

# Input and Output Inventory Dynamics

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Web Appendix

## Appendix (not for publication)

This appendix consider a standard one-sector RBC model with inventories—which is a modified version of the multi-sector inventory model in the paper. This alternative model abstracts from output inventories and features only input inventories. The essential differences between this modified model and the more complicated input-output inventory model is (i) the introduction of government spending shocks (which by design are *not* beneficial to consumers) and (ii) labor is not confined to the intermediate-goods sector with a linear technology. In the model, a social planner solves the following problem:

$$\max E \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - a \frac{N_t^{1+\gamma_n}}{1+\gamma_n} \right\} \quad (1)$$

subject to

$$C_t + G_t + K_{t+1} - (1 - \delta_k)K_t \leq \left[ \int_0^1 \theta(i)y(i)^\rho di \right]^{\frac{1}{\rho}} \quad (2)$$

$$y_t(i) + s_t(i) \leq (1 - \delta) s_{t-1}(i) + x_t(i) \quad (3)$$

$$s_t(i) \geq 0 \quad (4)$$

$$\int_0^1 x_t(i) di \leq A_t K_t^\alpha N_t^{1-\alpha}, \quad (5)$$

where  $G_t$  represents aggregate shocks to government spending, with the law of motion,  $\log G_t = \log G_{t-1} + \varepsilon_{gt}$ . The variables  $\{C_t, N_t, K_{t+1}, y_t(i), x_t(i), s_{t+1}(i)\}$  denote consumption, labor, capital, sales of intermediate good  $i$ , production of intermediate good  $i$ , and inventory of intermediate good  $i$ , respectively. Denoting  $\tilde{Y} \equiv \left[ \int_0^1 \theta(i)y(i)^\rho di \right]^{\frac{1}{\rho}}$ ,  $X \equiv A_t K_t^\alpha N_t^{1-\alpha}$ ,  $Y \equiv \int y(i) di$ , and  $S \equiv \int s(i) di$ , and using the same method outlined in the paper to solve intermediate-good firms' decision rules and aggregating by the law of large numbers, it can be shown that the aggregate variables satisfy the following decision rules:

$$Y_t = \tilde{Y}_t G(\theta_t^*)^{-\frac{1}{\rho}} D(\theta_t^*) \quad (6)$$

$$X_t + (1 - \delta) S_{t-1} = Y_t \frac{D(\theta_t^*) + H(\theta_t^*)}{D(\theta_t^*)} \quad (7)$$

$$S_t = Y_t \frac{H(\theta_t^*)}{D(\theta_t^*)}, \quad (8)$$

where

$$\begin{aligned}
D(\theta^*) &\equiv \int_{\theta(i) \leq \theta^*} \theta(i)^{\frac{1}{1-\rho}} dF(\theta) + \int_{\theta(i) > \theta^*} \theta^*{}^{\frac{1}{1-\rho}} dF(\theta) > 0, \\
H(\theta^*) &\equiv \int_{\theta(i) \leq \theta^*} \left[ \theta^*{}^{\frac{1}{1-\rho}} - \theta(i)^{\frac{1}{1-\rho}} \right] dF(\theta) > 0, \\
G(\theta^*) &\equiv \int_{\theta(i) \leq \theta^*} \theta(i)^{\frac{1}{1-\rho}} dF(\theta) + \int_{\theta(i) > \theta^*} \theta(i) \theta^*{}^{\frac{\rho}{1-\rho}} dF(\theta) > D(\theta^*).
\end{aligned} \tag{9}$$

These equations are analogous to equations (??)-(??) in the paper. The aggregate resource constraint (2) can then be rewritten as

$$C_t + K_{t+1} - (1 - \delta_k)K_t + P_t [S_t - (1 - \delta) S_{t-1}] = P_t [A_t K_t^\alpha N_t^{1-\alpha}], \tag{10}$$

where the right-hand side is GDP and  $P_t \equiv G(\theta_t^*)^{\frac{1}{\rho}} D(\theta_t^*)^{-1}$  measures the relative price of intermediate goods in terms of the final good. Recall that in a standard RBC model without inventories, the relative price of intermediate goods is constant (i.e.,  $\frac{\tilde{Y}}{\tilde{Y}} = 1$ ). Assume  $\theta(i)$  follows the Pareto distribution,  $F(\theta) = 1 - \left(\frac{1}{\theta}\right)^\sigma$ , and calibrate the common structural parameters of the model to the same values as in Table 1 in the paper (except  $\xi = 0$  because there are no capital adjustment costs here), which imply that the steady-state stock-to-sales ratio is 1 and the probability of stockout is 5% in the current model. The impulse responses of inventory investment ( $S_t - (1 - \delta) S_{t-1}$ ), inventory stock-to-sales ratio ( $\frac{S_t}{Y_t}$ ), the probability of stockout, and GDP to a positive one-standard-deviation government spending shock are graphed in Figure A (solid lines). Similar to the case of preference shocks, inventory investment is procyclical (upper-left panel) and the stock-to-sales ratio is countercyclical (lower-left panel). When inventories are eliminated from the model because of improved information technology, the volatility of GDP—measured by the standard deviation in  $P_t [A_t K_t^\alpha N_t^{1-\alpha}]$ —is increased visibly (see the dashed lines in the lower-right panel in Figure A). That is, the same amount of permanent increase in government expenditure would generate a permanently higher GDP without inventories than with inventories. As in the input-output inventory model, consumption and capital investment are less volatile and labor is more volatile with inventories than without.

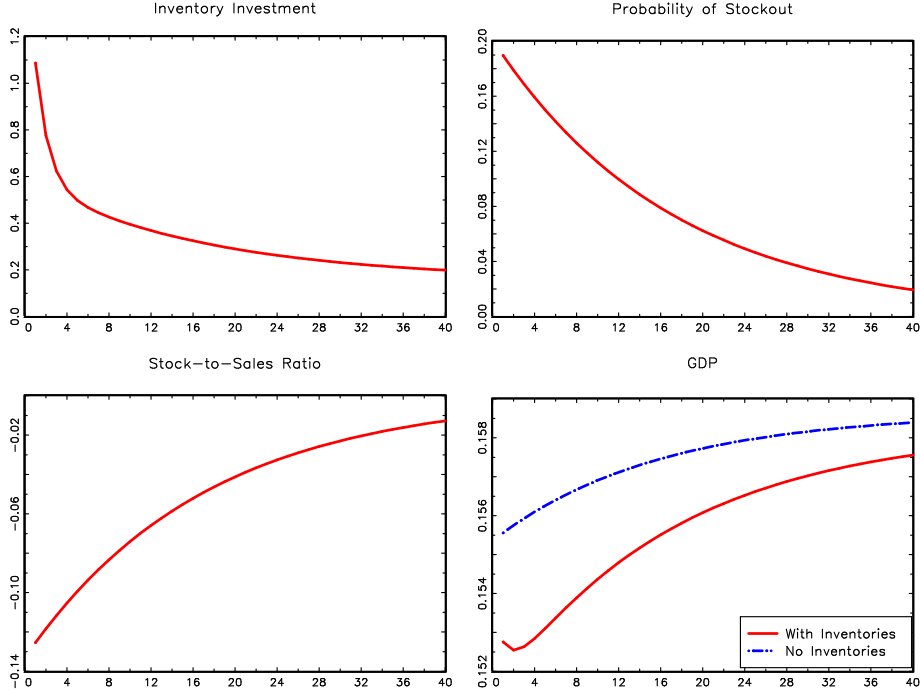


Figure A. Impulse Responses to Government Spending Shock.

The intuition behind the stabilizing effect of inventories is again that inventories stabilize final demand more than they destabilize production. This stabilizing effect is rooted in the procyclical probability of stockouts (see the upper-right panel in Figure 1). Define  $q_t = P_t^{-1}$  as the relative price of consumption goods in terms of inventory goods. Equation (10) can then be rewritten as  $q_t C_t + q_t [K_{t+1} - (1 - \delta)K_t] + S_t - S_{t-1} = A_t K_t^\alpha N_t^{1-\alpha}$ . Recall that a countercyclical stock-to-sales ratio requires the cutoff  $\theta_t^*$  to be countercyclical. Since the elasticity of  $q_t$  with respect to  $\theta_t^*$  is negative,  $q_t$  is thus procyclical. This implies that consumption and capital investment are more expensive in a boom and less expensive in a slump. Thus, the procyclical movements in  $q_t$  acts as an automatic stabilizer, which reduces the variability of final demand ( $C_t + K_{t+1} - (1 - \delta)K_t$ ) over the business cycle.