Decentralized Deterrence, with an Application to Labor Tax Auditing

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Online Appendix

A Details for example 2

Maximizing monetary returns of audits

Reporting Strategy: firms underreport by T if $y \geq T$, report 0 if $y \leq T$.

Auditing strategy:

$$p(r) = \frac{1}{2}\left(1 - \exp\left(\frac{r-(1-T)}{T}\right)\right)$$

The equilibrium is an application of results found in Erard and Feinstein (1994).

We now compute the parameter T so that the number of audits corresponds to 10% of firms audited:

$$\begin{aligned}\int_0^{1-T} \frac{1}{2}\left(1-\exp\left(\frac{x-(1-T)}{T}\right)\right)dx + \frac{T}{2}(\frac{1}{2}\left(1-\exp\left(\frac{T-1}{T}\right)\right)) &= 0.1 \\ T &= 0.6696.\end{aligned}$$

We now compute the revenue collected when this auditing strategy is used (tax collected + fines):

$$\begin{aligned}R &= \lambda t \int_0^1 x dx \\ &\quad + (1-\lambda) T \frac{T}{2}\frac{1}{2}\left(1-\exp\left(\frac{-(1-T)}{T}\right)\right) \\ &\quad + \int_0^{1-T} \frac{1}{2}\left(1-\exp\left(\frac{r-(1-T)}{T}\right)\right)(r/2+T)\,dr \\ &\quad + \int_0^{1-T} r/2\left(1-\frac{1}{2}\left(1-\exp\left(\frac{r-(1-T)}{T}\right)\right)\right)dr \\ &= 0.1742.\end{aligned}$$

Maximizing detection

Reporting strategy: firms with income y report $r = \frac{\alpha}{\alpha+1}y$.

Auditing strategy: $p(r) = \frac{1}{2}\left(1 - \left(\frac{\alpha+1}{a} r\right)^{\alpha}\right)$

The equilibrium is an application of results derived in section 5.

We now compute the parameter α so that the number of audits corresponds to 10% of firms audited

$$\begin{aligned} \int_0^{\alpha/(a+1)} \frac{1}{2}\left(1 - \left(\frac{\alpha+1}{\alpha} r\right)^{\alpha}\right)\left(\lambda + (1-\lambda)\frac{\alpha+1}{\alpha}\right) dr &= 0.1 \\ \alpha &= 0.441\,39 \end{aligned}$$

The revenue is calculated as follows:

$$\begin{aligned} R &= (1-\lambda)\int_0^{\frac{\alpha}{\alpha+1}} [p(r)\left(tr + (t+\theta)\left(\frac{\alpha+1}{\alpha} r - r\right)\right) + ((1-p(r))\, tr)] f(r)\, dr + \lambda t \int_0^1 x dx \\ R &= 0.179. \end{aligned}$$

B Proof of Proposition 3.

Proof. We first show that if X is distributed on $[a, b]$ according to $F(x) = \left(\frac{x-a}{b-a}\right)^{\beta}$, then $\mathbb{E}(X) = a + \frac{\beta}{\beta+1}(b-a)$.

$$\begin{aligned} \mathbb{E}(X) &= \int_a^b x dF(x) = a + \int_a^b (x-a)\, dF(x) = a + \int_a^b (x-a)\left(\frac{1}{b-a}\right)^{\beta} \beta (x-a)^{\beta-1} dx \\ &= a + \left(\frac{1}{b-a}\right)^{\beta} \beta \int_a^b (x-a)^{\beta} dx = a + \left(\frac{1}{b-a}\right)^{\beta} \beta \int_0^{b-a} y^{\beta} dy \\ &= a + \left(\frac{1}{b-a}\right)^{\beta} \beta \cdot \left(\left.\frac{y^{\beta+1}}{\beta+1}\right|_{y=0}^{b-a}\right) = a + \left(\frac{1}{b-a}\right)^{\beta} \frac{\beta}{\beta+1}(b-a)^{\beta+1} = a + \frac{\beta}{\beta+1}(b-a). \end{aligned}$$

Now, from the first-order conditions of the firm's problem, we get

$$\begin{aligned} \frac{p^{*\prime}(r)}{p^*(r) - \frac{t}{\theta+t}} &= \frac{1}{\gamma^{(1/\beta)}(r-a) + a - r} \\ &= \frac{1}{(r-a)}\frac{1}{\gamma^{(1/\beta)} - 1}, \end{aligned}$$

and integrating both sides yields

$$\ln\left(\frac{t}{\theta+t} - p^*(r)\right) = \ln(r-a)\frac{1}{\gamma^{(1/\beta)} - 1} + \kappa,$$

where κ denotes a constant of integration. Taking exponential on both sides leads to

$$p^*(r) = \frac{t}{\theta+t} - K(r-a)^{\alpha},$$

where $K = (\exp \kappa)$ is a constant that will be computed momentarily and we denote

$$\alpha = \frac{1}{\gamma^{(1/\beta)} - 1}.$$

Note that $\alpha > 0$ because $\gamma > 1$. Finally, we get the reporting strategy:

$$\rho^* (x) = a + \frac{1}{\gamma^{(1/\beta)}} (x - a) = a + \frac{\alpha}{\alpha + 1} (x - a) .$$

The constant K is computed using the fact that $p^* (\rho^* (b)) = 0$. Rewrite this condition as

$$\frac{t}{\theta + t} - K \left(\frac{\alpha}{\alpha + 1} (b - a) \right)^{\alpha} = 0,$$

whence $K = \frac{t}{\theta+t} \left(\frac{\alpha+1}{\alpha(b-a)} \right)^{\alpha}$. Substituting back into the probability of auditing yields

$$p^* (r) = \frac{t}{\theta + t} \left[1 - \left(\frac{\alpha + 1}{\alpha} \frac{r - a}{b - a} \right)^{\alpha} \right].$$

■

C Additional appendices for calibration

C.1 Numerical Solutions to the Nonlinear System of Equations (4)

C.1.1 Class below 10

Consider the audit class $(a = 0, M = 10)$, which is composed of all the firms who have true tax base between 1 and $b > 10$, and which are audited only when they report below 10. In this case we can compute that C_1, the ratio of honest to strategic firms among the firms which report less than 11 employees in our data, equals 0.6/0.4. We have C_2, the average number of employees reported by firms who report in $(1, 10)$, is given by 3.09 employees. Finally C_3, the total amount evaded conditional on evading a positive amount is 17,683 euros, which translated into employee-equivalents yields 0.67 employees.

$$\begin{aligned} \frac{\lambda}{(1-\lambda)\left(\frac{\alpha+1}{\alpha}\right)^{\beta}} &= \frac{0.6}{0.4} \\ (1-\lambda)\left(1+\frac{0.6}{0.4}\right)\alpha 0.67 &= 3.09 \\ \frac{10}{\alpha}\frac{\beta}{\beta+1} &= 0.67 \end{aligned}$$

Solution is: $\{[\lambda = 0.621\,51, \alpha = 4.\,874, \beta = 0.484\,91]\}$

C.1.2 Class 11-25

For the audit class identified by audited reports in $(a = 11, M = 25)$, which represent about 9 percent of our sample, we have that C_1, the ratio of honest to strategic firms equals 0.47/0.53. We have C_2, the average number of employees reported, is given by 15.39 employees. Finally C_3, the total amount evaded conditional on evading a positive amount is 29,950 euros, which translated into employee-equivalents yields 1.13 employees. Solving the system of equations (4) yields $[\lambda = 0.499\,37, \beta = 0.607\,88, \alpha = 4.\,684\,0]$. The highest true tax base in the audit class (unobserved because firms who report this much are not audited) equals $11 + (25 - 11)\,(\alpha + 1)/\alpha = 28$.

$$\begin{aligned}
\frac{\lambda}{(1-\lambda)\left(\frac{\alpha+1}{\alpha}\right)^{\beta}} &= \frac{0.47}{0.53} \\
(1-\lambda)\left(1+\frac{0.47}{0.53}\right)(11+\alpha\cdot 1.13) &= 15.39 \\
\frac{(25-11)}{\alpha}\frac{\beta}{\beta+1} &= 1.13
\end{aligned}$$

Rewrite slightly as

$$\begin{aligned}
\frac{\lambda}{(1-\lambda)\left(\frac{\alpha+1}{\alpha}\right)^{\beta}} &= 0.886\,79 \\
(1-\lambda)(1.886\,79)(11+\alpha(1.13)) &= 15.39 \\
14\frac{\beta}{\beta+1} &= \alpha(1.13)\,.
\end{aligned}$$

Solution is: $\{[\lambda = 0.499\,37, \beta = 0.607\,88, \alpha = 4.\,684\,0]\}$

C.1.3 Class 26-50

For the audit class identified by audited reports in $(a = 26, M = 50)$, which represent about 3 percent of our sample of audits, we have that C_1, the ratio of honest to strategic firms equals 0.43/0.57. We have C_2, the average number of employees reported, is 36 employees. Finally C_3, the total amount evaded conditional on evading a positive amount is 58,000 euros, which translated into employee-equivalents yields 2.19 employees. Solving the system of equations (4) yields $[\lambda = 0.551\,45, \alpha = 9.\,017\,1, \beta = 4.\,643\,8]$.

Solving the system of equations (4) yields

$$\begin{aligned}
\frac{\lambda}{(1-\lambda)\left(\frac{\alpha+1}{\alpha}\right)^{\beta}} &= \frac{0.43}{0.57} \\
(1-\lambda)\left(1+\frac{0.43}{0.57}\right)(26+\alpha 2.19) &= 36 \\
\frac{(50-26)}{\alpha}\frac{\beta}{\beta+1} &= 2.19.
\end{aligned}$$

Solution is: $\{[\lambda = 0.551\,45, \alpha = 9.017\,1, \beta = 4.643\,8]\}$.

The parameter β being greater than 1 will lead to a poor fit of (5) to the data. What we learn from this is that sometimes, matching the moments in (4) *perfectly* has a significant cost in terms of fit. One way around this problem is to relax the matching of the moments in (4). If we accept the second equation in (4) to equal 35.235 rather than 36, which seems like a small cost, then (4) yields a solution which produces a better fit of (5) to the data. This is done in the next subsection.

C.1.4 Class 26-50 relaxing $C_2 = 36$

Here we follow a different procedure. We fix $\beta = 1$, whereby from the third equation in (4) we get

$$24\frac{1}{1+1} = \alpha\,(2.19)$$

Solution is: $\alpha = 5.479\,5$. Substitute this into the first equation

$$\frac{\lambda}{(1-\lambda)\left(\frac{5.479\,5+1}{5.479\,5}\right)} = \frac{0.43}{0.57}$$

Solution is: $\lambda = 0.471\,48$. Substitute into the LHS of the second equation to get

$$(1 - 0.471\,48)\left(1 + \frac{0.43}{0.57}\right)(26 + (5.479\,5 \cdot 2.19)) = 35.235.$$

So if we allow C_2 to equal 35.235 rather than 36, then the parameter constellation we identifies solves the "relaxed" system (4). The solution $[\lambda = 0.471\,48, \alpha = 5.479\,5, \beta = 1]$ produces a better fit of (5) to the data. The implication is that, for this audit class, our model underpredicts slightly the reported number of employees.

C.1.5 Plotting Expression (5)

We want to work out expression (5) in the case of Power distribution. Since $\rho^{*-1}(r) = a + \frac{\alpha+1}{\alpha}(r-a)$, we have

$$\begin{aligned} & f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r} \\ = & \left(\frac{1}{b-a}\right)^{\beta}\beta\left(\frac{\alpha+1}{\alpha}(r-a)\right)^{\beta-1}\frac{\alpha+1}{\alpha} \\ = & \left(\frac{\alpha+1}{\alpha}\right)^{\beta}\left(\frac{1}{b-a}\right)^{\beta}\beta\,(r-a)^{\beta-1} = \left(\frac{\alpha+1}{\alpha}\right)^{\beta} f(r) \end{aligned} \tag{1}$$

Then

$$\begin{aligned} & p^*(r)\cdot\left[(1-\lambda)\,f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r} + \lambda f(r)\right] \\ = & \; p^*(r)\cdot\left[(1-\lambda)\left(\frac{\alpha+1}{\alpha}\right)^{\beta} + \lambda\right] f(r) \end{aligned}$$

where

$$p^* (r) = \frac{t}{\theta + t} \left[1 - \left(\frac{\alpha + 1}{\alpha} \frac{r - a}{b - a} \right)^{\alpha} \right],$$

and

$$f (r) = \left(\frac{1}{b - a} \right)^{\beta} \beta (r - a)^{\beta - 1}.$$

C.2 Details About Counterfactual Section I

In this appendix we describe how we compute $\widehat{\theta}$ and then to compute expressions 7 over 6.

$\widehat{\theta}$ is set so as to match the fraction of firms audited in our model to aggregate statistics available from INPS. In our model, the fraction of firms audited among those that report between a and M is constructed starting from (5) and is given by

$$\frac{\int_a^M p^*(r)\cdot\left[(1-\lambda)f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r}+\lambda f(r)\right]\,dr}{\int_a^M\left[(1-\lambda)f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r}+\lambda f(r)\right]\,dr}. \tag{2}$$

Crude statistics that are publicly available suggest that every year about 2-3 percent of firms who declare less than 10 reported employees are audited. When audited, we know from our data that the average firm's books are checked going back somewhat longer than 2 years. This "backward looking" span of the audit increases the deterrence power of auditing; we factor in this effect very crudely by setting the "effective" probability of auditing to 5 percent. Therefore we set expression (2) equal to 5 percent. We substitute out for $p^*(r)$ and write this condition as

$$\frac{\int_a^M\left[1-\left(\frac{\alpha+1}{\alpha}\frac{r-a}{b-a}\right)^{\alpha}\right]\cdot\left[(1-\lambda)f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r}+\lambda f(r)\right]\,dr}{\int_a^M\left[(1-\lambda)f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r}+\lambda f(r)\right]\,dr}=\frac{\theta+t}{t}0.05 \tag{3}$$

We set $t=0.4$ to capture an approximately 40% tax rate on gross wages, and we let θ range freely to achieve the desired equality. The parameter $\widehat{\theta}$ so obtained will capture the 33% penalty on the amount underreported, plus additional costs (psychological, legal, etc.) involved in being found in violation of the tax code. We expect therefore that $\widehat{\theta}\geq 0.33$.

C.2.1 Ancillary derivations used to compute expression 7

Expression (7) reads

$$\begin{aligned}
& \lambda\int_a^b tr\cdot f(r)\,dr \\
& +(1-\lambda)\int_a^M\left[tr+p^*(r)\left[(t+\theta)\left(\rho^{*-1}(r)-r\right)\right]\right]\cdot f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r}\,dr \\
= & \lambda\int_a^b tr\cdot f(r)\,dr \\
& +(1-\lambda)\left(\frac{\alpha+1}{\alpha}\right)^{\beta}\int_a^M\left[tr+p^*(r)\left[(t+\theta)\left(\frac{\alpha+1}{\alpha}(r-a)-(r-a)\right)\right]\right]\cdot f(r)\,dr
\end{aligned}$$

Since $\rho^{*-1}(r)=a+\frac{\alpha+1}{\alpha}(r-a)$ and $f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r}=\left(\frac{\alpha+1}{\alpha}\right)^{\beta}f(r)$,

(7) reads

$$\begin{aligned}
& \lambda \int_a^b tr \cdot f(r)\ dr \\
& + (1-\lambda) \int_a^M \left[tr + p^*(r) \left[(t+0.33) \left(\frac{\alpha+1}{\alpha}(r-a) - (r-a) \right) \right] \right] \cdot \left(\frac{\alpha+1}{\alpha} \right)^\beta f(r)\ dr \\
= & \lambda \int_a^b tr \cdot f(r)\ dr \\
& + (1-\lambda) \int_a^M \left[tr + p^*(r) \left[(t+0.33)(r-a) \left(\frac{\alpha+1}{\alpha} - 1 \right) \right] \right] \cdot \left(\frac{\alpha+1}{\alpha} \right)^\beta f(r)\ dr \\
= & \lambda \int_a^b tr \cdot f(r)\ dr \\
& + (1-\lambda) \int_a^M \left[tr + p^*(r) \left[(t+0.33)(r-a) \left(\frac{1}{\alpha} \right) \right] \right] \cdot \left(\frac{\alpha+1}{\alpha} \right)^\beta f(r)\ dr \\
= & \lambda t \int_a^b r \cdot f(r)\ dr + (1-\lambda) t \left(\frac{\alpha+1}{\alpha} \right)^\beta \int_a^M r \cdot f(r)\ dr \\
& + (1-\lambda)(t+0.33) \int_a^M p^*(r) \left[(r-a) \left(\frac{1}{\alpha} \right) \right] \cdot \left(\frac{\alpha+1}{\alpha} \right)^\beta f(r)\ dr
\end{aligned}$$

Lemma 1 $\int_a^M r \cdot f(r)\ dr = \left(\frac{\alpha}{\alpha+1} \right)^\beta \left[a + \frac{\alpha}{\alpha+1} \frac{\beta}{\beta+1} (b-a) \right]$

Proof.

$$\begin{aligned}
& \int_a^M r \cdot f(r)\ dr \\
= & \left(\frac{1}{b-a} \right)^\beta \beta \int_a^M r \cdot (r-a)^{\beta-1}\ dr \\
= & \left(\frac{1}{b-a} \right)^\beta \beta \left[\int_a^M (r-a)^\beta\ dr + \int_a^M a \cdot (r-a)^{\beta-1}\ dr \right] \\
= & \left(\frac{1}{b-a} \right)^\beta \beta \left[\int_0^{M-a} y^\beta\ dy + a \int_0^{M-a} y^{\beta-1}\ dy \right] \\
= & \left(\frac{1}{b-a} \right)^\beta \beta \left[\left. \frac{y^{\beta+1}}{\beta+1} \right|_{y=0}^{M-a} + a \left. \frac{y^\beta}{\beta} \right|_{y=0}^{M-a} \right] \\
= & \left(\frac{1}{b-a} \right)^\beta \beta \left[\frac{(M-a)^{\beta+1}}{\beta+1} + a \frac{(M-a)^\beta}{\beta} \right] \\
= & \left(\frac{1}{b-a} \right)^\beta \beta (M-a)^\beta \left[\frac{M-a}{\beta+1} + a \frac{1}{\beta} \right] \\
= & \left(\frac{M-a}{b-a} \right)^\beta \beta \left[\frac{M-a}{\beta+1} + a \frac{1}{\beta} \right].
\end{aligned}$$

Since $M = a + \frac{\alpha}{\alpha+1}(b-a)$, we can rewrite the above expression as

$$\begin{aligned} & \left(\frac{\alpha}{\alpha+1}\right)^{\beta} \beta \left[\frac{\frac{\alpha}{\alpha+1}(b-a)}{\beta+1} + a\frac{1}{\beta}\right] \\ = & \left(\frac{\alpha}{\alpha+1}\right)^{\beta} \left[\beta\frac{\frac{\alpha}{\alpha+1}(b-a)}{\beta+1} + a\right] \\ = & \left(\frac{\alpha}{\alpha+1}\right)^{\beta} \left[a + \frac{\alpha}{\alpha+1}\frac{\beta}{\beta+1}(b-a)\right]. \end{aligned}$$

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C.2.2 Audit class below 10

Solving equation (3) numerically based on the parameters calibrated in Section III., we get $\widehat{\theta} = 6.87$. This means that the *perceived* cost for being found cheating is estimated to be 6.87 times the amount underreported.

The first addend of (7) is the amount of taxes raised from honest firms; using Lemma 1, the first addend equals $\lambda t\left[a + \frac{\beta}{\beta+1}(b-a)\right] = \lambda t \cdot 3.9187 = \lambda \cdot 1.5675$. Combining Lemma 1 with (1), the second addend of (7) equals $(1-\lambda)\, t\left[a + \frac{\alpha}{\alpha+1}\frac{\beta}{\beta+1}(b-a)\right] = (1-\lambda)\, t \cdot 3.2516$; this is the amount of taxes paid by strategic firms.

Finally, the third addend of equation (7) is the money raised from audited cheaters and equals $(1-\lambda)(t+0.33)$ times the following:

$$\begin{aligned} & \int_a^{10} \frac{0.4}{6.87+0.4}\left(1 - \left(\frac{\alpha+1}{\alpha}\frac{r-a}{b-a}\right)^{\alpha}\right) \\ & \times (r-a)\left(\frac{1}{\alpha}\right) \cdot \left(\frac{\alpha+1}{\alpha}\right)^{\beta}\left(\frac{1}{b-a}\right)^{\beta} \beta\,(r-a)^{\beta-1}\; dr. \end{aligned}$$

We set the following values as definitions into the Scientific Word solver: $\alpha = 4.874; \beta = 0.48491; b = 12; a = 0$ and solve numerically. The integral is evaluated to be equal to 2.8132×10^{-2}. Therefore the third addend is equal to $(1-\lambda)(t+0.33)(0.02813)$. The total money raised from strategic firms is the sum of the second and third added, and after substituting $t = 0.4$ this amount equals

$$(1-\lambda)\, 1.3212.$$

C.2.3 Audit class 11-25

The relevant parameters for this class are $[a = 11, \lambda = 0.49937, \beta = 0.60788, \alpha = 4.6840]$. The highest true tax base in the audit class (unobserved because firms who report this much are not audited) equals $11 + (25-11)(\alpha+1)/\alpha = 28 = b$.

Probability of being audited about 2%, double it to 4%.

Replace 0.05 with 0.04 in equation (3) and solve numerically to get $\widehat{\theta} = 8.447$.

The first addend in (7) is the amount of taxes raised from honest firms; using Lemma 1, it equals $\lambda t\left[a+\frac{\beta}{\beta+1}(b-a)\right] = \lambda t \cdot 17.427 = \lambda \cdot 6.9708$. The second term is the amount of taxes paid by strategic firms, and it equals $(1-\lambda)\, t\left[a+\frac{\alpha}{\alpha+1}\frac{\beta}{\beta+1}(b-a)\right] = (1-\lambda)\, t \cdot 16.296 = (1-\lambda)\cdot 6.5184$. Finally, the the third term is the money raised from audited cheaters and is equal to $(1-\lambda)(t+0.33)$ times

$$\int_{11}^{25} \frac{0.4}{8.447+0.4}\left(1-\left(\frac{\alpha+1}{\alpha}\frac{r-a}{b-a}\right)^{\alpha}\right) \times (r-a)\left(\frac{1}{\alpha}\right)\cdot\left(\frac{\alpha+1}{\alpha}\right)^{\beta}\left(\frac{1}{b-a}\right)^{\beta}\beta\,(r-a)^{\beta-1}\ dr.$$

Solving numerically yields $(1-\lambda)(t+0.33)\, 0.038$.

The total money raised from strategic firms in equilibrium is

$$\begin{aligned} & (1-\lambda)\left[t\cdot 16.296+(t+0.33)(0.038)\right] \\ = \ & (1-\lambda)\left[(0.4)\cdot 16.296+(0.4+0.33)(0.038)\right] \\ = \ & (1-\lambda)\cdot 6.5461 \end{aligned}$$

C.2.4 Audit class 26-50

Solving equation (3) numerically based on the calibrated parameters in Section III., we get $\widehat{\theta} = 6.954$.

The first addend in (7) is the amount of taxes raised from honest firms; using Lemma 1, it equals $\lambda t\left[a+\frac{\beta}{\beta+1}(b-a)\right] = \lambda t\cdot 40.19 = \lambda\cdot 16.076$.

The second term is the amount of taxes paid by strategic firms, and it equals $(1-\lambda)\, t\left[a+\frac{\alpha}{\alpha+1}\frac{\beta}{\beta+1}(b-a)\right] = (1-\lambda)\, t\cdot 38.0 = (1-\lambda)\cdot 15.2$.

Finally, the third term is the money raised from audited cheaters and is equal to $(1-\lambda)(t+0.33)$ times

$$\int_{26}^{50} \frac{0.4}{6.954+0.4}\left(1-\left(\frac{\alpha+1}{\alpha}\frac{r-a}{b-a}\right)^{\alpha}\right) \times (r-a)\left(\frac{1}{\alpha}\right)\cdot\left(\frac{\alpha+1}{\alpha}\right)^{\beta}\left(\frac{1}{b-a}\right)^{\beta}\beta\,(r-a)^{\beta-1}\ dr.$$

Solving numerically yields $(1-\lambda)(t+0.33)\, 0.087$.

The total money raised from strategic firms in equilibrium is

$$\begin{aligned} & (1-\lambda)\left[t\cdot 38.0+(t+0.33)(0.087)\right] \\ = \ & (1-\lambda)\cdot 15.264 \end{aligned}$$

C.3 Details on Counterfactual Section II

Assume F has a Power distribution. Fix B and consider the class of "simple" audit strategies, each of which is characterized by $\widehat{x}$, T and p. We want to get to a closed form expression for the revenue (10) as a function of (known parameters and) p alone. To this end, we need to express $\widehat{x}$ and T as a function of p. From the budget constraint (9) we get

$$(1-\lambda)\left(\frac{\widehat{x}-a}{b-a}\right)^{\beta}+\lambda\left(\frac{T-a}{b-a}\right)^{\beta}=\frac{B}{p}$$

Using (8) to substitute for $T-a$ yields

$$\begin{aligned}
(1-\lambda)\left(\frac{\widehat{x}-a}{b-a}\right)^{\beta}+\lambda\left(\frac{p}{\widehat{\tau}}\frac{(\widehat{x}-a)}{b-a}\right)^{\beta} &= \frac{B}{p}\\
\left((1-\lambda)+\lambda\left(\frac{p}{\widehat{\tau}}\right)^{\beta}\right)\left(\frac{\widehat{x}-a}{b-a}\right)^{\beta} &= \frac{B}{p}\\
\left(\frac{\widehat{x}-a}{b-a}\right)^{\beta} &= \frac{B}{p\left((1-\lambda)+\lambda\left(\frac{p}{\widehat{\tau}}\right)^{\beta}\right)}
\end{aligned}$$

Hence, we have

$$\widehat{x}-a = (b-a)\left(\frac{B}{p\left((1-\lambda)+\lambda\left(\frac{p}{\widehat{\tau}}\right)^{\beta}\right)}\right)^{1/\beta} \tag{4}$$

$$\widehat{x} = a+(b-a)\left(\frac{B}{p\left((1-\lambda)+\lambda\left(\frac{p}{\widehat{\tau}}\right)^{\beta}\right)}\right)^{1/\beta} \tag{5}$$

$$T-a = \frac{p}{\widehat{\tau}}(b-a)\left(\frac{B}{p\left((1-\lambda)+\lambda\left(\frac{p}{\widehat{\tau}}\right)^{\beta}\right)}\right)^{1/\beta} \tag{6}$$

Now let's turn to the revenue. Expression (10) contains an integral which we want to solve for analytically. From the proof of Lemma 1 we have

$$\begin{aligned}
\int_a^x (y-a)\cdot f(y)\ dy &= \left(\frac{x-a}{b-a}\right)^{\beta}\left[\frac{x\beta+a}{(\beta+1)}\right]-a\left(\frac{x-a}{b-a}\right)^{\beta}\\
&= \left(\frac{x-a}{b-a}\right)^{\beta}\left[\frac{x\beta+a}{(\beta+1)}-a\right]\\
&= \left(\frac{x-a}{b-a}\right)^{\beta}\left[\frac{x\beta-a\beta}{(\beta+1)}\right]\\
&= \left(\frac{x-a}{b-a}\right)^{\beta}\frac{\beta}{(\beta+1)}(x-a)\,.
\end{aligned}$$

Substituting into (10) yields

$$R = (1-\lambda)\left[ta + p\cdot(t+\theta)\,F(\widehat{x})\,\frac{\beta}{(\beta+1)}(\widehat{x}-a) + t\,(T-a)\,(1-F(\widehat{x}))\right] + \lambda t\mathbb{E}(X).$$

Taking into account expressions (4)-(6), we have expressed the revenue as a function of p only. The term in brackets, corresponding to the revenue raised from strategic firms, is plotted in Figure 3.

Computing the revenue raised with the extremal strategy in the class below 10 The extremal strategy audits with probability $t/\left(t+\widehat{\theta}\right)$ all firms who report less than T (a threshold yet to be determined) and does not audit any firm which reports T or more. If $p(r) \geq t/\left(t+\widehat{\theta}\right)$ then a firm with true tax base r will report truthfully. Therefore, under the extremal strategy there is no cheating among firms who report below T, and the success rate of audits is zero. The threshold T is determined by the budget constraint. According to equation (2), in equilibrium a fraction equal to

$$0.05\int_a^M \left[(1-\lambda)\,f\left(\rho^{*-1}(r)\right)\frac{\partial\rho^{*-1}(r)}{\partial r} + \lambda f(r)\right]\,dr \qquad (7)$$

of all firms in the audit class is audited. This equation reduces to

$$\begin{aligned} & 0.05\left[(1-\lambda)\int_a^{10}\left(\frac{\alpha+1}{\alpha}\right)^{\beta}\left(\frac{1}{b-a}\right)^{\beta}\beta\,(r-a)^{\beta-1}\,dr + \lambda\left(\frac{M-a}{b-a}\right)^{\beta}\right] \\ = \;& 0.05\left[(1-\lambda)+\lambda\left(\frac{10}{12}\right)^{\beta}\right] \\ = \;& 0.05\left[(1-\lambda)+\lambda\,(0.915\,39)\right] = 0.05\cdot 0.947\,41 = 0.047, \end{aligned}$$

where we substituted $\lambda = 0.621\,5$. The number 0.047 needs to equal to the fraction of firms audited under the extremal strategy. Under the extremal strategy, all firms (strategic or not) with tax base below T will report truthfully and be audited with probability $\frac{t}{t+\theta}$. No other firm will report in that range. Therefore, the fraction of firms audited under the extremal strategy is given by the equation

$$\begin{aligned} \int_a^T \frac{t}{t+\widehat{\theta}} f(r)\,dr &= \frac{t}{t+\widehat{\theta}}F(T) \\ &= \frac{t}{t+\widehat{\theta}}\left(\frac{T-a}{b-a}\right)^{\beta}. \end{aligned}$$

Equating this to 0.047 and solving for T yields

$$T = a + (b-a)\left(\frac{t+\widehat{\theta}}{t}0.047\right)^{\frac{1}{\beta}} = 12\left(0.047\frac{0.4+6.87}{0.4}\right)^{\frac{1}{\beta}} = 8.671\,0.$$

The money raised from strategic firms under the extremal strategy equals the amount declared by firms with tax base below T, plus the amount declared by firms with tax base greater than T, which is exactly T. Formally,

$$\begin{aligned} & (1-\lambda)\left[\int_a^T trf(r)\,dr + tT(1-F(T))\right] \\ = \; & (1-\lambda)\left[t\int_a^T r\left(\frac{1}{b-a}\right)^{\beta}\beta(r-a)^{\beta-1}\,dr + tT\left(1-\left(\frac{T-a}{b-a}\right)^{\beta}\right)\right] \\ = \; & (1-\lambda)\left[t\left(\frac{T-a}{b-a}\right)^{\beta}\beta\left[\frac{T-a}{\beta+1}+a\frac{1}{\beta}\right] + tT\left(1-\left(\frac{T}{12}\right)^{\beta}\right)\right] \\ = \; & (1-\lambda)\,t\,[2.4188 + 1.2640] \\ = \; & (1-\lambda)\,1.4731. \end{aligned}$$

C.3.1 Other audit classes

We can repeat the same procedure for the other two audit classes. This is done in Appendix C.2.3 and C.2.4. For the audit class 26-50, The results for all audit classes are presented in the table below.

C.3.2 Audit class 11-25

According to equation (2), a fraction equal to

$$0.04\left((1-\lambda)+\lambda\left(\frac{M-a}{b-a}\right)^{\beta}\right) = 0.037$$

of all firms in the audit class is audited. This number needs to equal to the fraction of firms audited under the extremal strategy. Under the extremal strategy, the fraction of firms audited is given by the equation

$$\frac{t}{t+\widehat{\theta}}F(T) = \frac{t}{t+\widehat{\theta}}\left(\frac{T-a}{b-a}\right)^{\beta}.$$

Equating this to 0.037 and solving for T yields

$$T = a+(b-a)\left(\frac{t+\widehat{\theta}}{t}0.037\right)^{\frac{1}{\beta}} = a+(b-a)\left(\frac{0.4+8.447}{0.4}0.037\right)^{\frac{1}{\beta}} = 23.224.$$

The money raised from strategic firms under the extremal strategy equals

$$\begin{aligned} & (1-\lambda)\left[\int_a^T trf(r)\,dr + tT(1-F(T))\right] \\ = \; & (1-\lambda)\left[t\left(\frac{T-a}{b-a}\right)^{\beta}\beta\left[\frac{T-a}{\beta+1}+a\frac{1}{\beta}\right] + tT\left(1-\left(\frac{T-a}{b-a}\right)^{\beta}\right)\right] \\ = \; & (1-\lambda)\,t\,(12.784+4.219) = (1-\lambda)\,0.4\cdot(12.784+4.219) = (1-\lambda)\,6.8012 \end{aligned}$$

where the first equality follows from Lemma 1.

So, summing up, the money raised from honest firms is 6.9708. That raised from strategic firms in equilibrium is 6.5461. That raised from strategic firms under the extremal strategy is 6.8012.

C.3.3 Audit class 26-50

In this audit class we focus on the parameter constellation corresponding to $\beta = 1$. According to equation (2), a fraction equal to

$$0.046\left((1-\lambda)+\lambda\left(\frac{M-a}{b-a}\right)^{\beta}\right)=0.042$$

of all firms in the audit class is audited. This number needs to equal to the fraction of firms audited under the extremal strategy. Under the extremal strategy, the fraction of firms audited is given by the equation

$$\frac{t}{t+\widehat{\theta}}F(T)=\frac{t}{t+\widehat{\theta}}\left(\frac{T-a}{b-a}\right)^{\beta}.$$

Equating this to 0.042 and solving for T yields

$$T=a+(b-a)\left(\frac{t+\widehat{\theta}}{t}0.042\right)^{\frac{1}{\beta}}=a+(b-a)\left(\frac{t+6.954}{t}0.042\right)^{\frac{1}{\beta}}=47.914.$$

The money raised from strategic firms under the extremal strategy equals

$$\begin{aligned}
&(1-\lambda)\left[\int_a^T trf(r)\,dr+tT(1-F(T))\right]\\
=\;&(1-\lambda)\left[t\left(\frac{T-a}{b-a}\right)^{\beta}\beta\left(\frac{T-a}{\beta+1}+a\frac{1}{\beta}\right)+tT\left(1-\left(\frac{T-a}{b-a}\right)^{\beta}\right)\right]\\
=\;&(1-\lambda)\,t\,(28.537+10.917)=(1-\lambda)\,15.782
\end{aligned}$$

where the first equality follows from Lemma 1.

So, summing up, the money raised from honest firms is 16.076. That raised from strategic firms in equilibrium is 15.264. That raised from strategic firms under the extremal strategy is 15.782.