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Corrigendum: Pride and Diversity in Social Economies

By Fabio Maccheroni, Massimo Marinacci, and Aldo Rustichini

Due to typographical changes made by the author after the publication of "Pride and Diversity in Social Economies" by Fabio Maccheroni, Massimo Marinacci, and Aldo Rustichini in *The American Economic Journal: Microeconomics* 6 (4): 237–71, we are publishing the changes here. Where necessary, sections of the paper have been reproduced so that the math changes are in context.

On page 242–243:

$$\max_{(c_i,\, e_i)\in B_i} U_i(c_i,\, e_i), \tag{1}$$

where

$$U_i(c_i, e_i) \;=\; u_i(c_{i,0}, e_{i,0}) + \beta \sum_{s\in S} p_s\, u_i(c_{i,s}, e_{i,s}) \quad \forall (c_i, e_i) \in \mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1},$$

and B_i is the subset of $\mathbb{R}_+^{S+1} \times \mathbb{R}_+^{S+1}$ consisting of all (c_i, e_i) such that:

(i) $(c_i, e_i) \in \mathbb{R}_+^{S+1} \times \prod_{s=0}^{S}[0,\, h_{i,s}]$.

(ii) $c_{i,0} \leq F_{i,0}\,(e_{i,0})$.

(iii) $c_{i,s} = F_{i,s}(e_{i,s}) + R\big(F_{i,0}(e_{i,0}) - c_{i,0}\big)\ \forall\, s \in S$.

The set B_i is never empty since in every period and state each agent can consume all he produces. Next we make a first assumption on the economy.

On page 247:

Empirical evidence on this phenomenon can be found, for example, in Bowles and Park (2005) and the references therein. Anecdotal evidence is reported in Rivlin (2007),[1] who describes Silicon Valley workaholic executives as "working class millionaires."

[1] Rivlin, Gary. 2007. "In Silicon Valley, Millionaires Who Don't Feel Rich." *New York Times*, August 5.

On page 256:

$$V(x_o, x_o) \geq V(x_o, x_i E y_i) \tag{17}$$

On page 257:

Using these limits we can define the Dini derivatives. Let $f: [a, b] \to \mathbb{R}$. For all $c \in [a, b)$, set

$$D^+ f(c) \equiv \limsup_{h \to 0^+} \frac{f(c+h) - f(c)}{h}$$

$$\text{and } D_+ f(c) \equiv \liminf_{h \to 0^+} \frac{f(c+h) - f(c)}{h}.$$

On page 258:

Analogously, for all $c \in (a, b]$ set

$$D^- f(c) \equiv \limsup_{h \to 0^-} \frac{f(c+h) - f(c)}{h}$$

$$\text{and } D_- f(c) \equiv \liminf_{h \to 0^-} \frac{f(c+h) - f(c)}{h}.$$

On page 258–259:

Notice that

$$V_i(c, e) = \sum_{s \in S_0} \pi_s \, u_i(c_{i,s}, e_{i,s}) + \sum_{s \in S_0} \pi_s \, \gamma_i\Big(v(c_{i,s}) - \int_I (v \circ c_s) \, d\lambda\Big)$$

$$= W_i\Big(c_i, e_i, \Big[\int_I (v \circ c_s) \, d\lambda\Big]_{s \in S_0}\Big)$$

for all $(c, e) \in L_+^{S+1} \times L_+^{S+1}$.

On page 260:

Therefore, by Theorem 3.4.1 of Balder (1995) there exists a λ-measurable almost everywhere selection (c^*, e^*) of the correspondence $B : i \mapsto B_i$ such that for λ-almost all i, $(c_i^*, e_i^*) \in \arg\max_{(x,y)\in B_i}\left(\sum_{s\in S_0}\pi_s u_i(x_s, y_s) + \sum_{s\in S_0}\pi_s\left[\gamma_i\left(v(x_s) - m_s(c^*, e^*)\right)\right]\right)$ where $m_s(c^*, e^*) = \int_I g_s\left(\iota, (c_\iota^*, e_\iota^*)\right) d\lambda(\iota) = \int_I v\left(c_{\iota,s}^*\right) d\lambda(\iota)$.

On page 263–264:

First observe that Gateaux differentiability of W guarantees that for all $e \in \operatorname{int}(E)$ there exists a linear and continuous operator $\nabla W(e) : L \to L$ such that

$$\lim_{t\to 0}\frac{W(e + tk) - W(e)}{t} = \nabla W(e)(k) \in L \tag{29}$$

for all $k \in L$. Arbitrarily choose $e \in \operatorname{int}(E)$ and $k \in L$, (29) means that

$$\lim_{t\to 0}\left\|\frac{W(e + tk) - W(e)}{t} - \nabla W(e)(k)\right\|_{\sup} = 0.$$

On page 265:

Which plugged into (31) delivers,

$$\int v'\left(F_\iota(e_\iota^*)\right) F_\iota'(e_\iota^*)\, k_\iota\, d\lambda(\iota) \int_I - \gamma_i'\left(v\left(F_i(e_i^*)\right) - m^*\right) d\nu(i) = 0 \text{ for all } k \in L.$$

On page 265–266:

PROOF OF THEOREM 2:

Define $V : [0, \bar{x}_0] \to \mathbb{R}$ by

$$V(x) = U(x) + \gamma(v(x) - m_0^*) + \beta \sum_{s \in S} p_s \big[\gamma\big(v(\bar{x}_s + R(\bar{x}_0 - x)) - m_s^*\big)\big]$$

and set $y_s = \bar{x}_s + R(\bar{x}_0 - x)$ for all $s \in S$. For all $x \in [0, \bar{x}_0)$,

$$D_+ V(x) \geq \liminf_{h \to 0^+} \frac{U(x+h) - U(x)}{h}$$
$$+ \liminf_{h \to 0^+} \frac{\gamma\big(v(x+h) - m_0^*\big) - \gamma\big(v(x) - m_0^*\big)}{h}$$
$$+ \beta \sum_{s \in S} p_s \liminf_{h \to 0^+} \frac{\gamma(v(y_s - Rh) - m_s^*) - \gamma\big(v(y_s) - m_s^*\big)}{h}$$

Analogously, for all $x \in (0, \bar{x}_0]$,

$$D^- V(x) \leq \limsup_{h \to 0^-} \frac{U(x+h) - U(x)}{h}$$
$$\dot{+} \limsup_{h \to 0^-} \frac{\gamma\big(v(x+h) - m_0^*\big) - \gamma\big(v(x) - m_0^*\big)}{h}$$
$$\dot{+} \beta \sum_{s \in S} p_s \limsup_{h \to 0^-} \frac{\gamma(v(y_s - Rh) - m_s^*) - \gamma\big(v(y_s) - m_s^*\big)}{h}.$$

On page 266:

For $x^* \in [0, \bar{x}_0)$,

$$D_+ V(x^*) \geq U'_+(x^*) + \liminf_{h \to 0^+} \frac{\gamma\big(v(x^*+h) - v(x^*)\big) - \gamma\big(v(x^*) - v(x^*)\big)}{h}$$
$$+ \beta \sum_{s \in S} p_s \liminf_{h \to 0^+} \frac{\gamma\big(v(y_s^* - Rh) - v(y_s^*)\big) - \gamma\big(v(y_s^*) - v(y_s^*)\big)}{h}$$
$$= U'_+(x^*) + \liminf_{h \to 0^+} \frac{\gamma\big(v(x^*+h) - v(x^*)\big) - \gamma(0)}{h}$$
$$+ \beta \sum_{s \in S} p_s \liminf_{h \to 0^+} \frac{\gamma\big(v(y_s^* - Rh) - v(y_s^*)\big) - \gamma(0)}{h}.$$

On page 267–268:

For all $s \in S$, the function $v_s : [-y_s^*/R, +\infty) \to \mathbb{R}$ defined by $v_s(h) = v(y_s^* + Rh) - (y_s^*)$ is continuous, concave, differentiable on $(-y_s^*/R, +\infty)$ with $v_s' > 0$ and $v_s(0) = 0$. Moreover,

$$(v_s)'_-(0) = \lim_{h\to 0^-} \frac{v_s(h) - v_s(0)}{h} = \lim_{h\to 0^-} \frac{v(y_s^* + Rh) - v(y_s^*)}{h}$$

$$= R \lim_{h\to 0^-} \frac{v(y_s^* + Rh) - v(y_s^*)}{Rh} = R \lim_{t\to 0^-} \frac{v(y_s^* + t) - v(y_s^*)}{t} = Rv'_-(y_s^*).$$

Thus, since Assumption H.6 allows us to apply a chain rule for Dini derivatives, we have

$$\liminf_{h\to 0^+} \frac{\gamma(v(y_s^* - Rh) - v(y_s^*)) - \gamma(0)}{h}$$

$$= \lim_{\delta\to 0^+} \inf_{h\in(0,\delta)} \frac{\gamma(v(y_s^* - Rh) - v(y_s^*)) - \gamma(0)}{h}$$

$$= -\lim_{\delta\to 0^+} \sup_{h\in(0,\delta)} \frac{\gamma(v(y_s^* - Rh) - v(y_s^*)) - \gamma(0)}{-h}$$

$$= -\lim_{\delta\to 0^+} \sup_{h\in(-\delta,0)} \frac{\gamma(v(y_s^* - Rh) - v(y_s^*)) - \gamma(0)}{h}$$

$$= -\limsup_{h\to 0^-} \frac{(\gamma\circ v_s)(h) - (\gamma\circ v_s)(0)}{h}$$

$$= -D^-(\gamma\circ v_s)(0) = -(v_s)'_-(0)\, D^-\gamma(0) = -Rv'_-(y_s^*)\, D^-\gamma(0).$$

Hence,

$$(33) \qquad D_+V(x^*) \geq U'_+(x^*) + v'_+(x^*)\, D_+\gamma(0) - \beta R D^-\gamma(0) \sum_{s\in S} p_s v'_-(y_s^*)\,.$$

Analogously, for $x^* \in (0, \bar{x}_0]$

$$D^-V(x^*) \leq U'_-(x^*) + \limsup_{h\to 0^-} \frac{\gamma(v(x^* + h) - v(x^*)) - \gamma(0)}{h}$$

$$-\beta \sum_{s\in S} p_s \liminf_{h\to 0^+} \frac{\gamma(v(y_s^* + Rh) - v(y_s^*)) - \gamma(0)}{h}$$

The function $v_0 : [-x^*, +\infty) \to \mathbb{R}$ defined by $v_0(h) = v(x^* + h) - v(x^*)$ is continuous, concave, differentiable on $(-x^*, +\infty)$ with $v_0' > 0$. Moreover, $v_0(0) = 0$ and

$$(v_0)'_-(0) = \lim_{h \to 0^-} \frac{v_0(h) - v_0(0)}{h} = \lim_{h \to 0^-} \frac{v(x^* + h) - v(x^*)}{h} = v'_-(x^*).$$

Thus, since Assumption H.6 allows to apply a chain rule for Dini derivatives, we have

$$\begin{aligned} \limsup_{h \to 0^-} \frac{\gamma\big(v(x^* + h) - v(x^*)\big) - \gamma(0)}{h} &= D^-(\gamma \circ v_0)(0) \\ &= (v_0)'_-(0) D^- \gamma(v_0(0)) \\ &= v'_-(x^*) D^- \gamma(0). \end{aligned}$$

On page 270:

The strict concavity of u implies that of U. Along with the concavity of γ and v, this implies that V is strictly concave on $[0, \overline{x}_0]$.

REFERENCES

Anscombe, Frank J., and Robert J. Aumann. 1963. "A Definition of Subjective Probability." *Annals of Mathematical Statistics* 34 (1): 199–205.