Online Appendix:
"Uncertainty and Economic Activity: Evidence from Business Survey Data"

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This Appendix presents a simple model and some additional robustness checks to accompany "Uncertainty and Economic Activity: Evidence from Business Survey Data." What follows is intended only for online publication.

A A Simple Model

In the main text we rely upon forecast disagreement as a proxy for subjective uncertainty. The purpose of this section is to provide a highly stylized model to investigate the conditions under which this proxy is related to underlying uncertainty. The main conclusion is that, if firms receive signals about future conditions and those signals are neither perfectly informative nor perfectly uninformative, then disagreement and uncertainty will co-move.

To illustrate the relationship between concepts such as disagreement, uncertainty, and cross-sectional variance, we use the following simple two-period model: tomorrow's business situation of firms is unknown today. It can move in three directions. Business situations can improve $(+1)$, stay the same (0) or deteriorate (-1). For each firm, nature draws the change in business situation from the following probability distribution: $[0.5 \times (1-p), p, 0.5 \times (1-p)]$, which is assumed to be known to the firms. The cross-sectional variance of the future business situation is obviously $(1-p)$, a decreasing function of p. Furthermore, we assume that businesses receive a signal about the change in their business situation, with a structure illustrated in Table 1. For instance, if tomorrow's true state is $+1$, the signal can be $+1$ (with probability q) and 0 with probability $(1-q)$. q thus measures the informativeness of the signal.

Table 1: A Simple Two-Period Model of Firms' Business Situations

State Tomorrow

	$0.5 \times (1-p) \swarrow$			$\downarrow p$			$\searrow (1-p) \times 0.5$	
	+1			0			-1	
$q \swarrow$		$\searrow (1-q)$	$0.5 \times (1-q) \swarrow$	$q \downarrow$	$\searrow (1-q) \times 0.5$	$(1-q) \swarrow$		$\searrow q$
+1		0	+1	0	-1	0		-1

Signal

Using Bayes' Law we can compute the probabilities of the true state, conditional on a signal:

1. (a) $Prob(state = 1 | signal = 1) = \frac{q \times 0.5 \times (1-p)}{q \times 0.5 \times (1-p) + 0.5 \times (1-q) \times p}$

   (b) $Prob(state = 0 | signal = 1) = \frac{0.5 \times (1-q) \times p}{q \times 0.5 \times (1-p) + 0.5 \times (1-q) \times p}$

   (c) $Prob(state = -1 | signal = 1) = 0$

2. (a) $Prob(state = 1|signal = 0) = \frac{(1-q)\times 0.5\times(1-p)}{(1-q)\times 0.5\times(1-p)+q\times p+(1-q)\times 0.5\times(1-p)}$
   (b) $Prob(state = 0|signal = 0) = \frac{q\times p}{(1-q)\times 0.5\times(1-p)+q\times p+(1-q)\times 0.5\times(1-p)}$
   (c) $Prob(state = -1|signal = 0) = \frac{(1-q)\times 0.5\times(1-p)}{(1-q)\times 0.5\times(1-p)+q\times p+(1-q)\times 0.5\times(1-p)}$
3. (a) $Prob(state = 1|signal = -1) = 0$
   (b) $Prob(state = 0|signal = -1) = \frac{0.5\times(1-q)\times p}{q\times 0.5\times(1-p)+0.5\times(1-q)\times p}$
   (c) $Prob(state = -1|signal = -1) = \frac{q\times 0.5\times(1-p)}{q\times 0.5\times(1-p)+0.5\times(1-q)\times p}$

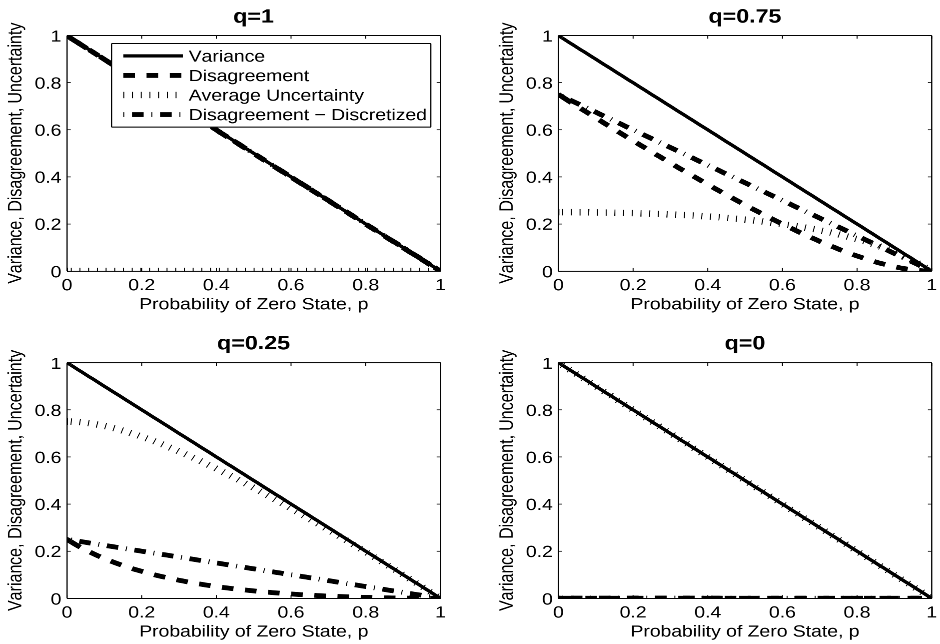
From these conditional probabilities, conditional expectations and variances can be computed. And these, in turn, allow us to calculate (i) the variance of the conditional expectations over the change in business situations, which is a measure of disagreement; and (ii) the average conditional variance over the change in the business situation of a firm, which is a measure of the average (subjective) uncertainty in the population of firms.

We begin with the case of perfectly informative signals: $q = 1$. In this case, obviously, survey disagreement moves one for one with the variance of tomorrow's state, but firms do not experience any subjective uncertainty about the change in their business situation. With $q = 1$ and in a two period set up disagreement and uncertainty do not comove. The fact that we find substantial forecast errors in the IFO-BCS suggests that this extreme case may not be realistic. But even if we assumed $q = 1$ and thus certainty for the immediate future, higher disagreement today indicates a higher cross-sectional variance in business situations tomorrow and thus higher uncertainty about business situations for periods beyond the immediate future, as long as the variance of future innovations to the business situation of firms has some persistence beyond the immediate period and signals are not perfectly informative about this farther future. Tables 4 and 5 in the main text show that all uncertainty measures are highly autocorrelated.

Next, we look at the cases with imperfectly informative signals, i.e. $q < 1$. We know from the conditional variance decomposition formula that if the variance of tomorrow's state increases either the variance of the conditional expectations over tomorrow's state (disagreement) or the average conditional variance over tomorrow's state (average subjective uncertainty) has to increase. Both may increase. The following Figure 1 shows for various levels of the signal precision, q, that the latter is indeed the case in this model. The actual cross-sectional variance of tomorrow's state is a (linearly declining) function of the probability of drawing the intermediate business state tomorrow, p, as depicted by the solid line; the variance of the conditional expectations over tomorrow's state (disagreement) is shown by the dashed line and the average conditional variance over tomorrow's state (subjective uncertainty) is the dotted line.

Notice that for intermediate signal qualities ($q = 0.75$ and $q = 0.25$), both disagreement and uncertainty decline in p, and move in the same direction as the actual variance of the state tomorrow. In short, in this simple example cross-sectional variance, disagreement and subjective uncertainty comove with each other, and, given a signal quality q are all determined by p, the probability of drawing the intermediate business state tomorrow. Since p and the cross-sectional variance are linearly and negatively related, we can equivalently say that both disagreement and subjective uncertainty comove with the actual cross-sectional variance. Of course, if the signal was such that it left everybody with the same conditional expectation, i.e. completely uninformative ($q = 0$), then disagreement would always be zero. Only subjective uncertainty would then be affected by p and equal the actual cross-sectional variance, which is

Figure 1: Cross-sectional Variance, Disagreement and Uncertainty



seen in the right lower panel of Figure 1. In this case, disagreement and its fluctuations would not be a good measure of either subjective uncertainty or cross-sectional variance and their fluctuations. Since in the IFO-BCS disagreement and cross-sectional error variance are highly correlated, the case of a completely uninformative signal is unlikely to hold.

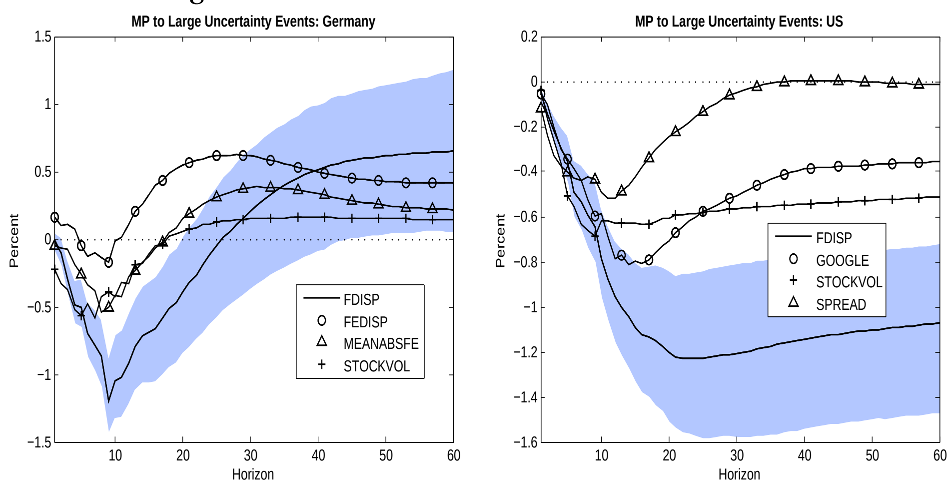
Finally, in order to translate the continuous disagreement measure – the variance of the conditional expectations over the change in business situations – into discrete disagreement in survey answers, where only $[-1,0,1]$ as an answer are possible, we assume that if the firm receives zero as a signal, it will answer zero, simply because the conditional expectation is zero in this case (by the symmetry of the model). Furthermore, if it receives a signal equal to 1, the probability of answering 1 in the survey equals the expectation conditional on the signal being 1, which ranges from 1 (if $p = 0$) to 0 (if $p = 1$). This conditional expectation, $E[state|signal = 1]$, is computed from the conditional probabilities above. This means that the closer the conditional expectation is to unity, the more likely firms are going to respond with 1 in the survey. Symmetrically this is also true for the case of receiving a signal that equals -1. With these assumptions, the variance of the survey answers is given by ($E[answer]$ is computed analogously):

$$
\begin{aligned}
VAR[answer] = \quad & (1-E[answer])^2 E[state|signal=1] \times Prob(signal=1) + \\
& (0-E[answer])^2 (1-E[state|signal=1]) \times Prob(signal=1) + \\
& (0-E[answer])^2 Prob(signal=0) + \\
& (0-E[answer])^2 (1-E[state|signal=-1]) \times Prob(signal=-1) + \\
& (-1-E[answer])^2 (E[state|signal=-1]) \times Prob(signal=-1)
\end{aligned}
$$

This discretized version of disagreement is also shown in Figure 1, by the dashed-dotted line. It closely follows the continuous disagreement measure, which gives us confidence that the discretized disagreement measure in the IFO-BCS and its fluctuations are good approximations to both the underlying continuous disagreement and subjective uncertainty and their fluctuations.

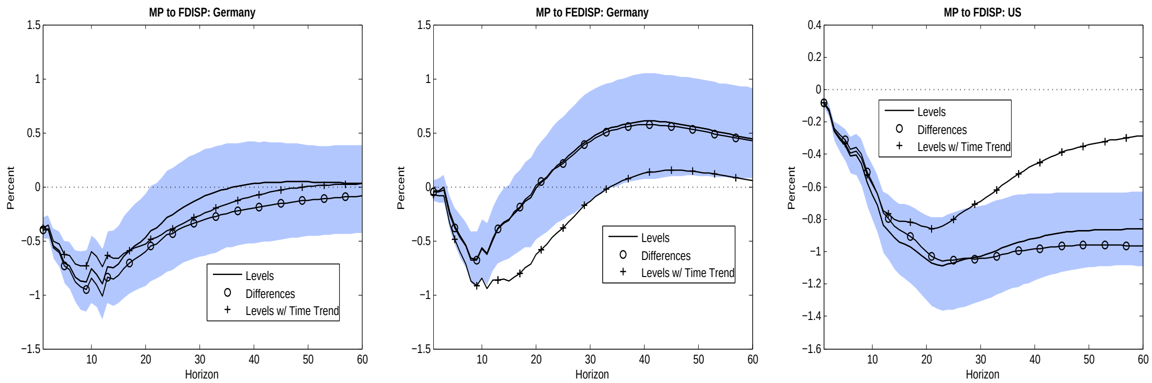
B Additional Robustness Checks

Figure 1: IRFs to "Large" Uncertainty Events



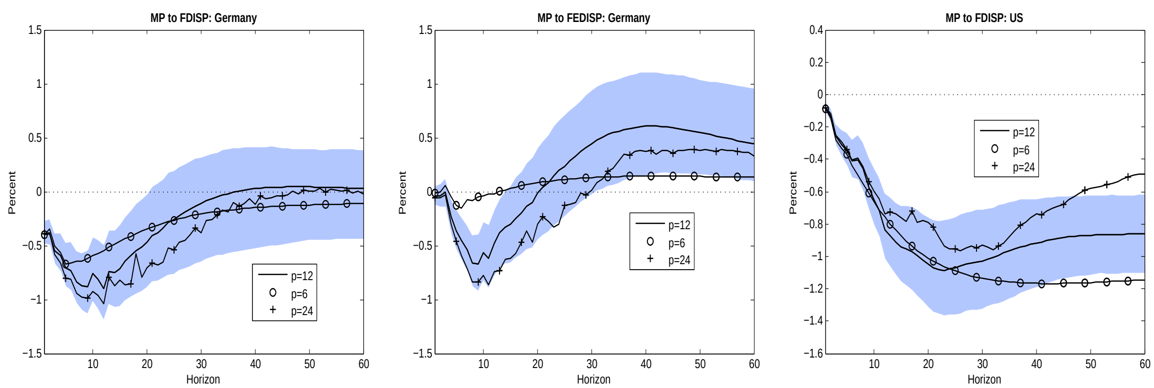
Notes: This figure plots in the left panel impulse responses of West German manufacturing production to innovations in various uncertainty measures. The responses are obtained from separately estimating a bivariate system with each different uncertainty measure and log manufacturing production. The frequency of the series in the VARs is monthly, the VARs are estimated with 12 lags, and manufacturing production enters the systems in levels. Instead of using the actual uncertainty measures in the VAR, we replaced them with a derived uncertainty series that takes on 1 in the months where the underlying uncertainty measure was one or more time series standard deviations above the mean of the underlying uncertainty series. $FDISP$ stands for the forecast disagreement index, based on Q1. $FEDISP$ stands for dispersion in forecast errors, constructed as described in the text. $MEANABSFE$ stands for the mean of the absolute value of forecast errors. $STOCKVOL$ stands for a stock market volatility index from Deutsche Börse, which combines realized volatility until 12/1991 and an implied volatility index from 1/1992 onward. The sample period for all VARs is common from 1/1980 - 12/2010. The VARs include an exogenous dummy after Germany's reunification in October 1990. The shaded gray region is the +/- one standard error confidence band from Kilian's (1998) bootstrap-after-bootstrap from the system using $FDISP$. The right panel does the same for the US uncertainty measures. $FDISP$ stands for the forecast disagreement index, based on Q3. $GOOGLE$ stands for the Google News subindex that is based on economic uncertainty from Baker et al. (2012). $STOCKVOL$ stands for the monthly measure of stock market volatility from Bloom (2009), which until 1986 is realized monthly stock market return volatility, and thereafter an implied volatility index. $SPREAD$ stands for the monthly spread of the 30-year Baa-rated corporate bond yield index over the 30-year treasury bond yield (in months where the 30-year treasury bond was missing we used the 20-year treasury bond instead). The sample period for the VAR with $FDISP$ is 5/1968 - 12/2011, for the one with $GOOGLE$ 1/1985 - 12/2011, and for the ones with $STOCKVOL$ and $SPREAD$ 7/1962 - 12/2011. The shaded gray region is the +/- one standard error confidence band from the system using $FDISP$.

Figure 2: ROBUSTNESS TO TREND ASSUMPTIONS



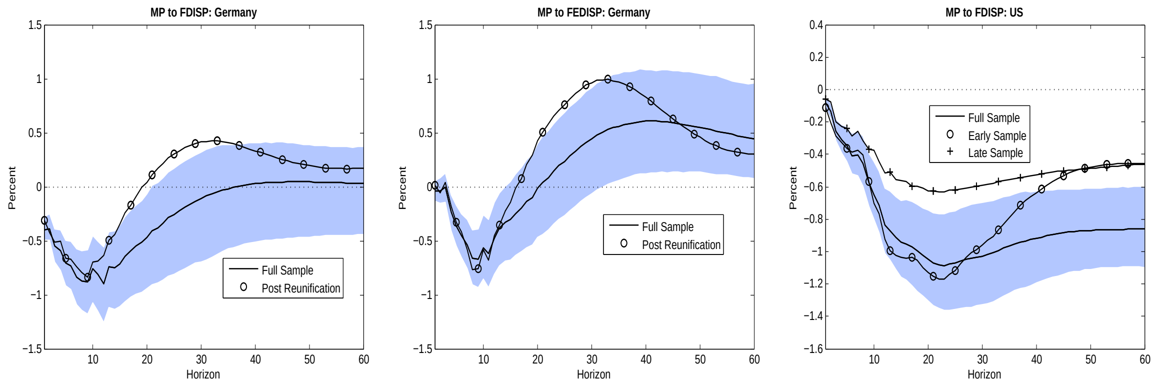
Notes: The VARs for *FDISP* and *FEDISP* in Germany and *FDISP* for the US here are estimated from bivariate VARs with an uncertainty measure and log manufacturing production, with the uncertainty series ordered first, under alternative trend assumptions: the solid lines show responses where production enters in levels, the lines with circles show cumulated responses when log production enters the VAR in first differences, and the lines with squares when a deterministic linear time trend is included in the VAR. The shaded gray regions are the +/- one standard error confidence bands from the system that is estimated in levels.

Figure 3: ROBUSTNESS TO LAG LENGTHS



Notes: The VARs for *FDISP* and *FEDISP* in Germany and *FDISP* for the US here are estimated from bivariate VARs with an uncertainty measure and log manufacturing under alternative lag structures. The shaded gray regions are the +/- one standard error confidence bands from the system that is estimated with 12 lags.

Figure 4: ROBUSTNESS: SUBSAMPLES



Notes: The VARs for *FDISP* and *FEDISP* in Germany and *FDISP* for the US here are estimated from bivariate VARs with an uncertainty measure and log manufacturing on different samples. For Germany we show results for both *FDISP* and *FEDISP* for the full sample, 1/1980-12/2010, as well as for a post-reunification sample, 1/1992-12/2010. For the US three sets of responses are shown: one for the full sample, 5/1968-12/2011; one for the pre "Great Moderation" sample, 5/1968-12/1983; and one for the period after the conventional dating of the "Great Moderation," 1/1984-12/2011. The shaded gray regions are the +/- one standard error confidence bands from the system that is estimated on the full sample.

C Small Business Economic Trend Survey (SBETS)

The Small Business Economic Trends Survey (SBETS) is a monthly survey conducted by the National Foundation of Independent Businesses (NFIB), which focuses on small companies across the US and across all sectors. Thus the SBETS is a good complement to the BOS which focuses on larger manufacturing firms in the Third FED District. To the extent that the SVAR results are similar this section lends additional support to our findings. The SBETS's monthly part starts in 1986. The survey on a quarterly basis is available since the mid 1970s. We prefer the highest possible frequency. None of our results depend on that choice of frequency. In terms of participation, the October 2009 issue of the SBETS (see Dunkelberg and Wade, 2009) reports that from January 2004 to December 2006 roughly 500 business owners responded, and that the number has subsequently increased to approximately 750.[1] Almost 25% of respondents are in the retail sector, 20% in construction and 15% in manufacturing, followed by services, which ranges well above 10%. All other one-digit sectors are represented with a single digit fraction. In terms of firm size, the sample contains much smaller enterprises than the BOS: the modal bin for the number of employees is "three to five", to which over 25% of respondents belong, followed by the "six to nine"-category with roughly 20%. The highest category is "forty or more", which contains just under 10% of firms.

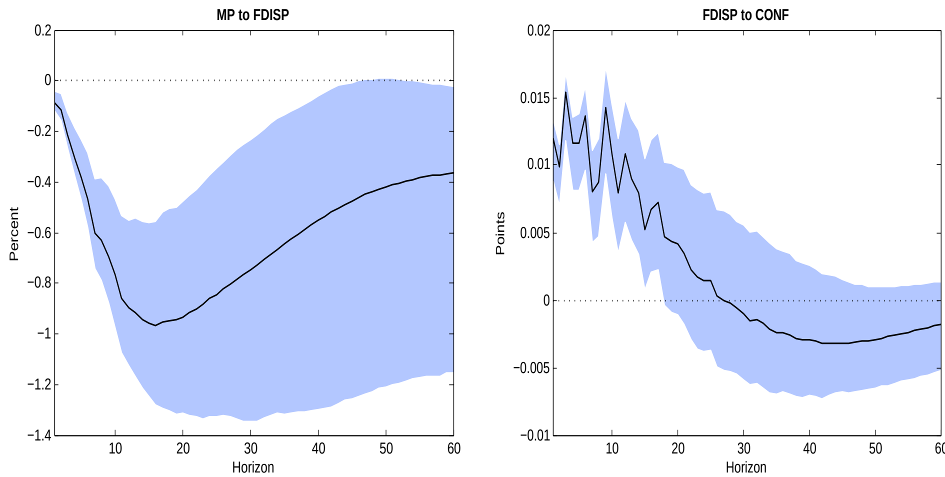
Our uncertainty index is based on an *FDISP* index derived from a question about general business conditions just like in the BOS (the box and the bold font are also used in the original):

Q *"About the* [*economy in general,*] *do you think that* ***six months from now*** *general business conditions will be better than they are now, about the same, or worse?:* [*1*] *Much better,* [*2*] *Somewhat better,* [*3*] *About the same,* [*4*] *Somewhat worse,* [*5*] *Much worse,* [*6*] *Don't know. "*

[1] The participation in the quarterly survey is higher, 1200 on average before January 2007 and 1750 thereafter.

One advantage of this question over its BOS cousin is that it is slightly more nuanced because it allows for two "increase" and two "decrease" categories. We quantify the extreme categories with -2 and 2, respectively.

Figure 1: SBETS RESULTS



Notes: The left panel plots an impulse response of log manufacturing production to an innovation in $FDISP$, obtained from estimating a bivariate VAR with the SBETS forecast dispersion index (**Q**, ordered first) and US manufacturing production. The frequency of the series in the VAR is monthly, the VAR is estimated with 12 lags, and log manufacturing production enters the systems in levels. The sample period for the VAR is from 1/1986 - 9/2009. Shaded regions are +/- one standard error confidence bands from Kilian's (1998) bootstrap-after-bootstrap. The right panel estimates a similar VAR, as in the left panel, augmented by a measure of "confidence" – based on the relative score, $Frac_t^+ - Frac_t^-$, from **Q** ($CONF$). It shows the response of $FDISP$ to a *negative* innovation in $CONF$, where the latter is ordered first.

Figure 1 displays the analogue of our results from Figures **??** and **??** in the main text. The left panel of Figure 1 plots the impulse response of log manufacturing production to an innovation in the SBETS-based $FDISP$ uncertainty measure. We use manufacturing production as the activity variable for comparability reasons with our main results, even though the SBETS covers more sectors than manufacturing. While somewhat less persistent than the results for the BOS-based $FDISP$ uncertainty measure, they are nevertheless qualitatively similar: the peak negative response is reached after well over a year and there is limited evidence of a strong rebound or even overshooting effect. The right panel of Figure 1 plots the impulse response of the SBETS-based $FDISP$ uncertainty measure to a negative innovation in a measure of "confidence" – based on the relative score, $Frac_t^+ - Frac_t^-$, from **Q**. This impulse response was obtained from the VAR that we used for the results in the left panel, augmented by this measure of confidence, ordered first. Similarly to what we find in Figure 8, uncertainty and confidence are negatively correlated in the SBETS, which is consistent with the "by-product" hypothesis.

REFERENCES

Dunkelberg, W. and H. Wade (2009). "NFIB Small Business Economic Trends." October.