Online Appendix for "Affirmative Action: One Size Does Not Fit All"

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Appendix A (For Online Publication)

In this Appendix, we consider a special case to illustrate the intuition in the model. Specifically, we parametrize the model to derive a closed-form solution and thereby compare the magnitudes of the effort effect and the selection effect. To simplify the analysis, we assume linearity so $s(a) = Sa$, $f(a, e) = a + e$, and $c(e) = Ce$, where S and C are parameters. In this case, the effort put in is given by

$$e^*(a, \tilde{P}) = \max\left(\tilde{P} - a, 0\right),$$

which means that agents with total ability greater than $\tilde{P}$ do not put in any effort. The equilibrium conditions under the non-discriminating quota (where the marginal agent exerts positive effort) are given by

$$\gamma_1\left(1 - H^1(a^*)\right) + \gamma_2\left(1 - H^2(a^*)\right) = \alpha, \tag{1}$$

$$Sa^* - T - C\left(\tilde{P} - a^*\right) = 0, \tag{2}$$

where the first equation determines the cutoff for the total ability, a^*, while the second one determines the performance cutoff:

$$\tilde{P} = \frac{(S + C)a^* - T}{C}.$$

Recall that γ_i is the share of group i in the total population.

We also assume that both types of ability are uniformly distributed across the agents. That is, $H_N(a_N) = a_N/a^N_{\max}$ on $\left[0, a^N_{\max}\right]$ and $H^i_A(a_A) = a_A/a^A_{\max,i}$ on $\left[0, a^A_{\max,i}\right]$ where $a^A_{\max,1} \geq a^A_{\max,2}$. Note that under this assumption, $H^1_A(a_A) \not\succeq_{LR} H^2_A(a_A)$ (but $H^1_A(a_A) \succeq_1 H^2_A(a_A)$) so that our assumption about the likelihood stochastic order does not hold anymore. However, as shown below, with uniform distributions of abilities, first-order stochastic dominance is sufficient for all the results formulated in the main body of the paper to hold.

In equilibrium, the cutoff ability a^* is pinned down by the number of seats α (see (1)). In our analysis, we consider the case that $a^* \geq a^N_{\max}$ and $a^* \geq a^A_{\max,i}$ for $i = 1, 2$. That is, the number of seats is so few that an agent needs both types of ability to get in. Next, we derive explicit expressions for the effort and selection effects evaluated at the non-discriminating quota θ^*.

Proposition 1 *The effort and selection effects evaluated at the non-discriminating quota are given by*

$$
\begin{aligned}
EE_{\theta=\theta^*} &= -\frac{\alpha\,(S+C)}{2}\left(a^A_{\max,1} - a^A_{\max,2} + \frac{\left(a_{\max,2} - \min(\tilde{P}, a_{\max,2})\right)^2}{a_{\max,2} - a^*} - \frac{\left(a_{\max,1} - \min(\tilde{P}, a_{\max,1})\right)^2}{a_{\max,1} - a^*}\right), \\
SE_{\theta=\theta^*} &= \frac{\alpha(1-\beta)S\left(a^A_{\max,1} - a^A_{\max,2}\right)}{2},
\end{aligned}
$$

where

$$
a_{\max,i} = a^A_{\max,i} + a^N_{\max}.
$$

Proof. In the subsection below. ∎

Given the performance cutoff, the magnitude of the effort effect positively depends on the parameters describing the returns to education and the cost of effort, S and C. Moreover, it is straightforward to see that the magnitude of the effort effect is increasing in the performance cutoff $\tilde{P}$: i.e. $\partial\left(|EE_{\theta=\theta^*}|\right)/\partial\tilde{P} \geq 0$. Indeed, if $\tilde{P} < a_{\max,i}$ for $i = 1, 2$, so that some of both abilities get in, then

$$
EE_{\theta=\theta^*} = -\frac{\alpha\,(S+C)}{2}\left(\tilde{P} - a^*\right)^2 \frac{a^A_{\max,1} - a^A_{\max,2}}{\left(a_{\max,2} - a^*\right)\left(a_{\max,1} - a^*\right)},
$$

which is negative and decreasing in $\tilde{P}$ as we assume that $a^* < \tilde{P}$. If $a_{\max,2} \leq \tilde{P} \leq a_{\max,1}$, then

$$
EE_{\theta=\theta^*} = -\frac{\alpha\,(S+C)}{2}\left(a^A_{\max,1} - a^A_{\max,2} - \frac{\left(a_{\max,1} - \tilde{P}\right)^2}{a_{\max,1} - a^*}\right),
$$

which is also decreasing in $\tilde{P}$. Finally, if $\tilde{P} > a_{\max,1}$, then the effort effect is given by

$$
EE_{\theta=\theta^*} = -\frac{\alpha\,(S+C)}{2}\left(a^A_{\max,1} - a^A_{\max,2}\right),
$$

and, therefore, does not depend on $\tilde{P}$. Thus, the effort effect is strictly decreasing in $\tilde{P}$ on $[a^*, a_{\max,1})$ and then is flat.

Note that if $\tilde{P} > a_{\max,1}$, the overall effect on social welfare is negative:

$$
\frac{\partial W}{\partial \theta}_{\theta=\theta^*} = EE_{\theta=\theta^*} + SE_{\theta=\theta^*} = -\frac{\alpha\left(a^A_{\max,1} - a^A_{\max,2}\right)}{2}(\beta S + C) < 0.
$$

In this case, the effort effect dominates the selection effect. Hence, we can conclude that, for sufficiently high values of the performance cutoff (which represents the level of competition for seats, which in turn depends on tuition and the availability of seats), the effort effect is stronger than the selection effect and, as a result, a quota in favor of disadvantaged results in welfare losses. Whereas, for sufficiently low values of $\tilde{P}$, the selection effect prevails over the effort effect and a reservation quota in favor of disadvantaged can increase the social welfare. Note that the tuition fee T affects the effort and selection effects only through $\tilde{P}$. Moreover, a rise in T reduces the performance cutoff $\tilde{P}$. Thus,

Proposition 2 *There exists a value of the tuition fee, T^{tr}, such that a marginal increase in the quota for the disadvantaged group when it is set to the non-discriminating level, θ^*, raises welfare if and only if $T > T^{tr}$. In other words,*

$$\frac{\partial W}{\partial \theta}_{\theta=\theta^*} > 0 \text{ if and only if } T > T^{tr}.$$

Intuitively, a higher tuition level reduces the magnitude of the effort effect and, as a result, a quota in favor of the disadvantaged is more likely to be welfare improving. For this reason, the model suggests that affirmative action is likely to reduce welfare in a setting where education is subsidized. In India, for example, backward castes and tribes have a share of seats (given by their population share) reserved for them in publicly funded higher education. These reservations result in the cutoff entrance exam scores that are much lower for these groups than for the general category.[1] As public higher education is much cheaper than private, and as the very best institutions are public and seats are scarce, competition to get in is extreme. In such a setting, reservations are likely to be welfare reducing. Higher education is also subsidized in many European countries. However, supply is abundant, and as a result, effort exerted to get in is far less than in the Indian context. In the U.S., State Universities tend to be cheaper than private ones of a similar quality. However, the emphasis on need blind admissions and the availability of financial aid significantly reduces the difference in price.

The Proof of Proposition 1

Recall that the effort effect evaluated at the non-discriminating quota can be written as follows:

$$EE_{\theta=\theta^*} = D\left(\frac{\int_{a^*}^{a_{\max,2}} c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}dH^2(a)}{h^2(a^*)} - \frac{\int_{a^*}^{a_{\max,1}} c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}dH^1(a)}{h^1(a^*)}\right),$$

where

$$D = \alpha\frac{s'(a^*) - c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}}.$$

[1] In the celebrated Indian Institutes of Technology, the entrance exam marks for the general category are in the high nineties, while they are in the low fifties for the reserved category.

Given the assumptions, the effort effect can be rewritten in the following way (recall that agents with total ability higher than $\tilde{P}$ put in zero effort and, therefore, the upper bound of the integrals is $\min(\tilde{P}, a_{\max,i})$):

$$EE_{\theta=\theta^*} = \alpha\,(S+C)\left(\frac{\int_{a^*}^{\min(\tilde{P},a_{\max,2})} h^2(a)da}{h^2(a^*)} - \frac{\int_{a^*}^{\min(\tilde{P},a_{\max,1})} h^1(a)da}{h^1(a^*)}\right).$$

Recall that

$$H^i(a) = \int_0^{a^A_{\max,i}} H_N(a-y)dH^i_A(y),$$

so

$$h^i(a) = \int_0^{a^A_{\max,i}} h_N(a-y)h^i_A(y)dy.$$

Since $H_N(\cdot)$ and $H^i_A(\cdot)$ are uniform,

$$h^i(a) = \frac{1}{a^A_{\max,i}a^N_{\max}}\int_{\max(0,a-a^N_{\max})}^{\min(a^A_{\max,i},a)} dy = \frac{\min(a^A_{\max,i},a) - \max(0,a-a^N_{\max})}{a^A_{\max,i}a^N_{\max}}.$$

This implies that if $a \geq a^* \geq \max\left[a^A_{\max,1}, a^A_{\max,2}, a^N_{\max}\right]$ (as assumed), then

$$\begin{aligned} h^i(a) &= \frac{a^A_{\max,i} + a^N_{\max} - a}{a^A_{\max,i}a^N_{\max}} \\ &= \frac{a_{\max,i} - a}{a^A_{\max,i}a^N_{\max}}, \end{aligned}$$

where $a_{\max,i} = a^A_{\max,i} + a^N_{\max}$. Hence,

$$\int_{a^*}^{\min(\tilde{P},a_{\max,i})} h^i(a)da = \frac{(a_{\max,i}-a^*)^2 - \left(a_{\max,i} - \min(\tilde{P},a_{\max,i})\right)^2}{2a^A_{\max,i}a^N_{\max}}.$$

Substituting this in the expression for the effort effect, we derive that

$$\begin{aligned} EE_{\theta=\theta^*} &= -\frac{\alpha\,(S+C)}{2}\frac{(a_{\max,1}-a^*)^2 - \left(a_{\max,1} - \min(\tilde{P},a_{\max,1})\right)^2}{a_{\max,1}-a^*} \\ &\quad +\frac{\alpha\,(S+C)}{2}\frac{(a_{\max,2}-a^*)^2 - \left(a_{\max,2} - \min(\tilde{P},a_{\max,2})\right)^2}{a_{\max,2}-a^*} \\ &= -\frac{\alpha\,(S+C)}{2}\left(a^A_{\max,1} - a^A_{\max,2} + \frac{\left(a_{\max,2} - \min(\tilde{P},a_{\max,2})\right)^2}{a_{\max,2}-a^*} - \frac{\left(a_{\max,1} - \min(\tilde{P},a_{\max,1})\right)^2}{a_{\max,1}-a^*}\right). \end{aligned}$$

Remember also that the selection effect is given by

$$\begin{aligned} SE_{\theta=\theta^*} &= \frac{\alpha}{h^1(a^*)}\int_0^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\,a_A)\right)h_N(a^*-a_A)dH^1_A(a_A) \\ &\quad -\frac{\alpha}{h^2(a^*)}\int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^* - (1-\beta)\,a_A)\right)h_N(a^*-a_A)dH^2_A(a_A). \end{aligned}$$

Using the functional forms assumed, the selection effect can be written as

$$SE_{\theta=\theta^*} = \alpha(1-\beta)S\left(\frac{\int_0^{a^A_{\max,1}} a_A h_N(a^*-a_A)dH^1_A(a_A)}{h^1(a^*)} - \frac{\int_0^{a^A_{\max,2}} a_A h_N(a^*-a_A)dH^2_A(a_A)}{h^2(a^*)}\right).$$

Notice that since $a^* > a^A_{\max,i}$

$$\begin{aligned}\frac{\int_0^{a^A_{\max,i}} a_A h_N(a^*-a_A)dH^i_A(a_A)}{h^i(a^*)} &= \frac{\int_{a^*-a^N_{\max}}^{a^A_{\max,i}} a_A da_A}{a^A_{\max,i}+a^N_{\max}-a^*} \\ &= \frac{a^A_{\max,i}+a^*-a^N_{\max}}{2}.\end{aligned}$$

Therefore,

$$\begin{aligned}SE_{\theta=\theta^*} &= \alpha(1-\beta)S\left(\frac{a^A_{\max,1}+a^*-a^N_{\max}}{2} - \frac{a^A_{\max,2}+a^*-a^N_{\max}}{2}\right) \\ &= \frac{\alpha(1-\beta)S\left(a^A_{\max,1}-a^A_{\max,2}\right)}{2}.\end{aligned}$$

Appendix B (For Online Publication)

In this Appendix, we consider the extension of the benchmark model with two universities of different quality.

The Model

We assume that the universities are different in that they offer education of different qualities, which affects the payoffs from being educated. As a result, in equilibrium, the performance cutoff for the better university is higher so that it needs more effort to be accepted to the higher quality university. The payoffs from being educated at university $i \in \{H, L\}$ are given by $q^i s(a)$, where q^i is the quality measure of university i (as before, a is the total ability). The net payoffs are given by

$$q^i s(a) - T_i - c(e^*(a, \tilde{P}^i)),$$

where T_i is the tuition fee, $\tilde{P}^i$ is the performance cutoff at university i ($\tilde{P}^H$ is assumed to be higher than $\tilde{P}^L$ (see the discussion below)), and $e^*(a, \tilde{P}^i)$ is the effort level put in to be accepted. $c(.)$ is weakly convex.

Lemma 1 shows that the difference between the net payoffs from studying in the better and worse university is increasing in ability. As a result, more able agents are matched with the better university.

Lemma 1 *For any given performance cutoffs, we define the difference between the net payoffs from studying in the better and worse university as*

$$\begin{aligned} D(a; \tilde{P}^H, \tilde{P}^L) &= \left(q^H s(a) - T_H - c(e^*(a, \tilde{P}^H))\right) - \left(q^L s(a) - T_L - c(e^*(a, \tilde{P}^L))\right) \\ &= \triangle q s(a) - \triangle T - c(e^*(a, \tilde{P}^H)) + c(e^*(a, \tilde{P}^L)) \end{aligned}$$

where $\triangle q = q^H - q^L > 0$ *and* $\triangle T = T_H - T_L$. *Then,*

$$\begin{aligned} \frac{\partial D(a; \tilde{P}^H, \tilde{P}^L)}{\partial a} &= \triangle q s'(a) + \left[-c'(e^*(a, \tilde{P}^H)) \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} - \left(-c'(e^*(a, \tilde{P}^L)) \frac{\partial e^*(a, \tilde{P}^L)}{\partial a} \right) \right] \\ &= \triangle q s'(a) - \left[c'(e^*(a, \tilde{P}^H)) - c'(e^*(a, \tilde{P}^L)) \right] \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} \\ &\quad + c'(e^*(a, \tilde{P}^L)) \left(\frac{\partial e^*(a, \tilde{P}^L)}{\partial a} - \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} \right) \\ &> 0. \end{aligned}$$

Proof. Using the fact that

$$f(a, e^*(a, \tilde{P})) = \tilde{P},$$

it is easy to see that

$$e^*_{\tilde{P}}(a, \tilde{P}) = \frac{1}{f_e(a, e^*(a, \tilde{P}))} > 0.$$

That is, for any given ability a, meeting a higher cutoff requires more effort. Thus, as $c(\cdot)$ is convex, $c'(e^*(a, \tilde{P}^H)) \geq c'(e^*(a, \tilde{P}^L))$. In addition,

$$e^*_a(a, \tilde{P}) = -\frac{f_a(a, e^*(a, \tilde{P}))}{f_e(a, e^*(a, \tilde{P}))} < 0. \tag{3}$$

This implies

$$-\left[c'(e^*(a, \tilde{P}^H)) - c'(e^*(a, \tilde{P}^L)) \right] \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} > 0.$$

Finally,

$$e^*_{a\tilde{P}}(a, \tilde{P}) = -\frac{f_{ea}(a, e^*(a, \tilde{P})) + f_{ee}(a, e^*(a, \tilde{P})) e^*_a(a, \tilde{P})}{\left(f_e(a, e^*(a, \tilde{P}))\right)^2} < 0,$$

as $f_{ee} < 0$, $e^*_a(a, \tilde{P}) < 0$, and $f_{ea} > 0$. This in turn means that $e^*_a(a, \tilde{P}^L) > e^*_a(a, \tilde{P}^H)$, implying that

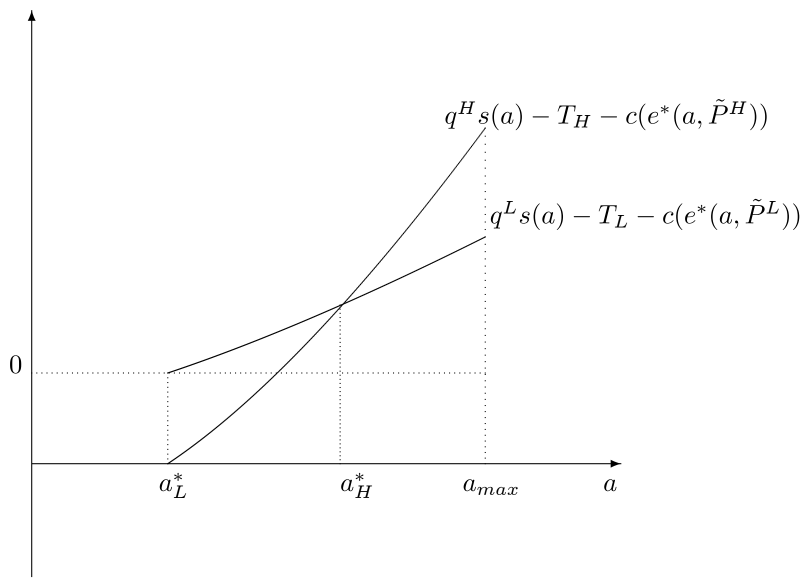
$$c'(e^*(a, \tilde{P}^L)) \left(\frac{\partial e^*(a, \tilde{P}^L)}{\partial a} - \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} \right) > 0.$$

Using the above finding and the fact that $s'(a) > 0$, it follows that

$$\frac{\partial D(a; \tilde{P}^H, \tilde{P}^L)}{\partial a} > 0.$$

■

Figure 1: Payoffs from Education: Two Universities



It would be helpful to see this lemma in a picture like Figure 1. $q^H s(a)$ is steeper than $q^L s(a)$, as $q^H > q^L$ and $s(a)$ is increasing in ability so that more able individuals earn more at any given education level, and this is more so at better institutions. In order to get into the university H (L), each agent must put in $e^*(a, \tilde{P}^H)$ $(e^*(a, \tilde{P}^L))$ and this needs the cost of $c(e^*(a, \tilde{P}^H))$ $(c(e^*(a, \tilde{P}^L)))$ to be incurred. These costs are decreasing in ability as the more able need to put in less effort to meet any given performance cutoff. Moreover, they decrease faster in ability when the cutoff is higher. This happens because the higher performance cutoff requires more effort from all individuals, but due to the complementarity between ability and effort in creating performance, more able agents need to put in less effort to attain the higher cutoff. As they are putting in less effort to get the lower performance cutoff anyway, this increased effort to meet a higher cutoff is also less costly for them.

Thus, the net surplus (the benefit net of the cost) from going to university is increasing in ability, and more so for the better university as depicted in Figure 1. This means that when we add the tuition cost which is independent of ability, the net benefit of going to the better university rises faster than that of the worse school so that these curves can cross each other at most once and better students will opt for the better university. Note this fact is independent of tuition, though a high enough tuition could make the payoff from that university lie entirely below that of the other so no one goes there.

Next we consider the equilibrium and the comparative statics of the model. In the equilibrium, there are two total ability cutoffs: a_H^* and a_L^*. Agents with abilities lower than a_L^* choose the outside option. The cutoffs are determined such that agents with abilities more than a_H^* fill the available seats in the better university and agents with abilities between a_L^* and a_H^* fill the worse university seats. This gives the equilibrium conditions:

$$
\begin{aligned}
1 - H(a_H^*) &= \alpha_H, &(4)\\
H(a_H^*) - H(a_L^*) &= \alpha_L, &(5)
\end{aligned}
$$

where α_i is the number of seats in university i and $H(.)$ is the cumulative distribution of total ability. As before, we assume that the natural and acquired abilities are independently distributed across agents. The distribution functions are given by $H_N(a_N)$ and $H_A(a_A)$ on $[0, a_{\max}^N]$ and $[0, a_{\max}^A]$, respectively. Then, the distribution function for the total ability a is $H(a)$ on $[0, a_{\max}]$, where

$$
H(a) = \int_0^{a_{\max}^A} H_N(a-y)dH_A(y) \qquad (6)
$$

and $a_{\max} = a_{\max}^N + a_{\max}^A$.

The agent at a_L^* is indifferent between the worse university and the outside option of zero which pins down $c(e^*(a_L^*, \tilde{P}^L))$ and therefore $\tilde{P}^L$. The agent at a_H^* is indifferent between the two universities which pins down $c(e^*(a_H^*, \tilde{P}^H))$ and therefore $\tilde{P}^H$. Thus

$$
\begin{aligned}
q^L s(a_L^*) - T_L - c(e^*(a_L^*, \tilde{P}^L)) &= 0, &(7)\\
\triangle q s(a_H^*) - \triangle T - c(e^*(a_H^*, \tilde{P}^H)) + c(e^*(a_H^*, \tilde{P}^L)) &= 0. &(8)
\end{aligned}
$$

Thus, we have four unknowns, a_H^*, a_L^*, $\tilde{P}^H$, and $\tilde{P}^L$ which can be pinned down by four equilibrium equations (4), (5), (7), and (8). Note that the condition, $\tilde{P}^H > \tilde{P}^L$, is equivalent to $c(e^*(a_H^*, \tilde{P}^H)) - c(e^*(a_H^*, \tilde{P}^L)) > 0$. Therefore, from the equilibrium condition (8), we can infer that $\tilde{P}^H > \tilde{P}^L$ if and only if $\triangle q s(a_H^*) - \triangle T > 0$. In other words, that the difference in the tuition levels is not set too high relative to the difference in quality.

Next, we explore how changes in T_i affect the equilibrium outcome. The following lemma holds.

Lemma 2 *1) The cutoffs, a_H^* and a_L^*, do not depend on the tuition fees, T_H and T_L.*

2) A rise in T_H does not affect $\tilde{P}^L$ and decreases $\tilde{P}^H$.

3) A rise in T_L decreases $\tilde{P}^L$ and increases $\tilde{P}^H$.

4) A rise in T_H and T_L (such that $\triangle T$ does not change) decreases $\tilde{P}^L$ and $\tilde{P}^H$.

Proof. 1), 2) and 4) follow directly from the equilibrium equations. Let us prove the third statement in the lemma. From the equilibrium condition (7), we have

$$
\frac{\partial \tilde{P}^L}{\partial T_L} = -\frac{1}{c'\left(e^*(a_L^*, \tilde{P}^L)\right) \frac{\partial e^*(a_L^*, \tilde{P}^L)}{\partial \tilde{P}}} < 0.
$$

In addition, differentiating (8) with respect to T_L gives

$$\frac{\partial \tilde{P}^H}{\partial T_L} = -\frac{-c'\left(e^*(a_H^*, \tilde{P}^L)\right)\frac{\partial e^*(a_H^*,\tilde{P}^L)}{\partial \tilde{P}}\frac{\partial \tilde{P}^L}{\partial T_L} - 1}{c'\left(e^*(a_H^*, \tilde{P}^H)\right)\frac{\partial e^*(a_H^*,\tilde{P}^H)}{\partial \tilde{P}}} = \frac{c'\left(e^*(a_H^*, \tilde{P}^L)\right)\frac{\partial e^*(a_H^*,\tilde{P}^L)}{\partial \tilde{P}}\frac{\partial \tilde{P}^L}{\partial T_L} + 1}{c'\left(e^*(a_H^*, \tilde{P}^H)\right)\frac{\partial e^*(a_H^*,\tilde{P}^H)}{\partial \tilde{P}}}.$$

The sign of the derivative is the same as the sign of the numerator (as the denominator is positive). The numerator is in turn equal to

$$c'\left(e^*(a_H^*, \tilde{P}^L)\right)\frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}}\frac{\partial \tilde{P}^L}{\partial T_L} + 1 = 1 - \frac{c'\left(e^*(a_H^*, \tilde{P}^L)\right)\frac{\partial e^*(a_H^*,\tilde{P}^L)}{\partial \tilde{P}}}{c'\left(e^*(a_L^*, \tilde{P}^L)\right)\frac{\partial e^*(a_L^*,\tilde{P}^L)}{\partial \tilde{P}}}.$$

Note that as $a_L^* < a_H^*$, $e^*(a_H^*, \tilde{P}^L) < e^*(a_L^*, \tilde{P}^L)$ implying that $c'\left(e^*(a_H^*, \tilde{P}^L)\right) < c'\left(e^*(a_L^*, \tilde{P}^L)\right)$. Moreover, since $\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}} = 1/f_e\left(a, e^*(a, \tilde{P})\right)$,

$$\frac{\frac{\partial e^*(a_H^*,\tilde{P}^L)}{\partial \tilde{P}}}{\frac{\partial e^*(a_L^*,\tilde{P}^L)}{\partial \tilde{P}}} = \frac{f_e\left(a_L^*, e^*(a_L^*, \tilde{P}^L)\right)}{f_e\left(a_H^*, e^*(a_H^*, \tilde{P}^L)\right)} < 1,$$

as $f_{ea} > 0$ and $f_{ee} < 0$. That is, the sign of the numerator is positive. This proves the third statement. ■

The intuition behind 1) and 2) in the lemma is straightforward. The idea behind 3) is as follows. Keep the performance cutoffs fixed. An increase in T_L shifts the payoff curve for L down and reduces a_H^* while raising a_L^*. As a result, more agents apply for the seats in the high-quality university and fewer for the low quality one. As the number of seats remains fixed, $\tilde{P}^H$ must rise and $\tilde{P}^L$ must fall.

The intuition behind 4 is simple. Suppose we increase tuition fees by the same amount. At any given performance cutoffs, while this change does not affect the intersection of the two curves a_H^* as it shifts down both curves by the same amount, it raises a_L^*. This makes the demand for the worse university seats less than its supply which in turn reduces the performance cutoff $\tilde{P}^L$. This fall in $\tilde{P}^L$ must shift the payoff curve for the worse university back to its original position so that it goes through the original level of a_L^*. However, the fall in L's performance cutoff has a smaller impact on the payoff for higher ability agents and so makes it flatter. This reduces a_H^* from its original level, requiring a fall in H's performance cutoff as well.[2]

Next, we explore the effects of changes in the number of available seats on the equilibrium outcomes. Using the equilibrium equations we see that a rise in α_L does not change a_H^* (as it is pinned down by α_H) and decreases a_L^*. This in turn means that $\tilde{P}^L$ and $\tilde{P}^H$ fall. Intuitively, more available seats in the low quality university reduces the performance cutoff in that university, making it more attractive compared to the high quality university. This absorbs some students from the high quality university to

[2] Formally, this result comes from the impact of T_H on $\tilde{P}^H$ being stronger than that of T_L.

the low quality university but since the number of seats in the high quality university does not change, the performance cutoff, $\tilde{P}^H$, falls to compensate for the decrease in the demand for seats in the high quality university.

A rise in α_H decreases both the ability cutoffs, a_L^* and a_H^*. The decrease in a_L^* in turn results in lower $\tilde{P}^L$. The low quality university has to reduce its performance cutoff in order to fill in all the available seats. The impact on $\tilde{P}^H$ is also straightforward. The direct effect of a rise in α_H is to decrease a_H^* and consequently $\tilde{P}^H$. In addition, the rise in α_H decreases $\tilde{P}^L$, which further reduces $\tilde{P}^H$ (see (8)). As can be seen, both effects work in the same direction. As a result, $\tilde{P}^H$ falls. The following lemma summarizes the above reasoning.

Lemma 3 *1) A rise in α_L does not change a_H^* and decreases a_L^*, $\tilde{P}^L$, and $\tilde{P}^H$.*

2) A rise in α_H decreases a_H^* and a_L^* and $\tilde{P}^L$ and $\tilde{P}^H$.

Next, we examine the welfare implications of changes in the parameters in the model.

Social Welfare

As before, we allow the private gains from education to be different from the social gains. Specifically, for an individual, natural and acquired abilities are of the same importance but for the society natural ability is more important than the acquired ability.

Social welfare is given by (the outside option is normalized to zero)

$$\begin{aligned} W &= \int_{a_N+a_A\geq a_H^*} \left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^H)) - F\right) dH_N(a_N)dH_A(a_A) \\ &+ \int_{a_L^*\leq a_N+a_A<a_H^*} \left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^L)) - F\right) dH_N(a_N)dH_A(a_A), \end{aligned} \tag{9}$$

where F is the social cost of education per student and $\beta < 1$. Note that as the tuition is a lump-sum transfer, T does not directly affect the welfare. It only affects welfare via the effort put in by agents.

Next, we explore the effects of the tuition fees on the social welfare. First, we examine how changes in T_L and T_H affect the welfare. Then, we find the values of T_L and T_H that maximize the social welfare function. It is straightforward to see that

$$\begin{aligned} \frac{\partial W}{\partial T_i} &= \int_{a_N+a_A\geq a_H^*} \frac{\partial\left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^H)) - F\right)}{\partial T_i} dH_N(a_N)dH_A(a_A) \\ &+ \int_{a_L^*\leq a_N+a_A<a_H^*} \frac{\partial\left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^L)) - F\right)}{\partial T_i} dH_N(a_N)dH_A(a_A). \end{aligned}$$

Here, we used the fact that the ability cutoffs do not depend on the tuition fees (see Lemma 2).

From the results stated in Lemma 2, we can conclude that (recall that $\partial\tilde{P}^L/\partial T_H = 0$)

$$\frac{\partial W}{\partial T_H} = -\frac{\partial \tilde{P}^H}{\partial T_H}\int_{a_N+a_A\geq a_H^*} c'(e^*(a_N+a_A,\tilde{P}^H))\frac{\partial e^*(a_N+a_A,\tilde{P}^H)}{\partial \tilde{P}}dH_N(a_N)dH_A(a_A) > 0,$$

while

$$\begin{aligned}\frac{\partial W}{\partial T_L} &= -\frac{\partial \tilde{P}^H}{\partial T_L}\int_{a_N+a_A\geq a_H^*} c'(e^*(a_N+a_A,\tilde{P}^H))\frac{\partial e^*(a_N+a_A,\tilde{P}^H)}{\partial \tilde{P}}dH_N(a_N)dH_A(a_A)\\ &\quad -\frac{\partial \tilde{P}^L}{\partial T_L}\int_{a_L^*\leq a_N+a_A< a_H^*} c'(e^*(a_N+a_A,\tilde{P}^L))\frac{\partial e^*(a_N+a_A,\tilde{P}^L)}{\partial \tilde{P}}dH_N(a_N)dH_A(a_A).\end{aligned}$$

The sign of the latter expression is ambiguous, as $\partial\tilde{P}^H/\partial T_L > 0$ and $\partial\tilde{P}^L/\partial T_L < 0$.

As can be seen, the impact of T_H on welfare is similar to that in the model with one university: $\partial W/\partial T_H > 0$. The intuition is similar as well. A rise in T_H reduces the effort put in by the agents who decide to apply for the high quality university and does not change the effort of the agents who apply for the low quality university. As a result, welfare rises. The impact of T_L is ambiguous in general. A rise in T_L reduces the effort put in by the agents applying for the low quality university and increases the effort put in by the agents applying for the high quality university. As a result, given T_H, there exists a certain optimal level of T_L such that $\partial W/\partial T_L = 0$ (unless the condition $\partial W/\partial T_L = 0$ delivers the minimum).

However, if the goal is to determine the value of the pair (T_H, T_L) that delivers the maximum social welfare, the outcome will be exactly the same as in the case with one university. In other words, the social welfare as a function of T_H and T_L is maximized when the effort put in by the marginal agent is equal to zero. That is, $e^*(a_H^*, \tilde{P}^H) = 0$ and $e^*(a_L^*, \tilde{P}^L) = 0$. This can be obviously seen from the expression for the social welfare (9), which is maximized when there is no wasted effort. The conditions of zero effort put in by the marginal agent can be written as follows:

$$s(a_H^*) = \frac{T_H - T_L}{\triangle q}, \tag{10}$$

$$s(a_L^*) = T_L/q^L. \tag{11}$$

Since the ability cutoffs are determined by the number of seats in the universities, we can find the optimal values for the tuition levels from the above equations.

Note that if we assume that $T_H = T_L = T$, then the welfare function will be increasing in T. However, the optimal value of T does not elicit the zero effort put in by all agents. Indeed, a rise in T reduces $\tilde{P}^L$ and, thereby, $\tilde{P}^H$ (recall that $\triangle T = 0$). In this case, it is straightforward to show that the social welfare is increasing in T. Therefore, we keep increasing T till the effort put in by the marginal agent a_L^* becomes zero (a further increase in T does not affect welfare, as $\tilde{P}^L$ is not affected anymore). The equilibrium conditions in this case are

$$\begin{aligned}\triangle q s(a_H^*) - c(e^*(a_H^*, \tilde{P}^H)) + c(e^*(a_H^*, \tilde{P}^L)) &= 0,\\ q^L s(a_L^*) - T &= 0.\end{aligned}$$

As $c(e^*(a_L^*, \tilde{P}^L)$ is equal to zero, $c(e^*(a_H^*, \tilde{P}^L))$ is equal to zero as well. Therefore, the equilibrium conditions can be written as follows:

$$\begin{aligned}\triangle q s(a_H^*) - c(e^*(a_H^*, \tilde{P}^H)) &= 0,\\ q^L s(a_L^*) - T &= 0.\end{aligned}$$

As can be seen, $e^*(a_H^*, \tilde{P}^H)$ is strictly positive in the equilibrium. That is, the marginal agent applying for the high quality university puts in some positive effort. The corresponding value of $\tilde{P}^H$ can be found from the first equation in the last system of equations.

Finally, similar to the benchmark case with one university, the distortion caused by selection into education can not be completely removed, as the social gains from education are different from the private gains.

The Case with Quotas

In this section, we introduce educational quotas in the above framework. We assume there are two groups of agents indexed by $i \in \{1, 2\}$, which have identical distributions of natural ability and potentially different distributions of acquired ability. The latter is motivated by the fact that agents with different social backgrounds have had different educational inputs prior to taking the exam, which in turn results in different acquired abilities. In particular, we assume that $H_N^1(a_N) = H_N^2(a_N) \equiv H_N(a_N)$, while $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ where $\succeq_{LR}$ stands for the likelihood stochastic order. Hence,

$$\frac{h_A^1(a_A)}{h_A^1(x)} > \frac{h_A^2(a_A)}{h_A^2(x)} \text{ for any } a_A, x : a_A > x.$$

This means that group 1 is more favored in terms of acquired ability than group 2. In addition, we assume that the distribution of natural ability has a log-concave density. This assumption is needed to ensure the likelihood stochastic order of the distributions of total ability: i.e. $H^1(a) \succeq_{LR} H^2(a)$.[3]

The share of each group in the total mass of agents (which is normalized to unity) is denoted by γ_i, where $\gamma_1 + \gamma_2 = 1$. We then define θ_{ik} as a share of available seats reserved for group i in university k: $\theta_{1k} + \theta_{2k} = 1$ for $k \in \{H, L\}$. If these quotas are binding, then the cutoffs for the two groups will differ.

[3] See Theorem 1.C.9 in Shaked and Shanthikumar (2007) for the proof. This assumption is not very restrictive, as a number of commonly used distributions such as the normal, uniform, Gamma, and Beta distributions satisfy it.

Note that the quota given to a certain group can be in general different in different universities. The equilibrium conditions can be then written as follows:

$$\begin{aligned}
\gamma_i \left(1 - H^i(a_{iH}^*)\right) &= \theta_{iH}\alpha_H, \\
\gamma_i \left(H^i(a_{iH}^*) - H^i(a_{iL}^*)\right) &= \theta_{iL}\alpha_L, \\
q^L s(a_{iL}^*) - T_L - c(e^*(a_{iL}^*, \tilde{P}^{iL})) &= 0, \\
\triangle q s(a_{iH}^*) - \triangle T - c(e^*(a_{iH}^*, \tilde{P}^{iH})) + c(e^*(a_{iH}^*, \tilde{P}^{iL})) &= 0,
\end{aligned}$$

where $i \in \{1, 2\}$.

We define θ_k^* a non-discriminating quota in university k (the quota assigned to group 1 so that the corresponding quota assigned to group 2 is $1 - \theta_k^*$) such that the quota leads to $\tilde{P}^{1k} = \tilde{P}^{2k}$, i.e. the performance cutoffs are the same for both groups. If both universities set the non-discriminating quotas, then it is straightforward to show that

$$\begin{aligned}
a_{1H}^* &= a_{2H}^*, \\
a_{1L}^* &= a_{2L}^*.
\end{aligned}$$

If in addition $H^1(a) \equiv H^2(a) = H(a)$, then

$$\theta_H^* = \theta_L^* = \gamma_1.$$

This is similar to the case with only one university.

Next, we write down the social welfare under the presence of two groups of agents:

$$\begin{aligned}
W &= \sum_i \gamma_i \int_{a_N + a_A \geq a_{iH}^*} \left(q^H s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iH})) - F\right) dH_N(a_N) dH_A^i(a_A) \qquad (12) \\
&\quad + \sum_i \gamma_i \int_{a_{iL}^* \leq a_N + a_A < a_{iH}^*} \left(q^L s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iL})) - F\right) dH_N(a_N) dH_A^i(a_A) \\
&= \sum_i \gamma_i \int_{a_N + a_A \geq a_{iH}^*} \left(q^H s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iH})) - T_H\right) dH_N(a_N) dH_A^i(a_A) \\
&\quad + \sum_i \gamma_i \int_{a_{iL}^* \leq a_N + a_A < a_{iH}^*} \left(q^L s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iL})) - T_L\right) dH_N(a_N) dH_A^i(a_A) \\
&\quad + \alpha_H (T_H - F) + \alpha_L (T_L - F).
\end{aligned}$$

In the next sections, we explore the behavior of social welfare (as a function of the quotas) around the non-discrimination quotas.

Symmetric Groups with no Selection Effect

In this subsection, we assume that the groups are symmetric, i.e. $H^1(a) \equiv H^2(a) = H(a)$, and examine how *uniform* changes in the quotas set by the universities locally affect the social welfare in the case of no selection effect. In particular, we assume that

$$\theta_{1k} = \mu\theta_k^*, \text{ implying that}$$
$$\theta_{2k} = 1 - \mu\theta_k^*.$$

This specification allows us consider uniform changes in the quotas set by the universities. Moreover, if $\mu = 1$, then both universities set the non-discrimination quotas θ_k^*. If $\mu = 0$, then in both universities all seats are given to the second group. Finally, if $\mu = 1/\gamma_1 > 1$, then all seats in both universities are given to the first group (recall that if $H^1(a) \equiv H^2(a)$, $\theta_H^* = \theta_L^* = \gamma_1$). Next, we consider the social welfare as a function of μ in the case of no selection effect, i.e. $\beta = 1$.

Taking into account (12), the derivative of the social welfare function with respect to μ can be written as

$$\begin{aligned}\frac{\partial W(.)}{\partial \mu} =& \sum_i \left(\frac{\partial W(.)}{\partial \tilde{P}^{iH}} \frac{\partial \tilde{P}^{iH}}{\partial \mu} + \frac{\partial W(.)}{\partial \tilde{P}^{iL}} \frac{\partial \tilde{P}^{iL}}{\partial \mu} \right) \\ &+ \sum_i \left(\frac{\partial W(.)}{\partial a_{iH}^*} \frac{\partial a_{iH}^*}{\partial \mu} + \frac{\partial W(.)}{\partial a_{iL}^*} \frac{\partial a_{iL}^*}{\partial \mu} \right).\end{aligned}$$

Since $\beta = 1$, we have

$$\begin{aligned}\frac{\partial W(.)}{\partial \mu} =& -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &- \sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ &- \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \left(q^H s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) - T_H \right) h^i(a_{iH}^*) \\ &+ \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \left(q^L s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iL})) - T_L \right) h^i(a_{iH}^*) \\ &- \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \left(q^L s(a_{iL}^*) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L \right) h^i(a_{iL}^*).\end{aligned}$$

Taking into account the equilibrium conditions for the marginal agent, these can be written as follows:

$$\begin{aligned}\frac{\partial W(.)}{\partial \mu} =& -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &- \sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ &- \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \left(q^L s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iL})) - T_L \right) h^i(a_{iH}^*) \\ &+ \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \left(q^L s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iL})) - T_L \right) h^i(a_{iH}^*) \\ &- \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} (0)\, h^i(a_{iL}^*).\end{aligned}$$

(as agent a_{iH}^* is indifferent between schools) so that

$$\begin{aligned}\frac{\partial W}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad - \sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a).\end{aligned}$$

Let us then find the derivative of $\tilde{P}^{ik}$ with respect to μ. From the equilibrium conditions, we have

$$\frac{\partial \tilde{P}^{iL}}{\partial \mu} = \frac{q_L s'(a_{iL}^*) - c'(e^*(a_{iL}^*, \tilde{P}^{iL})) \frac{\partial e^*(a_{iL}^*, \tilde{P}^{iL})}{\partial a}}{c'(e^*(a_{iL}^*, \tilde{P}^{iL})) \frac{\partial e^*(a_{iL}^*, \tilde{P}^{iL})}{\partial \tilde{P}}} \frac{\partial a_{iL}^*}{\partial \mu}.$$

Taking into account that

$$\theta_{iH}\alpha_H + \theta_{iL}\alpha_L = \gamma_i \left(1 - H^i(a_{iL}^*)\right),$$

we derive

$$\begin{aligned}\frac{\partial a_{1L}^*}{\partial \mu} &= -\frac{\theta_H^*\alpha_H + \theta_L^*\alpha_L}{\gamma_1 h(a_{1L}^*)} < 0, \\ \frac{\partial a_{2L}^*}{\partial \mu} &= \frac{\theta_H^*\alpha_H + \theta_L^*\alpha_L}{\gamma_2 h(a_{2L}^*)} > 0.\end{aligned}$$

This implies

$$\begin{aligned}\frac{\partial \tilde{P}^{1L}}{\partial \mu} &= -\frac{q_L s'(a_{1L}^*) - c'(e^*(a_{1L}^*, \tilde{P}^{1L})) \frac{\partial e^*(a_{1L}^*, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a_{1L}^*, \tilde{P}^{1L})) \frac{\partial e^*(a_{1L}^*, \tilde{P}^{1L})}{\partial \tilde{P}}} \frac{\theta_H^*\alpha_H + \theta_L^*\alpha_L}{\gamma_1 h(a_{1L}^*)} < 0, \\ \frac{\partial \tilde{P}^{2L}}{\partial \mu} &= \frac{q_L s'(a_{2L}^*) - c'(e^*(a_{2L}^*, \tilde{P}^{2L})) \frac{\partial e^*(a_{2L}^*, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a_{2L}^*, \tilde{P}^{2L})) \frac{\partial e^*(a_{2L}^*, \tilde{P}^{2L})}{\partial \tilde{P}}} \frac{\theta_H^*\alpha_H + \theta_L^*\alpha_L}{\gamma_2 h(a_{2L}^*)} > 0.\end{aligned}$$

As can be seen, at the non-discriminating quotas (when $\mu = 1$),

$$\gamma_1 \frac{\partial \tilde{P}^{1L}}{\partial \mu} = -\gamma_2 \frac{\partial \tilde{P}^{2L}}{\partial \mu},$$

implying that (recall that $H^1(a) \equiv H^2(a)$)

$$\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) = 0.$$

Next, we consider the derivative of $\tilde{P}^{iH}$ with respect to μ. From the equilibrium conditions,

$$\begin{aligned}\frac{\partial \tilde{P}^{iH}}{\partial \mu} &= \frac{\left[\triangle q s'(a_{iH}^*) - c'(e^*(a_{iH}^*, \tilde{P}^{iH})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial a} + c'(e^*(a_{iH}^*, \tilde{P}^{iL})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iL})}{\partial a}\right] \frac{\partial a_{iH}^*}{\partial \mu}}{c'(e^*(a_{iH}^*, \tilde{P}^{iH})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial \tilde{P}}} \\ &\quad + \frac{c'(e^*(a_{iH}^*, \tilde{P}^{iL})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iL})}{\partial \tilde{P}} \frac{\partial \tilde{P}^{iL}}{\partial \mu}}{c'(e^*(a_{iH}^*, \tilde{P}^{iH})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial \tilde{P}}}.\end{aligned}$$

In addition, we have that

$$\frac{\partial a_{1H}^*}{\partial \mu} = -\frac{\theta_H^* \alpha_H}{\gamma_1 h(a_{1H}^*)} < 0, \ \frac{\partial a_{2H}^*}{\partial \mu} = \frac{\theta_H^* \alpha_H}{\gamma_2 h(a_{2H}^*)} > 0.$$

Using all the previous results, we can see

$$\frac{\partial \tilde{P}^{1H}}{\partial \mu} < 0 \text{ and } \frac{\partial \tilde{P}^{2H}}{\partial \mu} > 0.$$

The latter follows from the fact that $\frac{\partial \tilde{P}^{1L}}{\partial \mu} < 0$, $\frac{\partial \tilde{P}^{2L}}{\partial \mu} > 0$, and (see Lemma 1)

$$\triangle q s'(a_{iH}^*) - c'(e^*(a_{iH}^*, \tilde{P}^{iH})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial a} + c'(e^*(a_{iH}^*, \tilde{P}^{iL})) \frac{\partial e^*(a_{iH}^*, \tilde{P}^{iL})}{\partial a} > 0.$$

Moreover, if $\mu = 1$, it is straightforward to see that

$$\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} = -\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu},$$

implying that

$$\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) = 0.$$

Thus, we have

$$\frac{\partial W}{\partial \mu}|_{\mu=1} = 0.$$

That is, non-discrimination delivers a local extremum. In the case of concave social welfare, non-discrimination is globally optimal. Next, we explore the case when the groups are asymmetric in terms of the distribution of total ability.

Intuitively, the logic is exactly the same. When the two groups are the same, the losses of one group exactly make up for the gains of the other for slight changes. Thus, if welfare is concave, this is a local maximum.

Asymmetric Groups with no Selection Effect

Assume now that $H^1(a) \succeq_{LR} H^2(a)$. Using the results derived in the above section, the derivative of social welfare with respect to μ is given by

$$\begin{aligned} \frac{\partial W}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad -\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a). \end{aligned}$$

First, consider the second term of the derivative:

$$\begin{aligned} &-\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ = \quad & (\theta_H^* \alpha_H + \theta_L^* \alpha_L) \left[\begin{array}{c} \frac{q_L s'(a_{1L}^*) - c'(e^*(a_{1L}^*, \tilde{P}^{1L})) \frac{\partial e^*(a_{1L}^*, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a_{1L}^*, \tilde{P}^{1L})) \frac{\partial e^*(a_{1L}^*, \tilde{P}^{1L})}{\partial \tilde{P}}} \int_{a_{1L}^* \leq a < a_{1H}^*} c'(e^*(a, \tilde{P}^{1L})) \frac{\partial e^*(a, \tilde{P}^{1L})}{\partial \tilde{P}} \frac{h^1(a)}{h^1(a_{1L}^*)} da \\ - \frac{q_L s'(a_{2L}^*) - c'(e^*(a_{2L}^*, \tilde{P}^{2L})) \frac{\partial e^*(a_{2L}^*, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a_{2L}^*, \tilde{P}^{2L})) \frac{\partial e^*(a_{2L}^*, \tilde{P}^{2L})}{\partial \tilde{P}}} \int_{a_{2L}^* \leq a < a_{2H}^*} c'(e^*(a, \tilde{P}^{2L})) \frac{\partial e^*(a, \tilde{P}^{2L})}{\partial \tilde{P}} \frac{h^2(a)}{h^2(a_{2L}^*)} da \end{array} \right]. \end{aligned}$$

At the non-discriminating quota, $a_{2L}^* = a_{1L}^*$ and $\tilde{P}^{2L} = \tilde{P}^{1L}$. This implies that

$$\frac{q_L s'(a_{2L}^*) - c'(e^*(a_{2L}^*, \tilde{P}^{2L}))\frac{\partial e^*(a_{2L}^*, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a_{2L}^*, \tilde{P}^{2L}))\frac{\partial e^*(a_{2L}^*, \tilde{P}^{2L})}{\partial \tilde{P}}} = \frac{q_L s'(a_{1L}^*) - c'(e^*(a_{1L}^*, \tilde{P}^{1L}))\frac{\partial e^*(a_{1L}^*, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a_{1L}^*, \tilde{P}^{1L}))\frac{\partial e^*(a_{1L}^*, \tilde{P}^{1L})}{\partial \tilde{P}}}.$$

Moreover, as $H^1(a) \succeq_{LR} H^2(a)$,

$$\frac{h^1(a)}{h^1(a_{1L}^*)} \geq \frac{h^2(a)}{h^2(a_{2L}^*)} \text{ for any } a > a_{1L}^* = a_{2L}^*.$$

Thus,

$$\int_{a_{1L}^* \leq a < a_{1H}^*} c'(e^*(a, \tilde{P}^{1L}))\frac{\partial e^*(a, \tilde{P}^{1L})}{\partial \tilde{P}}\frac{h^1(a)}{h^1(a_{1L}^*)}da > \int_{a_{2L}^* \leq a < a_{2H}^*} c'(e^*(a, \tilde{P}^{2L}))\frac{\partial e^*(a, \tilde{P}^{2L})}{\partial \tilde{P}}\frac{h^2(a)}{h^2(a_{2L}^*)}da,$$

implying that

$$-\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL}))\frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}}dH^i(a) > 0$$

when evaluated at the non-discriminating quota ($\mu = 1$).

Consider then the first term of the derivative, which is given by

$$-\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH}))\frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}}dH^i(a).$$

From the previous section, we have that

$$\begin{aligned}
\frac{\partial \tilde{P}^{iH}}{\partial \mu} &= \frac{\left[\triangle q s'(a_{iH}^*) - c'(e^*(a_{iH}^*, \tilde{P}^{iH}))\frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial a} + c'(e^*(a_{iH}^*, \tilde{P}^{iL}))\frac{\partial e^*(a_{iH}^*, \tilde{P}^{iL})}{\partial a}\right]\frac{\partial a_{iH}^*}{\partial \mu}}{c'(e^*(a_{iH}^*, \tilde{P}^{iH}))\frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial \tilde{P}}} \\
&\quad + \frac{c'(e^*(a_{iH}^*, \tilde{P}^{iL}))\frac{\partial e^*(a_{iH}^*, \tilde{P}^{iL})}{\partial \tilde{P}}\frac{\partial \tilde{P}^{iL}}{\partial \mu}}{c'(e^*(a_{iH}^*, \tilde{P}^{iH}))\frac{\partial e^*(a_{iH}^*, \tilde{P}^{iH})}{\partial \tilde{P}}}.
\end{aligned}$$

This means that at the non-discriminating quotas, we have

$$\begin{aligned}
\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} &= -\frac{\triangle q s'(a_{1H}^*) - c'(e^*(a_{1H}^*, \tilde{P}^{1H}))\frac{\partial e^*(a_{1H}^*, \tilde{P}^{1H})}{\partial a} + c'(e^*(a_{1H}^*, \tilde{P}^{1L}))\frac{\partial e^*(a_{1H}^*, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a_{1H}^*, \tilde{P}^{1H}))\frac{\partial e^*(a_{1H}^*, \tilde{P}^{1H})}{\partial \tilde{P}}}\frac{\theta_H^* \alpha_H}{h^1(a_{1H}^*)} \\
&\quad - \frac{c'(e^*(a_{1H}^*, \tilde{P}^{1L}))\frac{\partial e^*(a_{1H}^*, \tilde{P}^{1L})}{\partial \tilde{P}}\left(\frac{q_L s'(a_{1L}^*) - c'(e^*(a_{1L}^*, \tilde{P}^{1L}))\frac{\partial e^*(a_{1L}^*, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a_{1L}^*, \tilde{P}^{1L}))\frac{\partial e^*(a_{1L}^*, \tilde{P}^{1L})}{\partial \tilde{P}}}\right)}{c'(e^*(a_{1H}^*, \tilde{P}^{1H}))\frac{\partial e^*(a_{1H}^*, \tilde{P}^{1H})}{\partial \tilde{P}}}\frac{\theta_H^* \alpha_H + \theta_L^* \alpha_L}{h^1(a_{1L}^*)}, \\
\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu} &= \frac{\triangle q s'(a_{2H}^*) - c'(e^*(a_{2H}^*, \tilde{P}^{2H}))\frac{\partial e^*(a_{2H}^*, \tilde{P}^{2H})}{\partial a} + c'(e^*(a_{2H}^*, \tilde{P}^{2L}))\frac{\partial e^*(a_{2H}^*, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a_{2H}^*, \tilde{P}^{2H}))\frac{\partial e^*(a_{2H}^*, \tilde{P}^{2H})}{\partial \tilde{P}}}\frac{\theta_H^* \alpha_H}{h^2(a_{2H}^*)} \\
&\quad + \frac{c'(e^*(a_{2H}^*, \tilde{P}^{2L}))\frac{\partial e^*(a_{2H}^*, \tilde{P}^{i2})}{\partial \tilde{P}}\left(\frac{q_L s'(a_{2L}^*) - c'(e^*(a_{2L}^*, \tilde{P}^{2L}))\frac{\partial e^*(a_{2L}^*, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a_{2L}^*, \tilde{P}^{2L}))\frac{\partial e^*(a_{2L}^*, \tilde{P}^{2L})}{\partial \tilde{P}}}\right)}{c'(e^*(a_{2H}^*, \tilde{P}^{2H}))\frac{\partial e^*(a_{2H}^*, \tilde{P}^{2H})}{\partial \tilde{P}}}\frac{\theta_H^* \alpha_H + \theta_L^* \alpha_L}{h^2(a_{2L}^*)}.
\end{aligned}$$

Note that at the non-discrimination quota:

$$\begin{aligned}
& \theta^*_H \alpha_H \frac{\triangle q s'(a^*_{1H}) - c'(e^*(a^*_{1H},\tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1H})}{\partial a} + c'(e^*(a^*_{1H},\tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1H},\tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1H})}{\partial \tilde{P}}} \\
= \; & \theta^*_H \alpha_H \frac{\triangle q s'(a^*_{2H}) - c'(e^*(a^*_{2H},\tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial a} + c'(e^*(a^*_{2H},\tilde{P}^{2L}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2H},\tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial \tilde{P}}} \equiv A > 0.
\end{aligned}$$

Moreover, at the non-discriminating quota:

$$\begin{aligned}
& (\theta^*_H \alpha_H + \theta^*_L \alpha_L) \frac{c'(e^*(a^*_{1H},\tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1L})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L},\tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L},\tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{1H},\tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1H})}{\partial \tilde{P}}} \\
= \; & (\theta^*_H \alpha_H + \theta^*_L \alpha_L) \frac{c'(e^*(a^*_{2H},\tilde{P}^{2L}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2L})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L},\tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L},\tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{2H},\tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial \tilde{P}}} \equiv B > 0.
\end{aligned}$$

Thus, at the non-discriminating quota:

$$\begin{aligned}
\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} &= -\frac{A}{h^1(a^*_H)} - \frac{B}{h^1(a^*_L)}, \\
\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu} &= \frac{A}{h^2(a^*_H)} + \frac{B}{h^2(a^*_L)}.
\end{aligned}$$

As a result, at the non-discriminating quota:

$$\begin{aligned}
& -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a,\tilde{P}^{iH})) \frac{\partial e^*(a,\tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\
= \; & \left(\frac{A}{h^1(a^*_H)} + \frac{B}{h^1(a^*_L)} \right) \int_{a \geq a^*_H} c'(e^*(a,\tilde{P}^{H})) \frac{\partial e^*(a,\tilde{P}^{H})}{\partial \tilde{P}} h^1(a) da \\
& - \left(\frac{A}{h^2(a^*_H)} + \frac{B}{h^2(a^*_L)} \right) \int_{a \geq a^*_H} c'(e^*(a,\tilde{P}^{H})) \frac{\partial e^*(a,\tilde{P}^{H})}{\partial \tilde{P}} h^2(a) da.
\end{aligned}$$

Taking into account the stochastic order of the distributions of total ability, it is straightforward to see that

$$\begin{aligned}
A \int_{a \geq a^*_H} c'(e^*(a,\tilde{P}^{H})) \frac{\partial e^*(a,\tilde{P}^{H})}{\partial \tilde{P}} \frac{h^1(a)}{h^1(a^*_H)} da &> A \int_{a \geq a^*_H} c'(e^*(a,\tilde{P}^{H})) \frac{\partial e^*(a,\tilde{P}^{H})}{\partial \tilde{P}} \frac{h^2(a)}{h^2(a^*_H)} da, \\
B \int_{a \geq a^*_H} c'(e^*(a,\tilde{P}^{H})) \frac{\partial e^*(a,\tilde{P}^{H})}{\partial \tilde{P}} \frac{h^1(a)}{h^1(a^*_L)} da &> B \int_{a \geq a^*_H} c'(e^*(a,\tilde{P}^{H})) \frac{\partial e^*(a,\tilde{P}^{H})}{\partial \tilde{P}} \frac{h^2(a)}{h^2(a^*_L)} da.
\end{aligned}$$

In other words, at the non-discriminating quota:

$$-\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a,\tilde{P}^{iH})) \frac{\partial e^*(a,\tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) > 0.$$

To sum up, we have shown that the derivative of social welfare with respect to μ evaluated at the non-discriminating quota is positive. This means that discriminating in favor of group 1 locally increases the social welfare. This results is the same as that in the case of one university. That is, the effort effect works in favor of the advantaged group.

$$EE_{\mu=1} > 0.$$

This makes sense as weaker students need to put in more effort to get in and this effort is wasteful. So discriminating against the less advantaged group reduces the wasteful effort and thus raises welfare. Next, we explore the role of the selection effect.

The Selection Effect

Recall that the social welfare when $\beta < 1$ is given by

$$\begin{aligned}
W &= \sum_i \gamma_i \int_{a_N+a_A \geq a^*_{iH}} \left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A, \tilde{P}^{iH})) - F\right) dH_N(a_N) dH^i_A(a_A) \\
&\quad + \sum_i \gamma_i \int_{a^*_{iL} \leq a_N+a_A < a^*_{iH}} \left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A, \tilde{P}^{iL})) - F\right) dH_N(a_N) dH^i_A(a_A) \\
&= \sum_i \gamma_i \int_{a_N+a_A \geq a^*_{iH}} \left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A, \tilde{P}^{iH})) - T_H\right) dH_N(a_N) dH^i_A(a_A) \\
&\quad + \sum_i \gamma_i \int_{a^*_{iL} \leq a_N+a_A < a^*_{iH}} \left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A, \tilde{P}^{iL})) - T_L\right) dH_N(a_N) dH^i_A(a_A) \\
&\quad + \alpha_H(T_H - F) + \alpha_L(T_L - F).
\end{aligned}$$

The last expression can be written as follows:

$$\begin{aligned}
W &= \sum_i \gamma_i \int_0^{a^A_{\max,i}} \left(\int_{a^*_{iH}-a_A}^{a^N_{\max}} \left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A, \tilde{P}^{iH})) - T_H\right) dH_N(a_N)\right) dH^i_A(a_A) \\
&\quad + \sum_i \gamma_i \int_0^{a^A_{\max,i}} \left(\int_{a^*_{iL}-a_A}^{a^*_{iH}-a_A} \left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A, \tilde{P}^{iL})) - T_L\right) dH_N(a_N)\right) dH^i_A(a_A) \\
&\quad + \alpha_H(T_H - F) + \alpha_L(T_L - F).
\end{aligned}$$

When we explore only the selection effect, by definition we look at the effect which works just through the ability cutoffs and not through the performance cutoffs. Thus, the selection effect is as follows:

$$\begin{aligned}
SE &= -\sum_i \gamma_i \frac{\partial a^*_{iH}}{\partial \mu} \int_0^{a^A_{\max,i}} \left(q^H s(a^*_{iH} - a_A + \beta a_A) - c(e^*(a^*_{iH}, \tilde{P}^{iH})) - T_H\right) h_N(a^*_{iH} - a_A) dH^i_A(a_A) \\
&\quad + \sum_i \gamma_i \frac{\partial a^*_{iH}}{\partial \mu} \int_0^{a^A_{\max,i}} \left(q^L s(a^*_{iH} - a_A + \beta a_A) - c(e^*(a^*_{iH}, \tilde{P}^{iL})) - T_L\right) h_N(a^*_{iH} - a_A) dH^i_A(a_A) \\
&\quad - \sum_i \gamma_i \frac{\partial a^*_{iL}}{\partial \mu} \int_0^{a^A_{\max,i}} \left(q^L s(a^*_{iL} - a_A + \beta a_A) - c(e^*(a^*_{iL}, \tilde{P}^{iL})) - T_L\right) h_N(a^*_{iL} - a_A) dH^i_A(a_A) \\
&= -\sum_i \gamma_i \frac{\partial a^*_{iH}}{\partial \mu} \int_0^{a^A_{\max,i}} \begin{pmatrix} \triangle q s(a^*_{iH} - a_A + \beta a_A) - c(e^*(a^*_{iH}, \tilde{P}^{iH})) \\ + c(e^*(a^*_{iH}, \tilde{P}^{iL})) - \triangle T \end{pmatrix} h_N(a^*_{iH} - a_A) dH^i_A(a_A) \\
&\quad - \sum_i \gamma_i \frac{\partial a^*_{iL}}{\partial \mu} \int_0^{a^A_{\max,i}} \left(q^L s(a^*_{iL} - a_A + \beta a_A) - c(e^*(a^*_{iL}, \tilde{P}^{iL})) - T_L\right) h_N(a^*_{iL} - a_A) dH^i_A(a_A).
\end{aligned}$$

Taking into account the equilibrium conditions, we know that

$$\begin{aligned} \triangle q s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) + c(e^*(a_{iH}^*, \tilde{P}^{iL})) - \triangle T &= 0 \\ q^L s(a_{iL}^*) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L &= 0. \end{aligned}$$

Therefore,

$$\begin{aligned} & \triangle q s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) + c(e^*(a_{iH}^*, \tilde{P}^{iL})) - \triangle T \\ = \;& \triangle q \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*) \right), \text{ and} \\ & q^L s(a_{iL}^* - a_A + \beta a_A) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L \\ = \;& q^L \left(s(a_{iL}^* - a_A + \beta a_A) - s(a_{iL}^*) \right). \end{aligned}$$

Hence,

$$\begin{aligned} SE \;=\; & -\triangle q \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*) \right) h_N(a_{iH}^* - a_A) dH_A^i(a_A) \qquad (13) \\ & -q^L \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iL}^* - a_A + \beta a_A) - s(a_{iL}^*) \right) h_N(a_{iL}^* - a_A) dH_A^i(a_A). \end{aligned}$$

Note that in the case of one university we have only the second component of the above expression. However, we can apply the technique developed for the case with one university to both components, as they have similar functional forms.

Consider, for instance, the first term in the above expression given by:

$$SE_1 = -\triangle q \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*) \right) h_N(a_{iH}^* - a_A) dH_A^i(a_A).$$

Recall that

$$\frac{\partial a_{1H}^*}{\partial \mu} = -\frac{\theta_H^* \alpha_H}{\gamma_1 h^1(a_{1H}^*)} < 0, \;\; \frac{\partial a_{2H}^*}{\partial \mu} = \frac{\theta_H^* \alpha_H}{\gamma_2 h^2(a_{2H}^*)} > 0.$$

Hence, SE_1 (evaluated at the non-discriminating quota) can be written as follows:

$$\begin{aligned} SE_1|_{\theta=\theta^*} \;=\; & \triangle q \alpha_H \theta_H^* \int_0^{a_{\max,1}^A} \left(s(a_H^* - a_A + \beta a_A) - s(a_H^*) \right) \frac{h_N(a_H^* - a_A) h_A^1(a_A)}{h^1(a_H^*)} da_A \\ & - \triangle q \alpha_H \theta_H^* \int_0^{a_{\max,2}^A} \left(s(a_H^* - a_A + \beta a_A) - s(a_H^*) \right) \frac{h_N(a_H^* - a_A) h_A^2(a_A)}{h^2(a_H^*)} da_A. \end{aligned}$$

As in the benchmark case, we consider the following density functions:

$$\tilde{h}_A^i(a_A) \equiv \frac{h_A^i(a_A) h_N(a_H^* - a_A)}{\int_0^{a_{\max,i}^A} h_N(a_H^* - y) h_A^i(y) dy}.$$

Let $\tilde{H}_A^i(a_A)$ be its associated cumulative distribution function. As $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$,

$$\frac{\tilde{h}_A^1(a_A)}{\tilde{h}_A^1(x)} = \frac{h_A^1(a_A) h_N(a_H^* - a_A)}{h_A^1(x) h_N(a_H^* - x)} \geq \frac{h_A^2(a_A) h_N(a_H^* - a_A)}{h_A^2(x) h_N(a_H^* - x)} = \frac{\tilde{h}_A^2(a_A)}{\tilde{h}_A^2(x)} \text{ for any } a_A, \, x: \; a_A \geq x.$$

That is, $\tilde{H}_A^1(a_A) \succeq_{LR} \tilde{H}_A^2(a_A)$ which in turn implies $\tilde{H}_A^1(a_A) \succeq_1 \tilde{H}_A^2(a_A)$.

Then, SE_1 can be rewritten in the following way:

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q\alpha_H\theta_H^* \int_0^{a_{\max,1}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad + \triangle q\alpha_H\theta_H^* \int_0^{a_{\max,2}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^2(a_A). \end{aligned}$$

Equivalently, as $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ implies $a_{\max,1}^A \geq a_{\max,2}^A$

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q\alpha_H\theta_H^* \int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad - \triangle q\alpha_H\theta_H^* \int_0^{a_{\max,2}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\left(\tilde{H}_A^1(a_A) - \tilde{H}_A^2(a_A)\right). \end{aligned}$$

Integrating the second term above by parts implies that

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q\alpha_H\theta_H^* \int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad + \triangle q\alpha_H\theta_H^* \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_{\max,2}^A)\right) \left(1 - \tilde{H}_A^1(a_{\max,2}^A)\right) \\ &\quad - \triangle q\alpha_H\theta_H^* \left(1-\beta\right) \int_0^{a_{\max,2}^A} \left(\tilde{H}_A^2(a_A) - \tilde{H}_A^1(a_A)\right) s'(a_H^* - (1-\beta)\, a_A) da_A \\ &< 0, \end{aligned}$$

as $s'(\cdot)$ is positive and $\tilde{H}_A^2(a_A) \geq \tilde{H}_A^1(a_A)$ for any a_A (recall that $\tilde{H}_A^1(a_A) \succeq_1 \tilde{H}_A^2(a_A)$) and, moreover,

$$\int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) > \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_{\max,2}^A)\right) \left(1 - \tilde{H}_A^1(a_{\max,2}^A)\right).$$

Similarly, we can show that the second term in (13) evaluated at the non-discriminating quota:

$$SE_2|_{\theta=\theta^*} = -q^L \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_L^* - a_A + \beta a_A) - s(a_L^*)\right) h_N(a_L^* - a_A) dH_A^i(a_A),$$

is negative as well (the proof is exactly the same as that for SE_1). As a result, we can show that the selection effect evaluated at the non-discriminating quotas is negative, suggesting that we need to give quotas to the disadvantaged group. Moreover, if the groups are symmetric, then the selection effect is equal to zero at the non-discriminating quotas.