

# Public Goods, Social Pressure, and the Choice Between Privacy and Publicity

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## TECHNICAL APPENDIX

This technical appendix includes proofs of comparative statics results; the proof of the claim made in the text that if  $\beta' > \beta$ , then  $H^\delta(\delta; \beta')$  first-order stochastically dominates  $H^\delta(\delta; \beta)$ ; computational results, and detailed analysis of the *interim* preferences over policies.

### A. Comparative Statics

The functions  $g^P(\theta_i)$  and  $g^O(\theta_i)$  depend on  $\theta_i$ ,  $\beta$ ,  $\gamma$ , and  $p$ ; they are independent of  $\alpha$ .

*Comparative statics of  $g^P(\theta_i)$ .* Since  $g^P(\theta_i) = g_{min} + \theta_i$  and  $g_{min} = \gamma - p$ , it is obvious that  $g^P(\theta_i)$  is an increasing function of  $\theta_i$ , and that the function  $g^P(\theta_i)$  shifts upward with an increase in  $\gamma$ , and shifts downward with an increase in  $p$ . Finally, the function  $g^P(\theta_i)$  is always independent of  $\beta$ . Since the utility function is quasilinear,  $g^P(\theta_i)$  is independent of income,  $I$ .

*Comparative statics of  $g^O(\theta_i)$ .* Since  $g^O(0) = g^P(0)$ ,  $g^O(0)$  behaves as described above with respect to the parameters. Thus, in what follows, we will consider only  $\theta_i > 0$ . Let  $RHS \equiv g_{min} + \theta_i + \beta(1 - \exp[-(g^O(\theta_i) - g_{min})/\beta])$ . For any parameter  $m$ , the implicit function in Proposition 1(i) can be differentiated to obtain  $\partial g^O/\partial m = (\partial RHS/\partial m) + (\partial RHS/\partial g^O)(\partial g^O/\partial m)$ . Collecting terms implies that  $\partial g^O/\partial m = (\partial RHS/\partial m)/(1 - \exp[-(g^O(\theta_i) - g_{min})/\beta])$ . Since the denominator is positive, the sign of  $\partial g^O/\partial m$  is the same as the sign of  $(\partial RHS/\partial m)$ . To save on notation, it will be useful to define the function  $z^O(\theta_i) \equiv (g^O(\theta_i) - g_{min})/\beta$ , and to use  $z$  to denote an arbitrary (positive) value.

Since  $\partial RHS/\partial \theta_i = 1$ , it follows that  $g^{O'}(\theta_i) = 1/(1 - \exp[-z^O(\theta_i)]) > 0$ ; that is, the equilibrium action under a policy of publicity (openness) is increasing in type.

Since the parameters  $\gamma$  and  $p$  appear only in  $g_{min}$ , and  $(\partial RHS/\partial g_{min}) = (1 - \exp[-z^O(\theta_i)])$ , it is straightforward to show that  $\partial g^O(\theta_i)/\partial g_{min} = 1$ . Therefore  $\partial g^O(\theta_i)/\partial \gamma = 1$  and  $\partial g^O(\theta_i)/\partial p = -1$ .

Differentiating and collecting terms yields  $\partial g^O(\theta_i)/\partial \beta = (1 - \exp[-z^O(\theta_i)] - z^O(\theta_i)\exp[-z^O(\theta_i)])/(1 - \exp[-z^O(\theta_i)])$ . The function  $1 - \exp[-z] - z\exp[-z]$  is easily shown to be positive for  $z > 0$ ; thus,  $\partial g^O(\theta_i)/\partial \beta > 0$ .

*Comparative statics of the action differential  $g^O(\theta_i) - g^P(\theta_i)$ .*

Let  $\delta(\theta_i; \beta) \equiv g^O(\theta_i) - g^P(\theta_i) = \beta(1 - \exp[-z^O(\theta_i)])$  denote the action differential as a function of  $\theta_i$ . This difference is increasing in type; that is,  $\delta'(\theta_i; \beta) = \exp[-z^O(\theta_i)]g^{O'}(\theta_i) > 0$ . Thus, the highest type inflates his action the most. We have already seen that  $\partial g^O(\theta_i)/\partial g_{min} = 1$ ; this yields the immediate result that  $\partial z^O(\theta_i)/\partial g_{min} = (\partial g^O(\theta_i) - g_{min})/\partial g_{min}/\beta = 0$ . This implies that the action differential  $\delta(\theta_i; \beta)$  is independent of the parameters  $\gamma$  and  $p$ . Since  $g^P(\theta_i)$  is independent of  $\beta$ , then  $\partial \delta(\theta_i; \beta)/\partial \beta = \partial g^O(\theta_i)/\partial \beta > 0$ .

#### B. Proof of Claim that if $\beta' > \beta$ , then $H^\delta(\delta; \beta')$ First-order Stochastic Dominates $H^\delta(\delta; \beta)$

Recall that  $\delta(\theta; \beta) = \beta(1 - \exp[-(g^O(\theta) - g_{min})/\beta])$ , and let  $\bar{\alpha}(\beta) \equiv \delta(\bar{\theta}; \beta)$  for any given  $\beta$ ; since  $\delta(\bar{\theta}; \beta)$  is increasing in  $\beta$ , so is  $\bar{\alpha}(\beta)$ . Therefore the support of  $H^\delta(t; \beta)$  induced by  $H(\theta)$  and  $\delta(\theta; \beta)$  is  $[0, \bar{\alpha}(\beta)]$ . Then, fixing  $\beta$ :

$$H^\delta(t; \beta) \equiv \Pr\{\delta(\theta; \beta) \leq t\} = \Pr\{\theta \leq (g^O)^{-1}(\beta \ln(\beta/(\beta - t) + g_{min}))\} = H((g^O)^{-1}(\beta \ln(\beta/(\beta - t) + g_{min}))).$$

Thus,  $\partial H^\delta(t; \beta)/\partial \beta = h(t)[((g^O)^{-1}(t))'(\ln(\beta/(\beta - t) + g_{min}))][\ln \beta + 1 - \ln(\beta - t) - \beta/(\beta - t)]$ , so that  $\partial H^\delta(t; \beta)/\partial \beta < 0$  if and only if  $\ln \beta + 1 - \ln(\beta - t) - \beta/(\beta - t) < 0$ . Note that  $H^\delta(0; \beta) = 0$  and  $H^\delta(\bar{\alpha}(\beta); \beta) = \Pr\{\delta(\bar{\theta}; \beta) \leq \bar{\alpha}(\beta)\} = 1$  for any given value of  $\beta$ , so we are interested in  $\partial H^\delta(t; \beta)/\partial \beta$  for  $t \in (0, \bar{\alpha}(\beta))$ .<sup>1</sup>

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<sup>1</sup> Note that increasing  $\beta$  increases the right end-point, so this means we must extend  $H^\delta(t; \beta)$  to be 1 on the interval  $[\bar{\alpha}(\beta), \bar{\alpha}(\beta')]$  when we compare it to the distribution  $H^\delta(t; \beta')$ , so that they are on the same support.

Note that  $\ln\beta + 1 - \ln(\beta - t) - \beta/(\beta - t) < 0$  if and only if  $\ln(\beta/(\beta - t)) < 1 - \beta/(\beta - t)$  for  $t$  in this open interval. Note that  $t < \beta$  since  $(1 - \exp[-(g^O(\theta) - g_{min})/\beta]) < 1$ . Thus, we may restate the problem as: is  $\ln x < x - 1$  for  $x \geq 1$ ? In fact, the line  $x - 1$  is tangent to  $\ln x$  at  $x = 1$ , so  $\ln x < x - 1$  for  $x > 1$  and the two functions are equal at  $x = 1$ . Therefore,  $\partial H^\delta(t; \beta)/\partial \beta < 0$  for  $t \in (0, \bar{t}(\beta))$ , so that if  $\beta' > \beta$ , then  $H^\delta(t; \beta') < H^\delta(t; \beta)$  for  $t \in (0, \bar{t}(\beta))$ ; that is,  $H^\delta(t; \beta')$  FOSD  $H^\delta(t; \beta)$ .

### C. Computational Results on the Effect of $\beta$ on $\alpha^{PO}$

Table 1 below displays computational results for four density functions: 1) the Uniform density, with  $h(\theta) = 1$ ; 2) the Left Triangle density, with  $h(\theta) = 2 - 2\theta$ ; 3) the Middle Triangle density, with  $h(\theta) = 4\theta$  when  $\theta \leq 1/2$ , and  $h(\theta) = 4 - 4\theta$  when  $\theta > 1/2$ ; and 4) the Right Triangle density, with  $h(\theta) = 2\theta$ . Notice that the Uniform density is a mean-preserving spread of the Middle Triangle density.

TABLE 1 – EFFECT OF  $\beta$  ON  $E(\delta^2)/E(\delta)$  FOR ALTERNATIVE DENSITIES OF  $\theta$

density↓	$\beta \rightarrow$	0.5	1.0	2.0
Uniform		0.40859	0.69264	1.14159
Left Triangle		0.36996	0.61131	0.96546
Middle Triangle		0.41363	0.69296	1.10361
Right Triangle		0.43900	0.75341	1.22101

Table 1 suggests that, for a given density, increasing  $\beta$  increases  $\alpha^{PO}$ , so that  $\Phi^{PO}(\alpha^{PO})$  shifts up, associating more values of  $\alpha$  with privacy than were associated with the lower value of  $\beta$ . Also, note that, holding  $\beta$  constant, the computed values of  $E(\delta^2)/E(\delta)$  increase as we move from the Left to the Middle to the Right Triangle distributions. Thus, Table 1 is consistent with the conjecture that a shift in  $H$  to a new distribution  $H'$ , where  $H'$  first-order stochastically dominates  $H$ , results in higher

values of  $\alpha^{PO}$  as well (i.e., upward shifts of  $\Phi^{PO}$ , too).

#### D. Material on Interim Preferences over Policies $P$ and $O$

This material pertains to Proposition 4. Two results follow from equation (5). First, comparing with equation (3), we see that  $E(\Gamma^{PO}(\theta, \alpha)) = \Phi^{PO}(\alpha)$ , so that when evaluated at  $\alpha = \alpha^{PO}$ ,  $E(\Gamma^{PO}(\theta, \alpha^{PO}), \alpha^{PO}) = 0$ . Since differentiating  $\Gamma^{PO}(\theta_i, \alpha)$  shows that it is a monotonically decreasing function of  $\theta_i$  for each value of  $\alpha$ , this implies that  $\Gamma^{PO}(0, \alpha^{PO}) > 0$  and  $\Gamma^{PO}(\bar{\theta}, \alpha^{PO}) < 0$ , so that on an *interim* basis, if  $\alpha = \alpha^{PO}$ , then lower types will (*interim*) prefer  $P$  to  $O$  and higher types will (*interim*) prefer  $O$  to  $P$ . Define two other values of  $\alpha$ , namely  $\underline{\alpha}^{PO} \geq 0$  such that  $\Gamma^{PO}(\bar{\theta}, \underline{\alpha}^{PO}) = 0$  when  $\mu \geq \bar{\theta} - (\delta(\bar{\theta}; \beta))^2/2\beta$  (that is, the value of  $\alpha$  such that all types will *interim* prefer  $P$  to  $O$  for any  $\alpha \leq \underline{\alpha}^{PO}$ ; note that if  $\mu < \bar{\theta} - (\delta(\bar{\theta}; \beta))^2/2\beta$  then no such non-negative value exists), and  $\bar{\alpha}^{PO}$  such that  $\Gamma^{PO}(0, \bar{\alpha}^{PO}) = 0$  (that is, the value of  $\alpha$  such that all types will *interim* prefer  $O$  to  $P$  for any  $\alpha \geq \bar{\alpha}^{PO}$ ). By construction,  $\underline{\alpha}^{PO} < \alpha^{PO} < \bar{\alpha}^{PO}$ . Furthermore, when  $\alpha \leq \underline{\alpha}^{PO}$ , the *ex ante* social preference for  $P$  over  $O$  is therefore reinforced by *interim* unanimity for  $P$  over  $O$ , while when  $\alpha \geq \bar{\alpha}^{PO}$ , the *ex ante* social preference for  $O$  over  $P$  is reinforced by *interim* unanimity for  $O$  over  $P$ . However, when  $\alpha$  lies between  $\underline{\alpha}^{PO}$  and  $\bar{\alpha}^{PO}$ , lower types prefer  $P$  to  $O$  while higher types prefer  $O$  to  $P$ , so that for all  $\alpha$  in the interval  $(\underline{\alpha}^{PO}, \bar{\alpha}^{PO})$  there is disagreement about the preferred policy at the *interim* stage, and there will not be unanimous reinforcement of any *ex ante* policy choice.

#### E. Conflict Between Ex Ante and Interim Preferences

To see the possibility of conflict between *ex ante* and *interim* preferences in a case wherein  $O$  is *ex ante* preferred but  $P$  is *interim* preferred by the median type, let  $\theta^{PO}(\alpha)$  be the marginal type

such that  $\Gamma^{PO}(\theta^{PO}(\alpha), \alpha) = 0$ , for  $\alpha \geq 0$ . Note that  $\theta^{PO}(\alpha)$  is decreasing in  $\alpha$ , and that  $\theta^{PO}(0) > \mu$ , the mean (and median) type if  $H$  is symmetric. Thus, there is an  $\alpha^*$  such that  $\theta^{PO}(\alpha^*) = \mu$ . It is straightforward to show that  $\alpha^* \in (\alpha^{PO}, \bar{\alpha}^{PO})$ , so that for any value of  $\alpha$  in the interval  $(\alpha^{PO}, \alpha^*)$ , the *ex ante* social payoff-maximizing choice of policy is  $O$ , but on an *interim* basis, the median type would prefer  $P$  to  $O$ .

To see how the reverse conflict can occur, assume that  $\alpha = 0$ . Since  $\alpha^{PO} > 0$ , this means that society *ex ante* prefers  $P$  to  $O$ . Since  $\theta^{PO}(0) > \mu$ , then any density  $h$  whose median is to the right of  $\theta^{PO}(0)$  implies that the median type prefers  $O$  to  $P$ . Signaling type to gain esteem is sufficiently valuable to the median type (but is irrelevant in the case of the *ex ante* decision) for those types to *interim* prefer  $O$  to  $P$ . This conflict between the *ex ante* and *interim* settings is summarized below.

REMARK 2. Conflicting *Ex Ante* and *Interim* Preferences over Policies.

*h* symmetric: There are values of  $\alpha$  such that while a policy of publicity is *ex ante* socially preferred, the alternative policy of privacy is *interim*-preferred by the median type.

*h* sufficiently right-weighted: There are values of  $\alpha$  such that while a policy of privacy is *ex ante* socially preferred, a policy of publicity is *interim*-preferred by the median type.

PROOF OF PROPOSITION 6(a):

Proposition 6(a) provides the following ordering of the  $\alpha$ -values at which there is *ex ante* indifference between any two policies:  $0 < \alpha^{WO} < \alpha^{PO} < \alpha^{PW}$ . To see that  $0 < \alpha^{WO} < \alpha^{PO}$ , let:

$$\eta(t) \equiv \int_0^t (\delta(\theta; \beta))^2 h(\theta) d\theta / \int_0^t \delta(\theta; \beta) h(\theta) d\theta.$$

Then  $\alpha^{WO} = \eta(\theta^W)$ , which is clearly positive, while  $\alpha^{PO} = \eta(\bar{\theta})$ . It is straightforward to show that  $\text{sgn} \{\eta'(t)\} = \text{sgn} \{\delta(t; \beta) \int_0^t \delta(\theta; \beta) h(\theta) d\theta - \int_0^t (\delta(\theta; \beta))^2 h(\theta) d\theta\} > 0$  for all  $t > 0$ . Therefore, it follows that  $\alpha^{PO} = \eta(\bar{\theta}) > \eta(\theta^W) = \alpha^{WO}$ .

To see that  $\alpha^{PO} < \alpha^{PW}$ , let

$$v(s) \equiv \int_s^{\bar{\theta}} (\delta(\theta; \beta))^2 h(\theta) d\theta / \int_s^{\bar{\theta}} \delta(\theta; \beta) h(\theta) d\theta.$$

Then  $\alpha^{PO} = v(0)$ , while  $\alpha^{PW} = v(\theta^W)$ . It is straightforward to show that  $\text{sgn} \{v'(s)\} = \text{sgn} \{\int_s^{\bar{\theta}} (\delta(\theta; \beta))^2 h(\theta) d\theta - \delta(s; \beta) \int_s^{\bar{\theta}} \delta(\theta; \beta) h(\theta) d\theta\} > 0$  for all  $s < \bar{\theta}$ . Therefore, it follows that  $\alpha^{PO} = v(0) < v(\theta^W) = \alpha^{PW}$ .

#### F. Material on Interim Preferences over Policies $P$ , $O$ and $W$

Throughout this discussion we assume that  $\theta^W \in (0, \bar{\theta})$ ; if not, then the policy  $W$  coincides with either  $O$  or  $P$  and there are not three distinct policies to be compared.

Recall that the conditional mean is  $\mu(\theta^W) = \int_{\mathcal{J}} th(t) dt / H(\theta^W)$ , where  $\mathcal{J} = [0, \theta^W]$ . Furthermore, let  $E(g^O - g^P)$  denote the expected distortion under a policy of  $O$  versus a policy of  $P$ , and similarly for  $E(g^W - g^P)$  and  $E(g^O - g^W)$ . Then:

- (a)  $E(g^O - g^P) = \int \delta(t; \beta) h(t) dt$ , where the integral is taken over  $[0, \bar{\theta}]$ ;
- (b)  $E(g^W - g^P) = \int_{\mathcal{J}^c} \delta(t; \beta) h(t) dt$ , where the integral is taken over  $\mathcal{J}^c = [\theta^W, \bar{\theta}]$ ;
- (c)  $E(g^O - g^W) = \int_{\mathcal{J}} \delta(t; \beta) h(t) dt$ , where the integral is taken over  $\mathcal{J} = [0, \theta^W]$ .

The integral in part (a) reflects the fact that every type (except the lowest) distorts her action under a policy of  $O$  while no type distorts her action under a policy of  $P$ . The integral in part (b) reflects the fact that only those types in  $\mathcal{J}^c = [\theta^W, \bar{\theta}]$  distort their actions. Finally, the integral in part (c) reflects the fact that only those types in  $\mathcal{J} = [0, \theta^W]$  do not distort their actions.

These definitions allow us to summarize the type-specific value of one policy over another. Let  $I^{PO}(\theta_i, \alpha) \equiv V_i(g^P(\theta_i), \theta_i, \mu, G^P) - V_i(g^O(\theta_i), \theta_i, \theta_i, G^O)$  denote the type-specific value of a policy of privacy over a policy of publicity. Then:

$$\Gamma^{PO}(\theta_i, \alpha) = \beta(\mu - \theta_i) + (\delta(\theta_i; \beta))^2/2 - \alpha ME(g^O - g^P).$$

Similarly, let  $\Gamma^{PW}(\theta_i, \alpha) \equiv V_i(g^P(\theta_i), \theta_i, \mu, G^P) - V_i(g^W(\theta_i), \theta_i, \tilde{\theta}_i, G^W)$  denote the type-specific value of a policy of privacy over a policy of waiver. Then:

$$\begin{aligned} \Gamma^{PW}(\theta_i, \alpha) &= \beta(\mu - \mu(\theta^W)) - \alpha ME(g^W - g^P) \text{ for } \theta_i < \theta^W; \text{ and} \\ &= \beta(\mu - \theta_i) + (\delta(\theta_i; \beta))^2/2 - \alpha ME(g^W - g^P), \text{ for } \theta_i \geq \theta^W. \end{aligned}$$

Finally, let  $\Gamma^{WO}(\theta_i, \alpha) \equiv V_i(g^W(\theta_i), \theta_i, \tilde{\theta}_i, G^W) - V_i(g^O(\theta_i), \theta_i, \theta_i, G^O)$  denote the type-specific value of a policy of waiver over a policy of publicity. Then:

$$\begin{aligned} \Gamma^{WO}(\theta_i, \alpha) &= \beta(\mu(\theta^W) - \theta_i) + (\delta(\theta_i; \beta))^2/2 - \alpha ME(g^O - g^W), \text{ for } \theta_i < \theta^W; \text{ and} \\ &= -\alpha ME(g^O - g^W), \text{ for } \theta_i \geq \theta^W, \text{ for } \theta_i \geq \theta^W. \end{aligned}$$

The functions  $\Gamma^{PO}(\theta_i, \alpha)$ ,  $\Gamma^{PW}(\theta_i, \alpha)$ , and  $\Gamma^{WO}(\theta_i, \alpha)$  are continuous in both arguments and strictly decreasing in  $\alpha$ ; the latter two functions have portions that are constant with respect to  $\theta_i$ , but they are strictly decreasing in  $\theta_i$  over the non-constant regions.

We first determine conditions under which there will be non-trivial sets of types who prefer each policy in a binary comparison. In particular, let  $\bar{\alpha}^{IJ}$ , for  $IJ = PO, PW, WO$ , be the value of  $\alpha$  for which  $\theta_i = 0$  is indifferent between policy  $I$  and policy  $J$  (for this and any higher value of  $\alpha$ , policy  $J$  will be preferred to policy  $I$  for all types). Then  $\bar{\alpha}^{IJ}$  is defined uniquely by  $\Gamma^{IJ}(0, \bar{\alpha}^{IJ}) = 0$ , yielding:

$$\bar{\alpha}^{PO} = \beta\mu/(ME(g^O - g^P));$$

$$\bar{\alpha}^{PW} = \beta(\mu - \mu(\theta^W))/(ME(g^W - g^P));$$

$$\bar{\alpha}^{WO} = \beta\mu(\theta^W)/(ME(g^O - g^W)).$$

Provided that  $\alpha < \min \{\bar{\alpha}^{IJ}\}$ , there will be at least some (low) types who prefer policy  $I$  to policy  $J$  in a binary comparison. In order to have at least some (high) types who prefer policy  $J$  to policy  $I$  in a binary comparison, it must be that  $\Gamma^{IJ}(\bar{\theta}, \alpha) < 0$ ; our hypothesis that  $\theta^W < \bar{\theta}$  is enough to

guarantee that this holds for all  $\alpha > 0$ .

CLAIM 1: If  $0 < \alpha < \min \{\bar{\alpha}^{IJ}\}$ , then:

- (i) there exists a unique  $\theta^{IJ}(\alpha) \in (0, \bar{\theta})$  such that  $\Gamma^{IJ}(\theta^{IJ}(\alpha), \alpha) = 0$ ;
- (ii) moreover,  $\theta^{WO}(\alpha) < \theta^W < \theta^{PW}(\alpha)$  and  $\theta^{WO}(\alpha) < \theta^{PO}(\alpha) < \theta^{PW}(\alpha)$ .

PROOF OF CLAIM 1:

By construction, if  $0 < \alpha < \min \{\bar{\alpha}^{IJ}\}$ , then  $\Gamma^{IJ}(0, \alpha) > 0$  and  $\Gamma^{IJ}(\bar{\theta}, \alpha) < 0$ , for all  $IJ$ . First consider  $IJ = PO$ . The function  $\Gamma^{PO}(\theta, \alpha)$  is continuous and strictly decreasing in  $\theta$ ; therefore there exists a unique value  $\theta^{PO}(\alpha) \in (0, \bar{\theta})$  such that  $\Gamma^{PO}(\theta^{PO}(\alpha), \alpha) = 0$ . Next consider  $IJ = PW$ . The function  $\Gamma^{PW}(\theta, \alpha)$  is constant at a positive level for  $\theta_i < \theta^W$ , and  $\Gamma^{PW}(\theta, \alpha) = \Gamma^{PO}(\theta, \alpha) + E(g^O - g^W)$  for  $\theta_i \geq \theta^W$ . Since this is a continuous and strictly decreasing function, there is a unique value  $\theta^{PW}(\alpha) \in (\theta^W, \bar{\theta})$  such that  $\Gamma^{PW}(\theta^{PW}(\alpha), \alpha) = 0$ . Moreover, this implies that  $\Gamma^{PO}(\theta^{PW}(\alpha), \alpha) = -E(g^O - g^W) < 0$ , so  $\theta^{PO}(\alpha) < \theta^{PW}(\alpha)$ . Finally, consider  $IJ = WO$ . The function  $\Gamma^{WO}(\theta, \alpha)$  is constant at a negative level for  $\theta_i \geq \theta^W$ ; it is a continuous and strictly decreasing function for  $\theta_i < \theta^W$ . Therefore, there is a unique value  $\theta^{WO}(\alpha) \in (0, \theta^W)$  such that  $\Gamma^{WO}(\theta^{WO}(\alpha), \alpha) = 0$ . Moreover, evaluating  $\Gamma^{PO}$  at this level yields  $\Gamma^{PO}(\theta^{WO}(\alpha), \alpha) = \Gamma^{PW}(0, \alpha) > 0$ , so  $\theta^{WO}(\alpha) < \theta^{PO}(\alpha)$ .

Note that for the special case of  $\alpha = 0$  the claim above still holds with the following minor modifications. Now the function  $\Gamma^{WO}(\theta, \alpha)$  starts out positive and declines to zero at  $\theta^W$ ; moreover, it remains constant at zero for  $\theta_i \geq \theta^W$ . Thus, the equation  $\Gamma^{WO}(\theta^{WO}(\alpha), \alpha) = 0$  is satisfied by all members of the set  $[\theta^W, \bar{\theta}]$ ; we take the left-most element as  $\theta^{WO}(\alpha)$ , and thus  $\theta^{WO}(\alpha) = \theta^W$ . The rest of the claim continues to hold as stated.

Given the ordering  $\theta^{WO}(\alpha) < \theta^{PO}(\alpha) < \theta^{PW}(\alpha)$  derived above, it is straightforward to show that no type finds  $W$  to be the best policy. The preference orderings are as follows and are illustrated in Figure 3 in the main text:

$$\begin{aligned}
 \text{For } \theta \in [0, \theta^{WO}(\alpha)) & \quad P \succ W \succ O \quad (\text{with } W \sim O \text{ at } \theta^{WO}(\alpha)) \\
 \text{For } \theta \in (\theta^{WO}(\alpha), \theta^{PO}(\alpha)) & \quad P \succ O \succ W \quad (\text{with } P \sim O \succ W \text{ at } \theta^{PO}(\alpha)) \\
 \text{For } \theta \in (\theta^{PO}(\alpha), \theta^{PW}(\alpha)) & \quad O \succ P \succ W \quad (O \succ P \sim W \text{ at } \theta^{PW}(\alpha)) \\
 \text{For } \theta \in (\theta^{PW}(\alpha), \bar{\theta}] & \quad O \succ W \succ P
 \end{aligned}$$

Now we relax the assumption that  $\alpha < \min \{\bar{\alpha}^{IJ}\}$ ,  $IJ = PO, PW, WO$ . It is straightforward to show that  $\bar{\alpha}^{PO}$  must lie between  $\bar{\alpha}^{PW}$  and  $\bar{\alpha}^{WO}$ , but we are unable to determine in general whether  $\bar{\alpha}^{PW} < \bar{\alpha}^{WO}$  or  $\bar{\alpha}^{WO} < \bar{\alpha}^{PW}$  (however, if  $\bar{\alpha}^{WO} < \bar{\alpha}^{PW}$ , then  $W$  can never be *interim*-optimal for any type because  $\Gamma^{WO}(0, \alpha) < 0$ , implying that  $O$  is preferred to  $W$  for all types).

As claimed in the text, there are conditions under which some types will most-prefer a policy of  $W$ ; these conditions are now described. First, it can be shown that  $\bar{\alpha}^{PW} < \bar{\alpha}^{WO}$  for the case in which  $\theta$  is distributed uniformly on  $[0, \bar{\theta}]$ . For  $\bar{\alpha}^{PW} < \alpha < \bar{\alpha}^{WO}$ , all types strictly prefer  $P$  to  $W$ , while those in  $[0, \theta^{WO}(\alpha))$  also strictly prefer  $W$  to  $O$ . So it is possible for some types to *interim*-prefer  $W$  to both  $P$  and  $O$  (however, this set is limited by the fact that  $\theta^{WO}(\alpha) < \theta^W$  still holds). Notice that the types who *interim*-prefer  $W$  to both  $P$  and  $O$  will exercise privacy under a policy of  $W$  (since they are  $< \theta^W$ ), but hope to gain both from higher types who also choose privacy and from the disclosures and distortions of even higher types.