“Flip-Flopping, Primary Visibility and the Selection of Candidates”
Marina Agranov

ONLINE APPENDIX

Appendix A: Basic Election Model

Proof of Claim 1. Consider the challenger who is believed to be moderate with probability $p_2^{\text{Ch}}(s_2^{\text{Ch}}) \in [0,1]$ at the culmination of the general election stage and who competes with the incumbent with known type $t^{\text{Inc}} = R$. The challenger's chances of winning general election are determined by the preferences of the general-election median voter

$$\begin{aligned} W\left(p_2^{\text{Ch}}(s_2^{\text{Ch}})\right) &= \Pr\left[\text{Challenger with } p_2^{\text{Ch}}(s_2^{\text{Ch}}) \text{ wins}\right] = \Pr\left[\mathbb{E}u(m, p_2^{\text{Ch}}(s_2^{\text{Ch}})) > u(m,R)\right] = \\ &= \Pr\left[-p_2^{\text{Ch}}(s_2^{\text{Ch}})\cdot(m-M)^2 - (1-p_2^{\text{Ch}}(s_2^{\text{Ch}}))\cdot(m-L)^2 > -(m-R)^2\right] = \\ &= F\left[\frac{R^2 - p_2^{\text{Ch}}(s_2^{\text{Ch}})M^2 - (1-p_2^{\text{Ch}}(s_2^{\text{Ch}}))L^2}{2(R - p_2^{\text{Ch}}(s_2^{\text{Ch}})M - (1-p_2^{\text{Ch}}(s_2^{\text{Ch}}))L)}\right] \end{aligned}$$

It is straight-forward to check that the argument of F is strictly increasing in $p_2^{\text{Ch}}(s_2^{\text{Ch}})$ since $L < M < R$. Moreover, if $p_2^{\text{Ch}}(s_2^{\text{Ch}}) = 0$ then $W(0) = F\left[\frac{R+L}{2}\right] > 0$ and if $p_2^{\text{Ch}}(s_2^{\text{Ch}}) = 1$ then $1 > W(1) = F\left[\frac{R+M}{2}\right] > F\left[\frac{R+L}{2}\right] > 0$, **QED**.

Proof of Claim 2. Suppose that $p_1^{\text{Ch}} \in (0,1)$ and voters conjecture that, depending on her type, the challenger exerts efforts $\hat{e}_2^L$ and $\hat{e}_2^M$ in the general election stage. Then, expected payoffs of liberal and moderate challengers who exert efforts e_2^L and e_2^M, respectively, denoted by $\mathbb{E}\Pi^{t^{\text{Ch}}=L}(e_2^L)$ and $\mathbb{E}\Pi^{t^{\text{Ch}}=M}(e_2^M)$, can be written as

$$\mathbb{E}\Pi^{t^{\text{Ch}}=L}(e_2^L) = -e_2^L + W(p_2^{\text{Ch}}(\lambda)) + h(e_2^L, M, n_2)\cdot\left[W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda))\right]$$

$$\mathbb{E}\Pi^{t^{\text{Ch}}=M}(e_2^M) = -e_2^M + W(p_2^{\text{Ch}}(\mu)) - h(e_2^M, M, n_2)\cdot\left[W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda))\right]$$

where $p_2^{\text{Ch}}(\mu)$ and $p_2^{\text{Ch}}(\lambda)$ are specified in equations (1a) and (1b).

Assume that voters' beliefs after observing liberal and moderate signals during the general election campaign are the same, that is, $p_2^{\text{Ch}}(\mu) = p_2^{\text{Ch}}(\lambda)$ and $W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda)) = 0$. In this case, both types of challengers would choose zero effort, which means that signals are perfectly informative and we must have $p_2^{\text{Ch}}(\mu) = 1$ and $p_2^{\text{Ch}}(\lambda) = 0$. Thus, $p_2^{\text{Ch}}(\mu) \neq p_2^{\text{Ch}}(\lambda)$. Assume next that $p_2^{\text{Ch}}(\mu) < p_2^{\text{Ch}}(\lambda)$ then using Claim 1 we obtain $W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda)) < 0$. In this case, liberal challenger will exert zero effort, in which case moderate signal would fully reveal a moderate challenger $p_2^{\text{Ch}}(\mu) = 1$. In that case, we obtain $W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda)) > 0$, which contradicts our assumption above.

Thus, the only beliefs $(\hat{e}_2^L, \hat{e}_2^M)$ that might be consistent with equilibrium are

$$p_2^{\text{Ch}}(\mu) > p_2^{\text{Ch}}(\lambda) \Rightarrow W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda)) > 0$$

Given these beliefs, the moderate challenger would choose to exert no effort, $e_2^M = 0$, and liberal signal becomes perfectly informative of a liberal type, $p_2^{\text{Ch}}(\lambda) = 0$. For any pair of beliefs $(p_1^{\text{Ch}}, \hat{e}_2^L)$ define the best-response function of liberal challenger $e_2^L(\hat{e}_2^L, p_1^{\text{Ch}})$ as the one that maximizes her expected payoff

$$\frac{\mathrm{d}\mathbb{E}\Pi^{t^{\text{Ch}}=L}(e_2^L)}{\mathrm{d}e_2^L} = -1 + h_e(e_2^L, L, n_2) \cdot \Big[W(p_2^{\text{Ch}}(\mu)) - W(0)\Big] = 0$$

This best-response function is decreasing in $\hat{e}_2^L$ and satisfies $0 < e_2^L(1, p_1^{\text{Ch}}) < e_2^L(0, p_1^{\text{Ch}}) < 1$. Therefore, there exists a unique fixed point $e_2^{L^*}$ such that $e_2^L(\hat{e}_2^L, p_1^{\text{Ch}}) = \hat{e}_2^L \equiv e_2^{L^*}$. This optimal effort for the liberal challenger is determined by equation (3b) specified in Theorem 1, **QED**.

Proof of Claim 3. We will use the Implicit Function theorem to prove this claim. Define

$$S(p_1^{\text{Ch}}, e_2^{L^*}) = -1 + h_e(e_2^{L^*}, L, n_2) \cdot \Big[W\left(p_2^{\text{Ch}}(\mu)\right) - W(0)\Big] = 0$$

where $p_2^{\text{Ch}}(\mu) = \frac{p_1^{\text{Ch}}}{p_1^{\text{Ch}} + (1-p_1^{\text{Ch}}) \cdot h(e_2^{L^*}, L, n_2)}$.

$$\frac{\partial S(p_1^{\text{Ch}}, e_2^{L^*})}{\partial p_1^{\text{Ch}}} = h_e(e_2^{L^*}, L, n_2) \cdot \frac{\mathrm{d}W(p_2^{\text{Ch}}(\mu))}{\mathrm{d}p_2^{\text{Ch}}(\mu)} \cdot \frac{h(e^{L^*}, L, n_2)}{(p_1^{\text{Ch}} + (1-p_1^{\text{Ch}}) \cdot h(e_2^{L^*}, L, n_2))^2} > 0$$

$$\frac{\partial S(p_1^{\text{Ch}}, e_2^{L^*})}{\partial e_2^{L^*}} = h_{e_e}(e_2^{L^*}, L, n_2) \cdot \Big[W\left(p_2^{\text{Ch}}(\mu)\right) - W(0)\Big] - h_e(e_2^{L^*}, L, n_2) \cdot \frac{\mathrm{d}W(p_2^{\text{Ch}}(\mu))}{\mathrm{d}p_2^{\text{Ch}}(\mu)} \cdot \frac{(1-p_1^{\text{Ch}})h(e^{L^*}, L, n_2)}{(p_1^{\text{Ch}} + (1-p_1^{\text{Ch}}) \cdot h(e_2^{L^*}, L, n_2))^2} < 0$$

$$\Rightarrow \frac{\partial e_2^{L^*}}{\partial p_1^{\text{Ch}}} = -\frac{\frac{\partial S(p_1^{\text{Ch}}, e_2^{L^*})}{\partial p_1^{\text{Ch}}}}{\frac{\partial S(p_1^{\text{Ch}}, e_2^{L^*})}{\partial e_2^{L^*}}} > 0 \qquad \textbf{QED.}$$

Proof of Claim 4. Recall that $\mathbb{E}u(z_j, p_1^k(s_1^k))$ denotes expected utility of voter who has ideal point z_j when the winner of the primary stage is candidate k who generated signal s_1^k and, thus, believed to be moderate with probability $p_1^k(s_1^k)$:

$$\begin{aligned}
\mathbb{E}u(z_j, p_1^k(s_1^k)) &= p_1^k(s_1^k) \cdot \Big[W(p_2^{\text{Ch}}(\mu)) \cdot u(z_j, M) + (1 - W(p_2^{\text{Ch}}(\mu))) \cdot u(z_j, R)\Big] + \\
&+ (1 - p_1^k(s_1^k)) \cdot \begin{bmatrix} \Big[h(e_2^{L^*}, L, n_2)W(p_2^{\text{Ch}}(\mu)) + (1 - h(e_2^{L^*}, L, n_2))W(0)\Big] \cdot u(z_j, L) + \\ + \Big[h(e_2^{L^*}, L, n_2)(1 - W(p_2^{\text{Ch}}(\mu))) + (1 - h(e_2^{L^*}, L, n_2))(1 - W(0))\Big] \cdot u(z_j, R) \end{bmatrix} = \\
&= \Big[(1 - p_1^k(s_1^k))W(0) + \frac{p_1^k(s_1^k)(1 - p_2^{\text{Ch}}(\mu))}{p_2^{\text{Ch}}(\mu)}(W(p_2^{\text{Ch}}(\mu)) - W(0))\Big] \cdot u(z_i, L) + \\
&+ p_1^k(s_1^k)W(p_2^{\text{Ch}}(\mu)) \cdot u(z_j, M) + \Big[1 - W(0) - \frac{p_1^k(s_1^k)}{p_2^{\text{Ch}}(\mu)}(W(p_2^{\text{Ch}}(\mu)) - W(0))\Big]u(z_j, R)
\end{aligned}$$

On the other hand, a moderate politician who was revealed to be moderate in the primary stage and advances to the general election stage brings voter with ideal point z_j expected utility of $\mathbb{E}u(z_j, 1)$ where

$$\mathbb{E}u(z_j, 1) = W(1) \cdot u(z_j, M) + (1 - W(1)) \cdot u(z_j, R)$$

The voter with ideal point z_j will vote in the primary for a candidate k with belief $p_1^k(s_1^k) \in (0, 1)$ over candidate l with $p_1^l(s_1^l) = 1$ if and only if $\mathbb{E}u(z_j, p_1^k(s_1^k)) \geq \mathbb{E}u(z_j, 1)$. Define function $S(z_j)$ which captures the utility difference for a voter z_j between supporting an uncertain type

$p_1^k(s_1^k) \in (0,1)$ and a moderate type $p_1^l(s_1^l) = 1$. This function can be re-written as

$$S(z_j) = \mathbb{E}u(z_j, p_1^k(s_1^k)) - \mathbb{E}u(z_j, 1) = \gamma \cdot u(z_j, L) + (1-\gamma) \cdot u(z_j, R) - u(z_j, M)$$

where

$$\gamma = \frac{(1-p_1^k(s_1^k))W(0) + \frac{p_1^k(s_1^k)(1-p_2^{\mathrm{Ch}}(\mu))}{p_2^{\mathrm{Ch}}(\mu)} \cdot (W(p_2^{\mathrm{Ch}}(\mu)) - W(0))}{W(1) - p_1^k(s_1^k) \cdot W(p_2^{\mathrm{Ch}}(\mu))} \in (0,1)$$

First, we consider under which conditions $S(z_j) \geq 0$ for $z_j \in [0, \bar{d}]$. $S(z_j) \geq 0$ can be written as

$$\begin{aligned} & W(0) \cdot (u(z_j, L) - u(z_j, M)) + (1 - W(0)) \cdot (u(z_j, M) - u(z_j, R)) > \\ & > p_1^k(s_1^k) \cdot \begin{bmatrix} \left[W(0) - \frac{1-p_2^{\mathrm{Ch}}(\mu)}{p_2^{\mathrm{Ch}}(\mu)}(W(p_2^{\mathrm{Ch}}(\mu)) - W(0))\right] \cdot u(z_j, L) - \\ -W(p_2^{\mathrm{Ch}}(\mu)) \cdot u(z_i, M) + \frac{W(p_2^{\mathrm{Ch}}(\mu)) - W(0)}{p_2^{\mathrm{Ch}}(\mu)} \cdot u(z_j, R) \end{bmatrix} \end{aligned} \qquad (^*)$$

If right-hand side of the inequality (*) is negative, then we only need to make sure that the left-hand side is positive. If, however, the right-hand side is positive, then

$$\begin{aligned} & p_1^k(s_1^k) \cdot \left(\left[W(0) - \frac{1-p_2^{\mathrm{Ch}}(\mu)}{p_2^{\mathrm{Ch}}(\mu)}(W(p_2^{\mathrm{Ch}}(\mu)) - W(0))\right] \cdot u(z_j, L) - W(p_2^{\mathrm{Ch}}(\mu)) \cdot u(z_j, M) + \frac{W(p_2^{\mathrm{Ch}}(\mu)) - W(0)}{p_2^{\mathrm{Ch}}(\mu)} \cdot u(z_j, R) \right) < \\ & < \left[W(0) - \frac{1-p_2^{\mathrm{Ch}}(\mu)}{p_2^{\mathrm{Ch}}(\mu)}(W(p_2^{\mathrm{Ch}}(\mu)) - W(0))\right] \cdot u(z_j, L) - W(p_2^{\mathrm{Ch}}(\mu)) \cdot u(z_j, M) + \frac{W(p_2^{\mathrm{Ch}}(\mu)) - W(0)}{p_2^{\mathrm{Ch}}(\mu)} \cdot u(z_j, R) = \\ & = W(0) \cdot (u(z_j, L) - u(z_j, M)) - \frac{W(p_2^{\mathrm{Ch}}(\mu)) - W(0)}{p_2^{\mathrm{Ch}}(\mu)} \cdot \left[(1-p_2^{\mathrm{Ch}}(\mu)) \cdot u(z_j, L) + p_2^{\mathrm{Ch}}(\mu) \cdot u(z_j, M) - u(z_j, R)\right] \end{aligned}$$

Concavity of function $W(p_2^{\mathrm{Ch}}(s_2^{\mathrm{Ch}}))$ guarantees that

$$\frac{W(p_2^{\mathrm{Ch}}(\mu)) - W(0)}{p_2^{\mathrm{Ch}}(\mu)} \cdot \left[(1-p_2^{\mathrm{Ch}}(\mu)) \cdot u(z_j, L) + p_2^{\mathrm{Ch}}(\mu) \cdot u(z_j, M) - u(z_j, R)\right]$$

is decreasing in $p_2^{\mathrm{Ch}}(\mu)$.[1]

Collecting all the terms, we obtain that the right-hand side of inequality (*) is bounded above by $W(0) \cdot (u(z_j, L) - u(z_j, M)) - (W(1) - W(0)) \cdot (u(z_j, M) - u(z_j, R))$, which is exactly the left-hand side of inequality (*). Thus, inequality (*) is satisfied as long as function $W(p_2^{\mathrm{Ch}}(\mu))$ is (weakly) concave and the left-hand side of the inequality is non-negative, which simply means that voter z_j prefers to nominate a liberal over a moderate type because the gains from nominating a politician who is ideologically closer to him outweighs the risk (expressed in the utility terms) from this candidate losing the election:

$$W(0) \cdot (u(z_j, L) - u(z_j, M)) - (W(1) - W(0)) \cdot (u(z_j, M) - u(z_j, R)) \geq 0 \Leftrightarrow \mathbb{E}u(z_j, 0) \geq \mathbb{E}u(z_j, 1)$$

[1] To see this notice that for all $z_j \in [0, \bar{d}]$ we have assumed that $u(z_j, L) > u(z_j, M) > u(z_j, R)$, which ensures that $\left[(1-p_2^{\mathrm{Ch}}(\mu)) \cdot u(z_j, L) + p_2^{\mathrm{Ch}}(\mu) \cdot u(z_j, M) - u(z_j, R)\right]$ is decreasing in $p_2^{\mathrm{Ch}}(\mu)$. Moreover,

$$\frac{\mathrm{d}\frac{W(p_2^{\mathrm{Ch}}(\mu)) - W(0)}{p_2^{\mathrm{Ch}}(\mu)}}{\mathrm{d}p_2^{\mathrm{Ch}}(\mu)} = \frac{\frac{\mathrm{d}W(p_2^{\mathrm{Ch}})}{\mathrm{d}p_2^{\mathrm{Ch}}} p_2^{\mathrm{Ch}} - (W(p_2^{\mathrm{Ch}}) - W(0))}{p_2^2} \leq 0$$

as long as $\frac{\mathrm{d}W(p_2^{\mathrm{Ch}}(\mu))}{\mathrm{d}p_2^{\mathrm{Ch}}(\mu)} \leq \frac{W(p_2^{\mathrm{Ch}}(\mu)) - W(0)}{p_2^{\mathrm{Ch}}(\mu)}$, which is guaranteed for all $p_2^{\mathrm{Ch}}(\mu) \in (0,1)$ since $\frac{\mathrm{d}^2 W(p_2)}{\mathrm{d}(p_2^{\mathrm{Ch}}(\mu))^2} \leq 0$.

Second, consider

$$\frac{\mathrm{d}S(z_j)}{\mathrm{d}z_j} = \gamma \cdot \frac{\mathrm{d}u(z_j, L)}{\mathrm{d}z_j} + (1-\gamma) \cdot \frac{\mathrm{d}u(z_j, R)}{\mathrm{d}z_j} - \frac{\mathrm{d}u(z_j, M)}{\mathrm{d}z_j} = -2 \cdot [M - \gamma L - (1-\gamma)R]$$

Therefore, if $M \geq \gamma L + (1-\gamma)R$ then $\frac{\mathrm{d}S(z_j)}{\mathrm{d}z_j} \leq 0$ and it is enough to make sure that $S(\bar{d}) \geq 0$ to guarantee that all voters with $z_j \in [0, \bar{d}]$ vote for the uncertain over moderate type in the primary election. If, however, $M < \gamma L + (1-\gamma)R$ then $\frac{\mathrm{d}S(z_j)}{\mathrm{d}z_j} > 0$ and it is enough to make sure that $S(0) \geq 0$ to guarantee that all voters with $z_j \in [0, \bar{d}]$ vote for the uncertain over moderate type in the primary election, **QED**.

Proof of Claim 5. Assume that voters believe that $\hat{e}_1^M \in (0, 1]$, $\hat{e}_1^L = 0$ and candidate A follows this strategy. We will show that candidate B wants to follows this strategy as well. First consider what liberal candidate B would do:

$$\mathbb{E}\Pi^{t^B=L}(e_1^L) = -e_1^L + W(1) \cdot \frac{h(e_1^L, L, n_1)(1 - h(\hat{e}_1^M, M, n_1))}{4} + \\ + \Big(W(\hat{p}_2(\mu)) - e_2^{L^*}(\hat{p}_1(\lambda))\Big) \cdot (1 - h(e_1^L, L, n_1)) \cdot \frac{3 - h(\hat{e}_1^M, M, n_1)}{4}$$

where

$$\hat{p}_2(\mu) = \frac{\hat{p}(\lambda)}{\hat{p}_1(\lambda) + (1 - \hat{p}_1(\lambda)) \cdot h(e_2^{L^*}(\hat{p}_1(\lambda)), L, n_2)} \text{ and } \hat{p}_1(\lambda) = \frac{h(\hat{e}_1^M, M, n_1)}{h(\hat{e}_1^M, M, n_1) + 1}$$

$$\frac{\mathrm{d}\mathbb{E}\Pi^{t^B=L}(e_1^L)}{\mathrm{d}e_1^L} = -1 + h_e(e_1^L, L, n_1) \cdot \left[\frac{1 - h(\hat{e}_1^M, M, n_1)}{4} \cdot W(1) - \frac{3 - h(\hat{e}_1^M, M, n_1)}{4} \cdot \Big(W(\hat{p}_2(\mu)) - e_2^{L^*}(\hat{p}_1(\lambda))\Big)\right] < 0$$

because

$$\frac{1 - h(\hat{e}_1^M, M, n_1)}{4} \cdot W(1) - \frac{3 - h(\hat{e}_1^M, M, n_1)}{4} \cdot \Big(W(\hat{p}_2(\mu)) - e_2^{L^*}(\hat{p}_1(\lambda))\Big) < \\ < \frac{1 - h(\hat{e}_1^M, M, n_1)}{4} \cdot W(1) - \frac{3 - h(\hat{e}_1^M, M, n_1)}{4} \cdot W(0) < \frac{-1 - h(\hat{e}_1^M, M, n_1)}{4} \cdot W(0) < 0$$

The first inequality follows from the fact that $W(\hat{p}_2(\mu)) - e_2^{L^*}(\hat{p}_1(\lambda)) \geq 0$ since liberal challenger could choose zero effort in the general election stage and preferred not to. The second inequality follows from the condition $2W(0) > W(1)$. Therefore, liberal candidate B prefers to put no effort in the primary campaign.

Consider now incentives of moderate candidate B:

$$\mathbb{E}\Pi^{t^B=M}(e_1^M) = -e_1^M + W(1) \cdot \frac{(1 - h(e_1^M, M, n_1))(1 - h(\hat{e}_1^M, M, n_1))}{4} + W(\hat{p}_2(\mu)) \cdot h(e_1^M, L, n_1) \cdot \frac{3 - h(\hat{e}_1^M, M, n_1)}{4}$$

$$\frac{\mathrm{d}\mathbb{E}\Pi^{t^B=M}(e_1^M)}{\mathrm{d}e_1^M} = -1 + h_e(e_1^M, M, n_1) \cdot \left[\frac{3 - h(\hat{e}_1^M, M, n_1)}{4} \cdot W(\hat{p}_2(\mu)) - \frac{1 - h(\hat{e}_1^M, M, n_1)}{4} \cdot W(1)\right]$$

Define best-response function of moderate candidate B, $e_1^{M^*}(\hat{e}_1^M)$. This is the effort level of moderate candidate B, $e_1^{M^*} \in (0, 1)$, that solves $\frac{\mathrm{d}\mathbb{E}\Pi^{t^B=M}(e_1^M)}{\mathrm{d}e_1^M}|_{e_1^{M^*}} = 0$. Notice that best-response exists and it is unique for all $\hat{e}_1^M \in (0, 1]$.

We are left to show that there exists a unique fixed point such that $e_1^{M^*}(\hat{e}_1^M) = \hat{e}_1^M$, which is determined by the equation (3a). This follows from three observations: (1) $e_1^{M^*}(0) > 0$, (2) $e_1^{M^*}(1) < 1$, and (3) $\frac{\mathrm{d}e_1^{M^*}(\hat{e}_1^M)}{\mathrm{d}\hat{e}_1^M} > 0$ and $\frac{\mathrm{d}^2 e_1^{M^*}(\hat{e}_1^M)}{\mathrm{d}(\hat{e}_1^M)^2} < 0$ by the Implicit Function theorem and assumptions imposed on the scrutiny function h. We, therefore, conclude that there exists a unique fixed point $e_1^{M^*}(\hat{e}_1^M) = \hat{e}_1^M$ that constitutes part of the equilibrium strategy, **QED**.

Proofs of Claims 6 and 8.

As Theorem 1 asserts, the optimal efforts of candidates $(e_1^{M^*}, e_2^{L^*})$ are determined by equations (3a) and (3b), and depend upon n_1 and n_2, which are exogenous parameters that capture prominence levels of the primary and the general election stages, respectively. To simplify exposition, we abuse notation and use the following shortcuts

$$e_1^{M^*} = X(n_1, n_2) \equiv x \text{ and } h(e_1^{M^*}, M, n_1) = g(x, n_1) \equiv g$$

$$e_2^{L^*} = Y(n_1, n_2) \equiv y \text{ and } h(e_2^{L^*}, L, n_2) = h(y, n_2) \equiv h$$

Then equations (3a) and (3b) can be re-written as

$$\begin{cases} V(x, y, n_1, n_2) = -1 + g_x \cdot \left[\frac{3-g}{4} \cdot W\left(\frac{g}{h+g}\right) - \frac{1-g}{4} \cdot W(1)\right] = 0 \\ U(x, y, n_1, n_2) = -1 + h_y \cdot \left[W\left(\frac{g}{h+g}\right) - W(0)\right] = 0 \end{cases}$$

Finding $\frac{\partial x}{\partial n_1}$, $\frac{\partial x}{\partial n_2}$, $\frac{\partial y}{\partial n_1}$ and $\frac{\partial y}{\partial n_2}$ is a straight-forward application of Cramer's rule:

$$\frac{\partial x}{\partial n_1} = \frac{\begin{vmatrix} -\frac{\partial U}{\partial n_1} & \frac{\partial U}{\partial y} \\ -\frac{\partial V}{\partial n_1} & \frac{\partial V}{\partial y} \end{vmatrix}}{\begin{vmatrix} \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} \end{vmatrix}} \qquad \frac{\partial x}{\partial n_2} = \frac{\begin{vmatrix} -\frac{\partial U}{\partial n_2} & \frac{\partial U}{\partial y} \\ -\frac{\partial V}{\partial n_2} & \frac{\partial V}{\partial y} \end{vmatrix}}{\begin{vmatrix} \frac{\partial U}{\partial x} & \frac{\partial U}{\partial y} \\ \frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} \end{vmatrix}} \qquad \frac{\partial y}{\partial n_1} = \frac{\begin{vmatrix} -\frac{\partial U}{\partial n_1} & \frac{\partial U}{\partial x} \\ -\frac{\partial V}{\partial n_1} & \frac{\partial V}{\partial x} \end{vmatrix}}{\begin{vmatrix} \frac{\partial U}{\partial y} & \frac{\partial U}{\partial x} \\ \frac{\partial V}{\partial y} & \frac{\partial V}{\partial x} \end{vmatrix}} \qquad \frac{\partial y}{\partial n_2} = \frac{\begin{vmatrix} -\frac{\partial U}{\partial n_2} & \frac{\partial U}{\partial x} \\ -\frac{\partial V}{\partial n_2} & \frac{\partial V}{\partial x} \end{vmatrix}}{\begin{vmatrix} \frac{\partial U}{\partial y} & \frac{\partial U}{\partial x} \\ \frac{\partial V}{\partial y} & \frac{\partial V}{\partial x} \end{vmatrix}}$$

where

$$\frac{\partial U}{\partial x} = \frac{h \cdot h_y \cdot g_x}{(h+g)^2} \cdot W'\left(\frac{g}{h+g}\right) > 0$$

$$\frac{\partial U}{\partial y} = h_{y_y} \cdot \left[W\left(\frac{g}{h+g}\right) - W(0)\right] - W'\left(\frac{g}{h+g}\right) \frac{h_y^2 \cdot g}{(h+g)^2} < 0$$

$$\frac{\partial U}{\partial n_1} = h_y \cdot W'\left(\frac{g}{h+g}\right) \cdot \frac{g_{n_1} \cdot h}{(h+g)^2} < 0$$

$$\frac{\partial U}{\partial n_2} = h_{y_{n_2}} \cdot \left[W\left(\frac{g}{h+g}\right) - W(0)\right] - h_y \cdot \frac{h_{n_2} \cdot g}{(h+g)^2} \cdot W'\left(\frac{g}{h+g}\right)$$

$$\frac{\partial V}{\partial x} = g_{x_x} \cdot \left[\frac{3-g}{4} \cdot W\left(\frac{g}{h+g}\right) - \frac{1-g}{4} \cdot W(1)\right] + g_x \cdot \left[\frac{3-g}{4} \cdot W'\left(\frac{g}{h+g}\right) \cdot \frac{h \cdot g_x}{(h+g)^2} + \frac{g_x}{4} \cdot \left(W(1) - W\left(\frac{g}{h+g}\right)\right)\right]$$

$$\frac{\partial V}{\partial y} = -\frac{g_x \cdot h_y \cdot g}{(h+g)^2} \cdot \frac{3-g}{4} \cdot W'\left(\frac{g}{h+g}\right) < 0$$

$$\begin{aligned} \frac{\partial V}{\partial n_1} =& g_{x_{n_1}} \cdot \left[\frac{3-g}{4} \cdot W\left(\frac{g}{h+g}\right) - \frac{1-g}{4} \cdot W(1)\right] + \\ &+ g_x \cdot \left[\frac{g_{n_1}}{4} \cdot \left[W(1) - W\left(\frac{g}{h+g}\right)\right] + \frac{3-g}{4} \cdot \frac{g_{n_1} \cdot h}{(h+g)^2} \cdot W'\left(\frac{g}{h+g}\right)\right] < 0 \end{aligned}$$

$$\frac{\partial V}{\partial n_2} = -\frac{g_x \cdot g \cdot h_{n_2}}{(h+g)^2} \cdot \frac{3-g}{4} \cdot W'\left(\frac{g}{h+g}\right) > 0$$

If scrutiny function satisfies condition (C3), which asserts that $-g_{xx} \cdot g \geq g_x^2$ for all $x \in [0,1]$, then the sign of $\frac{\partial V}{\partial x}$ is negative. To see why this is the case, notice that

$$\frac{\partial G}{\partial x} \leq g_{x_x} \cdot \left[\frac{3-g}{4} \cdot W\left(\frac{g}{h+g}\right) - \frac{1-g}{4} \cdot W(1)\right] + \frac{g_x^2}{4} \cdot \left[W(1) - W\left(\frac{g}{h+g}\right)\right] + \frac{g_x^2}{4} \cdot \frac{W\left(\frac{g}{h+g}\right) - W(0)}{\frac{g}{h+g}} \cdot \frac{h(3-g)}{(h+g)^2} =$$

$$= \frac{g_{x_x}}{4} \cdot \left[(3-g)W\left(\frac{g}{h+g}\right) - (1-g)W(1)\right] + \frac{g_x^2}{4g} \cdot \frac{h(3-g)}{h+g} \cdot \left[W\left(\frac{g}{h+g}\right) - W(0)\right]$$

First inequality follows from concavity of function W. Further, $2W(0) > W(1)$ guarantees that

$$(3-g)W\left(\frac{g}{h+g}\right) - (1-g)W(1) > \frac{h(3-g)}{h+g} \cdot \left[W\left(\frac{g}{h+g}\right) - W(0)\right]$$

Once, the sign of $\frac{\partial V}{\partial x}$ is determined, so is the signs of the denominators

$$\frac{\partial U}{\partial x} \cdot \frac{\partial V}{\partial y} - \frac{\partial V}{\partial x} \cdot \frac{\partial U}{\partial y} < 0 \text{ and } \frac{\partial U}{\partial y} \cdot \frac{\partial V}{\partial x} - \frac{\partial V}{\partial y} \cdot \frac{\partial U}{\partial x} > 0$$

Thus, both liberal challengers in the general election and moderate candidates in the primaries exert less effort when primaries are more visible:

$$\frac{\partial e_2^{L^*}}{\partial n_1} \equiv \frac{\partial y}{\partial n_1} = \frac{-\frac{\partial U}{\partial n_1} \cdot \frac{\partial V}{\partial x} + \frac{\partial V}{\partial n_1}\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y} \cdot \frac{\partial V}{\partial x} - \frac{\partial V}{\partial y}\frac{\partial U}{\partial x}} = \frac{\text{negative}}{\text{positive}} < 0$$

$$\frac{\partial e_1^{M^*}}{\partial n_1} \equiv \frac{\partial x}{\partial n_1} = \frac{-\frac{\partial U}{\partial n_1} \cdot \frac{\partial V}{\partial y} + \frac{\partial V}{\partial n_1}\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial x} \cdot \frac{\partial V}{\partial y} - \frac{\partial V}{\partial x}\frac{\partial U}{\partial y}} = \frac{\text{positive}}{\text{negative}} < 0$$

In addition, we obtain

$$\frac{d\left(\frac{g}{h+g}\right)}{dn_1} = \frac{h \cdot \frac{dg}{dn_1} - g \cdot \frac{dh}{dn_1}}{(h+g)^2} < 0$$

by substituting $\frac{\partial x}{\partial n_1}$ and $\frac{\partial y}{\partial n_1}$ into the derivative above and performing algebraic manipulations given the assumptions imposed on scrutiny and winning general election functions. Thus, voters believe that the likelihood that moderate type generates signal λ in the primary and signal μ in the general election is lower when the primary race is more prominent. This completes the proof of Claim 6.

Further, higher prominence level of the general election incentivizes moderate candidates in the primary to exert higher effort and results in more likely revelations of the liberal challengers in the general election stage:

$$\frac{\partial e_1^{M^*}}{\partial n_2} \equiv \frac{\partial x}{\partial n_2} = \frac{-\frac{\partial U}{\partial n_2} \cdot \frac{\partial V}{\partial y} + \frac{\partial V}{\partial n_2}\frac{\partial U}{\partial y}}{\frac{\partial U}{\partial x} \cdot \frac{\partial V}{\partial y} - \frac{\partial V}{\partial x}\frac{\partial U}{\partial y}} = \frac{\text{negative}}{\text{negative}} > 0$$

$$\frac{\mathrm{d}h(e_2^{L^*},L,n_2)}{\mathrm{d}n_2} \equiv \frac{\mathrm{d}h(y,n_2)}{\mathrm{d}n_2} = h_{n_2} + h_y \cdot \frac{\partial y}{\partial n_2} = \frac{1}{\frac{\partial V}{\partial x}\frac{\partial U}{\partial y} - \frac{\partial V}{\partial y}\frac{\partial U}{\partial x}} \cdot \left[h_{n_2}\frac{\partial U}{\partial y}\frac{\partial V}{\partial x} - h_{n_2}\frac{\partial U}{\partial x}\frac{\partial V}{\partial y} - h_y\frac{\partial U}{\partial n_2}\frac{\partial V}{\partial x} + h_y\frac{\partial U}{\partial x}\frac{\partial V}{\partial n_2}\right] =$$
$$= \frac{1}{\frac{\partial V}{\partial x}\frac{\partial U}{\partial y} - \frac{\partial V}{\partial y}\frac{\partial U}{\partial x}} \cdot \frac{\partial V}{\partial x} \cdot \left[\frac{\partial U}{\partial y}h_{n_2} - \frac{\partial U}{\partial n_2}h_y\right] < 0$$

which completes the proof of Claim 8, **QED.**

Proof of Claim 7. We will prove that the likelihood of the candidate from the Democratic party winning the election is decreasing in the prominence level of the primary selection process. We will continue using the notation introduced in Proofs of Claims 6 and 8 to save trees:

$$\begin{aligned}
\text{Pr[Democrat wins general election]} &= \frac{1}{4} \cdot \left(h \cdot W\left(\frac{g}{h+g}\right) + (1-h)\cdot W(0)\right) + \\
&+ \frac{1}{2}\cdot\left(\left(1-\frac{g}{2}\right)\cdot\left[h\cdot W\left(\frac{g}{h+g}\right) + (1-h)\cdot W(0)\right] + \frac{g}{2}\cdot W\left(\frac{g}{h+g}\right)\right) + \\
&+ \frac{1}{4}\cdot\left(\left(2g-g^2\right)\cdot W\left(\frac{g}{h+g}\right) + (1-g)^2\cdot W(1)\right) = \\
&= \frac{(1-h)(3-g)}{4}\cdot W(0) + \frac{(3-g)(h+g)}{4}\cdot W\left(\frac{g}{h+g}\right) + \frac{(1-g)^2}{4}\cdot W(1)
\end{aligned}$$

$$\begin{aligned}
\frac{d}{dn_1} &= \frac{-\frac{dh}{dn_1}(3-g) - \frac{dg}{dn_1}(1-h)}{4}\cdot W(0) + \frac{-2(1-g)\frac{dg}{dn_1}}{4}\cdot W(1) + \\
&+ \frac{-\frac{dg}{dn_1}(h+g) + (3-g)\left(\frac{dh}{dn_1}+\frac{dg}{dn_1}\right)}{4}\cdot W\left(\frac{g}{h+g}\right) + \frac{(3-g)(h+g)}{4}\cdot W'\left(\frac{g}{h+g}\right)\cdot\frac{\frac{dg}{dn_1}h - g\frac{dh}{dn_1}}{(h+g)^2} = \\
&= \frac{dh}{dn_1}\cdot\frac{3-g}{4}\cdot\left[W\left(\frac{g}{h+g}\right) - W(0) - \frac{g}{h+g}\cdot W'\left(\frac{g}{h+g}\right)\right] + \\
&+ \frac{dg}{dn_1}\cdot\left[\frac{1-h}{4}\cdot W(0) - \frac{2(1-g)}{4}\cdot W(1) + \frac{3-2g-h}{4}\cdot W\left(\frac{g}{h+g}\right) + \frac{h(3-g)}{4(h+g)}\cdot W'\left(\frac{g}{h+g}\right)\right]
\end{aligned}$$

First, note that $W(\cdot)$ is weakly concave, and, therefore, $W'(x) \le \frac{W(x)-W(0)}{x} \quad \forall x \in (0,1)$.

Thus,

$$\frac{3-g}{4}\cdot\left[W\left(\frac{g}{h+g}\right) - W(0) - \frac{g}{h+g}\cdot W'\left(\frac{g}{h+g}\right)\right] \ge 0$$

If $\left[\frac{1-h}{4}\cdot W(0) - \frac{2(1-g)}{4}\cdot W(1) + \frac{3-2g-h}{4}\cdot W\left(\frac{g}{h+g}\right) + \frac{h(3-g)}{4(h+g)}\cdot W'\left(\frac{g}{h+g}\right)\right] > 0$ then the proof is complete, since both $\frac{dh}{dn_1} = h_y \cdot \frac{\partial y}{\partial n_1} < 0$ as well as $\frac{dg}{dn_1} = g_{n_1} + g_x\frac{\partial x}{\partial n_1} < 0$, which guarantees that the whole derivative is negative.

Assume that $\left[\frac{1-h}{4}\cdot W(0) - \frac{2(1-g)}{4}\cdot W(1) + \frac{3-2g-h}{4}\cdot W\left(\frac{g}{h+g}\right) + \frac{h(3-g)}{4(h+g)}\cdot W'\left(\frac{g}{h+g}\right)\right] < 0$. Then, we will use the fact that

$$h\cdot\frac{dg}{dn_1} < g\cdot\frac{dh}{dn_1}$$

and we will re-write the whole derivative as

$$\frac{d}{dn_1} < \frac{dh}{dn_1}\cdot\left[\begin{array}{l}\frac{3-g}{4}\cdot W(\frac{g}{h+g}) - \frac{3-g}{4}\cdot W(0) - \frac{3-g}{4}\cdot\frac{g}{h+g}\cdot W'(\frac{g}{h+g}) - \frac{g(1-h)}{4h}\cdot W(0) - \\ -\frac{2g-2g^3}{4h}\cdot W(1) + \frac{3g-2g^2-hg}{4h}\cdot W\left(\frac{g}{h+g}\right) + \frac{3-g}{4}\cdot\frac{g}{h+g}\cdot W'(\frac{g}{h+g})\end{array}\right]$$

$$= \frac{dh}{dn_1}\cdot\left[\frac{3h+3g-2hg-2g^2}{4h}\cdot W\left(\frac{g}{h+g}\right) - \frac{3h+g-2hg}{4h}\cdot W(0) - \frac{2g-2g^2}{4h}\cdot W(1)\right]$$

We will show now that the last bracket is positive:

$$\frac{3g+3h-2g^2-2hg}{4h} \cdot W\left(\frac{g}{h+g}\right) \geq \frac{3h+g-2hg}{4h} \cdot W(0) + \frac{2g-2g^2}{4h} \cdot W(1) \Leftrightarrow$$

$$(3g+3h-2g^2-2hg) \cdot W\left(\frac{g}{h+g}\right) \geq (3h+g-2hg) \cdot W(0) + (2g-2g^2) \cdot W(1)$$

This last inequality can be re-written as $W(x) \cdot (a+b) \geq W(0) \cdot a + W(1) \cdot b$ where $a = 3h+g-2hg$, $b = 2g-2g^2$ and $x = \frac{g}{h+g}$. Notice that

$$x = \frac{g}{h+g} > \frac{b}{a+b} = \frac{2g-2g^2}{3g+3h-2g^2-2hg} \Leftrightarrow$$

$$3g^2+3hg-2g^3-2hg^2 > 2hg+2g^2-2hg^2-2g^3 \Leftrightarrow g^2+hg > 0 - \text{TRUE!}$$

Thus, since function $W(\cdot)$ is strictly increasing and weakly concave we get

$$W(x) > W\left(\frac{b}{a+b}\right) \geq \frac{a}{a+b} \cdot W(0) + \frac{b}{a+b} \cdot W(1) \qquad \textbf{QED.}$$

Appendix B: Properties of Scrutiny Function

In this section we discuss the role of the assumption (A1c) which states that a candidate that exerts no effort generates signal which coincides with her type for sure, that is, $h(0,t,n)=0$. This assumption plays an important part in Claim 3. In particular, when $h(0,t,n)=0$, we show in Claim 3, that the liberal challenger exerts more effort in pretending to be moderate during the general election campaign when prior on her being moderate at the beginning of the general election campaign is higher, $\frac{de_2^{L^*}}{dp_1^{\text{Ch}}}>0$.

Consider the basic election model in which candidate that exerts no effort has a very small but positive chance of generating signal opposite from her true type. To capture this, we will modify assumption (A1c):

$$\text{(A1c}^*\text{)} \qquad h(0,t,n)=\epsilon>0 \qquad \text{where } \epsilon=h(0,t,n)<\min\left\{\frac{1}{2},1-h(1,t,n)\right\}$$

In the remainder of this section, we characterize optimal behavior of challenger in the general election stage depending on her type (Claim 2*) and then study how it varies with prior belief about challenger's type, p_1^{Ch} (Claim 3*).

Claim 2*. Assume that the parameters of the election game satisfy assumptions (A1a), (A1b), (A1c*), (A1d)-(A1f), (A2)-(A6). If the belief about challenger's type at the beginning of the general election stage is degenerate, $p_1^{\text{Ch}}=0$ or $p_1^{\text{Ch}}=1$, then she exerts no effort irrespectively of her type. If, however, voters are uncertain about challenger's type after the primary race, i.e. $p_1^{\text{Ch}}\in(0,1)$, then the unique equilibrium in the general election subgame prescribes the moderate challenger to exert no effort and the liberal challenger to put positive effort in mimicking the moderate type, where the amount of mimicking $e_2^{L^*}$ is determined by equation (*) below

$$h_e(e_2^{L^*},L,n_2)\cdot\left[W\left(p_2^{\text{Ch}}(\mu)\right)-W\left(p_2^{\text{Ch}}(\lambda)\right)\right]=1 \qquad ...(*)$$

where

$$p_2^{\text{Ch}}(\mu)=\frac{p_1^{\text{Ch}}\cdot(1-\epsilon)}{p_1^{\text{Ch}}\cdot(1-\epsilon)+(1-p_1^{\text{Ch}})\cdot h(e_2^{L^*},L,n_2)} \text{ and } p_2^{\text{Ch}}(\lambda)=\frac{p_1^{\text{Ch}}\cdot\epsilon}{p_1^{\text{Ch}}\cdot\epsilon+(1-p_1^{\text{Ch}})\cdot(1-h(e_2^{L^*},L,n_2))}$$

Proof of Claim 2*. First note that Claim 1 and its proof remain unchanged with modified assumption (A1c*), as they do not depend on the properties of the scrutiny function h.

Suppose that $p_1^{\text{Ch}}\in(0,1)$ and voters conjecture that, depending on her type, the challenger exerts efforts $\hat{e}_2^L$ and $\hat{e}_2^M$ in the general election stage. Then, expected payoffs of liberal and moderate challengers who exert efforts e_2^L and e_2^M, respectively, denoted by $\mathbb{E}\Pi^{t^{\text{Ch}}=L}(e_2^L)$ and $\mathbb{E}\Pi^{t^{\text{Ch}}=M}(e_2^M)$, can be written as

$$\mathbb{E}\Pi^{t^{\text{Ch}}=L}(e_2^L)=-e_2^L+W(p_2^{\text{Ch}}(\lambda))+h(e_2^L,M,n_2)\cdot\left[W(p_2^{\text{Ch}}(\mu))-W(p_2^{\text{Ch}}(\lambda))\right]$$

$$\mathbb{E}\Pi^{t^{\text{Ch}}=M}(e_2^M)=-e_2^M+W(p_2^{\text{Ch}}(\mu))-h(e_2^M,M,n_2)\cdot\left[W(p_2^{\text{Ch}}(\mu))-W(p_2^{\text{Ch}}(\lambda))\right]$$

where

$$p_2^{\text{Ch}}(\mu) = \frac{p_1^{\text{Ch}} \cdot (1 - h(\hat{e}_2^M, M, n_2))}{p_1^{\text{Ch}} \cdot (1 - h(\hat{e}_2^M, M, n_2)) + (1 - p_1^{\text{Ch}}) \cdot h(\hat{e}_2^L, L, n_2)}$$

$$p_2^{\text{Ch}}(\lambda) = \frac{p_1^{\text{Ch}} \cdot h(\hat{e}_2^M, M, n_2)}{p_1^{\text{Ch}} \cdot h(\hat{e}_2^M, M, n_2) + (1 - p_1^{\text{Ch}}) \cdot (1 - h(\hat{e}_2^L, L, n_2))}$$

Assume that voters' beliefs after observing liberal and moderate signals during the general election campaign are the same, that is, $p_2^{\text{Ch}}(\mu) = p_2^{\text{Ch}}(\lambda)$ and $W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda)) = 0$. In this case, both types of challengers would choose zero effort since $\frac{d\mathbb{E}\Pi^{t^{\text{Ch}}=L}}{de_2^L} = \frac{d\mathbb{E}\Pi^{t^{\text{Ch}}=M}}{de_2^M} = -1 < 0$. Thus, we must have

$$\frac{p_1^{\text{Ch}} \cdot (1 - \epsilon)}{p_1^{\text{Ch}} \cdot (1 - \epsilon) + (1 - p_1^{\text{Ch}}) \cdot \epsilon} = \frac{p_1^{\text{Ch}} \cdot \epsilon}{p_1^{\text{Ch}} \cdot \epsilon + (1 - p_1^{\text{Ch}}) \cdot (1 - \epsilon)}$$

which is satisfied only for $\epsilon = \frac{1}{2}$. In other words, as long as $\epsilon < \frac{1}{2}$, $W(p_2^{\text{Ch}}(\mu)) = W(p_2^{\text{Ch}}(\lambda))$ is not part of the equilibrium system of beliefs.

Assume next that $p_2^{\text{Ch}}(\mu) < p_2^{\text{Ch}}(\lambda)$ then using Claim 1 we obtain $W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda)) < 0$. In this case, liberal challenger will exert zero effort since

$$\frac{d\mathbb{E}\Pi^{t^{\text{Ch}}=L}}{de_2^L} = -1 + h_e(e_2^L, L, n_2) \cdot \left[W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda))\right] < 0$$

and we must have

$$\frac{p_1^{\text{Ch}} \cdot (1 - h(e_2^M, M, n_2))}{p_1^{\text{Ch}} \cdot (1 - h(e_2^M, M, n_2)) + (1 - p_1^{\text{Ch}}) \cdot \epsilon} < \frac{p_1^{\text{Ch}} \cdot h(e_2^M, M, n_2)}{p_1^{\text{Ch}} \cdot h(e_2^M, M, n_2) + (1 - p_1^{\text{Ch}}) \cdot (1 - \epsilon)}$$

The inequality above is false as long as

$$\epsilon = h(0, t, n) < 1 - h(1, t, n) \leq 1 - h(e, t, n) \text{ for all } e \in [0, 1]$$

Thus, the only beliefs $(\hat{e}_2^L, \hat{e}_2^M)$ that might be consistent with equilibrium are

$$p_2^{\text{Ch}}(\mu) > p_2^{\text{Ch}}(\lambda) \Rightarrow W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda)) > 0$$

Given these beliefs, the moderate challenger would choose to exert no effort, since

$$\frac{d\mathbb{E}\Pi^{t^{\text{Ch}}=M}}{de_2^M} = -1 - h_e(e_2^M, M, n_2) \cdot \left[W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda))\right] < 0$$

For any pair of beliefs $(p_1^{\text{Ch}}, \hat{e}_2^L)$ define the best-response function of liberal challenger $\bar{e}_2^L = e_2^L(\hat{e}_2^L, p_1^{\text{Ch}})$ as the one that maximizes her expected payoff

$$\frac{\mathrm{d}\mathbb{E}\Pi^{t^{\text{Ch}}=L}(e_2^L)}{\mathrm{d}e_2^L} = -1 + h_e(\bar{e}_2^L, L, n_2) \cdot \left[W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda))\right] = 0$$

where

$$p_2^{\text{Ch}}(\mu) = \frac{p_1^{\text{Ch}} \cdot (1 - \epsilon)}{p_1^{\text{Ch}} \cdot (1 - \epsilon) + (1 - p_1^{\text{Ch}}) \cdot h(\hat{e}_2^L, L, n_2)} \text{ and } p_2^{\text{Ch}}(\lambda) = \frac{p_1^{\text{Ch}} \cdot \epsilon}{p_1^{\text{Ch}} \cdot \epsilon + (1 - p_1^{\text{Ch}}) \cdot (1 - h(\hat{e}_2^L, L, n_2))}$$

We will show that this best-response function is decreasing in $\hat{e}_2^L$. Define function $S\left(\bar{e}_2^L, \hat{e}_2^L\right) = 0$ and use Implicit Function Theorem to obtain the required derivative. For the simplicity of exposition, we will use the following shortcuts in this part: $p_1 \equiv p_1^{\text{Ch}}$, $h(\bar{e}_2^L, L, n_2) \equiv h(\bar{e}_2^L)$ and $h(\hat{e}_2^L, L, n_2) \equiv h(\hat{e}_2^L)$.

$$S\left(\bar{e}_2^L, \hat{e}_2^L\right) = -1 + h_e(\bar{e}_2^L) \cdot \left[W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda))\right] = 0$$

$$\frac{\partial S}{\partial \bar{e}_2^L} = h_{ee}(\bar{e}_2^L) \cdot \left[W(p_2^{\text{Ch}}(\mu)) - W(p_2^{\text{Ch}}(\lambda))\right] < 0$$

$$\frac{\partial S}{\partial \hat{e}_2^L} = h_e(\bar{e}_2^L) \cdot \left[-W'(p_2^{\text{Ch}}(\mu)) \cdot \frac{p_1(1-p_1)(1-\epsilon) \cdot h_e(\hat{e}_2^L)}{[p_1(1-\epsilon) + (1-p_1)h(\hat{e}_2^L)]^2} - W'(p_2^{\text{Ch}}(\lambda)) \cdot \frac{p_1(1-p_1)\epsilon \cdot h_e(\hat{e}_2^L)}{[p_1\epsilon + (1-p_1)(1-h(\hat{e}_2^L))]^2}\right] < 0$$

$$\Rightarrow \frac{\partial \bar{e}_2^L}{\partial \hat{e}_2^L} < 0$$

Therefore, there exists a unique fixed point $e_2^{L^*}$ such that $\bar{e}_2^L = e_2^L(\hat{e}_2^L, p_1^{\text{Ch}}) = \hat{e}_2^L \equiv e_2^{L^*}$. This optimal effort for the liberal challenger is determined by equation (1) specified above, **QED**.

Claim 3*. Assume that the parameters of the election game satisfy assumptions (A1a), (A1b), (A1c*), (A1d)-(A1f), (A2)-(A6) and $p_1^{\text{Ch}} \in [0, 1)$. Then for all $\delta < 1$, there exists $\epsilon^* > 0$ such that $\forall \epsilon < \epsilon^*$ we have $\frac{de_2^{L^*}}{dp_1^{\text{Ch}}} > 0$ on the domain $[0, \delta]$.

Proof of Claim 3*. We will use the Implicit Function theorem to prove this claim. Use the following shortcuts to simplify the exposition: $p_1 \equiv p_1^{\text{Ch}}$, $p_2(\mu) \equiv p_2^{\text{Ch}}(\mu)$, $p_2(\lambda) \equiv p_2^{\text{Ch}}(\lambda)$ and $h(e_2^{L^*}) \equiv h(e_2^{L^*}, L, n_2)$. Define

$$S(p_1, e_2^{L^*}) = -1 + h_e(e_2^{L^*}) \cdot [W(p_2(\mu)) - W(p_2(\lambda))] = 0$$

where $p_2(\mu)$ and $p_2(\lambda)$ are described in equation (*) above.

$$\frac{\partial S(p_1, e_2^{L^*})}{\partial e_2^{L^*}} = h_{ee}(e_2^{L^*}) \cdot [W(p_2(\mu)) - W(p_2(\lambda))] + h_e(e_2^{L^*}) \cdot \left[W'(p_2(\mu))\frac{dp_2(\mu)}{de_2^{L^*}} - W'(p_2(\lambda))\frac{dp_2(\lambda)}{de_2^{L^*}}\right] < 0$$

because

$$W'(p_2(\mu))\frac{dp_2(\mu)}{de_2^{L^*}} - W'(p_2(\lambda))\frac{dp_2(\lambda)}{de_2^{L^*}} = -W'(p_2(\mu))\frac{p_1(1-p_1)(1-\epsilon)h_e(e_2^{L^*})}{(p_1(1-\epsilon) + (1-p_1)h(e_2^{L^*}))^2} - W'(p_2(\lambda))\frac{p_1(1-p_1) \cdot \epsilon \cdot h_e(e_2^{L^*})}{(p_1\epsilon + (1-p_1)(1-h(e_2^{L^*})))^2} < 0$$

$$\frac{\partial S(p_1, e_2^{L^*})}{\partial p_1} = h_e(e_2^{L^*}) \cdot \left[W'(p_2(\mu))\frac{dp_2(\mu)}{dp_1} - W'(p_2(\lambda))\frac{dp_2(\lambda)}{dp_1}\right]$$

where

$$\frac{dp_2(\mu)}{dp_1} = \frac{(1-\epsilon)h(e_2^{L^*})}{(p_1(1-\epsilon) + (1-p_1)h(e_2^{L^*}))^2} \text{ and } \frac{dp_2(\lambda)}{dp_1} = \frac{\epsilon(1-h(e_2^{L^*}))}{(p_1 \cdot \epsilon + (1-p_1)(1-h(e_2^{L^*})))^2}$$

Under what conditions $\frac{\partial S(p_1, e_2^{L^*})}{\partial p_1} > 0$?

$$\frac{\partial S(p_1, e_2^{L^*})}{\partial p_1} > 0 \Leftrightarrow W'(p_2(\mu))\frac{(1-\epsilon)h(e_2^{L^*})}{(p_1(1-\epsilon) + (1-p_1)h(e_2^{L^*}))^2} > W'(p_2(\lambda))\frac{\epsilon(1-h(e_2^{L^*}))}{(p_1 \cdot \epsilon + (1-p_1)(1-h(e_2^{L^*})))^2}$$

Recall that the winning function $W(p_2)$ is strictly increasing in p_2 (Claim 1). Thus, $W'(p_2) > 0$

for all $p_2 \in [0,1]$. In particular, $\frac{W'(p_2(\mu))}{W'(p_2(\lambda))} \geq \frac{W'(1)}{W'(0)} > 0$.

Now, consider the following inequality

$$\frac{W'(1)}{W'(0)} > \frac{\epsilon(1-h(e_2^{L^*}))}{(1-\epsilon)h(e_2^{L^*})} \cdot \left(\frac{p_1(1-\epsilon)+(1-p_1)h(e_2^{L^*})}{p_1\epsilon+(1-p_1)(1-h(e_2^{L^*}))}\right)^2$$

The left-hand side of this inequality is a positive constant, while the right-hand side approaches zero from above when ϵ approaches zero. Therefore, for any $(p_1, e_2^{L^*})$ there exists $\epsilon^*\left(p_1, e_2^{L^*}\right) > 0$ such that for all $\epsilon < \epsilon^*\left(p_1, e_2^{L^*}\right)$ the inequality above is satisfied. This means that for all $\epsilon < \epsilon^*\left(p_1, e_2^{L^*}\right)$ we have $\frac{\partial S(p_1, e_2^{L^*})}{\partial p_1} > 0$, which is enough to guarantee that $\frac{de_2^{L^*}}{dp_1^{\text{Ch}}} > 0$, **QED**.

Example. To intuit Claim 3* and appreciate the role of assumption (A1c*) consider the basic election game with the scrutiny function

$$h(e,t,n) = \epsilon + (1-n)\cdot\sqrt{e}$$

probability winning function

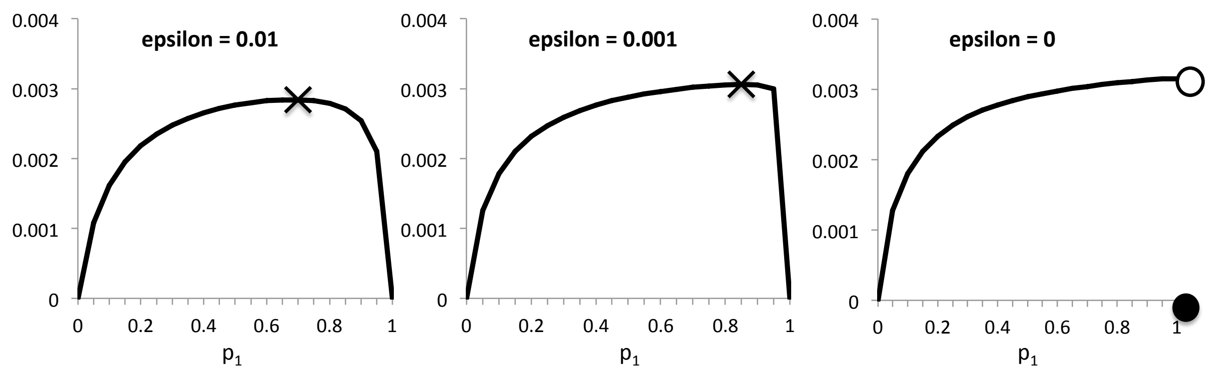
$$W\left(p_2^{\text{Ch}}\right) = \frac{1}{8}\cdot\left(p_2^{\text{Ch}}\right)^{\frac{19}{20}} + \frac{1}{4}$$

and the prominence level of the general election is $n_2 = \frac{1}{10}$.

First note that these functions satisfy assumptions (A1a), (A1b), (A1d)-(A1f) and (A2)-(A6). Moreover, if $\epsilon = 0$, then assumption (A1c) is satisfied, while if $\epsilon > 0$ then assumption (A1c*) is satisfied.

Figure 1 depicts optimal effort of the liberal challenger in the general election stage as a function of prior belief about her type, p_1^{Ch}, for various values of ϵ: 0.01 and 0.001 and 0. Symbol X marks optimum when the peak is interior.

Figure 1: Optimal effort of liberal challenger in the general election as a function of prior belief, p_1^{Ch}.



As Figure 1 illustrates, for any positive value of ϵ, the peak of the function is interior. However, as ϵ approaches zero, the peak shifts to the right, that is p_1^{Ch} that maximizes $e_2^{L^*}(p_1^{\text{Ch}})$ approaches 1. When $\epsilon = 0$, the optimal effort of liberal challenger is strictly increasing in p_1^{Ch} on the domain $(0,1)$ but drops discontinuously at $p_1^{\text{Ch}} = 1$ to zero.

Appendix C: Election Game with Partially Informative Primaries

Proof of Theorem 2.

Behavior of the challenger in the general election stage.

Similar to the basic election model, the challenger's chances of winning general election are determined by the preferences of the general-election median voter. Since all registered Democrats vote for the challenger irrespectively of his type, the median voter in the general election stage has an ideal point $z_j \in (\bar{z}, 1)$ and, consequently, believes that any Democratic nominee is equally likely to be a liberal or a moderate type. The argument presented in Claim 2 and its proof holds here as well. That is, the unique equilibrium in the general election stage is for moderate challenger to exert no effort, $\tilde{e}_2^{M^*} = 0$, and for liberal challenger to exert effort level $\tilde{e}_2^{L^*} \in (0,1)$ which is determined by the equation

$$h_e(\tilde{e}_2^{L^*}, L, n_2) \cdot \Big[W(\tilde{p}_2^{\text{Ch}}(\mu) - W(0))\Big] = 1 \text{ where } \tilde{p}_2^{\text{Ch}}(\mu) = \frac{1}{1 + h(\tilde{e}_2^{L^*}, L, n_2)}$$

Behavior of registered Democrats in the primary election.

At the end of the primary campaign, registered Democrats contemplate candidate l who is moderate for sure, $p^l = 1$, and candidate k whose type is uncertain and who is believed to be moderate with probability $p_1^k \in (0,1)$:

$$\mathbb{E}u(z_j, 1) = W(\tilde{p}_2^{\text{Ch}}(\mu)) \cdot u(z_j, M) + (1 - W(\tilde{p}_2^{\text{Ch}}(\mu)) \cdot u(z_j, R)$$

$$\mathbb{E}u(z_j, p_1^k) = p_1^k \cdot \mathbb{E}u(z_j, 1) + (1 - p_1^k) \cdot \begin{bmatrix} \Big(h(\tilde{e}_2^{L^*}, L, n_2) \cdot W(\tilde{p}_2^{\text{Ch}}(\mu)) + (1 - h(\tilde{e}_2^{L^*}, L, n_2)) \cdot W(0)\Big) \cdot u(z_j, L) + \\ + \Big(h(\tilde{e}_2^{L^*}, L, n_2) \cdot (1 - W(\tilde{p}_2^{\text{Ch}}(\mu))) + (1 - h(\tilde{e}_2^{L^*}, L, n_2)) \cdot (1 - W(0))\Big) \cdot u(z_j, R) \end{bmatrix}$$

$$\mathbb{E}u(z_j, p_1^k) \geq \mathbb{E}u(z_j, 1) \Leftrightarrow$$

$$\Big(W(0) + h(\tilde{e}_2^{L^*}, L, n_2) \cdot (W(\tilde{p}_2^{\text{Ch}}(\mu)) - W(0))\Big) \cdot (u(z_j, L) - u(z_j, R)) \geq W(\tilde{p}_2^{\text{Ch}}(\mu)) \cdot (u(z_j, M) - u(z_j, R))$$

The last inequality holds true for the majority of the registered Democrats, because it is implied by $\mathbb{E}u(z_j, 0) \geq \mathbb{E}u(z_j, 1)$ which is guaranteed by conditions (C1) and (C2).

Behavior of candidates in the primary stage.

Assume that voters believe that $\hat{e}_1^M \in (0,1]$, $\hat{e}_1^L = 0$ and candidate A follows this strategy. We will show that candidate B wants to follows this strategy as well. First consider what liberal candidate B would do:

$$\mathbb{E}\Pi^{t^B = L}(e_1^L) = -e_1^L + \frac{3 - h(\hat{e}_1^M, M, n_1) - 2h(e_1^L, L, n_1)}{4} \cdot \Big(-\tilde{e}_2^{L^*} + h(\tilde{e}_2^{L^*}, L, n_2) \cdot W(\tilde{p}_2^{\text{Ch}}(\mu)) + (1 - h(\tilde{e}_2^{L^*}, L, n_2)) \cdot W(0)\Big)$$

Notice that the expected payoff of the liberal challenger (last brackets) does not depend on behavior in the primary stage. Therefore,

$$\frac{\mathrm{d}\mathbb{E}\Pi^{t^B = L}(e_1^L)}{\mathrm{d}e_1^L} = -1 - \frac{h_e(e_1^L, L, n_1)}{2} \cdot \Big(-\tilde{e}_2^{L^*} + h(\tilde{e}_2^{L^*}, L, n_2) \cdot W(\tilde{p}_2^{\text{Ch}}(\mu)) + (1 - h(\tilde{e}_2^{L^*}, L, n_2)) \cdot W(0)\Big) < 0$$

Thus, liberal candidate B prefers to exert no effort in the primary campaign.

Consider now the incentives of the moderate candidate B:

$$\mathbb{E}\Pi^{t^B=M}(e_1^M) = -e_1^M + \frac{1 + 2h(e_1^M, M, n_1) - h(\hat{e}_1^M, M, n_1}{4} \cdot W(\tilde{p}_2^{\text{Ch}}(\mu))$$

$$\Rightarrow \frac{\text{d}\mathbb{E}\Pi^{t^B=M}(e_1^M)}{\text{d}e_1^M} = -1 + \frac{1}{2} h_e(e_1^M, M, n_1) \cdot W(\tilde{p}_2^{\text{Ch}}(\mu))$$

Define best-response function of moderate candidate B, $\tilde{e}_1^{M^*}(\hat{e}_1^M)$. This is the effort level of moderate candidate B, $\tilde{e}_1^{M^*} \in (0,1)$, that solves $\frac{\text{d}\mathbb{E}\Pi^{t^B=M}(e_1^M)}{\text{d}e_1^M}|_{\tilde{e}_1^{M*}} = 0$. Notice that best-response exists and it is unique for all $\hat{e}_1^M \in (0,1]$. It is easy to see that there exists a unique fixed point such that $\tilde{e}_1^{M^*}(\hat{e}_1^M) = \hat{e}_1^M$, which is determined by the equation (7a). This completes the proof of Theorem 2, **QED**.

Proof of Claim 9. We will start by showing that conditional on observing a moderate signal μ in the general election campaign, belief about the challenger in the election game with partially informative primaries is higher than the one in the basic election game in which the challenger won the nomination after generating liberal signal, $\tilde{p}_2^{\text{Ch}}(\mu) > p_2^{\text{Ch}}(\mu)$. Assume that it is not the case, and, in fact, $\tilde{p}_2^{\text{Ch}}(\mu) \le p_2^{\text{Ch}}(\mu)$. Thus, we must have

$$\frac{1}{1 + h(\tilde{e}_2^{L^*}, L, n_2)} \le \frac{p_1^{\text{Ch}}}{p_1^{\text{Ch}} + (1 - p_1^{\text{Ch}}) h(e_2^{L^*}, L, n_2)} \Leftrightarrow (1 - p_1^{\text{Ch}}) \cdot h(e_2^{L^*}, L, n_2) \le p_1^{\text{Ch}} \cdot h(\tilde{e}_2^{L^*}, L, n_2)$$

But at the same time we know that if $\tilde{p}_2^{\text{Ch}}(\mu) \le p_2^{\text{Ch}}(\mu)$ then $\tilde{e}_2^{L^*} \le e_2^{L^*}$. This follows from the properties of the scrutiny function ($h_{ee} < 0$) and the equations (3a) and (7a). Thus, $h(\tilde{e}_2^{L^*}, L, n_2) \le h(e_2^{L^*}, L, n_2)$, and coupled with condition that $p_1^{\text{Ch}} < \frac{1}{2}$ contradicts inequality above. Therefore, it must be that $\tilde{p}_2^{\text{Ch}}(\mu) > p_2^{\text{Ch}}(\mu)$, which implies that $\tilde{e}_2^{L^*} > e_2^{L^*}$.

To show that $\tilde{e}_1^{M^*} > e_1^{M^*}$ we consider equations (3b) and (7b) and notice that

$$\frac{1}{2} W(\tilde{p}_2^{\text{Ch}}(\mu)) > \frac{1}{2} W(p_2^{\text{Ch}}(\mu)) > \frac{1}{2} W(p_2^{\text{Ch}}(\mu)) - \frac{1 - h(e_1^{M^*}, M, n_1)}{4} \cdot \Big(W(1) - W(p_2^{\text{Ch}}(\mu)) \Big)$$

which coupled with the properties of the scrutiny function guarantees that $\tilde{e}_1^{M^*} > e_1^{M^*}$, **Q.E.D.**

Proof of Claim 10. To simplify the exposition, we will use the following notation $\tilde{h} \equiv h(\tilde{e}_2^{L^*}, L, n_2)$ and $\tilde{g} \equiv h(\tilde{e}_1^{M^*}, M, n_1)$. Then, the probability that a Democrat wins the election in the model with partially informative primaries is

$$\begin{aligned} \Pr[\text{Democrat wins general election}] &= \frac{1}{4} \cdot \left(\tilde{h} \cdot W\left(\tilde{p}_2^{\text{Ch}}(\mu)\right) + (1 - \tilde{h}) \cdot W(0) \right) + \frac{1}{4} \cdot W\left(\tilde{p}_2^{\text{Ch}}(\mu)\right) + \\ &+ \frac{1}{2} \cdot \left(\left(1 - \frac{\tilde{g}}{2} \right) \cdot \left[\tilde{h} \cdot W\left(\tilde{p}_2^{\text{Ch}}(\mu)\right) + (1 - \tilde{h}) \cdot W(0) \right] + \frac{g}{2} \cdot W\left(\tilde{p}_2^{\text{Ch}}(\mu)\right) \right) = \\ &= \frac{(1 - \tilde{h})(3 - \tilde{g})}{4} \cdot W(0) + \frac{4 - (3 - \tilde{g})(1 - \tilde{h})}{4} \cdot W\left(\tilde{p}_2^{\text{Ch}}(\mu)\right) \end{aligned}$$

We compare this expression to the probability of a Democrat winning the general election in the basic model, which is specified in Proof of Claim 7. To show that Democrats enjoy higher chance of winning the election in the game with partially informative primaries compared with the basic game, we use properties of the winning function $W(\cdot)$ as well as Claim 9 which ranks effort levels of candidates in these two versions of the game. In particular, the proof follows from algebraic manipulations using the following observations:

$$W(1) > W\left(\tilde{p}_2^{\mathrm{Ch}}(\mu)\right) = W\left(\frac{1}{1+\tilde{h}}\right) > W\left(\frac{g}{h+g}\right) = W\left(p_2^{\mathrm{Ch}}(\mu)\right) > W(0)$$

$$2W(0) > W(1)$$

$$W(x) > (1-x)\cdot W(0) + x \cdot W(1)$$

$$1 > \tilde{h} > h > 0 \quad 1 > \tilde{g} > g > 0 \quad \tilde{g} > \tilde{h} \quad g > h$$ **Q.E.D.**

Appendix D: Election Game with Endogenous Primary Prominence

Proof of Theorem 3. The election game with endogenous primary prominence (as defined in Section 5.2) is the same as the basic election model studied in Sections 2 - 4 except for the investment decisions of primary candidates. Therefore, to prove Theorem 3, it suffices to show that expected payoff of a moderate candidate is decreasing and expected payoff of a liberal candidate is increasing in the primary prominence. This would guarantee that liberal candidates are happy to invest in boosting primary visibility, while moderate ones refrain from doing so.

Ex-ante expected payoff of candidate k who has type $t^k = L$ can be written as

$$\mathbb{E}\Pi^{t^k=L} = \frac{3-h(e_1^{M^*},M,n_1)}{4}\cdot\left[-e_2^{L^*}+W(0)+h(e_2^{L^*},L,n_2)\cdot\left[W\left(\frac{h(e_1^{M^*},M,n_1)}{h(e_2^{L^*},L,n_2)+h(e_1^{M^*},M,n_1)}\right)-W(0)\right]\right]$$

$$\frac{\mathrm{d}\mathbb{E}\Pi^{t^k=L}}{\mathrm{d}n_1} = -\frac{1}{4}\cdot\frac{\mathrm{d}h(e_1^{M^*},M,n_1)}{\mathrm{d}n_1}\cdot\left[-e_2^{L^*}+W(0)+h(e_2^{L^*},L,n_2)\cdot\left[W\left(\frac{h(e_1^{M^*},M,n_1)}{h(e_2^{L^*},L,n_2)+h(e_1^{M^*},M,n_1)}\right)-W(0)\right]\right]+$$
$$+\frac{3-h(e_1^{M^*},M,n_1)}{4}\cdot\left[-\frac{\partial e_2^{L^*}}{\partial n_1}+h_e(e_2^{L^*},L,n_2)\cdot\frac{\partial e_2^{L^*}}{\partial n_1}\cdot\left[W\left(\frac{h(e_1^{M^*},M,n_1)}{h(e_2^{L^*},L,n_2)+h(e_1^{M^*},M,n_1)}\right)-W(0)\right]\right]+$$
$$+\frac{3-h(e_1^{M^*},M,n_1)}{4}\cdot h(e_2^{L^*},L,n_2)\cdot W'\left(\frac{h(e_1^{M^*},M,n_1)}{h(e_2^{L^*},L,n_2)+h(e_1^{M^*},M,n_1)}\right)\cdot\frac{\mathrm{d}\frac{h(e_1^{M^*},M,n_1)}{h(e_1^{M^*},M,n_1)+h(e_2^{L^*},L,n_2)}}{\mathrm{d}n_1}$$

To show that this derivative is positive, $\frac{\mathrm{d}\mathbb{E}\Pi^{t^k=L}}{\mathrm{d}n_1}>0$, we use equilibrium condition (3b), the fact that if liberal challenger chose to exert positive effort in the general election stage this means he prefers this action to exerting no effort at all

$$-e_2^{L^*}+W(0)+h(e_2^{L^*},L,n_2)\cdot\left[W\left(\frac{h(e_1^{M^*},M,n_1)}{h(e_2^{L^*},L,n_2)+h(e_1^{M^*},M,n_1)}\right)-W(0)\right]>W(0)$$

as well as the comparative static results obtained in Claim 6 after substituting derivatives $\frac{\partial e_1^{M^*}}{\partial n_1}$ and $\frac{\partial e_2^{L^*}}{\partial n_2}$ into the expression above, and, finally, the fact that sequence of signals λ in the primary and μ in the general election is less likely to come from the moderate candidates when primaries are more visible, $\frac{\mathrm{d}\frac{h(e_1^{M^*},M,n_1)}{h(e_1^{M^*},M,n_1)+h(e_2^{L^*},L,n_2)}}{\mathrm{d}n_1}<0$, which is the last part of Claim 6.

Ex-ante expected payoff of candidate k who has type $t^k = M$ can be written as

$$\mathbb{E}\Pi^{t^k=M} = -e_1^{M^*}+\frac{h(e_1^{M^*},M,n_1)(3-h(e_1^{M^*},M,n_1))}{4}\cdot W\left(\frac{h(e_1^{M^*},M,n_1)}{h(e_2^{L^*},L,n_2)+h(e_1^{M^*},M,n_1)}\right)+\frac{(1-h(e_1^{M^*},M,n_1))^2}{4}\cdot W(1)$$

$$\frac{\mathrm{d}\mathbb{E}\Pi^{t^k=M}}{\mathrm{d}n_1} = -\frac{\partial e_1^{M^*}}{\partial n_1}+\frac{3-2h(e_1^{M^*},M,n_1)}{4}\frac{\mathrm{d}h(e_1^{M^*},M,n_1)}{\mathrm{d}n_1}\cdot W\left(\frac{h(e_1^{M^*},M,n_1)}{h(e_2^{L^*},L,n_2)+h(e_1^{M^*},M,n_1)}\right)+$$
$$+\frac{h(e_1^{M^*},M,n_1)(3-h(e_1^{M^*},M,n_1))}{4}\cdot W'\left(\frac{h(e_1^{M^*},M,n_1)}{h(e_2^{L^*},L,n_2)+h(e_1^{M^*},M,n_1)}\right)\cdot\frac{\mathrm{d}\frac{h(e_1^{M^*},M,n_1)}{h(e_1^{M^*},M,n_1)+h(e_2^{L^*},L,n_2)}}{\mathrm{d}n_1}-$$
$$-\frac{1-h(e_1^{M^*},M,n_1)}{2}\cdot\frac{\mathrm{d}h(e_1^{M^*},M,n_1)}{\mathrm{d}n_1}\cdot W(1)$$

Similarly, to show that this derivative is negative, $\frac{\mathrm{d}\mathbb{E}\Pi^{t^k=M}}{\mathrm{d}n_1}<0$, we use equilibrium conditions as well as comparative statics results obtained in Claim 6, **QED**.