Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap

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Online Appendix: Non-cooperative Loss Function

Section 7 of the text reports the results for optimal monetary policy for the foreign policy-maker when policy is chosen non-cooperatively, in contrast to the main results of the paper, which pertain to the cooperative equilibrium policy choice.

There are some difficult technical aspects of the specification of non-cooperative policy making environment in a multi-country DSGE environment. As is well known, in an infinite horizon strategic setting there is generally a large number of equilibria indexed by threats, or trigger strategies (e.g. Chari and Kehoe, 1990). We narrow the focus down from this by concentrating on discretionary policy-making for each authority, so that all expectations of future variables are taken as given (as in the previous section). In addition, we closely follow Clarida et al. (2002) in deriving a loss function for non-cooperative policy-making, except that we focus on time preference shocks, as in the main text, and, also in contrast to Clarida et al. (2002) we allow for home bias in preferences. As in Clarida et al. (2002), the key strategic assumption we make is that the each country chooses its policy rate to influence its domestic output and inflation rate, taking output and inflation of the foreign county as given. In fact, because the home country is constrained by the zero lower bound, effectively, only the foreign country makes a policy choice when policy is made under discretion.

In evaluating the loss function for monetary policy in New Keynesian models, it is well known (see Woodford 2003) that first order terms arise due to the incentive for monetary authorities to manipulate policy so as to eliminate distortions. In the closed economy, or the cooperative two-country economy, for instance, there is a markup distortion due to monopoly pricing. In the main text, we followed Engel (2011) and Clarida et al. (2002) in removing this distortion with the assumption of offsetting constant input subsidies that are given to producers. This ensures that the steady state level of production is efficient. In the non-cooperative case, the situation gets more complicated. This is because, as shown in Corsetti and Pesenti (2001) and Clarida et al. (2002), there are two competing deviations from efficiency. The first is the monopoly distortion as before, while the second is the incentive of any individual country's monetary authority to manipulate the terms of trade in its favor. Again, we follow Clarida et al. (2002) in choosing a steady state subsidy so as to just offset the combination of these two distortions. That is, we derive the loss function approximation around a steady state which balances the incentives for an individual monetary authority to manipulate the terms of trade against the distortions involved in monopoly price setting. In this, our analysis differs from that of Benigno and Benigno (2006), who derive a cooperative loss function by approximating around the *distorted* steady state of the competitive equilibrium [1] .

Proceeding in this manner, then, we can outline the details as follows. For simplicity, in this analysis we will ignore the fiscal policy decision, abstracting away from the role of public goods, and hence assuming that all output goes to private consumption. First, we identify the incentive for the home country government to manipulate the terms of trade, and we choose a subsidy policy to offset this distortion, combined with the monopoly pricing distortion.

Say that the home country government chooses steady state output subject to the steady state equilibrium conditions. These conditions are described by (**??**)-(**??**) for both economies, as well as (**??**) and (**??**), but under the condition that all prices are flexible, so that $\kappa = 1$, and therefore the economy is in steady state. In the steady state, utility is given by

$$U(\bar{C}, 1) - V(\bar{N}).$$

Then using that fact that in the steady state, $N = Y$, the optimal output level for the home government will satisfy[2]

$$U_C(\bar{C}, 1)\frac{\partial \bar{C}}{\partial \bar{Y}} - V_N(\bar{Y}) = 0$$

Then, using the log linear approximations to (**??**), (**??**), and (**??**), we can write this as:

$$U_C(\bar{C}, 1)\bar{C}\delta_1 - V_N(\bar{Y})\bar{Y} = 0$$

where $\delta_1 = \frac{D+v-1}{2D}$

In a steady state of the competitive equilibrium, optimal pricing and labor supply satisfies:

$$1 = \frac{\theta}{\theta - 1}(1 - s)\frac{V_N}{U_C} = \frac{\theta}{\theta - 1}(1 - s)\frac{V_N(\bar{Y})\bar{Y}}{U_C(\bar{C})\bar{C}} \tag{1}$$

[1]If the objective was to compare welfare across non-cooperative and cooperative equilibria, then it would be desirable to follow the approach of Benigno and Benigno (2006), since by eliminating the terms of trade externality, a source of welfare loss from non-cooperative policy-making is ignored. In the current paper however, we do not focus on welfare comparisons. Benigno and Benigno (2006) also focus on a more general specification of strategy spaces in a non-cooperative equilibrium than that of Clarida et al. (2002). A final and important difference from our simplifying environment and that of Benigno and Benigno (2006) is that, in order to negate the presence of initial conditions in the welfare loss, they describe monetary policy from a 'timeless perspective' as in Benigno and Woodford (2006). In our specification, we avoid these initial conditions by the use of the steady state production subsidy.

[2]In this appendix, we differentiate between labor and output, because their responses are only the same up to a *first* order approximation. But in the use of the non-cooperative loss function of section 7 in the main text, we revert to the n_t notation, since in that case, only first order terms are relevant.

where $(\frac{\theta}{\theta-1})$ represents the optimal monopoly markup, and s represents a production subsidy. The second equality follows from the fact that in a symmetric steady state, $\bar{C} = \bar{Y}$. In order to remove the incentive to implement monetary policy so as to eliminate this combination of distortions, then the optimal subsidy must satisfy

$$1 = \frac{\theta}{\theta - 1}(1 - s)\delta_1 \tag{2}$$

This ensures that in the steady state, we have the condition $U_C(\bar{C}, 1)\bar{C}\delta_1 = V_N(\bar{Y})\bar{Y}$.

With this condition, we can now define the objective function for the monetary authority in a non-cooperative policy environment. First, take the period utility function

$$U(C, \epsilon) - V(N)$$

. Take a second order approximation of this around the steady state. Start with the term $U(C, \epsilon)$. Then we have:

$$U(C, \epsilon) \approx U(\bar{C}, 1) + U_C\bar{C}(c + \frac{(1 - \sigma)}{2}c^2 + c\varepsilon) + \text{t.i.p.} + \parallel o \parallel^3 \tag{3}$$

where $c \equiv \log(\frac{C}{\bar{C}})$, and t.i.p. indicates terms independent of policy decisions.

Now define $\hat{c} = c - \tilde{c}$, where $\tilde{c} \equiv \log(\frac{\tilde{C}}{\bar{C}})$, the log of the deviation of the efficient, flexible price consumption level from the non-stochastic steady state consumption level. We may then re-write (**??**) as:

$$U(C, \epsilon) \approx U(\bar{C}, 1) + U_C\bar{C}(\hat{c} + \frac{(1 - \sigma)}{2}\hat{c}^2 + (1 - \sigma)\hat{c}\tilde{c} + \hat{c}\varepsilon) + \text{t.i.p.} + \parallel o \parallel^3 \tag{4}$$

Likewise, we may approximate the term $V(N)$ as follows

$$V(N) \approx V(\bar{N}, 1) + V_N\bar{N}(n + \frac{(1 + \phi)}{2}n^2) = V(\bar{N}, 1) + V_N\bar{N}(\hat{n} + \frac{(1 + \phi)}{2}\hat{n}^2 + (1 + \phi)\tilde{n}\hat{n}) + \text{t.i.p.} + \parallel o \parallel^3 \tag{5}$$

These two expressions contain linear terms in $\hat{c}$ and $\hat{n}$ which are 2nd-order accurate. A second order expansion for $\hat{c}$ gives:

$$\hat{c} = \delta_1\hat{y} + \delta_2\hat{y}^* - \frac{(\hat{y} - \hat{y}^*)^2(2 - v)v(1 - v)^2(1 - \sigma)^2}{8D^2} + \parallel o \parallel^3, \tag{6}$$

where $\delta_2 \equiv \frac{D+1-v}{2D}$.

A second order expansion for n can be derived using the results of Clarida et al. (2002)

and Woodford (2003), and gives:

$$\tilde{n} = \tilde{y} + \frac{\theta}{2}\sigma^2_{pHt} + \parallel o \parallel^3 \tag{7}$$

σ^2_{pHt} is the cross sectional variance of the home country good price. Woodford (2003) shows that in the presence of Calvo pricing, this can be rewritten in the following way:

$$\sum_{t=0}^{\infty} \beta^t \sigma^2_{pHt} = E_t \sum_{t=0}^{\infty} \beta^t \frac{\pi_{Ht}}{k}$$

Now substituting (??) and (??) into (??) and (??), and using (??), we obtain the present value loss function:

$$V^{NC*} = -E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(\hat{y}_t - \hat{y}_t{}^*)^2 (2-v) v (1-v)^2 (1-\sigma)^2}{8\delta_1 D^2} - \frac{(1-\sigma)}{2} (\delta_1 \hat{y}_t + \delta_2 \tilde{y}_t{}^*)^2 \right.$$

$$\left. + \frac{(1+\phi)}{2} \tilde{y}_t{}^2 + \frac{\theta}{2k} \pi^2_{Ht} + ((1-\sigma)\tilde{c}_t + \varepsilon_t - (1+\phi)\tilde{y}_t)\hat{y}_t \right] + \text{t.i.p.} + \parallel o \parallel^3 \tag{8}$$

The non-cooperative loss is a function of the domestic and foreign output gap, domestic inflation, and a cross term, given by the last expression in (??). This captures an interaction between the welfare effects of movements in consumption and output in the efficient flexible price equilibrium and the output gap. In Clarida et al. (2002), this interaction term is zero. But with home bias in preferences, it is in general non-zero, when $\varepsilon \neq 0$.

The assumptions on strategy choices in Clarida et al. (2002) are followed here. Each country's monetary authority chooses its level of output, taking the output level of the other country as given. This choice is implemented by an interest rate policy. As in the cooperative case, we focus on an equilibrium where the shock ε is negative and large enough that the home country if unconstrained would always choose a negative interest rate. As a result, the home country is always at the zero bound. But the foreign country may or may not be at the zero bound, depending on the size of the shock, and the importance of home bias in preferences, as before.

The optimal policy for the foreign government in a noncooperative game is described as the solution to

$$\max_{\widehat{y}_t^*,\pi_t^*,r_t^*} L_t^{NC} = v_t^{NC*} + \lambda_t \left[\pi_t^* - k(\phi + s\tfrac{(1+D)}{2}\widehat{y}_t^* + s\tfrac{(D-1)}{2}\widehat{y}_t) - \beta E_t \pi_{t+1}^*\right] \tag{9}$$

$$+\psi_t \left[sE_t(\widehat{y}_{t+1}^* - \widehat{y}_t^*)(D+1) + sE_t(\widehat{y}_{t+1} - \widehat{y}_t)(D-1) - E_t\left(r_t^* - \widetilde{r}_t^* - \pi_{t+1}^*\right)\right] + \gamma_t r_t^*$$

where

$$v_t^{NC*} = -[\frac{(\hat{y}_t - \hat{y}_t{}^*)^2(2-v)v(1-v)^2(1-\sigma)^2}{8\gamma_1 D^2} - \frac{(1-\sigma)}{2}(\delta_1 \hat{y}_t{}^* + \delta_2 \tilde{y}_t)^2$$

$$+\frac{(1+\phi)}{2}\tilde{y}_t{}^{*2} + \frac{\theta}{2k}\pi_t^{*2} + ((1-\sigma)\tilde{c}_t{}^* - (1+\phi)\tilde{y}_t{}^*)\hat{y}_t{}^*]$$

A discretionary equilibrium of the non-cooperative game in the asymmetric world economy (where the home country is stuck in a liquidity trap) is obtained by the first order conditions to problem (**??**), together the with inflation and Euler equations for the home country, constrained by the condition $r = 0$, and closed by the condition that all expectational variables adjust with persistence μ, following the persistence of the underlying preference shock. The solution for the non-cooperative interest rate rule is illustrated in Figure 6 of the main text.

Appendix C

Figure A1 illustrates the trade-offs implicit in the comparison of the non-cooperative outcome with the policy under which the foreign monetary authority sets the foreign interest rate equal to the foreign conditional natural rate of interest. Targeting the conditional natural rate of interest (when it is above zero) involves setting the interest rate so that foreign inflation is zero - which sustains the foreign *conditional* flexible price equilibrium. But in the Figure, we see that involves a significantly positive output gap. By contrast, in the non-cooperative outcome, the foreign policy maker sets a higher policy interest rate (when the optimal rate exceeds the zero bound), which precipitates a deflation, but attains a lower level of the output gap.

Figure A1: Policy Rates: Noncooperative and Conditional Natural Rates