# Online Appendix to:
# "A Pitfall with DSGE-Based, Estimated, Government Spending Multipliers"

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## I Alternative Transmission Mechanisms

In the body of the paper, we considered Edgeworth complementarity/substitutability as the transmission mechanism of government spending. In this section, we propose other mechanisms that yield the exact same reduced form. We still consider that aggregate output is equal to the the sum of private and public consumptions ($y_t = c_t + g_t$). It follows that a multiplier exceeding one is equivalent to a positive multiplier on private consumption. In our simple setup, the multiplier is a decreasing function of $\alpha_g$ and it exceeds unity when $\alpha_g < \nu(1 - s_g)/(1 + \nu s_g)$.

In all the following alternative specifications, we show that the log-linear approximation of the economy takes the form

$$\hat{y}_t = \alpha \hat{g}_t + \xi z_t, \tag{I.1}$$

as in the simple model of section 2. In what follow, we focus exclusively on the parameter $\alpha$, since the parameter $\xi$ can be normalized to unity by rescaling appropriately the standard error of the technology shock. Moreover, we discuss conditions on deep parameters representing preferences or technology for the long-run GSM to exceed unity. We also show that the value of this multiplier is unambiguously related to a single parameter summarizing the transmission mechanism of public spending.

### I.1 Non-Separable Preferences

Following Linnemann (2006) and Bilbiie (2009), we consider a more general utility function. The utility function in equation (1) rewrites more generally as $u(c_t, l_t)$, where leisure $l_t$ satisfies $l_t = 1 - n_t$. Combining the optimality conditions of the households' problem yields $u_l(c_t, l_t) = u_c(c_t, l_t) w_t$.

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Firms produce a homogenous good $y_t$ using the same constant returns-to-scale technology as in 2, so that profit maximization yields $w_t = \mathrm{e}^{z_t}$. It immediately comes that $u_l(c_t, l_t) = u_c(c_t, l_t)\mathrm{e}^{z_t}$. After log-linearizing this equation, we get

$$\varphi \hat{n}_t = z_t - \psi \hat{c}_t,$$

where $\varphi \equiv (u_{cl} n / u_c) - (u_{ll} n / u_l)$ and $\psi \equiv -(u_{cc} c / u_c) + (u_{cl} c / u_l)$. Using the other equilibrium conditions, we exactly obtain equation (I.1), where the parameter $\alpha$ is now defined by

$$\alpha \equiv \frac{\psi s_g}{\psi + \varphi(1 - s_g)}$$

The long-run government spending multiplier is then

$$\frac{\Delta y}{\Delta g} = \frac{\psi}{\psi + \varphi(1 - s_g)}.$$

The multiplier exceeds unity when $\psi > 0$ and $\varphi < 0$. As stated in Bilbiie (2009), this imposes that consumption be an inferior good (see also Bilbiie, 2011). The multiplier is a decreasing function of $\varphi$. Assuming all the other parameters constant (in particular $\psi$), the long-run multiplier is then directly linked to the key parameter $\varphi$ summarizing households' preferences.

### I.2 Externalities

We consider two types of externalities yielding equation (I.1) exactly. One relates to the specification of preferences, whereas the second relates to the production technology.

#### I.2.1 Externality in Labor Supply

We adapt Benhabib and Farmer (2000) to our simple setup. Let us rewrite the instantaneous utility function in equation (1) as

$$\log(c_t) - \frac{\eta}{1+\nu} \left( \frac{n_t}{\bar{n}_t^{\vartheta}} \right)^{1+\nu},$$

where $\bar{n}_t$ represents the average labor supply in the economy. The parameter $\vartheta$ measures the external effect of other households' labor on individual utility. For example, when $\vartheta > 0$, individual and aggregate labor supplies are complement. Using the firm's first-order condition and the aggregate resources constraint, we obtain $\eta (y_t / z_t)^{\nu - \vartheta(1+\nu)} = \mathrm{e}^{z_t} / (y_t - g_t)$. Loglinearizing this yields

$$\alpha = \frac{s_g}{1 + (1 - s_g)(\nu - \vartheta(1+\nu))}$$

The long-run GSM is then

$$\frac{\Delta y}{\Delta g} = \frac{1}{1 + (1 - s_g)(\nu - \vartheta(1+\nu))}$$

The multiplier exceeds unity whenever $\vartheta > \nu/(1+\nu)$. Notice that for these values of $\vartheta$, the constant-consumption, aggregate labor supply is downward slopping. The multiplier is an increasing function of $\vartheta$. For $s_g$ and $\nu$ set to given values, the multiplier is unambiguously linked to the labor supply externality.

### I.2.2 Externality in production

We now adapt Benhabib and Farmer (1994) and Devereux, Head and Lapham (1996) to our simple setup. Let us assume that the technology takes the form

$$y_t = \mathrm{e}^{z_t} n_t s_t.$$

Here $s_t$ is an externality on production specified as $s_t = \bar{n}_t^{\theta_n}$. As before, $\bar{n}_t$ represents the average level of labor. The parameter $\theta_n$ governs the productive externality. Notice that the technology displays constant returns-to-scale at the private level, but increasing returns at the social level when $\theta_n > 0$. In equilibrium, the real wage obeys $w_t = \mathrm{e}^{z_t} n_t^{\theta_n}$. Plugging this equation into the households' optimality condition and using the aggregate resources constraint, we obtain finally $\eta(y_t/z_t)^{(\nu-\theta_n)/(1+\theta_n)} = \mathrm{e}^{z_t}/(y_t - g_t)$. Loglinearizing this yields equation (I.1), where the parameter $\alpha$ is now given by

$$\alpha = \frac{s_g(1+\theta_n)}{1+\theta_n(1-s_g)(\nu-\theta_n)}$$

The long-run GSM is

$$\frac{\Delta y}{\Delta g} = \frac{1+\theta_n}{1+\theta_n(1-s_g)(\nu-\theta_n)}$$

The multiplier exceeds unity whenever $\theta_n > \nu$. Under this restriction, the aggregate labor demand is more upward sloping than the constant-consumption labor supply (see Bilbiie, 2011). The multiplier is an increasing function of $\theta_n$. Keeping all the other parameters constant, the multiplier is unambiguously related to the size of the production externality.

## I.3 Deep Subsistence Point

We now adapt Ravn, Schmitt-Grohé and Uribe (2006) and Ravn, Schmitt-Grohé and Uribe (2008) to our simple setup. The instantaneous utility rewrites as

$$\log(x_t^c) - \frac{\eta}{1+\nu} n_t^{1+\nu},$$

where $x_t^c$ represents a composite consumption good, composed of a continuum of differentiated goods indexed by $j \in [0,1]$. The composite good $x_t^c$ is given by

$$x_t^c = \left[\int_0^1 \left(c_{j,t} - c_j^\star\right)^{1-1/\rho} \mathrm{d}j\right]^{1/(1-1/\rho)}, \tag{I.2}$$

where $c_j^\star$ is the subsistence level of consumption of good $j$. The parameter $\rho > 0$ is the elasticity of substitution across varieties. Minimizing total consumption expenditures $\int_0^1 P_{j,t} c_{j,t} \mathrm{d}j$, where $P_{j,t}$ denotes the price of good $j$, subject to the aggregation constraint (I.2) yields the demand for each good $j$

$$c_{j,t} = (P_{j,t}/P_t)^{-\rho} x_t^c + c_j^\star,$$

where $P_t = [\int_0^1 P_{j,t}^{1-\rho} \mathrm{d}j]^{1/(1-\rho)}$ is the price index. Notice that the price elasticity is not constant as soon as $c_j^\star > 0$. The optimality condition of households' problem is given by $\eta n_t^\nu = w_t / x_t^c$.

Symmetrically with the households'problem, the government allocates spending among individual varieties of goods, $g_{j,t}$, so as to maximize the quantity $x_t^g$ of a composite good

$$x_t^g = \left[\int_0^1 \left(g_{j,t} - g_j^\star\right)^{1-1/\rho} \mathrm{d}j\right]^{1/(1-1/\rho)},$$

where $g_j^\star$ denotes the subsistence level of consumption of public good $j$. Given the budget constraint $\int_0^1 P_{j,t} g_{j,t} \mathrm{d}j \leq P_t g_t$, the government demand for each good is given by $g_{j,t} = (P_{j,t}/P_t)^{-\rho} x_t^g + g_j^\star$.

Finally, each good $j$ is produced by a monopolist using the technology $y_{j,t} = \mathrm{e}^{z_t} n_{j,t}$. Each firm sets its price and satisfies demand, $\mathrm{e}^{z_t} n_{j,t} \geq c_{j,t} + g_{j,t}$. After substituting for the expressions for the demands $c_{j,t}$ and $g_{j,t}$ into the previous constraint, we obtain the first order conditions $\mathrm{mc}_{j,t} = w_t/\mathrm{e}^{z_t}$ and $P_{j,t}/P_t = \mathrm{mku}_{j,t} \times \mathrm{mc}_{j,t}$, where $\mathrm{mc}_{j,t}$ denotes the marginal cost of labor and $\mathrm{mku}_{j,t}$ the associated markup. This markup is given by

$$\mathrm{mku}_{j,t} = \left[1 - \frac{1}{\rho\left(1 - \frac{c_j^\star + g_j^\star}{y_{j,t}}\right)}\right]^{-1}$$

For simplicity, we impose $c_j^\star = c^\star$ and $g_j^\star = g^\star$. In a symmetric equilibrium, $P_{j,t}/P_t = 1, \forall j \in [0,1]$, so that $w_t = \mathrm{e}^{z_t}/\mathrm{mku}_t$. The equilibrium of this economy is then given by

$$(y_t/\mathrm{e}^{z_t})^\nu = \left(1 - \frac{1}{\rho\left(1 - \frac{c^\star + g^\star}{y_t}\right)}\right) \frac{\mathrm{e}^{z_t}}{y_t - g_t - c^\star}$$

Denoting $c^\star = \omega c$ and $g^\star = \omega g$, the steady-state share of subsistence consumption $c^\star + g^\star$ in output, we exactly obtain equation (I.1), where the parameter $\alpha$ is defined by

$$\alpha = \frac{s_g}{(\nu(1-s_g)(1-\omega)+1) - (1-s_g)\omega(\mathrm{mku}-1)},$$

where mku $> 1$ is the steady-state markup, provided $\rho(1-\omega) - 1 \equiv (\mu - 1)^{-1} > 0$. The long-run GSM is

$$\frac{\Delta y}{\Delta g} = \frac{1}{(\nu(1-s_g)(1-\omega)+1) - (1-s_g)\omega(\mathrm{mku}-1)}$$

The multiplier exceeds unity when $\omega > \nu/(\nu + \mu - 1)$. The multiplier is an increasing function of $\omega$. Keeping all the other parameters constant (for example, mku is fixed), the multiplier unambiguously depends on the relative size of the deep subsistence point.

## II Consistent Estimator of the Long-Run GSM

An easy way to obtain a consistent estimator of $\alpha$ relies on indirect estimation using the following representation of the reduced form

$$\hat{y}_t = \pi_1 \hat{y}_{t-1} + \epsilon_{1,t}$$
$$\hat{g}_t = \pi_2 \hat{y}_{t-1} + \epsilon_{2,t}$$

The *plim* estimators of $\pi_1$ and $\pi_2$ are given by

$$\hat{\pi}_1 = \frac{\mathrm{E}\{\hat{y}_t \hat{y}_{t-1}\}}{\mathrm{E}\{\hat{y}_t^2\}} \quad \text{and} \quad \hat{\pi}_2 = \frac{\mathrm{E}\{\hat{g}_t \hat{y}_{t-1}\}}{\mathrm{E}\{\hat{y}_t^2\}}$$

from which we deduce

$$\hat{\alpha} = \frac{\hat{\pi}_1}{\hat{\pi}_2} = \frac{\mathrm{E}\{\hat{y}_t \hat{y}_{t-1}\}}{\mathrm{E}\{\hat{g}_t \hat{y}_{t-1}\}}$$

From (7)-(8), we obtain:

$$\mathrm{E}\{\hat{y}_t \hat{y}_{t-1}\} = \frac{\alpha \varphi_g}{1 + \alpha \varphi_g} \mathrm{E}\{\hat{y}_t^2\} \quad \text{and} \quad \mathrm{E}\{\hat{g}_t \hat{y}_{t-1}\} = \frac{\varphi_g}{1 + \alpha \varphi_g} \mathrm{E}\{\hat{y}_t^2\}$$

The indirect estimator $\hat{\alpha}$ of $\alpha$ is thus consistent. Similarly, $\hat{\varphi}_g$ is also a consistent estimator of $\varphi_g$.

## III Additional Real Frictions

A central ingredient of our preferred specification is the presence of dynamic complementarities in labor supply. Importantly, output dynamics inherit the built-in persistence of hours worked generated by this mechanism.

The recent DSGE literature, however, has emphasized alternative real frictions capable of generating very strong aggregate persistence. Important such mechanisms are habits in consumption and dynamic investment adjustment costs, see Christiano, Eichenbaum and Evans (2005). We considered versions of our preferred model augmented with these additional mechanisms.

When either of these are included, they do not significantly contribute to the model's fit. A standard likelihood ratio test would not reject the restriction of no habits in consumption and/or no dynamic adjustment costs. For example, in the case of specification (4) augmented with habits in consumption and dynamic adjustment costs, the log-likelihood is equal to 2702.22, to be compared to our reference specification (4) where the log-likelihood is equal to 2701.32. The habits in consumption parameter is equal to 0.11 (not significantly different from zero at conventional levels) and the adjustment cost parameter is almost zero. In addition, we redo the specification tests (normality and serial correlation). Including habits in consumption and dynamic adjustment costs does not improve upon the model performance: the normality test statistic is almost the same for each innovation and the serial correlation coefficients are very similar.

More importantly for our purpose, the empirical interaction between $\alpha_g$ and $\varphi_g$ still holds under this more complete framework. In particular, when $\varphi_g$ is constrained to zero, we obtain $\alpha_g = -0.21$, yielding a multiplier $\Delta y/\Delta g = 0.85$. In contrast, when $\varphi_g$ is freely estimated, government policy turns out to be countercyclical ($\varphi_g = 0.60$) and the parameter $\alpha_g = -0.79$, implying a multiplier equal to 1.19. This confirms our main result.

## IV News Shocks in the Government Spending Rule

As emphasized by Ramey (2009) and Schmitt-Grohé and Uribe (2008), the expected component in public expenditures constitutes an important element of government policy. We accordingly modify our benchmark specification to allow for news shocks in the government spending rule, according to

$$\epsilon_{g,t} = \rho_g \epsilon_{g,t-1} + \sum_{i=0}^{q} \sigma_{g,i} \zeta_{g,t-i}, \quad \forall i \in \{0, \ldots, q\}, \sigma_{g,i} \geq 0,$$

where the $\zeta_{g,t}$ is *iid* with $\zeta_{g,t} \sim N(0,1)$.

We first imposed $q = 4$. According to our estimation results, we obtain that lags $i = 1,2,3$ are not significant. This specification delivers a significantly better fit to the data than our preferred model (4), according to the likelihood ratio test (in this case, the log-likelihood is equal to 2717.39). However, the parameter estimates do not change too much compared to specification (4). In particular, the parameter $\alpha_g$ is now equal to $-0.86$, whereas the feedback rule parameter is equal to 0.59. In addition, allowing for news shocks in government spending does not improve upon the specification tests of our reference model (4).

Importantly, adding news shocks does not modify our main conclusion. When policy is exogenous ($\varphi_g = 0$), we obtain $\alpha_g = -0.21$, with an associated multiplier equal to 0.56. In contrast, when $\varphi_g$ is freely estimated, we obtain $\alpha_g = -0.86$, resulting in a higher multiplier $\Delta y/\Delta g = 1.11$.

# V Results from Simulated Data

In this section, we expound additional results based on simulated data.

## V.1 Simulation results on the estimation bias for the degree of Edgeworth complementarity

Table V.1. Simulation Results

| | DGP: Specification (4) $\alpha_g \neq 0, \varphi_g = 0$ | | | DGP: Specification (3) $\alpha_g \neq 0, \varphi_g \neq 0$ | | |
|---|---|---|---|---|---|---|
| | | Estimated Models | | | Estimated Models | |
| Parameters | True Value | (4) | (3) | True Value | (4) | (3) |
| $\alpha_g$ | -0.9452 | -0.9095 | -0.5945 | -0.6340 | -0.6100 | -0.6134 |
| $\varphi_g$ | 0.6117 | 0.6140 | — | — | -0.0075 | — |
| $\phi$ | 0.4110 | 0.4052 | 0.3848 | 0.3766 | 0.3845 | 0.3845 |
| $\gamma_z$ | 1.0044 | 1.0045 | 1.0043 | 1.0043 | 1.0044 | 1.0044 |
| $\rho_g$ | 0.9756 | 0.9729 | 0.9181 | 0.9535 | 0.9455 | 0.9459 |
| $\rho_c$ | 0.9834 | 0.9736 | 0.9572 | 0.9795 | 0.9698 | 0.9699 |
| $\rho_n$ | 0.8399 | 0.8302 | 0.8093 | 0.8366 | 0.8213 | 0.8215 |
| $\sigma_z$ | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 |
| $\sigma_g$ | 0.0119 | 0.0118 | 0.0137 | 0.0129 | 0.0128 | 0.0129 |
| $\sigma_c$ | 0.0133 | 0.0123 | 0.0096 | 0.0123 | 0.0113 | 0.0113 |
| $\sigma_n$ | 0.0266 | 0.0265 | 0.0277 | 0.0273 | 0.0272 | 0.0272 |

**Notes:** Simulation results obtained from 1000 replications. Model (4): $\alpha_g \neq 0$ and $\varphi_g \neq 0$; Model (3): $\alpha_g \neq 0$ and $\varphi_g = 0$. In each case, we report the average value of parameters across simulations.

Based on our previous results, one can suspect that the greater $\alpha_g$ obtained under model (3) is the outcome of a misspecification bias. Indeed, we previously saw that omitting $\varphi_g$ always increases the estimated value of $\alpha_g$. In the simple model considered in the first section, we were able to formally show the existence of such a bias. In our DSGE framework, no such analytical results is available, though the same economic forces seem to be at play when we estimate our model on actual data.

In this appendix, we resort to simulation techniques in order to investigate whether the negative link between $\alpha_g$ and $\varphi_g$ is idiosyncratic to our sample. In addition, resorting to simulation enables us to investigate whether $\varphi_g$ can be estimated to non-zero values even in a world where no such mechanism exists (an exercise that we can hardly perform on actual data).

To investigate these points, we develop a controlled experiment in which we use our model as our DGP, using the estimated values reported in table 1. More specifically, using model (4) as our DGP we first want to make sure that (i) estimating specification (4) on simulated data delivers consistent parameter estimates and (ii) estimating specification (3) on the exact same simulated data yields severely biased estimates of $\alpha_g$. To complement on this, we also run the symmetric estimations in which we use model (3) as our DGP and successively estimate specifications (3) and (4) on simulated data. In this case, the crucial point is to check whether our estimation procedure is able to properly reject a policy feedback rule when no such rule exists in simulated data. Our Monte Carlo simulations are run as follows: using either model (4) or model (3), we generate 1000 samples of observables $(\Delta \log(y_t), \log(n_t), \Delta \log(c_t), \Delta \log(g_t)$, with the same sample size as actual data, after having eliminated 800 initial observations, thus ensuring that initial conditions do not contaminate our estimation results. To do so, the four structural shocks innovations are drawn from independent Gaussian distributions with zero mean and unit variance. On each simulated sample, we estimate specifications (3) and (4) and thus generate a population of estimated parameters.

Table V.1 reports the simulation results when using either specifications (4) or (3) as DGP and/or estimated model. We first check whether estimating model (4) on data simulated from model (4) yields consistent parameter estimates. It turns out that this is the case. Indeed, we see from table V.1 that the average parameters estimates almost coincide with the true ones. Now, consider what happens when estimating model (3) on data simulated from model (4). In this case, all the parameters linked to government policy ($\alpha_g$, $\rho_g$, $\sigma_g$) turn out to be biased. This is particularly striking when it comes to $\alpha_g$, the average value of which is almost twice as small (in absolute term) as the true one. Interestingly, the average estimated value of $\alpha_g$ from our simulation experiment is very similar to what obtains from actual data when estimating model (3).

Consider now what happens when using specification (3) as our DGP. The results are reported in table V.1. We first check whether estimating specification (3) on data simulated from model (3) yields consistent estimates. This again turns out to be the case. Now, consider what happens when estimating model (4) on data simulated from model (3). Basically, this procedure is able to recover the true parameters on average. This is particularly striking when it comes to the feedback parameter $\varphi_g$, which is zero on average. Recall that the latter does not exist in model (3), the DGP used for this simulation experiment, and appears only in model (4). This implies that a significant $\varphi_g$ on actual data does not seem to be an artifact of our particular sample.

### V.2 Simulation results on countercyclicality of the endogenous component of policy

In our robustness analysis, we considered several alternative government spending rules and used simple likelihood comparisons to motivate why we selected our benchmark specification. In particular, we used this approach to discriminate between our benchmark specification and an alternative rule in which the endogenous component of detrended government expenditures reacts to detrended output (in relative deviation from its steady-state value). This alternative rule is widespread in the literature and has a very important implication: using it, we would most definitely conclude that the endogenous component of governement spending is procyclical, in stark contrast with what obtains under our preferred specification. In the remainder, our benchmark is labelled (4) and the alternative specification is labelled (A), as in our analysis of alternative fiscal rules.

In this section, we consider an alternative empirical validation exercise which consists in resorting to an encompassing criterion.[1] Under this criterion, a specification must not just be judged based on the associated likelihood. It must also be able to predict the results based on an opposing model (specification (A) in our case). If one of the two views fails this encompassing test, the one that passes should be preferred.

Table V.2. Simulation Results – Encompassing test

| | DGP: Specification (4) | | | DGP: Specification (4′) | | |
|---|---|---|---|---|---|---|
| | | Estimated Models | | | Estimated Models | |
| Parameters | True Value | (4) | (A) | True Value | (4) | (A) |
| $\alpha_g$ | -0.9452 | -0.9095 | -0.5498 | -0.5773 | -0.8950 | -0.5676 |
| $\varphi_g$ | 0.6117 | 0.6140 | -0.3791 | -0.4866 | 0.0681 | -0.4854 |
| $\phi$ | 0.4110 | 0.4052 | 0.4051 | 0.4072 | 0.3702 | 0.4149 |
| $\gamma_z$ | 1.0044 | 1.0045 | 1.0043 | 1.0043 | 1.0043 | 1.0043 |
| $\rho_g$ | 0.9756 | 0.9729 | 0.9116 | 0.9547 | 0.9730 | 0.9478 |
| $\rho_c$ | 0.9834 | 0.9736 | 0.9564 | 0.9776 | 0.9703 | 0.9686 |
| $\rho_n$ | 0.8399 | 0.8302 | 0.8024 | 0.8396 | 0.8425 | 0.8158 |
| $\sigma_z$ | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 | 0.0107 |
| $\sigma_g$ | 0.0119 | 0.0118 | 0.0134 | 0.0123 | 0.0128 | 0.0122 |
| $\sigma_c$ | 0.0133 | 0.0123 | 0.0094 | 0.0119 | 0.0131 | 0.0112 |
| $\sigma_n$ | 0.0266 | 0.0265 | 0.0275 | 0.0273 | 0.0268 | 0.0271 |

**Notes:** Simulation results obtained from 1000 replications. Fiscal rule under model (4): $\log(\tilde{g}_t) = -\varphi_g(\Delta \log(y_t) - \log(\gamma_z))$; Fiscal rule under model (A): $\log(\tilde{g}_t) = -\varphi_g(\log(y_t^s) - \log(\bar{y}^s))$, where, $y_t^s = y_t e^{-z_t}$ and $\bar{y}^s$ is the steady-state value of $y_t^s$. In each case, we report the average value of parameters across simulations.

We proceed as before. Using either (4) or (A) as our DGP, we estimate specifications (4) and (A) on simulated data. Our Monte Carlo simulations are conducted in the exact same way as before.

[1]See Christiano, Eichenbaum and Vigfusson (2003) for an early example of this approach.

These simulation results are reported in table V.2. For each DGP, the table reports the "true value", i.e. the parameter values obtained when estimating the model on actual data and used to simulate the model. Consider first what happens when specification (4) is our DGP. As before, we see that when estimating (4) on data simulated from (4), our procedure recovers parameter values close to their true values (albeit with a small sample bias). More interestingly, when we estimate specification (A) on data simulated from (4), we obtain parameter values close to what obtains when estimating (A) on actual data. Thus, specification (4) encompasses specification (A). Now, let us proceed the other way round and use (A) as our DGP. Clearly, if (A) were to be estimated on data simulated from (A), we would on average recover the true parameter values (once again, with a small sample bias). However, if (4) were to be estimated on data simulated from (A), we would not recover the parameter values obtained when estimating (4) on actual data. Thus, (A) fails to encompass (4).

To sum up, specification (4) encompasses specification (A) while the converse is not true. This is yet another confirmation that our benchmark specification is a good description of the data. It also serves the purpose of reinforcing our empirical results on the countercyclicality of the government spending rule.

# VI Specification Tests

We complement the above results by performing specification tests for the innovation of each variables used for estimation in equation (16), *i.e.* $\Delta \log(y_t)$ output growth, $\log(n_t)$ the log of hours, $\Delta \log(c_t)$ private consumption growth, and $\Delta \log(g_t)$ government consumption growth. The innovations are obtained as the difference between the observed variables and their predicted value at convergence of the estimation stage. The specification tests, reported in table VI.1, are conducted for the four model's specifications. The first column reports the Shapiro and Wilk (1965) test statistic. The null hypothesis being tested is that the innovation of the variables listed on the left is normally distributed. A small value of the test statistic indicates a rejection of the null, whereas a value close to unity favors the normality assumptions. On the right, we report the $P$-value (in %) of the test statistic. Except for consumption growth, normality is rejected in all cases. However, rejection is essentially driven by a few outliers. Given the parametric parsimony of the model, such a rejection is hard to interpret.

More interestingly, table VI.1 also includes serial correlation tests. We report the least-squares coefficient obtained by projecting each innovation on its own lag. For each coefficient, we report the associated 95% confidence interval. We find that omitting the feedback rule deteriorates the results. Indeed, comparing specification (3) and (4) shows that consumption and government spending innovations display less serial correlation when the policy rule coefficient is not constrained to zero.

Table VI.1. Specification Tests

| Specification | Innovation in | Normality: Shapiro-Wilk Statistic | Normality: $P$-value (in %) | Serial Correlation: Coefficient | Serial Correlation: Confidence Interval (at 95%) |
|---|---|---|---|---|---|
| (1) | $\Delta\log(y_t)$ | 0.9657 | 0.0531 | 0.1883 | [0.0482 , 0.3284] |
| | $\log(n_t)$ | 0.9737 | 0.3513 | 0.0055 | [−0.1372 , 0.1482] |
| | $\Delta\log(c_t)$ | 0.9847 | 7.2771 | 0.3146 | [0.1783 , 0.4508] |
| | $\Delta\log(g_t)$ | 0.9686 | 0.1019 | 0.1685 | [0.0267 , 0.3103] |
| (2) | $\Delta\log(y_t)$ | 0.9668 | 0.0673 | 0.1821 | [0.0419 , 0.3224] |
| | $\log(n_t)$ | 0.9738 | 0.3577 | -0.0170 | [−0.1597 , 0.1256] |
| | $\Delta\log(c_t)$ | 0.9850 | 7.8055 | 0.3125 | [0.1762 , 0.4489] |
| | $\Delta\log(g_t)$ | 0.9672 | 0.0741 | 0.0393 | [−0.1045 , 0.1831] |
| (3) | $\Delta\log(y_t)$ | 0.9655 | 0.0510 | 0.1948 | [0.0549 , 0.3348] |
| | $\log(n_t)$ | 0.9746 | 0.4386 | 0.0354 | [−0.1071 , 0.1778] |
| | $\Delta\log(c_t)$ | 0.9860 | 10.7253 | 0.2851 | [0.1476 , 0.4225] |
| | $\Delta\log(g_t)$ | 0.9699 | 0.1405 | 0.1504 | [0.0081 , 0.2926] |
| (4) | $\Delta\log(y_t)$ | 0.9681 | 0.0917 | 0.1787 | [0.0384 , 0.3191] |
| | $\log(n_t)$ | 0.9744 | 0.4246 | -0.0223 | [−0.1647 , 0.1201] |
| | $\Delta\log(c_t)$ | 0.9865 | 12.3860 | 0.0924 | [−0.0505 , 0.2352] |
| | $\Delta\log(g_t)$ | 0.9693 | 0.1211 | -0.0008 | [−0.1447 , 0.1431] |

**Notes:** Sample period: 1960:1-2007:4. $\Delta\log(y_t)$ denotes output growth, $\log(n_t)$ hours, $\Delta\log(c_t)$ private consumption growth, and $\Delta\log(g_t)$ government consumption growth. Specification (1): $\alpha_g = \varphi_g = 0$, Specification (2): $\alpha_g = 0$ , $\varphi_g \neq 0$, Specification (3): $\alpha_g \neq 0$ , $\varphi_g = 0$, Specification (4): $\alpha_g \neq 0$ , $\varphi_g \neq 0$. Coefficients are obtained by projecting each innovation on its own lag.

# VII The Smets-Wouters model

This section describes our medium-scale DSGE model, which is similar to Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). The model combines a neoclassical growth core with several shocks and frictions. It includes features such as habit formation, investment adjustment costs, variable capital utilization, monopolistic competition in goods and labour markets, and nominal price and wage rigidities.

The economy is populated by five classes of agents: producers of a final good, intermediate goods producers, households, employment agencies, and the government.

## VII.1 Household sector

### VII.1.1 Employment agencies

As in Erceg, Henderson and Levin (2000), each household, indexed by $j \in [0, 1]$, is a monopolistic supplier of specialized labor $n_t(j)$. At every point in time $t$, a large number of competitive "employment agencies" combine households' labor into a homogenous labor input $n_t$ sold to intermediate firms, according to

$$n_t = \left( \int_0^1 n_t(j)^{\frac{1}{\lambda_{w,t}}} \mathrm{d}j \right)^{\lambda_{w,t}}, \tag{VII.3}$$

where $\lambda_{w,t} = \lambda_w e^{\epsilon_{w,t}}$, $\lambda_w$ is the desired steady-state wage markup over the marginal rate of substitution between consumption and leisure, and the markup shock $\epsilon_{w,t}$ is assumed to evolve according to

$$\epsilon_{w,t} = \rho_w \epsilon_{w,t-1} + \sigma_w \zeta_{w,t}, \quad \zeta_{w,t} \sim Niid(0, 1).$$

Profit maximization by the perfectly competitive employment agencies implies the labor demand function

$$n_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\frac{\lambda_{w,t}}{\lambda_{w,t}-1}} n_t, \tag{VII.4}$$

where $W_t(j)$ is the wage paid by the employment agencies to the household supplying labor variety $j$, while

$$W_t \equiv \left( \int_0^1 W_t(j)^{\frac{1}{\lambda_{w,t}-1}} \mathrm{d}j \right)^{\lambda_{w,t}-1} \tag{VII.5}$$

is the wage paid to homogenous labor.

#### VII.1.2 Household's preferences

Household $j$ has preferences given by

$$\mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \mathrm{e}^{\epsilon_{\beta,t+s}} \left[ \log(\tilde{c}_{t+s} - h\tilde{c}_{t+s-1}) - \frac{\eta}{1+\nu} n_{t+s}(j)^{1+\nu} \right], \tag{VII.6}$$

where $\mathrm{E}_t$ denotes the mathematical expectation operator conditional upon information available at $t$, $\tilde{c}_t \equiv c_t + \alpha_g g_t$, $c_t$ denotes consumption, $g_t$ is government expenditures, $n_t(j)$ is labor of type $j$, $\beta$ is the subjective discount factor, $h \in [0,1]$ denotes the degree of habit formation, $1/\nu > 0$ is the Frisch elasticity of labor supply, $\eta$ is a scale parameter, and $\epsilon_{c,t}$ is a preference shock evolving according to

$$\epsilon_{\beta,t} = \rho_\beta \epsilon_{\beta,t-1} + \sigma_\beta \zeta_{\beta,t}, \quad \zeta_{\beta,t} \sim Niid(0,1).$$

As we explain below, households are subject to idosyncratic shocks about whether they are able to re–optimize their wage. Hence, the above described problem makes the choices of wealth accumulation contingent upon a particular history of wage rate decisions, thus leading to households heterogeneity. For the sake of tractability, we assume that the momentary utility function is separable across consumption and leisure. Combining this with the assumption of a complete set of contingent claims market, all the households will make the same choices regarding consumption and will only differ by their wage rate and supply of labor. This is directly reflected in our notations.

Household $j$'s period budget constraint is given by

$$P_t(c_t + x_t) + T_t + B_t = B_{t+1}/R_t + Q_t(j) + D_t + W_t(j)n_t(j) + P_t r_t^k u_t \bar{k}_{t-1} - P_t \vartheta(u_t)\bar{k}_{t-1}, \tag{VII.7}$$

where $x_t$ is investment, $T_t$ denotes nominal lump–sum taxes (transfers if negative), $B_t$ is the quantity one-period riskless nominal bond acquired at $t$ and maturing at $t+1$, $R_t$ is the nominal interest rate on bonds, $Q_t(j)$ is the net cash flow from household $j$'s portfolio of state contingent securities, $D_t$ is the equity payout received from the ownership of firms, and $r_t^k$ is the real rental rate of capital. The capital utilization rate $u_t$ transforms physical capital $\bar{k}_t$ into the service flow of effective capital $k_t$ according to

$$k_t = u_t \bar{k}_{t-1}, \tag{VII.8}$$

and the effective capital is rented to intermediate firms at the real rental rate $r_t^k$. The costs of capital utilization per unit of capital is given by the convex function $\vartheta(u_t)$. We assume that $u = 1$, $\vartheta(1) = 0$, and we define

$$\eta_u \equiv \frac{\vartheta''(1)/\vartheta'(1)}{1 + \vartheta''(1)/\vartheta'(1)}.$$

Later, we estimate $\eta_u$ rather than the elasticity $\vartheta''(1)/\vartheta'(1)$ to avoid convergence issues.

The physical capital accumulates according to

$$\bar{k}_t = (1-\delta)\,\bar{k}_{t-1} + \mathrm{e}^{\epsilon_{x,t}}\left(1 - S\left(\frac{x_t}{x_{t-1}}\right)\right)x_t \tag{VII.9}$$

where $\delta$ is the depreciation rate of capital, $S(\cdot)$ is an adjustment cost function which satisfies $S(\gamma_z) = S'(\gamma_z) = 0$ and $S''(\gamma_z) = \eta_k > 0$, $\gamma_z$ is the steady-state growth rate of technology, and $\epsilon_{x,t}$ is an investment shock, evolving according to

$$\epsilon_{x,t} = \rho_x \epsilon_{x,t-1} + \sigma_x \zeta_{x,t}, \quad \zeta_{x,t} \sim Niid(0,1).$$

Households set nominal wages in a staggered fashion, following Calvo (1983). In each period, a fraction $\alpha_w$ of households cannot choose their wage optimally, but adjust it to keep up with the increase in the general wage level in the previous period according to the indexation rule

$$W_t(j) = \gamma_z \pi^{1-\gamma_w} \pi_{t-1}^{\gamma_w} W_{t-1}(j), \tag{VII.10}$$

where $\pi_t \equiv P_t/P_{t-1}$ represents the gross inflation rate, $\pi$ is steady–state (or trend) inflation and the coefficient $\gamma_w \in [0,1]$ is the degree of indexation to past wages. The remaining fraction of households chooses instead an optimal wage, to maximize (VII.6), subject to the labor demand function (VII.4).

## VII.2 Business sector

### VII.2.1 Final good producers

At every point in time $t$, perfectly competitive firms produce a final good $y_t$ by combining a continuum of intermediate goods $y_t(i)$, $i \in [0,1]$, according to the technology

$$y_t = \left(\int_0^1 y_t(i)^{\frac{1}{\lambda_{p,t}}}\,\mathrm{d}i\right)^{\lambda_{p,t}}, \tag{VII.11}$$

where $\lambda_{p,t} = \lambda_p \mathrm{e}^{\epsilon_{p,t}}$, $\lambda_p$ is the desired steady–state price markup over the marginal cost of intermediate firms, and the markup shock $\epsilon_{p,t}$ is assumed to evolve according to

$$\epsilon_{p,t} = \rho_p \epsilon_{p,t-1} + \sigma_p \zeta_{p,t}, \quad \zeta_{p,t} \sim Niid(0,1).$$

Final good producing firms take their output price, $P_t$, and their input prices, $P_t(i)$, as given and beyond their control. Profit maximization implies the following first-order condition

$$y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\lambda_{p,t}}{\lambda_{p,t}-1}} y_t. \tag{VII.12}$$

Integrating (VII.12) and imposing (VII.11), we obtain the following relationship between the final good and the prices of the intermediate goods

$$P_t \equiv \left( \int_0^1 P_t(i)^{\frac{1}{\lambda_{p,t}-1}} \mathrm{d}i \right)^{\lambda_{p,t}-1}. \tag{VII.13}$$

### VII.2.2 Intermediate-goods firms

Intermediate good $i \in [0,1]$ is produced by a monopolist firm using the following production function

$$y_t(i) = k_t(i)^{\theta} \left( \mathrm{e}^{z_t} h_t(i) \right)^{1-\theta} - \mathrm{e}^{z_t} F, \tag{VII.14}$$

where $\theta$ denotes the capital share, $k_t(i)$ and $h_t(i)$ denote the amounts of capital and effective labor used by firm $i$, $F$ is a fixed cost of production that ensures that profits are zero in steady state, and $z_t$ is an exogenous labor–augmenting productivity, evolving according to

$$z_t = \log(\gamma_z) + z_{t-1} + \epsilon_{z,t}$$

where $\epsilon_{z,t}$ is a productivity shock

$$\epsilon_{z,t} = \rho_z \epsilon_{z,t-1} + \sigma_z \zeta_{z,t}, \quad \zeta_{z,t} \sim Niid(0,1).$$

In addition, we assume that intermediate firms rent capital and labor in perfectly competitive factor markets.

Intermediate firms set prices in a staggered fashion, following Calvo (1983). In each period, a fraction $\alpha_p$ of firms cannot choose their price optimally, but adjust it to keep up with the increase in the general price level in the previous period according to the indexation rule

$$P_t(i) = \pi^{1-\gamma_p} \pi_{t-1}^{\gamma_p} P_{t-1}(i), \tag{VII.15}$$

where the coefficient $\gamma_p \in [0,1]$ indicates the degree of indexation to past prices. The remaining fraction of firms chooses its price $P_t^{\star}(i)$ optimally, by maximizing the present discounted value of future profits

$$\mathrm{E}_t \sum_{s=0}^{\infty} (\beta \alpha_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left\{ \Pi^p_{t,t+s} P_t^{\star}(i) y_{t+s}(i) - \left[ W_{t+s} h_{t+s}(i) - r^k_{t+s} k_{t+s}(i) \right] \right\} \tag{VII.16}$$

where $\Lambda_t$ denotes the Lagrange multiplier on household $j$'s nominal budget constraint and

$$\Pi^p_{t,t+s} \equiv \begin{cases} \prod_{\nu=1}^{s} \pi^{1-\gamma_p} \pi_{t+\nu-1}^{\gamma_p} & s > 0 \\ 1 & s = 0, \end{cases} \tag{VII.17}$$

subject to the demand from final goods firms given by equation (VII.12) and the production function (VII.14).

### VII.3 Public sector

The government faces the budget constraint

$$P_t g_t + B_t = T_t + \frac{B_{t+1}}{R_t}.$$

The government spending rule is, as before, given by

$$g_t e^{-z_t} = \bar{g}^s \tilde{g}_t e^{\epsilon_{g,t}},$$

where $\bar{g}^s$ denotes the deterministic steady–state value of $g_t e^{-z_t}$. The endogenous component of policy $\tilde{g}_t$ obeys

$$\log(\tilde{g}_t) = -\varphi_g \left(\Delta \log(y_t) - \log(\gamma_z)\right), \tag{VII.18}$$

and the exogenous component evolves according to

$$\epsilon_{g,t} = \rho_g \epsilon_{g,t-1} + \sigma_g \zeta_{g,t}, \quad \zeta_{g,t} \sim Niid(0,1).$$

The short–run nominal interest rate $R_t$ is set according to

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{y_t}{y_{p,t}}\right)^{\phi_y}\right]^{(1-\phi_R)} \left(\frac{y_t y_{p,t-1}}{y_{p,t} y_{t-1}}\right)^{\phi_{\Delta y}} e^{\sigma_R \zeta_{R,t}} \tag{VII.19}$$

where $R$ is the steady state of the gross nominal interest rate and $\zeta_{R,t}$ is $Niid\,(0,1)$. The monetary authorities follow a generalized Taylor rule by gradually adjusting the nominal rate in response to inflation and the output gap, defined as the ratio of actual to potential output (*i.e.* the level of output that would prevail under flexible prices and wages and constant elasticity of substitution among intermediate goods and labor types). In addition, there is a short-run feedback from the change in the output gap.

### VII.4 Market clearing

Market clearing condition on final goods market is given by

$$y_t = c_t + x_t + g_t + \vartheta\left(u_t\right) \bar{k}_{t-1}. \tag{VII.20}$$

$$\Delta_{p,t} y_t = \left(u_t \bar{k}_{t-1}\right)^{\theta} \left(e^{z_t} n_t\right)^{1-\theta} - e^{z_t} F \tag{VII.21}$$

where $\Delta_{p,t} = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\lambda_p \epsilon_{p,t}}{\lambda_p \epsilon_{p,t-1}}} \mathrm{d}i$ is a measure of the price dispersion.

## VII.5 Non-Ricardian Agents

As in Cogan et al. (2010), we also consider a version of the SW model featuring a fraction $\omega$ of non-Ricardian agents. These agents do not have access to financial markets and simply consume their disposable income in each and every period. Disposable income, in turn, equals wage receipts net of lump-sum taxes.

## VII.6 Detailed Estimation Results

Four parameters are calibrated before estimation. These are: the discount factor $\beta = 0.99$, the depreciation rate $\delta = 0.025$, the steady-state wage markup $\lambda_w = 0.15$, and the steady-state share of government spending in output $s_g = 0.20$.

In table VII.1, we report our estimation results in the SW model. As before, we consider analogs to specifications (3) and (4). In (3), we impose $\varphi_g = 0$ while in (4) this parameter is freely estimated. In both specifications, we also impose that the mass of non-Ricardian agents be zero, i.e. $\omega = 0$. In specifications (3′) and (4′), we freely estimate $\omega$, while considering $\varphi_g = 0$ in (3′) and $\varphi_g \neq 0$ in (4′).

Table VII.1. Estimated parameters in the SW model

| | Prior | Posterior | | | |
|---|---|---|---|---|---|
| | | SW Specification | | SW with NR agents | |
| | | Model 3 | Model 4 | Model 3′ | Model 4′ |
| $\omega$ | $\mathscr{B}$ [0.50,0.2] | – | – | 0.165 [0.095,0.231] | 0.149 [0.085,0.211] |
| $\varphi_g$ | $\mathscr{U}$ [0,0.65] | – | 0.752 [0.545,0.964] | – | 0.728 [0.528,0.919] |
| $\alpha_g$ | $\mathscr{U}$ [0,1.30] | −0.199 [−0.454,0.040] | −0.626 [−0.914,−0.332] | −0.870 [−1.201,−0.552] | −1.133 [−1.443,−0.819] |
| $\nu$ | $\mathscr{N}$ [2,0.75] | 2.829 [1.882,3.782] | 2.952 [1.991,3.895] | 3.054 [2.062,4.030] | 3.156 [2.184,4.094] |
| $h$ | $\mathscr{B}$ [0.6,0.1] | 0.642 [0.579,0.709] | 0.634 [0.557,0.709] | 0.571 [0.484,0.666] | 0.585 [0.480,0.693] |
| $\eta_u$ | $\mathscr{B}$ [0.5,0.1] | 0.544 [0.395,0.699] | 0.665 [0.487,0.852] | 0.587 [0.440,0.736] | 0.608 [0.444,0.770] |
| $\eta_k$ | $\mathscr{N}$ [4,1] | 6.066 [4.698,7.438] | 5.450 [3.886,6.979] | 5.747 [4.392,7.081] | 5.624 [4.260,6.973] |
| $\theta$ | $\mathscr{N}$ [0.33,0.05] | 0.174 [0.144,0.204] | 0.204 [0.173,0.234] | 0.179 [0.154,0.205] | 0.200 [0.172,0.229] |
| $\log(\gamma_z)$ | $\mathscr{N}$ [0.5,0.1] | 0.485 [0.399,0.574] | 0.458 [0.360,0.548] | 0.476 [0.392,0.559] | 0.461 [0.373,0.548] |
| $\alpha_p$ | $\mathscr{B}$ [0.66,0.1] | 0.910 [0.884,0.941] | 0.876 [0.834,0.925] | 0.906 [0.865,0.947] | 0.884 [0.844,0.927] |
| $\alpha_w$ | $\mathscr{B}$ [0.66,0.1] | 0.906 [0.867,0.945] | 0.889 [0.852,0.929] | 0.921 [0.884,0.959] | 0.907 [0.872,0.944] |
| $\gamma_p$ | $\mathscr{B}$ [0.5,0.15] | 0.083 [0.028,0.135] | 0.089 [0.032,0.144] | 0.074 [0.026,0.121] | 0.083 [0.029,0.135] |
| $\gamma_w$ | $\mathscr{B}$ [0.5,0.15] | 0.703 [0.566,0.842] | 0.725 [0.592,0.864] | 0.712 [0.583,0.851] | 0.724 [0.595,0.854] |

Continued on next page

| | Prior | Posterior | | | |
|---|---|---|---|---|---|
| | | SW Specification | | SW with NR agents | |
| | | Model 3 | Model 4 | Model 3′ | Model 4′ |
| $\lambda_p$ | $\mathscr{N}$ [0.15,0.1] | 0.117 [0.031,0.204] | 0.226 [0.136,0.322] | 0.107 [0.021,0.192] | 0.196 [0.109,0.283] |
| $\phi_R$ | $\mathscr{B}$ [0.6,0.2] | 0.805 [0.769,0.841] | 0.800 [0.767,0.835] | 0.820 [0.784,0.856] | 0.819 [0.788,0.852] |
| $\phi_\pi$ | $\mathscr{N}$ [1.7,0.3] | 1.589 [1.364,1.805] | 1.528 [1.338,1.720] | 1.657 [1.421,1.902] | 1.597 [1.380,1.788] |
| $\phi_y$ | $\mathscr{N}$ [0.125,0.05] | 0.067 [0.026,0.113] | 0.018 [−0.011,0.056] | 0.063 [0.003,0.108] | 0.018 [−0.012,0.051] |
| $\phi_{\Delta y}$ | $\mathscr{N}$ [0.125,0.05] | 0.287 [0.239,0.338] | 0.252 [0.199,0.302] | 0.281 [0.232,0.331] | 0.252 [0.201,0.301] |
| $\rho_w$ | $\mathscr{B}$ [0.6,0.2] | 0.253 [0.144,0.366] | 0.244 [0.133,0.352] | 0.277 [0.159,0.387] | 0.272 [0.159,0.384] |
| $\rho_\beta$ | $\mathscr{B}$ [0.6,0.2] | 0.759 [0.689,0.832] | 0.788 [0.710,0.872] | 0.824 [0.746,0.899] | 0.842 [0.757,0.931] |
| $\rho_x$ | $\mathscr{B}$ [0.6,0.2] | 0.758 [0.662,0.839] | 0.888 [0.754,0.997] | 0.780 [0.683,0.873] | 0.873 [0.764,0.977] |
| $\rho_p$ | $\mathscr{B}$ [0.6,0.2] | 0.858 [0.781,0.934] | 0.871 [0.783,0.964] | 0.898 [0.830,0.987] | 0.873 [0.788,0.958] |
| $\rho_z$ | $\mathscr{B}$ [0.3,0.2] | 0.251 [0.129,0.377] | 0.180 [0.062,0.298] | 0.184 [0.059,0.297] | 0.132 [0.023,0.228] |
| $\rho_g$ | $\mathscr{B}$ [0.6,0.2] | 0.987 [0.977,0.998] | 0.978 [0.958,0.998] | 0.989 [0.980,0.999] | 0.984 [0.970,0.999] |
| $\sigma_w$ | $\mathscr{IG}$ [0.1,2] | 0.231 [0.195,0.267] | 0.239 [0.202,0.275] | 0.223 [0.186,0.260] | 0.230 [0.193,0.266] |
| $\sigma_\beta$ | $\mathscr{IG}$ [0.1,2] | 0.041 [0.029,0.052] | 0.041 [0.026,0.057] | 0.046 [0.032,0.060] | 0.046 [0.027,0.064] |
| $\sigma_x$ | $\mathscr{IG}$ [0.1,2] | 0.429 [0.353,0.507] | 0.424 [0.310,0.537] | 0.437 [0.359,0.512] | 0.407 [0.325,0.482] |
| $\sigma_p$ | $\mathscr{IG}$ [0.1,2] | 0.049 [0.033,0.065] | 0.055 [0.036,0.073] | 0.042 [0.027,0.056] | 0.053 [0.034,0.070] |
| $\sigma_z$ | $\mathscr{IG}$ [0.1,2] | 0.860 [0.772,0.951] | 0.933 [0.824,1.041] | 0.848 [0.762,0.935] | 0.905 [0.804,1.004] |
| $\sigma_g$ | $\mathscr{IG}$ [0.1,2] | 1.116 [1.012,1.214] | 1.207 [1.078,1.330] | 1.114 [1.013,1.213] | 1.187 [1.069,1.307] |
| $\sigma_R$ | $\mathscr{IG}$ [0.1,2] | 0.229 [0.208,0.249] | 0.222 [0.202,0.243] | 0.227 [0.207,0.247] | 0.222 [0.202,0.241] |
| $\mathscr{L}$ | | −1298.946 | −1277.683 | −1296.944 | −1275.890 |

**Notes:** Sample period: 1960:1-2007:4. Notice that all the data have been multiplied by 100. In the column labelled Prior, $\mathscr{B}$, $\mathscr{N}$, $\mathscr{U}$, $\mathscr{IG}$ denote Beta, Normal, Uniform, and Inverse Gamma prior densities, respectively. The figures in brackets are the prior mean and the prior standard deviation. In the columns labelled Posterior, the figures correspond to the posterior mean and the figures in brackets below the posterior mean indicate the posterior 90% interval. The model specification are as follows. In (3), we impose $\omega = 0$, $\varphi_g = 0$, and $\alpha_g \neq 0$; in (4), we impose $\omega = 0$, $\varphi_g \neq 0$, and $\alpha_g \neq 0$; in (3′) we impose $\omega \neq 0$, $\varphi_g = 0$, and $\alpha_g \neq 0$; in (4′), we impose $\omega \neq 0$, $\varphi_g \neq 0$, and $\alpha_g \neq 0$. Finally, $\mathscr{L}$ denotes the marginal likelihood.

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