Grading Standards and Education Quality: Supplementary Material

Raphael Boleslavsky & Christopher Cotton

June 10, 2014

B ONLINE APPENDIX

B.1 FORMAL ANALYSIS OF THE GAME WITH STUDENT EFFORT

We characterize Perfect Bayesian equilibrium in which students are expected to exert effort for certain, $e_i = e_j = \widehat{e}_i = \widehat{e}_j = 1$, under both strategic grading and fully informative grading.

STRATEGIC GRADING

Evaluation. When the evaluator conjectures that students exert effort $\widehat{e}_i = \widehat{e}_j = 1$, the evaluators prior belief about quality in the evaluation stage, generally equal to $\widehat{Q}_i = q_i\widehat{e}_i$, reduces to $\widehat{Q}_i = q_i$. Given school i's grading policy (H_i, L_i), with densities $(f_H(\cdot), f_L(\cdot))$, and transcript realization x, the posterior is given by Bayes' rule:

$$\Pr(t = h|x) = \frac{\widehat{Q}_i f_H(x)}{\widehat{Q}_i f_H(x) + (1 - \widehat{Q}) f_L(x)} = \frac{q_i f_H(x)}{q_i f_H(x) + (1 - q_i) f_L(x)} \tag{1}$$

which is identical to the belief about graduate ability arising in the game without student effort, with investment level q_i and grading policy (H_i, L_i). The evaluator's sequentially rational strategy is identical to the case without student effort.

Posterior Belief Random Variables. The following lemma describes the connection between posterior belief random variables and grading policies in the game with student effort.

Lemma B.1 *(Modified Bayesian Persuasion Representation)*

- *Consider any valid random variable Γ_i with support confined to the unit interval and expectation q_i. If the prior belief that a student is high ability is q_i and schools and evaluator expect students to exert effort for certain $\widehat{e}_i = 1$, then there exists a grading policy (H_i, L_i) for which the ex ante posterior belief is Γ_i.*

- *Suppose that the prior belief that a student is high ability is q_i and schools and evaluator conjecture that students exert effort for certain $\widehat{e}_i = 1$. Suppose also that under these conditions grading policy (H_i, L_i) generates posterior belief random variable $\widehat{\Gamma}_i$ with density $\widehat{g}_i(x)$. If the student secretly shirked, then grading policy (H_i, L_i) generates a posterior belief random random variable with density*
$$g_i(x) = \widehat{g}_i(x)\frac{1 - x}{1 - q_i}$$

Proof of Lemma B.1.

Bullet Point 1. Once investments have been determined (stage one), both the schools and the evaluator share a common prior that each graduate has high ability: $\widehat{Q}_i = q_e\widehat{e}_i$. Given that $\widehat{e}_i = 1$, school and evaluator belief that the graduate has high ability is simply q_i. The result follows from Lemma **??**.

Bullet Point 2. Let grading policy (H_i, L_i) generate posterior belief random variable $\widehat{\Gamma}_i$, under the conjecture that $\widehat{e}_i = 1$, implying $\widehat{Q}_i = q_i$. If (H_i, L_i) is an admissible grading policy it must satisfy the monotone likelihood ratio property. Therefore, the posterior belief is monotonic increasing in the transcript realization. Hence, a single transcript realization generates any evaluator posterior belief inside the support of $\widehat{\Gamma}_i$. For any $x \in \text{support}[\widehat{\Gamma}_i]$, let $\tau(x)$ represent the unique transcript realization that generates it as the posterior:

$$x = \frac{q_i f_H(x)(\tau(x))}{q_i f_H(\tau(x)) + (1-q_i) f_L(\tau(x))} \tag{2}$$

According to the assumption of the proposition, when $\widehat{e}_i = 1$, the density of the evaluator posterior x is $\widehat{g}_i(x)$ and therefore:

$$\widehat{g}_i(x) = q_i f_H(\tau(x)) + (1-q_i) f_L(\tau(x)) \tag{3}$$

If the graduate secretly shirks, the evaluator's posterior belief conditional on transcript $\tau(x) = x$. Because she forms beliefs based on her conjecture $\widehat{e}_i = 1 \Rightarrow \widehat{Q}_i = q_i$ equation (2) describes her posterior belief for realization $\tau(x)$, whether or not her conjecture is correct. However, if he shirks, the graduate is low ability for certain, and his transcript is always a realization of random variable L_i, with density $f_L(\cdot)$.

$$g_i(x) = f_L(\tau(x))$$

Observe that equations (2, 3) imply that

$$f_H(\tau(x)) = \frac{x\widehat{g}_i(x)}{q_i} \qquad f_L(\tau(x)) = \frac{(1-x)\widehat{g}_i(x)}{1-q_i}$$

and thus:

$$g_i(x) = \frac{1-x}{1-q_i}\widehat{g}_i(x)$$

■

The first part of this lemma is a straightforward extension of the Bayesian Persuasion Representation, showing that the representation applies provided students are expected to exert effort for certain. The second part of the lemma describes the posterior belief random variable, arising after an unobserved deviation—from working to shirking—by the student. This characterization is important, because it determined the student's payoff from secretly shirking in the investment stage. Intuitively, because the shirking is unobserved both evaluator and schools believe that student i is high ability with probability q_i, though his ability is actually low. The evaluator therefore draws the same inference from any observed transcript realization. Meanwhile, schools design grading policies that would generate a posterior random variable with density $g_i(x)$, under the assumption that the student is high ability with probability q_i; that is, his transcript is a realization of H_i with probability q_i and L_i with probability $1 - q_i$. However, if the student shirked, he is low ability for certain and his transcript is *always* a realization of L_i. This reduces the probability of generating high posterior belief realizations, and increases the probability of generating low posterior belief realizations, "shifting" the posterior belief distribution toward low realizations.

Grading policies. At the beginning of the grading stage, schools observe investments undertaken in stage one (q_a, q_b) with $q_a \geq q_b$. Schools and evaluator believe graduate i to be high ability with probability $\widehat{Q}_i = q_i \widehat{e}_i$, which reduces to $\widehat{Q}_i = q_i$ because the students are expected to work hard for certain. In this stage of the game, schools simultaneously design grading policies $\{(H_a, L_a), \{H_b, L_b\}$, that generate posterior belief random variables (Γ_a, Γ_b). Because their conjecture is that students exert effort, $\widehat{e}_i = 1$, schools design their grading policies to generate their desired posterior belief random variables (Γ_a, Γ_b) assuming that the transcript will be a realization of H_i with probability q_i and a realization of L_i with probability $1 - q_i$. A school receives payoff 1 when the realization of its posterior belief random variable is higher than the other school's realization and 1/2 when the realizations are identical. Therefore, under the assumption that students exert effort for sure, the grading stage game is identical to the one in Section 4. Hence, with the expectation that students exert effort, schools will choose grading policies to generate the posterior belief random variables described in Lemma 3.

Lemma B.2 *If schools expect effort to exert effort for certain $\widehat{e}_i = \widehat{e}_j = 1$, then given school investments (q_a, q_b) with $q_a \geq q_b$, in equilibrium schools design grading policies that they anticipate will generate the posterior belief random variables described in Lemma 3. Schools expected payoffs following investment levels (q_a, q_b) are also identical to those in Lemma 3. If a student secretly shirks in stage one, then these grading policies will generate a posterior belief random variable with the following density:*

- *When $q_a \leq 1/2$:*

$$g_a(x) = \Big(\frac{1}{2q_a}\Big)\Big(\frac{1-x}{1-q_a}\Big)$$
$$g_b(x) = \Big(\delta_0(x)\Big(1 - \frac{q_b}{q_a}\Big) + \Big(\frac{q_b}{q_a}\Big)\frac{1}{2q_a}\Big)\Big(\frac{1-x}{1-q_b}\Big)$$

supported on $[0, 2q_a]$.

- *When $q_a > 1/2$:*

$$g_a(x) = \Big(\frac{1}{2q_a} + \delta_1(x)(2 - \frac{1}{q_a})\Big)\Big(\frac{1-x}{1-q_a}\Big)$$
$$g_b(x) = \Big(\delta_0(x)\Big(1 - \frac{q_b}{q_a}\Big) + \frac{q_b}{q_a}\frac{1}{2q_a} + \delta_1(x)\frac{q_b}{q_a}(2 - \frac{1}{q_a})\Big)\Big(\frac{1-x}{1-q_b}\Big)$$

supported on $[0, 2(1-q_a)] \cup 1$.

School Investments. If schools expect that students exert effort for certain, then in the second stage they design grading policies that generate posterior belief random variables identical to the ones in Lemma 3 on the equilibrium path—where students actually exert effort. Hence, anticipating student effort, schools expect the payoffs given in Lemma 3. Consequently, when they conjecture that students exerting effort, school investments are identical to the ones described in Lemma 4, which reduce to $q_\alpha = q_\beta = \rho/2$ when schools are symmetric.

These lemmas imply that in any equilibrium in which students exert effort for certain, schools and the evaluator behave exactly as they do in the game without student effort. School investments are identical in both cases, and in both cases, each school designs grading policies that (under the conjecture that students exerted effort) generate posterior belief random variables described in Lemma 3. In equilibrium, the conjectured strategy for the students must also be sequentially rational: the conjecture that students exert effort must agree with the student's sequentially rational decision, given the strategies of the other players.

Student Effort. Suppose that student i expects schools to each invest $\rho/2$ and design grading policies that (under the conjecture that students exerted effort) generate posterior belief random variables described in Lemma 3. Suppose that student i also expects student j to exert effort. If student i complies with expectations and exerts effort, his expected payoff is $1/2 - \kappa$. It is possible to calculate this explicitly, but it follows immediately from symmetry between schools and students and equilibrium uniqueness. Next, consider student i's expected payoff from secretly shirking. When $\rho \leq 1$, Lemma C.2 implies that the graduates posterior belief densities are

$$g_i(x) = \frac{2(1-x)}{\rho(2-\rho)} \quad g_j(x) = \frac{1}{\rho}$$

supported on $[0, \rho]$. Hence, the payoff of shirking for student i is:

$$\int_0^\rho g_i(x) G_j(x)\, dx = \int_0^\rho \frac{2(1-x)}{\rho(2-\rho)} \frac{x}{\rho}\, dx = \frac{3-2\rho}{3(2-\rho)}$$

Comparing the payoff of shirking with the payoff of exerting effort, we find that for $1 \leq \rho \leq 2$, student effort is incentive whenever

$$\kappa \leq \frac{\rho}{6(2-\rho).}$$

Whenever $1 \leq \rho \leq 2$ Lemma C.2 implies that the graduates posterior belief densities are

$$g_i(x) = \Big(\frac{1}{\rho} + \delta_1(x)\frac{2(\rho-1)}{\rho})\Big)\Big(\frac{2(1-x)}{2-\rho}\Big) = \frac{2(1-x)}{\rho(2-\rho)}$$
$$g_j(x) = \frac{1}{\rho} + \delta_1(x)\frac{2(\rho-1)}{\rho}$$

where $g_i(x)$ is supported on $[0, 2-\rho]$ and $g_j(x)$ is supported on $[0, 2-\rho] \cup 1$.[1] Hence, the student's expected payoff of shirking is equal to

$$\int_0^{2-\rho} g_i(x) G_j(x)\, dx = \int_0^{2-\rho} \frac{2(1-x)}{\rho(2-\rho)} \frac{x}{\rho}\, dx = \frac{5\rho - 2\rho^2 - 2}{3\rho^2}.$$

Comparing the payoff of shirking with the payoff of exerting effort, we find that effort is incentive compatible for the student whenever

$$\kappa \leq \frac{7\rho^2 - 10\rho + 4}{6\rho^2}.$$

We therefore find the following result, referenced in the text.

Lemma B.3 (Full effort equilibrium: Strategic grading) *If grading is strategic, a symmetric equilibrium in which both students exert effort for certain, $e_i = e_j = \widehat{e}_i = \widehat{e}_j = 1$ exists if and only if $\kappa \leq \phi(\rho)$ where*

$$\phi(\rho) \equiv \begin{cases} \frac{\rho}{6(2-\rho)} & \text{if} \quad \rho \leq 1 \\ \frac{7\rho^2 - 10\rho + 4}{6\rho^2} & \text{if} \quad \rho > 1 \end{cases}$$

In any such equilibrium, school invest $q_i = q_j = \rho/2$. In the grading subgame schools choose any grading policies (H_i, L_i) and (H_j, L_j) that they anticipate will generate the posterior belief random variables described in Lemma 3, given $\widehat{e}_i = \widehat{e}_j = 1$. If $\kappa > \phi(\rho)$, a unique equilibrium exists in which neither school invests and students shirk.

Aside: Student Best Responses. Here we calculate student i's best response to a symmetric investment level $q_i = q_j = q$ and $e_j = 1$ in the case of strategic grading. We first calculate the student payoff of secretly deviating from the expected level of effort. If student i exerts effort as expected, his payoff is $1/2 - \kappa$. Suppose that student i shirks. In this case $\widehat{Q}_i = q$ and $Q_i = 0$. We calculate student i's expected payoff from shirking, as above.

When $q \leq 1/2$, if student i shirks, then the posterior belief distributions are:

$$g_j(x) = \frac{1}{2q} \quad g_i(x) = \frac{1}{2q} \frac{1-x}{1-q}$$

on support $[0, 2q]$, and hence the payoff of shirking is:

$$v_i = \int_0^{2q} \left(\frac{x}{2q}\right)\left(\frac{1}{2q}\right) \frac{1-x}{1-q} \mathrm{d}x = \frac{3-4q}{6(1-q)}$$

[1] Note that strictly speaking $g_i(x) = \frac{2(1-x)}{\rho(2-\rho)} + \delta_1(x)(0)$. In the calculation of the expected payoff, the second term disappears.

When $q > 1/2$, is student i shirks then the posterior belief distributions are:

$$g_j(x) = (\frac{1}{q} - 1)\frac{1}{2(1-q)} + \delta_1(x)(2 - \frac{1}{q}) = \frac{1}{2q} + \delta_1(x)(2 - \frac{1}{q})$$
$$g_i(x) = ((\frac{1}{q} - 1)\frac{1}{2(1-q)} + \delta_1(x)(2 - \frac{1}{q}))\frac{1-x}{1-q} = \frac{1}{2q}$$

where $g_i(\cdot)$ is supported on $[0, 2(1-q)] \cup 1$ and $g_j(\cdot)$ is supported on $[0, 2(1-q)]$. Hence, the payoff of shirking is

$$v_i = \int_0^{2(1-q)} \Big(\frac{1}{2q}\Big)\Big(\frac{x}{2q}\Big)\Big(\frac{1-x}{1-q}\Big)\mathrm{d}x = \frac{1+5q-4q^2}{6q^2}$$

Comparing the shirking payoff to the payoff of exerting effort, we find that student i's sequentially rational effort choice is

$$e^* = \begin{cases} 1 & \text{if} \quad \kappa < f(q) \\ [0,1] & \text{if} \quad \kappa = f(q) \\ 0 & \text{if} \quad \kappa > f(q) \end{cases} \tag{4}$$

where

$$f(q) \equiv \begin{cases} \frac{q}{6(1-q)} & \text{if} \quad q \leq 1/2 \\ \frac{(7q^2-5q+1)}{6q^3} & \text{if} \quad q > 1/2 \end{cases}$$

This sequentially rational effort choice will be discussed in a later section.

FULLY INFORMATIVE GRADING WITH EFFORT

Fully Informative Grading. When grades are fully informative, beliefs about student effort play no role in the second stage of the game, as the schools are constrained to grade in a way that completely reveals graduate abilities. Thus, schools invest and students exert effort to increase the probability that the student has high ability, but these play no direct role in evaluator inference. As in the case of strategic grading, complete coordination failure is always possible: students expect schools to invest nothing and therefore shirk, schools expect students to shirk and therefore invest nothing. In the next lemma, we characterize equilibria in which students are expected to exert effort for certain.[2]

Lemma B.4 (Full effort equilibrium: Fully informative Grading) *If grading is fully informative, an equilibrium in which students exert effort for certain, $t_i = t_j = 1$ exists if and only if $\kappa \leq \phi^*(\rho)$, where*

$$\phi^*(\rho) \equiv \rho^2/8$$

In this equilibrium, schools invest $s_i = s_j = \rho^2/4$. If $\kappa > \phi^*(\rho)$, a unique equilibrium exists in which schools invest nothing and students shirk.

[2] As in the previous case, whenever such an equilibrium exists, a third equilibrium exists in which students exert effort according to a mixed strategy. We ignore this equilibrium, as we focus on the evaluator-preferred equilibrium with full student effort.

Proof to Lemma B.4. We derive the equilibrium with certain effort from students, when grading is informative. With fully informative grading, evaluator conjectures about student effort are irrelevant, as the grading policies reveal the student's type for certain. If students exert efforts (e_i, e_j) and schools invest (q_i, q_j), then school i's expected payoff is

$$u_i(q_i, q_j, e_i, e_j) = \frac{1}{2}(1 + e_i q_i - e_j q_j) - \frac{q_i^2}{\rho^2}$$

Hence, school i's best response investment level is

$$q_i^* = \frac{\rho^2}{4} e_i.$$

Meanwhile, student i's payoff of exerting effort $e_i = 1$ is

$$v_i(q_i, q_j, 1, e_j) = \frac{1}{2}(1 + q_i - e_j q_j) - \kappa$$

and student i's payoff from shirking is

$$v_i(q_i, q_j, 0, e_j) = \frac{1}{2}(1 - e_j q_j)$$

Hence, student i's best response is

$$e_i^* = \begin{cases} 1 & \text{if} \quad \kappa < q_i/2 \\ \{0,1\} & \text{if} \quad \kappa = q_i/2 \\ 0 & \text{if} \quad \kappa > q_i/2 \end{cases} \tag{5}$$

Therefore, combining these best responses implies that if $e_i = 1$ then $q_i = \rho^2/4$, but given $q_i = \rho^2/4$, full effort is only incentive compatible if $\kappa \leq \rho^2/4$. Hence, an equilibrium with $t_i = t_j = 1$ exists if and only if $\kappa \leq \rho^2/8$. ■

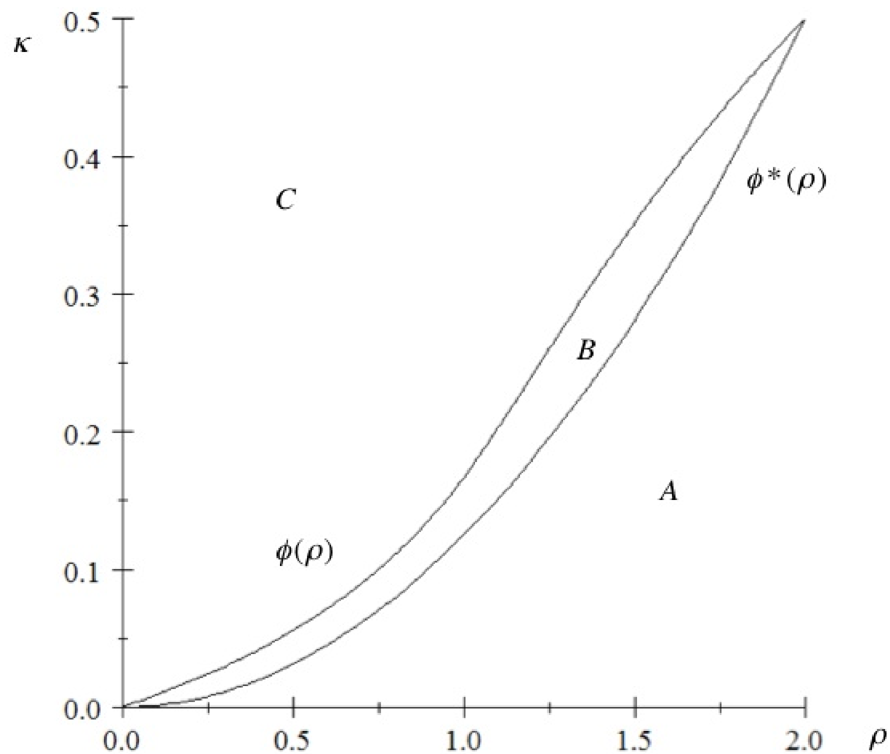
STRATEGIC VERSUS FULLY REVEALING GRADING WITH EFFORT

In this subsection we compare the equilibria arising under strategic and fully informative grading, beginning with a comparison of student effort. If multiple equilibria exist (for a particular grading regime), we focus on the equilibrium that delivers the evaluator the highest payoff: when an equilibrium with certain student effort exists, it is focal, otherwise the equilibrium with complete coordination failure (shirking and no investment) equilibrium is focal.

A simple calculation reveals that $\rho \in (0, 2) \Rightarrow \phi^*(\rho) < \phi(\rho)$. The implication is surprising: if students exert effort for certain with fully revealing grading, then they will certainly exert effort with strategic grading. The reverse, however, is not true: for any $\kappa \in (\phi^*(\rho), \phi(\rho))$ an equilibrium exists in which both students exert full effort if grading is strategic, but the students shirk for certain in the (unique) equilibrium with fully informative grading. This observations brings us to the first proposition of this section, which compares student effort in the two scenarios.

Proposition B.5 (Student Effort Comparison) *In any evaluator-preferred equilibrium, requiring schools to assign fully informative grades rather than engage in strategic grading never increases student effort and sometimes decreases it.*

Proof to Proposition B.5. The following graph illustrates that $\phi^*(\rho) < \phi(\rho)$ for all $\rho \in (0, 2)$. In region A, $\kappa < \phi^*(\rho) < \phi(\rho)$ hence an equilibrium exists in which students exert effort for certain in both scenarios and these are evaluator-preferred. In region B an equilibrium exists in which students exert effort for certain under strategic grading, but shirk for certain with fully informative grading. Thus, in the evaluator preferred equilibrium, student effort is higher with strategic grading. In region C students shirk for certain in both scenarios.



Hence, in any evaluator preferred equilibrium, student effort is weakly higher when grading is strategic. ■

Student effort with fixed investment. In the text, we mention that if students expect the same level of school investment in both cases, students have stronger incentives to shirk with strategic grading. This follows by comparing $f(q)$ defined in equation 4, the threshold κ below which a student exerts effort given investment q and $\widehat{e}_i = \widehat{e}_j = 1$ in the strategic grading equilibrium, and $q/2$, the corresponding threshold in the fully informative grading equilibrium. It is possible to establish that $f(q) < q/2$, hence, *given the same investment levels*, students are more inclined to shirk in the strategic grading equilibrium.

The next proposition follows from the combination of the previous result and the results of Section V. As long as students exert effort for certain in equilibrium, school investments and the evaluator payoff are identical to the case analyzed in Sections III and V, for both grading regimes. Because $\rho_i = \rho_j = \rho$, the propositions of Section V establish that when students are expected to exert effort for certain, both schools invest more, and that the evaluator's payoff is higher, when grading is strategic. Furthermore, Proposition B.5 establishes that in any pure strategy equilibrium, imposing fully informative grading only changes student effort by sometimes inducing students to *shirk*, reducing likely graduate ability (to zero) and with it, evaluator welfare.

Proposition B.6 (Evaluator Welfare Comparison) *In any evaluator-preferred equilibrium, requiring schools to assign fully informative grades rather than engage in strategic grading never benefits and sometimes hurts the evaluator.*

Proof to Proposition B.6. When parameters are in region A of the above figure (when $\kappa < \phi^*(\rho)$) both students exert effort for certain and schools grade and invest as they do in Section V. Because we focus on identical schools, Proposition 5 implies that the evaluator's payoff is strictly higher with grade inflation. In region B (when $\kappa \in (\phi^*(\rho), \phi(\rho))$) the equilibrium with fully informative grading induces full shirking, while the strategic grading equilibrium does not; hence strategic grading is preferred. In region C (where $\kappa > \phi(\rho)$) students always shirk in either case. ■

Therefore, for the symmetric version of the model, the main insights developed in earlier sections are robust (and even strengthened) to the inclusion of unobserved student as an essential input in the education process.

B.2 ENDOGENOUS SCHOOL COMMITMENT TO FULLY INFORMATIVE GRADING POLICIES

In this section, we solve for the equilibrium of the strategic grading game when schools have the ability to commit to fully informative grading strategies before the investment stage. Schools select whether to commit simultaneously. Any school that commits must choose a fully informative grading policy. Any school that does not commit will strategically chooses its grading policy in stage two, after observing school investments. Commitment decisions are publicly observed. To simplify the analysis, we solve for commitment for the game with symmetric schools (i.e. $\rho_1 = \rho_2 = \rho$) and without student effort.

When neither school commits, the game no-commitment subgame is identical to the model analyzed in the body of the paper. In this subgame, each school expects a payoff equal to 1/4, (consult proof of Lemma 4.3, recalling $\rho_1 = \rho_2 = \rho$).

When both schools commit to fully informative grading policies, then the subgame is identical to the fully informative grading benchmark analyzed in the body of the paper. In this subgame, each school expects a payoff equal to $1/2 - \rho^2/16$, as we show in the earlier analysis.

When only one school commits (for the explanation, let this be school 1) to a fully informative grading policy, then the other school (school 2) will choose the grading policy that is a best response to fully informative grading. In the grading stage (stage 2), school 2 chooses a grading policy that results in a posterior belief distribution made up of a mass point on 1, a mass point "just above" 0, and no other possible realizations. In other words, school 2 responds to a fully informative grading policy by school 1 by itself adopting a grading policy that is "almost fully informative." Such a grading policy would be achieved by always assigning low grades to low ability students, and almost always assigning high grades to high ability students (but with very small probability assigning a high ability student a low grade). Such a grading policy allows school 2 to always place its graduate when the graduate from school 1 is low ability, and to place its student half of the time when both schools produce high ability graduates.[3]

School 2's best response grading strategy is the same regardless of school investments q_1 and q_2 made in stage one of the subgame. In the subgame equilibrium when only school 1 commits to a fully informative grading policy:

$$E[u_1] = q_1(1 - q_2) + \frac{q_1 q_2}{2} - \frac{q_1^2}{\rho^2}$$

$$E[u_2] = 1 - q_1 + \frac{q_1 q_2}{2} - \frac{q_2^2}{\rho^2}$$

In stage one of the school 1 commitment subgame, schools choose their investment levels (q_1 or q_2) to maximize their respective payoff ($E[u_1]$ and $E[u_2]$) given the equilibrium investment of the other school. Taking first order conditions of $E[u_1]$ with respect to q_1 and $E[u_2]$ with respect to q_2, and solving the system of equations for q_1 and q_2 gives the following unique Nash Equilibrium

$$q_1^* = \frac{8\rho^2}{16 + \rho^4} \quad \text{and} \quad q_2^* = \frac{2\rho^4}{16 + \rho^4}$$

Plugging these expressions for q_1^* and q_2^* in to the expected payoffs gives

$$u_C^* \equiv E[u_1(q_1^*, q_2^*)] = \frac{64\rho^2}{(16 + \rho^4)^2}$$

[3]School 2 would like to choose a strategy of the following type, $\Pr(\Gamma_2 = 1) = q_2 - \epsilon_1$ and $\Pr(\Gamma_2 = \epsilon_2) = 1 - q_2$, for (ϵ_1, ϵ_2) approaching zero (but chosen to satisfy the mean constraint). Formally, this best response is not well-defined because of an open set problem. To eliminate this open set problem, the evaluator's equilibrium tie-breaking rule in the subgame where only a single school commits must adjust, ensuring that school 2's payoff is continous as $\epsilon_1, \epsilon_2 \to 0$. Ensuring this continuity requires that whenever both graduates generate a *zero* posterior belief, the evaluator selects the graduate of the school that *did not commit* (school 2)—otherwise she randomizes fairly. This strategy is sequentially rational for the evaluator; it eliminates school 2's incentive to make the ϵ deviations described above, and it generates exactly the same payoff as presented above.

$$u^*_{NC} \equiv E[u_2(q^*_1, q^*_2)] = 1 - \frac{4\rho^2(32+\rho^4)}{(16+\rho^4)^2}$$

Anticipating the subgame equilibria that follow from any commitment strategies, schools play the following two-by-two game during the commitment stage. Strategy C represents commitment to fully informative grading, and strategy NC represents no committment.

		School 2: C	School 2: NC
School 1	C	$1/2 - \rho^2/16$, $1/2 - \rho^2/16$	u^*_C , u^*_{NC}
	NC	u^*_{NC} , u^*_C	$1/4$, $1/4$

The Commitment Stage

Note that (ignoring the evaluator) for $\rho \in [0, 2)$, (C,C) Pareto dominates (NC,NC). Because schools are symmetric, their graduates are equally likely to win the prize under both the fully informative (C,C) and strategic grading (NC,NC) games. However, as discussed in the body of the paper, investment is higher with strategic grading.

Comparing these payoffs, NC is a best response to C when $u^*_{NC} \geq 1/2 - \rho^2/16$. Define the ρ which solves this with equality as $\bar{\rho}_1$, approximately 1.18. When $\rho \leq \rho_1$, NC is a best response to C. When $\rho \geq \bar{\rho}_1$, C is a best response to C.

Similarly, NC is a best response to NC when $1/4 \geq u^*_C$. Define the ρ which solves this with equality as $\bar{\rho}_2$, approximately 1.08. When $\rho \leq \bar{\rho}_2$, NC is a best response to NC. When $\rho \geq \bar{\rho}_2$, C is a best response to NC.

Hence, when $\rho \leq \bar{\rho}_2$, NC is a dominant strategy and the commitment stage game is a prisoner's dilemma, where (NC,NC) is the unique equilibrium. In this case, the ability of schools to commit to a fully informative grading policy up front they never choose to do so. Hence, on the equilibrium path the four stage (commitment, investment, grading, evaluation) generates identical outcomes as the game without the commitment stage.

When $\bar{\rho}_2 < \rho < \bar{\rho}_1$, NC is a best response to C, and C is a best response to NC. Therefore, two pure strategy equilibria exist, each involving up front commitment by one school and no commitment by the other school.

When $\bar{\rho}_1 \leq \rho$, C is a dominant strategy. In this case, (C,C) is the unique equilibrium of the commitment stage game. Here the four stage game leads to identical outcomes as the fully revealing benchmark.