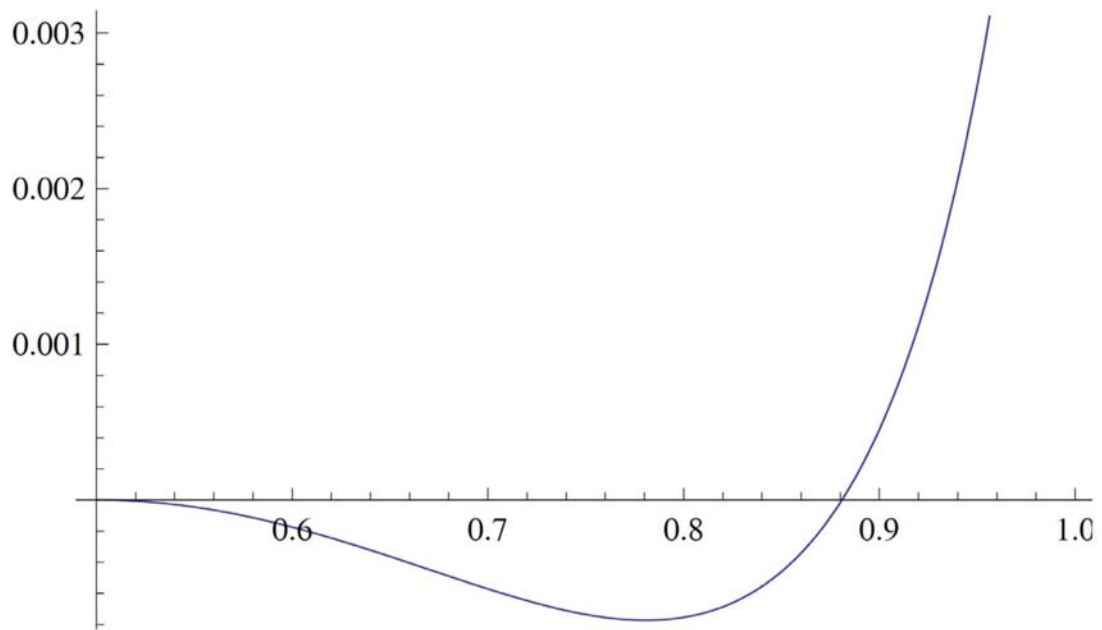


Figure 1: $R^{Before} - R^{None}$ as z varies and $\alpha = 0.24$