Online Appendix for

"Set-Asides and Subsidies in Auction,"

by Susan Athey, Dominic Coey and Jonathan Levin

Appendix A. Classifying Firms by Their Small Business Status

There are two types of bidders in our analysis: small bidders, who are eligible for set-asides, and big bidders, who are not. ALS use a different categorization. They distinguish between loggers, who do not have manufacturing capacity, and mills, who do. Appendix Table A1 shows how these classifications compare. Virtually all logging companies are small firms. These companies are responsible for most small bidder participation. There are, however, a number of mills that are eligible for small business set-aside sales.

One possibility would be to treat the small mills as a separate category of bidder. The main disadvantage, and the reason we did not do this, is the increased complexity, particularly in computing counterfactual equilibria. More generally, there is inevitably a trade-off between incorporating greater heterogeneity in the model and maintaining a tractable framework for analysis. Because the focus of the current analysis is on policies to aid small businesses, the large/small business distinction is the crucial dimension of heterogeneity to include, even though it means some departure from ALS.

Appendix B. Computing Sealed Bid Equilibria

To simulate outcomes under a counterfactual logger subsidy, we need to compute the (type-symmetric) bidding equilibrium of the auction when such a subsidy is in place. As noted in the text, we approximate the equilibrium with the equilibrium of an auction in which the bid space is discrete. This appendix describes how we find the type-symmetric equilibrium of this latter game.

Consider an auction with tract characteristics (X, u) and $n = (n_S, n_B)$ entrants. We use the value distributions $F_S(\cdot|X, u)$ and $F_B(\cdot|X, u)$, estimated in Section 4. We define a discrete approximation to the continuous auction game by assuming that each bidder must choose its bid from the set $\{b^0 = 0, b^1, ..., b^K = \bar{b}\}$. We set $K = 40$; finer grids did not have much impact on the results. We choose the maximum bid $\bar{b}$ to be the bid of a strong bidder with the highest possible value, $\overline{v} = \$400$ per mbf, in an unsubsidized auction with $n = (2, 2)$. Following Athey (2001), we look for a type-symmetric nondecreasing pure-strategy equilibrium of the discrete game. A strategy is nondecreasing if a higher value realization leads to a weakly higher bid.

We search for an equilibrium by iterating the best-response mapping. The key step in this process is to compute the best response for a bidder of type $\tau = S, B$, given that each type of opponent is using a nondecreasing strategy. Bidder j's optimal strategy given the strategies of its opponents can be represented as the vector of values $(v^0, v^1 ..., v^K = \overline{v})$, where j bids b^{k+1} for $v \in [v^k, v^{k+1})$, and 0 for $v < v^0$ (we suppress dependence on j in the notation).

To find j's best response, we first search for the values $v^{k,k+1}$ at which j is indifferent between bidding b^k and b^{k+1}, for $k = 0, 1, ..., K - 1$. Provided b^{k+1} has positive probability of winning, the structure of j's bidding problem then means that if $v < v^{k,k+1}$, j strictly prefers to bid b^k over b^{k+1}, while if $v > v^{k,k+1}$, j strictly prefers b^{k+1} over b^k.

Now, if $0 \leq v^{0,1} \leq v^{1,2} \leq ... \leq v^{K-1,K} \leq \overline{v}$, then j's optimal strategy is given by the vector of indifference points: $(v^0, v^1, ..., v^K) = (v^{0,1}, ..., v^{K-1,K}, \overline{v})$. To see why, note that if $v \in \left(v^{k-1,k}, v^{k,k+1}\right)$, j strictly prefers b^k to b^{k+1}, and transitively to any higher bid, and also prefers b^k to b^{k-1} and transitively to any lower bid.

If $0 \leq v^{0,1} \leq v^{1,2} \leq ... \leq v^{K-1,K} \leq \overline{v}$ fails, the best response strategy "skips" some bids. That is, some bids are not optimal for any values. In this case the algorithm works as follows: If $v^{0,1} < \overline{v}$ and if bidding b^1 is optimal given value $v^{0,1}$, set $v^0 = v^{0,1}$. Otherwise, bid b^1 is skipped and we move to b^2. If $v^{0,2} < \overline{v}$ and if b^2 is optimal given value $v^{0,2}$, set $v^0 = v^1 = v^{0,2}$. Otherwise, bid b^2 is also skipped, and we move to b^3. We continue in this way until we find a bid that isn't skipped. Once we find such a bid b^ℓ we start again, by checking that $v^{\ell,\ell+1} < \overline{v}$ and that bidding $b^{\ell+1}$ is optimal given value $v^{\ell,\ell+1}$. When complete, this process gives the best response vector $(v^0, v^1, ..., v^K = \overline{v})$.

Having identified the thresholds $v^0, ..., v^K$, we take a linear interpolation through the implied step function in order to produce a "best-response" bidding function without mass points. (As the grid gets fine, this linear interpolation converges to the continuous bid space best response function.)

Now, denote the best response correspondence by BR, where $BR : (\mathbb{R}_+^K)^2 \longrightarrow (\mathbb{R}_+^K)^2$. The type-symmetric Nash equilibrium $E \in (\mathbb{R}_+^K)^2$ satisfies $BR(E) = E$. We approximate E by iteration. Initially, opponents are assumed to play a naive linear strategy, in which they bid a fraction $(n_L + n_M - 1)/(n_L + n_M)$ of their value. Thereafter we define $E_t = \lambda E_{t-1} + (1-\lambda) BR(E_{t-1})$ and iterate until E_t and E_{t-1} are sufficiently close together, so that the differences in expected outcomes implied by E_t and E_{t-1} are small (the implied change in prices is less than $\frac{1}{10}\%$). If the "damping parameter" $\lambda = 0$, this reduces to iterating best responses. One test of this algorithm is to verify that it finds the equilibrium estimated in the data. Indeed, the bidding distribution estimated directly from the data matches the equilibrium the algorithm computes, given the estimated value distributions.

Note that our simulation procedure requires us to compute an equilibrium for each set of observed tract characteristics X, each possible entry vector n, and each value of u. Several simplifications reduce the computational burden. Because we are interested in expected outcomes that average across values of u, we compute equilibria for eight values of u, and use quadrature to compute expected outcomes. Our model in Section 4 also has the convenient feature that the observable tract characteristics X affect bidder values (and equilibrium bids) as a multiplicative shifter. So given u, n, once we compute an equilibrium for an auction with characteristics X, we can find the equilibrium for an auction with characteristics X' (and identical u, n) simply by multiplying the equilibrium bids by $\exp((X - X')\beta_X)$.

Appendix C. Additional Tables and Figures

Table A1 contains cross-tabulations comparing our classification of small and large firms to ALS's classification of logging companies and mills.

Table A2 contains summary statistics for the sample of sealed bid auctions used to estimate the structural model.

Table A3 reports a logistic regression in which the dependent variable is a dummy equal to one if a small firm won the auction and the explanatory variables are sale characteristics and the set-aside propensity score. The data sample for estimation consists of unrestricted auctions.

Figure A1 shows the value distributions for small and big bidders for a sale with average characteristics ($X = \overline{X}$ and $u = 1$), along with the equilibrium sealed bid functions assuming two small and two large firms participate in the auction.

Figure A2 shows the empirical distribution of small bidder entrants across all unrestricted sales in our data, as well as the distribution implied by the fitted binomial entry model.

Appendix Table A1: Bidder Definitions

	By Participation			By Identity		
	Small Bidder	Big Bidder	Total	Small Bidder	Big Bidder	Total
Logger	2659	17	2676	530	4	534
Mill	1240	1796	3036	47	30	77
Total	3899	1813	5712	577	34	611

Note: In the "By participation" columns, an observation is an instance of participation in an auction by a bidder. In the "By identity" columns, an observation is a bidder. A small bidder is a bidder eligible to participate in a small business set-aside sale. A logger is a bidder without manufacturing capacities.

Appendix Table A2: Summary Statistics - Estimation Sample

	Small Sales		Big Sales	
N	162		154	
	Mean	Std. Dev.	Mean	Std. Dev.
Auction Outcomes				
Prices ($/mbf)	92.42	59.64	105.29	72.51
Entrants	4.38	2.55	5.18	2.42
# Small Firms Entering	4.17	2.49	4.08	2.28
# Big Firms Entering	0.22	0.49	1.09	1.20
Small Firm Wins Auction	0.94	0.24	0.70	0.46
Appraisal Variables				
Volume of timber (100 mbf)	2.90	1.37	18.71	22.05
Small Sale Dummy	1.00	0.00	0.00	0.00
Reserve Price ($/mbf)	41.23	33.83	39.49	33.72
Selling Value ($/mbf)	2.59	1.12	264.56	106.46
Road Construction ($/mbf)	0.32	2.43	2.44	6.33
Road Costs Missing	0.00	0.00	0.01	0.08
Logging Costs ($/mbf)	98.06	46.86	94.42	41.31
Manufacturing Costs ($/mbf)	109.39	60.32	118.56	47.92
Sale Characteristics				
Contract Length (months)	8.44	4.60	15.11	8.35
Species Herfindal	0.57	0.22	0.55	0.24
Density of Timber (10,000 mbf/acre)	1156.86	1391.30	1566.86	2056.82
Sealed Bid Sale	1.00	0.00	1.00	0.00
Scale Sale	0.56	0.50	0.77	0.42
Quarter of Sale	2.57	0.88	2.49	0.95
Year of Sale	84.98	2.01	85.58	2.25
Housing Starts	1601.64	261.59	1567.91	254.99
Local Industry Activity				
Logging companies in county	14.06	13.24	20.32	19.83
Sawmills in County	5.27	5.67	6.32	8.15
Small Firms Active in Last Year	14.54	7.38	12.11	6.33
Big Firms Active in Last Year	2.79	1.63	2.88	1.46

Note: The bidding model (presented in Table 6) is estimated on unrestricted sealed sales with more than one entrant. This table summarizes outcomes and tract characteristics for this sample, broken down by size.

Appendix Table A3: Small Bidder Wins

	Marginal Effect	Std. Err.
Appraisal Controls		
Ln(Reserve Price)	-0.070	(0.996)
Ln(Selling Value)	0.024	(0.346)
Ln(Manufacturing Costs)	-0.509	(7.275)
Ln(Logging Costs)	-0.072	(1.040)
Ln(Road Costs)	-0.038	(0.542)
Road Costs Missing (Dummy)	-0.521	(2.294)
Appraisal Missing	-0.843	(4.807)
Other Sale Characteristics		
SBA Propensity Score	0.280	(4.012)
ln(Contract Length/volume)	-8.455	(120.920)
Species Herfindal	-0.059	(0.855)
Density of Timber (10,000 mbf/acres)	-0.272	(3.886)
Sealed Bid (Dummy)	0.049	(0.747)
Scale Sale (Dummy)	-0.128	(2.218)
ln(Monthly US House Starts)	-0.449	(6.416)
Volume Controls (Dummy Variables):		
Volume: 1.5-3 hundred mbf	-	-
Volume: 3-5	-0.888	9.891
Volume: 5-8	-0.924	8.020
Volume: 8-12	-0.885	9.424
Volume: 12-20	-0.919	8.101
Volume: 20-40	-0.955	5.734
Volume: 40-65	-0.984	2.795
Volume: 65-90	-0.971	4.186
Volume: 90+	-0.997	0.801
Local Industry Activity		
ln(Loggers in County)	-0.025	0.364
ln(Sawmills in County)	-0.023	0.324
ln(Active Small Firms)	0.048	0.680
ln(Active Big Firms)	-0.135	1.932
Additional Controls (Dummy Variables)		
Chi-Squared Statistics (p-value in parenthesis)		
Years	14.860	(0.038)
Quarters	2.750	(0.432)
Species	2.900	(0.822)
Location	32.800	(0.000)
	N=1064	
	LR chi2 (52)	492.59
	P-value	0.00
	Pseudo-R2	0.33

Note: Table reports results from a logit regression on unrestricted sales where the dependent variable is equal to one if a small bidder wins the sale. The estimates are reported as marginal probability effects at the mean of the independent variables. SBA Propensity Score is the predicted probability of the sale being held as a set-aside sale. Volume: 1.5-3 hundred mbf predicts success perfectly and is dropped.

Figure A1: Estimated Value Distributions and Bid Functions for the Case of Two Small and Two Big Bidders in a Big Sale

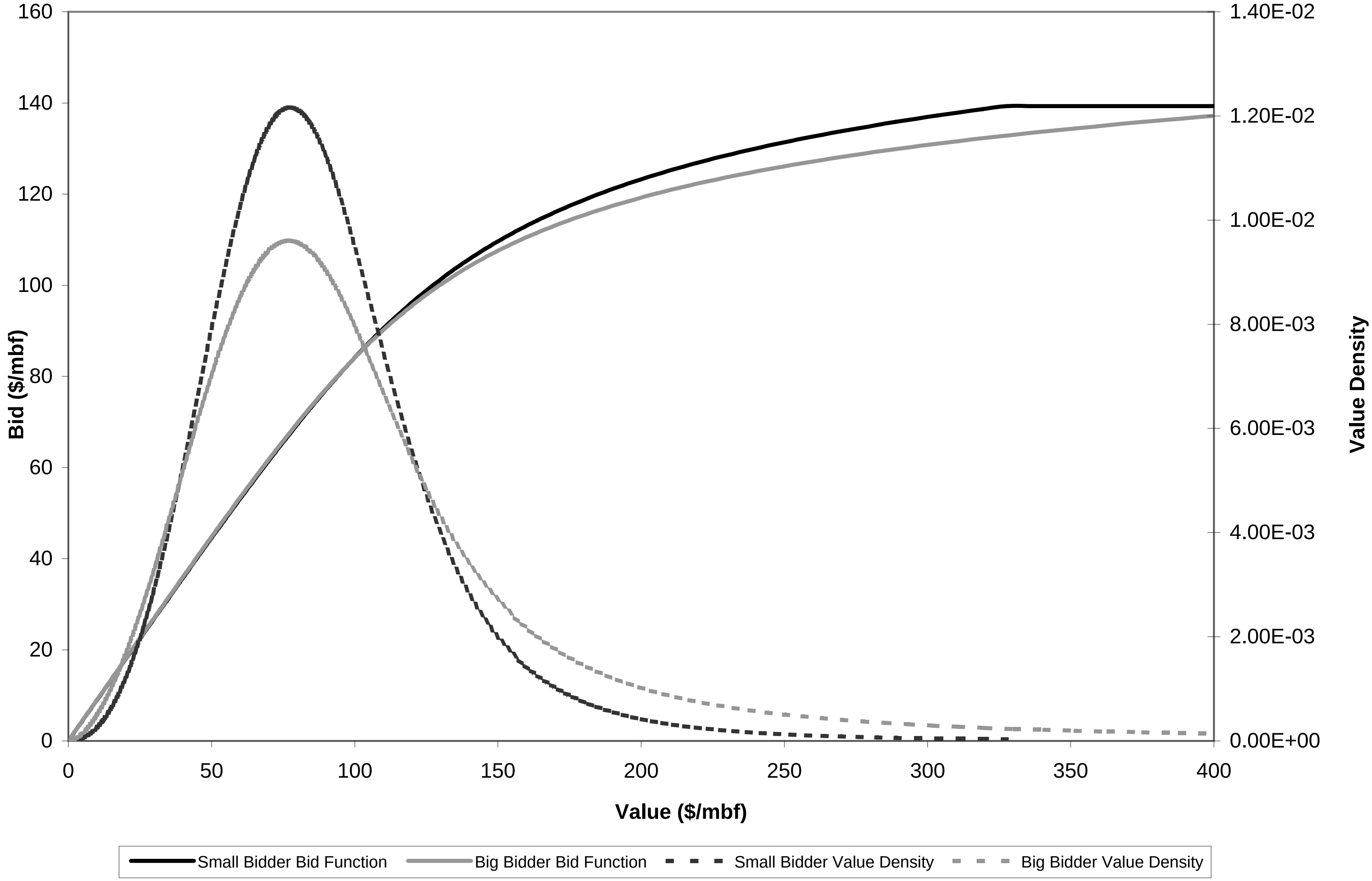

Figure A2: Actual and Simulated Entry