Web-Appendix: General, Non-separable Contracts

General non-separable contracts specify a wage for each realization of $(\tilde{z}_1, \tilde{z}_2)$. For a non-integrated firm, the contract given by Lemma 1remains optimal, since there is no reason to condition wage payments on the other firm's payoff, and since in each firm realized payoff is only μ or zero, requiring only one non-zero wage variable.

In the integrated firm, limited liability and a non-binding participation constraint implies that it is optimal to pay each manager zero if both divisions have a zero payoff. The managers' (symmetric) contracts can then be characterized by the triple (β, γ, δ), where manager i is paid β (as a share of μ) if only $\tilde{z}_i = \mu$, γ if only $\tilde{z}_j = \mu$, and δ if both units have a high payoff. Separable incentive contracts are then a special case corresponding to the restriction $\delta = \beta + \gamma$.

As explained in the text, with general contracts it is no longer the case that the CEO's ex-post optimal resource allocation automatically leads to the efficient allocation k^*. Consequently, in the latter case, solving the firm's program (4) in the main text assuming (but not imposing as constraint) that the CEO implements k^* may lead to wages for which the CEO would rather misallocate resources in order to save on wage costs. However, the only way in which integration can possibly improve over nonintegration is through the ability to shift all resources to one division if its project is good and the other's bad, while any other deviation from $k_1 = k_2 = 1$ cannot create any benefit. Our conjecture hence is that any optimal solution to (4) in the main text is also a solution to the same program with the added constraint that the CEO implements k^* in allocating resources. Propositions 8-10 below are therefore stated with this additional constraint imposed.

The firm's expected net profit is μ times

$$\begin{aligned} & \left[p^2\varphi^2 + (1-p)^2\right] \frac{y_1^2}{\mu^2} (2 - 2\delta) \\ (1) \qquad & +2(1-\beta-\gamma) \left[p^2 \frac{\varphi y_1}{\mu} \left(1 - \frac{\varphi y_1}{\mu}\right) + (1-p)^2 \frac{y_1}{\mu} \left(1 - \frac{y_1}{\mu}\right) + p(1-p)\varphi \frac{y_2}{\mu}\right]. \end{aligned}$$

This expression is obtained as follows. If both divisions have a high payoff, the firm's profit is $(2 - 2\delta)\mu$. This can occur only if both projects are good or both bad, which leads to the first term in (1). Otherwise, if only one division has high payoff, the firm's profit is $(1 - \beta - \gamma)\mu$; cf. the coefficient of the second term in (1). This occurs if each division gets one unit of resources but only one has a high payoff (the first two terms in []-brackets in (1)), or if only one division has a good project and gets all resources (the last term in []-brackets in (1)).

Our first result generalizes Proposition 1:

PROPOSITION 8: *In an integrated firm in which the CEO has perfect information about $\boldsymbol{\theta}$ and allocates resources efficiently, the optimal contract for each division manager entails $\gamma = 0$, and the expected total wage bill is lower than under non-integration.*

Proof: Under perfect information, if manager 1 has a good project, then with probability p manager 2 has a good project as well and each is allocated one unit of resources.

Manager 1 can then earn either $\delta\mu$, $\beta\mu$ or $\gamma\mu$ (with appropriate probabilities), depending on which of the two divisions has a high payoff. With probability $1-p$, manager 2 has a bad project, all resources are allocated to division 1, and manager 1 earns $\beta\mu$ with probability $\varphi y_2/\mu$. Overall, manager 1's expected wage from having a good project is

$$W(G)=\left\{p\left[\frac{\varphi^2 y_1^2}{\mu^2}\delta+\frac{\varphi y_1}{\mu}\left(1-\frac{\varphi y_1}{\mu}\right)(\beta+\gamma)\right]+(1-p)\frac{\varphi y_2}{\mu}\beta\right\}\mu. \tag{2}$$

If manager 1 has a bad project, then with probability p, manager 2 has a good one, and all resources go to division 2; whereas with probability $1-p$, manager 2 has a bad project as well. Manager 1's expected wage from having a bad project then is

$$\left\{p\frac{\varphi y_2}{\mu}\gamma+(1-p)\left[\frac{y_1^2}{\mu^2}\delta+\frac{y_1}{\mu}\left(1-\frac{y_1}{\mu}\right)(\beta+\gamma)\right]\right\}\mu. \tag{3}$$

By symmetry, these expressions are the same for manager 2, and so each manager will exert high effort if $pW(G)+(1-p)W(B)-c\geq qW(G)+(1-q)W(B)$, or equivalently,

$$(p-q)\left\{\left(p\varphi^2-1+p\right)\frac{y_1^2}{\mu}\delta+\left[(1-p)\left(\varphi y_2-y_1\left(1-\frac{y_1}{\mu}\right)\right)+p\varphi y_1\left(1-\frac{\varphi y_1}{\mu}\right)\right]\beta\right.$$

$$\left.-\left[p\varphi\left(y_2-y_1\left(1-\frac{\varphi y_1}{\mu}\right)\right)+(1-p)y_1\left(1-\frac{y_1}{\mu}\right)\right]\gamma\right\}\ \geq\ c. \tag{4}$$

As in the separable case, the left-hand side of (4) is decreasing in γ, meaning that γ has a negative effect on effort incentives. It is also clear that the firm's profit is decreasing in γ. It is therefore optimal to set $\gamma=0$. Moreover, since the owner cannot do worse with general than with separable contracts, it follows from Proposition 1 that the wage bill must lower than under non-integration. ■

It is possible but not helpful to derive the precise expressions for β and δ for the optimal contract. Note that given $\gamma=0$, separability of the contract imposes the restriction $\delta=\beta$, whereas in the general case there is no such restriction. With two variables to choose, only one relevant constraint (IC-e), and a linear program, the optimum is typically a corner solution with one of β or δ set to zero and the other positive. The main conclusion from Proposition 1, however, remains intact in the more general case: with perfect information, the *competition effect* of centralized resource allocation *improves* effort incentives relative to non-integration, as reflected in a lower wage bill.

Next, Proposition 2 already covers the general case; there is nothing further to show: the information-rent always dominates the competition effect in the sense that integration with high effort always leads to a wage bill at least as high as under non-integration.

Let us now turn to the case where project types are communicated strategically by the managers. As in Section III D., additional constraints come into play. In the following, we first derive these constraints formally, and then generalize Proposition 4.

First, it must be optimal for each manager to report his type truthfully. For a manager 1 with a *good* project, the expected payoff from reporting truthfully, and under the assumption that manager 2 reports truthfully too, is given by $\bar{w_1}(G,G) = W(G)$ as given by (2). Suppose manager 1 reports "B" instead. Then with probability p, manager 2 has a good project, in which case all resources go to division 2 and manager 1 earns γ if division 2 has high payoff. With probability $1-p$, manager 2 has a bad project, each division is allocated one unit of resources, and the manager can earn δ, β or γ times μ, depending on both divisions' payoffs. The resulting expected wage for manager 1 is

$$\bar{w_1}(G,B) = \left\{ p\frac{\varphi y_2}{\mu}\gamma + (1-p)\left[\varphi\frac{y_1^2}{\mu^2}\delta + \frac{\varphi y_1}{\mu}\left(1-\frac{y_1}{\mu}\right)\beta + \left(1-\frac{\varphi y_1}{\mu}\right)\frac{y_1}{\mu}\gamma\right]\right\}\mu. \tag{5}$$

The truthtelling constraint (IC-G) given by $\bar{w_1}(G,G) \geq \bar{w_1}(G,B)$ therefore is

$$\varphi\frac{y_1^2}{\mu}(p\varphi - 1 + p)\delta + \varphi\left[(1-p)y_2 - (1-p)y_1\left(1-\frac{y_1}{\mu}\right) + py_1\left(1-\frac{\varphi y_1}{\mu}\right)\right]\beta$$

$$+\left[(p\varphi - 1 + p)y_1\left(1-\frac{\varphi y_1}{\mu}\right) - p\varphi y_2\right]\gamma \;\geq\; 0. \tag{6}$$

For a manager 1 with a *bad* project, the expected payoff from reporting truthfully is $\bar{w_1}(B,B) = W(B)$ as given by (3). Suppose manager 1 reports "G" instead. Then with probability p, manager 2 has a good project too, in which case each division gets one unit of resources. With probability $1-p$, manager 2 has a bad project, and all resources go to division 1. The resulting expected wage for manager 1 is

$$\bar{w_1}(B,G) = \left\{ p\left[\frac{\varphi y_1^2}{\mu^2}\delta + \left(1-\frac{y_1}{\mu}\right)\frac{\varphi y_1}{\mu}\gamma + \frac{y_1}{\mu}\left(1-\frac{\varphi y_1}{\mu}\right)\beta\right] + (1-p)\frac{y_2}{\mu}\beta\right\}\mu. \tag{7}$$

The truthtelling constraint (IC-B) given by $\bar{w_1}(B,B) \geq \bar{w_1}(B,G)$ then is

$$-(p\varphi - 1 + p)\frac{y_1^2}{\mu}\delta - \left[(1-p)\left(y_2 - y_1\left(1-\frac{y_1}{\mu}\right)\right) + py_1\left(1-\frac{\varphi y_1}{\mu}\right)\right]\beta$$

$$+\left[p\varphi\left(y_2 - y_1\left(1-\frac{y_1}{\mu}\right)\right) + (1-p)y_1\left(1-\frac{y_1}{\mu}\right)\right]\gamma \;\geq\; 0. \tag{8}$$

Second, based on our arguments at the beginning of this Appendix, we will look at contracts that induce the CEO to allocate resources efficiently if he assumes that project types are reported truthfully. Suppose first that both projects are good. If the CEO allocates the resources equally (the efficient allocation), the expected profit for the firm is

$$2(1-\delta)\frac{\varphi^2 y_1^2}{\mu^2} + 2(1-\beta-\gamma)\frac{\varphi y_1}{\mu}\left(1-\frac{\varphi y_1}{\mu}\right) \tag{9}$$

times μ (in the following equations, all profit expressions are stated as shares of μ). If instead the CEO allocates all resources to one division, then the expected profit would be

$$(1-\beta-\gamma)\frac{\varphi y_2}{\mu}. \tag{10}$$

For the CEO to make the efficient choice requires that (9) be at least as large as (10), or

$$2(\beta+\gamma-\delta)\varphi\frac{y_1^2}{\mu} + (1-\beta-\gamma)(2y_1-y_2) \;\geq\; 0. \tag{11}$$

By similar reasoning, it can be shown that the condition for the CEO to allocate resources efficiently if both projects are bad is given by

$$2(\beta+\gamma-\delta)\frac{y_1^2}{\mu} + (1-\beta-\gamma)(2y_1-y_2) \;\geq\; 0. \tag{12}$$

Finally, suppose that division 1's project is good and division 2's bad. If the CEO allocates all resources division to 1 (the efficient allocation), the firm's expected profit is

$$(1-\beta-\gamma)\frac{\varphi y_2}{\mu} \tag{13}$$

If instead the CEO allocates the resources equally, then the expected profit would be

$$2(1-\delta)\frac{\varphi y_1^2}{\mu^2} + (1-\beta-\gamma)\left[\frac{\varphi y_1}{\mu}\left(1-\frac{y_1}{\mu}\right)+\left(1-\frac{\varphi y_1}{\mu}\right)\frac{y_1}{\mu}\right]. \tag{14}$$

For the CEO to choose efficiently requires that (13) be at least as large as (14), or equivalently

$$(1-\beta-\gamma)[\varphi(y_2-y_1)-y_1] - 2(\beta+\gamma-\delta)\frac{\varphi y_1^2}{\mu} \;\geq\; 0. \tag{15}$$

Of these three constraints, (12) is redundant. To see why, notice that since both (11) and (15) must hold, the sum of their left-hand sides, which yields $\mu(\varphi-1)(y_2-y_1)(1-\beta-\gamma)$, must be positive. This in turn requires that $\beta+\gamma<1$. Next, given the last result, both (11) and (12) can be binding only if $\delta>\beta+\gamma$; but in that case (10) is the more restrictive condition. We can therefore ignore (12).

The problem we are concerned with, therefore, is that of maximizing (1) with respect to β, γ and δ, subject to the effort incentive constraint (4), the truthtelling constraints (6) and (8), the resource allocation constraints (11) and (15), and the nonnegativity constraints $\beta, \gamma, \delta \geq 0$.

PROPOSITION 9: *In an integrated firm with cheap-talk communication, the optimal non-separable contract for each division manager that leads to high effort, truthful reports about investment projects, and an efficient resource allocation, is given by*

$$\begin{aligned} \beta &= c\,\frac{1-p(1+\varphi)}{(1-p)(p-q)(\varphi-1)[(1-p)y_2-p(\varphi-1)y_1]} \\ \gamma &= 0, \\ \delta &= c\,\frac{(\mu-\varphi y_1)[p(2y_1-y_2)+y_2-y_1]+(1-p)y_1[\varphi(y_2-y_1)+y1]}{(1-p)(p-q)(\varphi-1)y_1^2[(1-p)y_2-p(\varphi-1)y_1]}. \end{aligned} \tag{16}$$

if $p \leq 1/(1+\varphi)$ and c sufficiently small. In this case, the resulting expected wage per agent is <u>*the same*</u> *as under non-integration. Otherwise, the resulting expected wage per agent is* <u>*strictly higher*</u> *than under non-integration. In particular, if $p > 1/(1+\varphi)$, the optimal contract entails $\gamma > 0$.*[1]

Proof: The contract (16) is the unique solution for which $\gamma = 0$ and both (4) and (8) are binding. Feasibility of this solution requires $\beta, \delta \geq 0$. Since the numerator of δ in (16) is positive, we need $(1-p)y_2 > p(\varphi-1)y_1$ for the denominator of δ to be positive. Since the same term appears in the denominator of β, we need $p < 1/(1+\varphi)$ for β to be positive as well. Conversely, if $p < 1/(1+\varphi)$ or $1-p > p\varphi$, then it follows that $(1-p)y_2 > p\varphi y_1 > p(\varphi-1)y_1$, i.e. the same condition we started with. We can conclude that $p < 1/(1+\varphi)$ is necessary for the stated solution to be feasible.

However, the resource allocation constraints (11) and (15) need to be satisfied too. It can be shown that $\delta > \beta$ for the contract (16). It follows that (15) is always satisfied, but (11) may not be. As c decreases to zero, so do β and δ in (16), in which case (11) reduces to $2y_1 - y_2 \geq 0$, which means that (16) is overall feasible. However, for larger c, condition (11) is easily violated.

The total expected wage bill for the integrated firm is μ times

$$\begin{aligned} &2\delta\left[p^2\varphi^2+(1-p)^2\right]\frac{y_1^2}{\mu^2} \\ &+2(\beta+\gamma)\left[p^2\frac{\varphi y_1}{\mu}\left(1-\frac{\varphi y_1}{\mu}\right)+p(1-p)\frac{\varphi y_2}{\mu}+(1-p)^2\frac{y_1}{\mu}\left(1-\frac{y_1}{\mu}\right)\right], \end{aligned} \tag{17}$$

cf. the firm's net profit in (1). Substituting the contract (16) into (17) and simplifying leads to $2c(p\varphi-1+p)/[(p-q)(\varphi-1)]$, which is the same as the total wage bill for both

[1] With three variables to specify and eight linear constraints, there are as many as $8!/(3!\ 5!) = 56$ different corner points as possible candidates for an optimal solution in the case $p > 1/(1+\varphi)$. It is possible to narrow the set of possible solutions down to only six; however, there is little to gain from a more complete characterization of the solution.

firms under non-integration, cf. (20) in the main text. Optimality of the solution (16) then follows from Proposition 2.

The contract (16) is not feasible if either $p > 1/(1+\varphi)$, or if $p \leq 1/(1+\varphi)$ but c is too large. In the latter case, we just saw that a contract that satisfies (4) and (8) with equality leads to the lowest possible wage cost. If c is too large, then (11) is violated, and the optimal contract that satisfies (11) while still satisfying (4) and (8) must lead to a higher wage bill.

If $p > 1/(1+\varphi)$, the truthtelling constraint (8) is decreasing in δ. Since it is also decreasing in β, the only way to satisfy (8) is to set $\gamma > 0$. Evaluating the difference $\varphi\bar{w}(B,G) - \bar{w}(G,G)$, using the expressions in (2) and (7), simplifies to $p\varphi(\varphi-1)y_1\gamma$. This means that if $\gamma > 0$, then $\varphi\bar{w}(B,G)$ strictly exceeds $\bar{w}(G,G)$. In this case, it follows from Proposition 2 that the wage bill is *strictly* higher than under non-integration. ■

Proposition 9 states that unless both p and c are small, the conclusions of Proposition 4 carry over to the non-separable case: any *feasible* solution leads to a wage bill higher than under non-integration, typically involving a contract with $\gamma > 0$.

Only if $p \leq 1/(1+\varphi)$ and c is small, the managers' information can be elicited without additional cost relative to the case of non-integration, similar to the message-contingent contract of Proposition 3. What makes this possible is that if $p \leq 1/(1+\varphi)$, the truthtelling constraint (8) is increasing in δ.[2] It is thus in principle possible to establish truthtelling without requiring $\gamma > 0$, by setting δ high enough. The problem is that the required δ may be too high to satisfy (11).

Our last result generalizes Proposition 6. Like in Section V, we allow for asymmetric contracts for the managers. The managers' wages for the different possible payoff outcomes can therefore be described by $\delta_1, \beta_1, \gamma_1$ and $\delta_2, \gamma_2, \beta_2$, respectively.

PROPOSITION 10: *Assume that the owner of an integrated firm wants to induce high effort and an efficient resource allocation. Then the incentive constraints in the CEO hierarchy are unambiguously less restrictive than those in the skewed hierarchy.*

Proof: As in the separable case, in the skewed hierarchy all incentive constraints for manager 2, as well as the effort incentive constraint for manager 1, are the same as in the CEO hierarchy, cf. the proof of Proposition 6. It remains to show how the resource allocation constraints for a manager 1 with a bad project compare to his truthtelling constraint in the hierarchy with CEO. Suppose manager 1 has a bad project. If manager 2's project is bad too, and manager 1 allocates the resources equally as would be efficient, his expected wage is

$$\left[\frac{y_1^2}{\mu^2}\delta_1 + \frac{y_1}{\mu}\left(1 - \frac{y_1}{\mu}\right)(\beta_1 + \gamma_1)\right]\mu.$$

[2] To understand the sign of this derivative, observe first that a manager can earn δ only if both divisions have high payoffs, which requires that the CEO allocates the resources equally between the divisions (if one division has no resources, it cannot attain a high payoff). If manager 1 has a bad project and reports truthfully, he earns δ if manager 2 also has a bad project (the probability of which is $1-p$), and both have high payoff, which occurs with probability y_1^2/μ^2. In contrast, if manager 1 claims to have a good project, he earns δ if manager 2 has a good project (the probability of which is p), and both have high payoff, which occurs with probability $\varphi y_1^2/\mu^2$. Thus, the effect of δ to report truthfully is given by $(1-p-p\varphi)y_1^2/\mu$, which is positive if and only if $p \leq 1/(1+\varphi)$.

If instead he allocates all resources to himself, his expected wage is $\beta_1 y_2$. Manager 1 therefore allocates resources efficiently if

$$\left[\frac{y_1^2}{\mu^2}\delta_1 + \frac{y_1}{\mu}\left(1 - \frac{y_1}{\mu}\right)(\beta_1 + \gamma_1)\right]\mu - \beta_1 y_2 \;\geq\; 0. \tag{18}$$

If manager 2's project is good and manager 1 allocates all resources to division 2 as is efficient, his expected wage is $\gamma_1 \varphi y_2$. If instead he allocates the resources equally, his expected wage is

$$\left[\frac{\varphi y_1^2}{\mu^2}\delta_1 + \frac{y_1}{\mu}\left(1 - \frac{\varphi y_1}{\mu}\right)\beta_1 + \left(1 - \frac{y_1}{\mu}\right)\frac{\varphi y_1}{\mu}\gamma_1\right]\mu.$$

Manager 1 therefore allocates resources efficiently if

$$\gamma_1 \varphi y_2 - \left[\frac{\varphi y_1^2}{\mu^2}\delta_1 + \frac{y_1}{\mu}\left(1 - \frac{\varphi y_1}{\mu}\right)\beta_1 + \left(1 - \frac{y_1}{\mu}\right)\frac{\varphi y_1}{\mu}\gamma_1\right]\mu \;\geq\; 0. \tag{19}$$

It can then be shown that left-hand side of (8) is equal to $(1-p)$ times the left-hand side of (18) plus p times the left-hand side of (19), which completes the proof (see the proof of Proposition 6 for further details). ■

As mentioned in the main text, an attempt to generalize Proposition 7 on horizontal exchange to non-separable contracts leads to several cases that need to be considered. Numerical simulations suggest that Proposition 7 continues to hold for all cases, but we have been able to prove this formally only for some of the cases. Thus, we do not have a counterpart of Proposition 10 for horizontal exchange.