Online Appendix for

Intermediate Goods and Weak Links: A Theory of Economic Development

Charles I. Jones
Stanford GSB and NBER

This appendix contains outlines of the proofs of the propositions reported in the paper.

Proof of Proposition 1. The Symmetric Allocation, Given Capital

Follows directly from the fact that $Q_i = A_i m$, where $m = (K^\alpha H^{1-\alpha})^{1-\sigma} X^\sigma$ is constant across activities. QED.

Proof of Proposition 2. The Competitive Equilibrium, Given Capital

1. The first order conditions from the Variety i Problem are

$$(1-\tau_i)p_i \alpha(1-\sigma)\frac{Q_i}{K_i} = r + \delta$$

$$(1-\tau_i)p_i(1-\alpha)(1-\sigma)\frac{Q_i}{H_i} = w$$

$$(1-\tau_i)p_i \sigma \frac{Q_i}{X_i} = q.$$

Substituting these conditions back into the production function yields an equation that characterizes the price of good i:

$$p_i = \frac{mc}{A_i(1-\tau_i)\epsilon}, \tag{1}$$

where $mc \equiv ((r+\delta)^\alpha w^{1-\alpha})^{1-\sigma} q^\sigma$ is a key piece of the marginal cost and $\epsilon \equiv (\alpha^\alpha(1-\alpha)^{1-\alpha})^{1-\sigma}(1-\sigma)^{1-\sigma}\sigma^\sigma$.

2. Integrating the Variety i first order conditions above gives

$$(r+\delta)K = \alpha(1-\sigma)\int(1-\tau_i)p_iQ_idi \tag{2}$$

$$wH = (1-\alpha)(1-\sigma)\int(1-\tau_i)p_iQ_idi \tag{3}$$

$$qX = \sigma\int(1-\tau_i)p_iQ_idi \tag{4}$$

where the limits of the integration are understood to be 0 to 1. Note that

$$\int p_ic_idi = Y, \quad \int p_iz_idi = qX, \quad \int p_iQ_idi = Y + qX.$$

Define $\tau \equiv \frac{E}{Y+qX}$ to be distortion revenues as a share of gross output. Then

$$\int(1-\tau_i)p_iQ_idi = (1-\tau)(Y+qX).$$

Substituting this expression into (2), (3), and (4) gives

$$(r+\delta)K = \alpha(1-\sigma)\frac{1-\tau}{1-\sigma(1-\tau)}Y \tag{5}$$

$$wH = (1-\alpha)(1-\sigma)\frac{1-\tau}{1-\sigma(1-\tau)}Y \tag{6}$$

$$qX = \frac{\sigma(1-\tau)}{1-\sigma(1-\tau)}Y. \tag{7}$$

These expressions allow us to solve for mc (see the definition under (1)) as

$$mc = \frac{1-\tau}{1-\sigma(1-\tau)}\cdot\epsilon\cdot\frac{Y}{(K^\alpha H^{1-\alpha})^{1-\sigma}X^\sigma}. \tag{8}$$

3. Next, consider the first-order conditions from the Final Good and Intermediate Good Problems. For each of these problems, take the first order condition and then integrate it back into the firm's production function. For the final good, this gives

$$\left(\int p_i^{-\frac{\theta}{1-\theta}}di\right)^{-\frac{1-\theta}{\theta}} = 1 \tag{9}$$

and for the intermediate good

$$\left(\int p_i^{-\frac{\rho}{1-\rho}} di\right)^{-\frac{1-\rho}{\rho}} = q \tag{10}$$

Now substitute (1) into (9) to get

$$mc = \epsilon B_\theta \tag{11}$$

where

$$B_\theta \equiv \left(\int_0^1 (A_i(1-\tau_i))^{\frac{\theta}{1-\theta}} di\right)^{\frac{1-\theta}{\theta}}. \tag{12}$$

Combining (1) with this expression, we can solve (10) to find

$$q = \frac{B_\theta}{B_\rho} \tag{13}$$

where B_ρ is defined analogously to B_θ. Combining (8), (11), (7), and (13) yields the main result in the proposition.

4. Finally, we need to solve for τ. From the first-order conditions for the Final Goods Problem and the Intermediate Goods Problem we get

$$p_i Q_i = p_i c_i + p_i z_i = p_i^{-\frac{\theta}{1-\theta}} Y + (p_i/q)^{-\frac{\rho}{1-\rho}}(qX).$$

Multiplying this expression by τ_i, integrating, and then using (1), (11), and (13) leads to the solution for τ:

$$\tau = (1-\sigma(1-\tau))T_\theta + \sigma(1-\tau)T_\rho \tag{14}$$

where $T_\rho \equiv \int_0^1 \tau_i \left(\frac{A_i(1-\tau_i)}{B_\rho}\right)^{\frac{\rho}{1-\rho}} di$. That is, T_ρ is a weighted average of the sector-specific distortions, where the weights depend on ρ; T_θ is defined analogously. QED.

Proof of Proposition 3. The Competitive Equilibrium in Steady State

Straightforward using (5) and the Euler equation from the Household Problem. QED.

Proof of Proposition 4. Symmetric Wedges

Straightforward evaluation given earlier results. QED.

Proof of Proposition 5. Random Productivity and Wedges

1. Define $B(\eta) \equiv \left(\int (A_i(1-\tau_i))^\eta di\right)^{1/\eta}$. Define $m_i \equiv \eta(a_i + \omega_i)$. Then $m_i \sim N(\eta(\mu_m + \mu_a), \eta^2\nu^2)$, where $\nu^2 \equiv \nu_a^2 + \nu_\omega^2 + 2\nu_{a\omega}$. Therefore,

$$\begin{aligned} B(\eta) &= (E(e^{m_i}))^{1/\eta} \\ &= e^{\mu_a + \mu_\omega + \frac{1}{2}\eta\nu^2}. \end{aligned} \tag{15}$$

   Let $\bar{B} \equiv [B(\frac{\theta}{1-\theta})]^{1-\sigma}[B(\frac{\rho}{1-\rho})]^\sigma$. Then

$$\log \bar{B} = \mu_a + \mu_w + \frac{1}{2} \cdot \left((1-\sigma)\frac{\theta}{1-\theta} + \sigma\frac{\rho}{1-\rho} \right) \nu^2.$$

   Making the substitution $1 - \bar{\tau} \equiv e^{\mu_w + \nu_w^2/2}$ then yields term ②.

2. To get term ①, we need to solve for τ. From equation (14), one can obtain

$$1 - \tau = \frac{1 - T_\theta}{1 - \sigma(T_\theta - T_\rho)}.$$

   Evaluating the integrals in T_θ and T_ρ as above gives

$$T_\theta = 1 - \exp\{\mu_\omega + \frac{1}{2} \cdot \frac{1+\theta}{1-\theta}\nu_\omega^2 + \frac{\theta}{1-\theta}\nu_{a\omega}\}$$

   and T_ρ is the analogous expression. Straightforward algebra then delivers term ①. QED.