WEB Appendix to "Contrasting Trends in Firm Volatility" by David Thesmar and Mathias Thoenig

February 8, 2011

1 A CARA - Gaussian Version of the Model

We consider hereafter the CARA-Gaussian version of the model (see Thesmar and Thoenig 2004, for more details). The static economy is endowed with L risk-averse workers. There are three markets: the financial market, the labor market and the product market on which n firms compete imperfectly. These firms are initially owned by some workers that we call entrepreneurs. Among these firms, a share μ_L is listed on the stockmarket while the remaining share μ_N is non listed (ie. privately held). To simplify exposition, we assume n, μ_L and μ_N to be exogenously fixed.

The sequence of events has three periods. At date 1, each entrepreneur, whether her firm is listed or not, chooses a strategy indexed by $0 \leq s \leq 1$. A strategy defines both the mean and the variance of the demand addressed to the firm, and we assume that a larger average demand comes at the expense of more uncertainty. At date 2, the financial market clears and risk sharing takes place: $\mu_L n$ entrepreneurs sell the shares of their firms to investors. At date 3, demand uncertainty is revealed and production takes place; the product and labor markets clear; the savers obtain the earnings from the securities they hold; everyone consumes.

1.1 Setup

Demand Side

Each agent $k \in (0, L)$ in the economy has a CARA utility

$$U_k = -e^{-aC_k} \tag{1}$$

where C_k is a consumption index which depends on the consumption levels $y_{k,i}$ of the n different goods i which are produced under monopolistic competition. The consumption index is given by the usual

Dixit Stiglitz formulation:

$$C_k = \left(\sum_{j=0}^{n} (1+\tilde{\delta}_i)^{\frac{1}{\sigma}} . y_{k,i}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where we assume n to be large and $\sigma \geq 2$. The difference with the standard Dixit Stiglitz framework is that consumers experience taste shocks, modelled by random coefficients $\tilde{\delta}_i$. These shocks are the *only* source of uncertainty in the model and for the sake of simplicity we will assume that they are Gaussian.[1] The $\tilde{\delta}_i$'s are assumed to be good-specific, small and uncorrelated: hence all agents k experience the same taste shocks on the good i. This extreme form of correlation structure is not necessary; what matters here is that there is some scope for risk sharing among entrepreneurs. Finally the specific mean-variance profile of the taste shock $\tilde{\delta}_i$ is a choice variable of the firm i (cf. infra).

Given these assumptions about preferences, the total demand y_i addressed to each industry i, can be easily derived by aggregating the individual demand functions $y_{k,i}$ over the whole population $k \in (0, L)$:

$$y_i = (1+\tilde{\delta}_i). \frac{E}{P} . (\frac{p_i}{P})^{-\sigma} \tag{3}$$

where p_i is the price charged by the monopoly producing the good i, $E \equiv \sum_0^L E_k$ is the aggregate nominal expenditure and P is a price index equal to:

$$P \equiv \left(\sum_{j=0}^{n} (1+\tilde{\delta}_j). p_j^{1-\sigma} \right)^{1/1-\sigma} \tag{4}$$

The Strategy: Standardization vs Customization

Each good i is produced by a monopoly owned by an entrepreneur. At date 1, the entrepreneur chooses her marketing strategy $0 \leq s \leq 1$; this choice impacts the distribution of demand shocks $\tilde{\delta}_i$ that the firm experiences at date 3. We posit that the demand shock is drawn from a Gaussian distribution with mean s and variance $s^2\Sigma$:

$$\tilde{\delta}_i \sim N(s, s^2\Sigma) \tag{5}$$

While such strategies could receive many alternative interpretations, we will refer to the choice of s as the "customization policy". Under this interpretation, $s = 0$ can be thought of as the design of

[1] More rigourously, $\tilde{\delta}_i$ should be drawn from a *truncated* gaussian distribution since demand cannot be negative. To focus on the economics of the model, $\tilde{\delta}_i$ is assumed to be small such that we can ignore the fact that its distribution is truncated on the left tail.

a standardized good whose market demand is fully safe but remains small. On the contrary, $s = 1$ corresponds to the design of a fully customized, potentially highly valuable, good but whose demand is difficult to predict because of erratic trends and fashions.

Finally, whatever the choice of s, each firm exploits a constant return to scale technology; it uses labor l paid at a wage w to produce:

$$y = l \tag{6}$$

The Financial Market

The $\mu_L n$ entrepreneurs who own a listed corporation may sell equity (claims on their firms' profits) on the financial market. The pool of investors consists of an exogenous number $\phi L < L$ of agents that are given access to security trading on a domestic financial market. To make the analysis tractable, we follow Pagano (1993) and assume that these agents are also given the right to borrow or buy an infinite amount of savings from international capital markets at a given risk free rate R.[2]

The financial market allows to trade two types of securities. First, it allows the ϕL investors to issue bonds at the exogenous risk free rate R: hence there is no restriction on short sales or borrowing, like in Pagano (1993). Second, claims on listed firms' profits can be sold by entrepreneurs, and bought by investors with access to the financial markets. These securities give to their owners a right to a fraction of profits. In the following, we use for these securities the labels "equity" or "shares" interchangeably, although the exercise of control rights usually attached to the possession of equity is not explicitly modelled here. In addition, entrepreneurs do not need external capital to produce. Hence, in this model, the sole role of financial markets is to share risk.

Given our assumptions, there are ϕL investors on the demand side of the stockmarket. On the supply side, there are $\mu_L n$ firms going public. In the rest of the analysis, we will interpret ϕ, our main comparative statics variable, as the *degree of risk sharing* between investors.

1.2 Partial Equilibrium

We solve the model backwards. At period 3, after observing its idiosyncratic demand shock $\tilde{\delta}$, each firm maximizes its own monopoly profit. At period 2, trade of financial assets takes place. At period 1, both listed and privately held firms choose their degree of customization s.

[2] A perfectly equivalent alternative assumption is that (1) investors have large endowments of savings and (2) that they can invest either in stocks or in a riskless security of return R (Pagano [1993]).

Firm Profits

At date 3, after the idiosyncratic demand shock $\tilde{\delta}$ is revealed, each entrepreneur chooses the amount of production in order to maximize her monopoly profit:[3]

$$\tilde{\pi} = \max_{l}(1+\tilde{\delta})^{1/\sigma}.(P^{\sigma-1}E)^{1/\sigma}l^{\frac{\sigma-1}{\sigma}} - wl \tag{7}$$

which depends on demand shock $\tilde{\delta}$. Solving this maximization problem gives the following reduced form for the firm's profits:

$$\tilde{\pi} = (1+\tilde{\delta}).\underbrace{\frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}}.\left(\frac{P}{w}\right)^{\sigma}.\frac{E}{P}}_{\equiv\pi_0} \tag{8}$$

where π_0 corresponds to the profits of a fully standardized firm (ie. with no uncertainty):

Given the mean-variance trade-off [5], the mean and variance of the firm's profits are increasing in s and are given by:

$$\begin{cases} E\tilde{\pi}(s) = (1+s).\pi_0 \\ VAR\tilde{\pi}(s) = \Sigma s^2.\pi_0^2 \end{cases} \tag{9}$$

In this set-up, a risk neutral firm owner would always choose the largest s; what prevents it to happen is that owners are risk averse.

Portfolio Selection by Investors

On the supply side there are $\mu_L n$ listed firms, indexed by j; each one issuing a measure 1 of shares. Each share j is traded at price ρ_j. On the demand side there are ϕL investors, indexed by k: each one borrows on international markets b_k units of savings at rate R in order to buy x_{kj} shares of each firm j. Investor k's budget constraint thus writes as:

$$\sum_{j=0}^{\mu_L n} x_{kj}\rho_j \leqslant b_k \tag{10}$$

Her ex post consumption is equal to the labor and net financial incomes taken in real terms:

$$\tilde{C}_k = \frac{w}{P} + \frac{1}{P}\left[\sum_{j=0}^{\mu_L n} x_{kj}\widetilde{\pi}_j - (1+R)b_k\right]$$

[3] We implicitly assume here that, even when the firm is widely held, there are no agency costs of separation of ownership and control. Our view in this paper is that past changes in financial markets have had much more impact on the degree of risk sharing between investors than on the separation of ownership and control. Put otherwise, risk sharing has improved much more than corporate governance.

Plugging back the budget constraint [10] into the consumption expression leads to:

$$\tilde{C}_k = \frac{1}{P}.\left[\sum_{0}^{\mu_L n} x_{kj}\left(\tilde{\pi}_j - \rho_j R\right) + w\right] \tag{11}$$

The program of investor k consists of maximizing her expected CARA utility [1] with respect to her portfolio $\{x_{kj}\}_{j=0}^{\mu_L n}$, taking equity prices ρ_j, the risk free rate R and ex post deterministic wage w as given. As the $\tilde{\delta}$ demand shocks are Gaussian, so are the profits $\tilde{\pi}_j$ and therefore the consumption level $\tilde{C}_k$. As it is standard in such a CARA -Gaussian framework, solving this investor's problem amounts to maximizing the following mean-variance criterion:

$$\max_{\{x_{kj}\}_{j\in(0,\mu_L n)}} \quad \frac{w}{P} + \sum_{j=0}^{\mu_L n}\left[x_{kj}\frac{E\tilde{\pi}(s_j) - \rho_j R}{P} - \frac{a}{2}\frac{x_{kj}^2.VAR\tilde{\pi}(s_j)}{P^2}\right] \tag{12}$$

Thus, the demand for share j by investor k is given by:

$$x_{kj} = P.\left\{\frac{E\tilde{\pi}(s_j) - R\rho_j}{a.VAR\tilde{\pi}(s_j)}\right\} \tag{13}$$

where $E\tilde{\pi}(s_j)$ and $VAR(\tilde{\pi}(s_j))$ as functions of s_j are given by equations [9]. Demand for risky asset j is a decreasing function of risk aversion a, its risk $VAR(\tilde{\pi}(s_j))$ and of its price ρ_j. It is, of course, an increasing function of its expected return $E\tilde{\pi}(s_j)$.

Equilibrium on the Stockmarket

We assume that entrepreneurs taking their firms to the public do not behave like monopolies when they supply securities to the stockmarket.[4] The price of their firm is therefore taken as given and lies at the intersection of the demand and supply curves of share of firm j. X_j^d is the aggregate demand for shares j and can be easily obtained through adding individual demands given by [13] for all ϕL investors:

$$X_j^d = \phi L.P.\left\{\frac{E\tilde{\pi}(s_j) - R\rho_j}{a.VAR\tilde{\pi}(s_j)}\right\}$$

As the overall supply of shares is equal to one, we get that in equilibrium:

$$\rho_j = \frac{1}{R}\left\{E\tilde{\pi}(s_j) - \frac{a}{\phi L}\frac{VAR\tilde{\pi}(s_j)}{P}\right\} \tag{14}$$

This equilibrium condition illustrates the benefits of risk sharing for the entrepreneur: by selling her firm to the market, she will receive the amount ρ_j of savings. In doing so, she will behave as if she had reduced her risk aversion from a to $a/\phi L$.

[4] This assumption allows us to suppress an unwanted market imperfection, but does not affect our qualitative results (see Pagano [1993]).

The Choice of s

At period 1, entrepreneurs choose their marketing strategies s_j in order to maximize their own utilities. However, this utility takes a very different form whether the entrepreneur will take her firm to the public or not. For future listed firms, the choice of s_j will affect the sales price of the firm ρ_j. For firms remaining privately held, this choice of s_j will affect the entrepreneur's utility through both expectation and variance of her ex post income, as can be seen from equation [12].

Given our CARA assumption, the utility of an entrepreneur of a non listed firm writes as:

$$U_j^N = \frac{w}{P} + \underbrace{\left(\frac{E\tilde{\pi}(s_j)}{P} - \frac{a}{2}\frac{VAR\tilde{\pi}(s_j)}{P^2}\right)}_{\text{utility derived from own firm}} \tag{15}$$

As her firm j is not listed, the entrepreneur bears all the specific risk specific linked to it. She chooses a degree of customization s such as to maximize her utility U_j^N. Given the definitions of $E\tilde{\pi}$ and $VAR(\tilde{\pi})$ (see equations [9]), we have:

$$s_N = \frac{1}{a\Sigma}\frac{1}{\pi_0/P} \tag{16}$$

When the entrepreneur sells her firm to the market, her utility writes:

$$U_j^L = \frac{w}{P} + \underbrace{\left(\frac{E\tilde{\pi}(s_j)}{P} - \frac{a}{2\phi L}\frac{VAR\tilde{\pi}(s_j)}{P^2}\right)}_{\text{utility derived from own firm, after IPO}} \tag{17}$$

In contrast to an entrepreneur that did not list, she is able to partly diversify her risk with other entrepreneurs and ϕL investors. Hence, she acts as if her risk aversion were smaller. Again, given the definitions of $E\tilde{\pi}$ and $VAR(\tilde{\pi})$, the strategy maximizing her utility is:

$$s_L = \frac{\phi L}{a\Sigma}\frac{1}{\pi_0/P} \tag{18}$$

We thus have that entrepreneurs of listed firms choose riskier projects than private ones:

$$s_N < s_L = \phi L s_N \tag{19}$$

1.3 Financial Market Development in General Equilibrium

We now study the impact of financial market development, which the model describes as a broadening of the pool of investors ϕL. From the previous analysis, we know that in partial equilibrium - for given wage w and price P - a larger ϕ will make owners of listed firms behave more like risk neutral investors. As a consequence, the risk taken by public firms s_L will increase. An increase in ϕ has, however, no direct effect on s_N since owners of privately held firms are not directly affected by changes on the stock market. However, a look at [16] and [18] shows that s_L and s_N also depend on aggregate variables (the term π_0/P in each equation) which are affected by a change in ϕ. Therefore, we expect stockmarket development to have an *indirect effect* on both listed *and* private firms through its general equilibrium effect on wages w and price P. This is what the following analysis will make clear.

To compute the values of s_L and s_N in equilibrium, we need to obtain π_0/P, the real profit of a firm adopting a riskless strategy. This requires to clear both product and labor markets. At the firm level, labor demand is easily derived from program [7]. By aggregating on the whole set of firms, the labor market clearing condition writes:

$$L = \frac{(\sigma-1)^\sigma}{\sigma^\sigma}.\frac{E}{P}.(\frac{P}{w})^\sigma.\,[n\mu_L(1+s_L)+n\mu_N(1+s_N)] \tag{20}$$

Similarly profit maximization [7] leads to the price charged by each monopoly. Using its definition [4], we derive the consumption price index:

$$P = \frac{\sigma}{\sigma-1}.w.\,[n\mu_L(1+s_L)+n\mu_N(1+s_N)]^{1/(1-\sigma)} \tag{21}$$

Given these two equations, profits are easily given by:

$$\frac{\pi_0}{P} = \frac{L}{\sigma}.\,[n\mu_L(1+s_L)+n\mu_N(1+s_N)]^{(2-\sigma)/(\sigma-1)} \tag{22}$$

As $\sigma \geq 2$, the profit π_0/P is decreasing with respect to the average degree of customization within the economy, $(\mu_L s_L + \mu_N s_N)$. The intuition for this is that a standard pro competitive effect is at work. Firms compete for resources on the labor market: an increase in $(\mu_L s_L + \mu_N s_N)$ means that the average demand shock $\tilde{\delta}$ is larger. This props up aggregate labor demand, and as labor supply is inelastic, wages go up and profits fall.[5]

Now that we computed the equilibrium level of riskless profits π_0/P, we obtain the values of s_L and s_N from equations [18] and [16]. They are implicitly given by the following two equations:

$$s_N = \frac{\sigma}{L.a\Sigma}.\,[n\mu_L(1+s_L)+n\mu_N(1+s_N)]^{(\sigma-2)/(\sigma-1)} \tag{23}$$

$$s_L = \frac{\sigma}{L.a\Sigma}.\phi L.\,[n\mu_L(1+s_L)+n\mu_N(1+s_N)]^{(\sigma-2)/(\sigma-1)} \tag{24}$$

[5] In fact two countervailing effects are at work. As usual in monopolistically competitive frameworks, more production reduces prices through the standard demand externality of Dixit Stiglitz models, which props up aggregate demand and therefore profits. Given that $\sigma \geq 2$, this demand externality effect is however dominated by the pro competitive effect described in the text.

Using these equations, we immediately obtain that:

Result 1: *After a broadening of the shareholder base ϕ, both listed and non listed firms adopt more risky strategies.*

$$\frac{ds_N}{d\phi} > 0 \; ; \; \frac{ds_L}{d\phi} > 0 \text{ and } \frac{d(s_L - s_N)}{d\phi} > 0$$

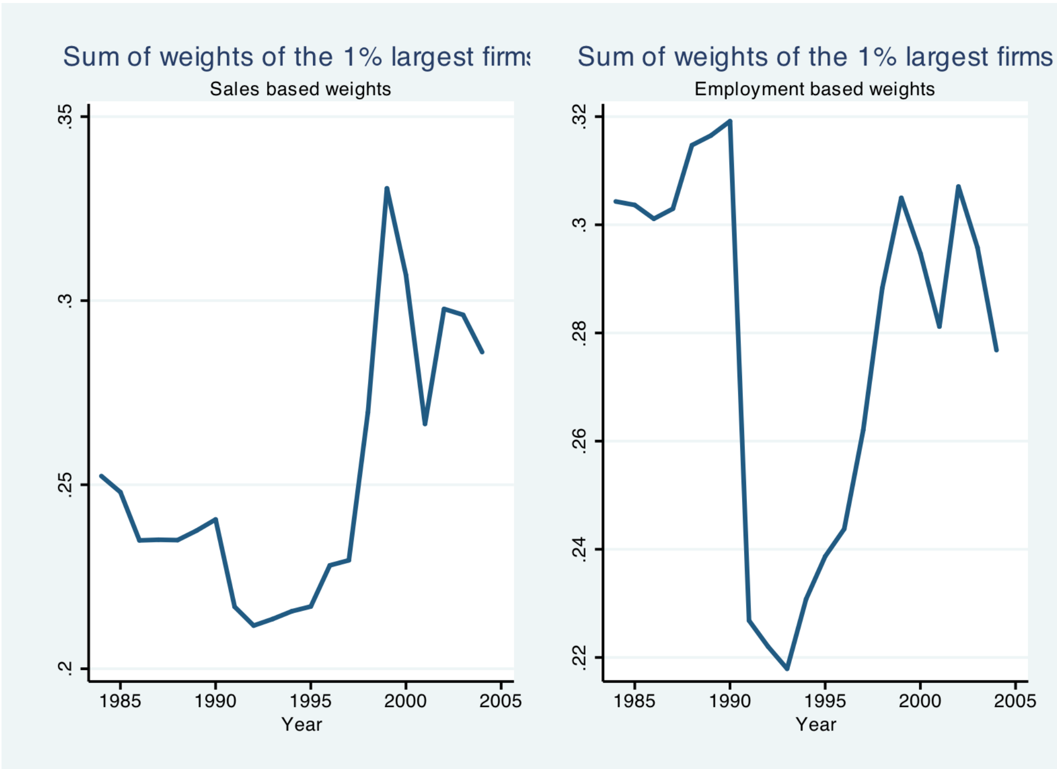
As is apparent from [23-24], financial development affects the economy through two channels. The direct channel, acting only on listed firms s_L corresponds to the *improvement of risk sharing*: more numerous investors are on average smaller equity holders, who are therefore willing to pay more for a larger s_L. The indirect effect corresponds to the s_L terms on the right hand sides of [23-24]: both s_L and s_N are increasing functions of s_L. More risk taking s_L implies that the average listed firm will want to produce more and therefore hire more. Real wages go up and profits π_0/P decline. As firms' profits become smaller, owners of all firms are willing to bear more risk because their risk aversion is constant. This increases the willingness of privately held firms to take on risk and amplifies the direct effect for listed firms. Finally $d(s_L - s_N)/d\phi > 0$ means that the difference between listed and non listed firms in term of risk taking increases with ϕ.

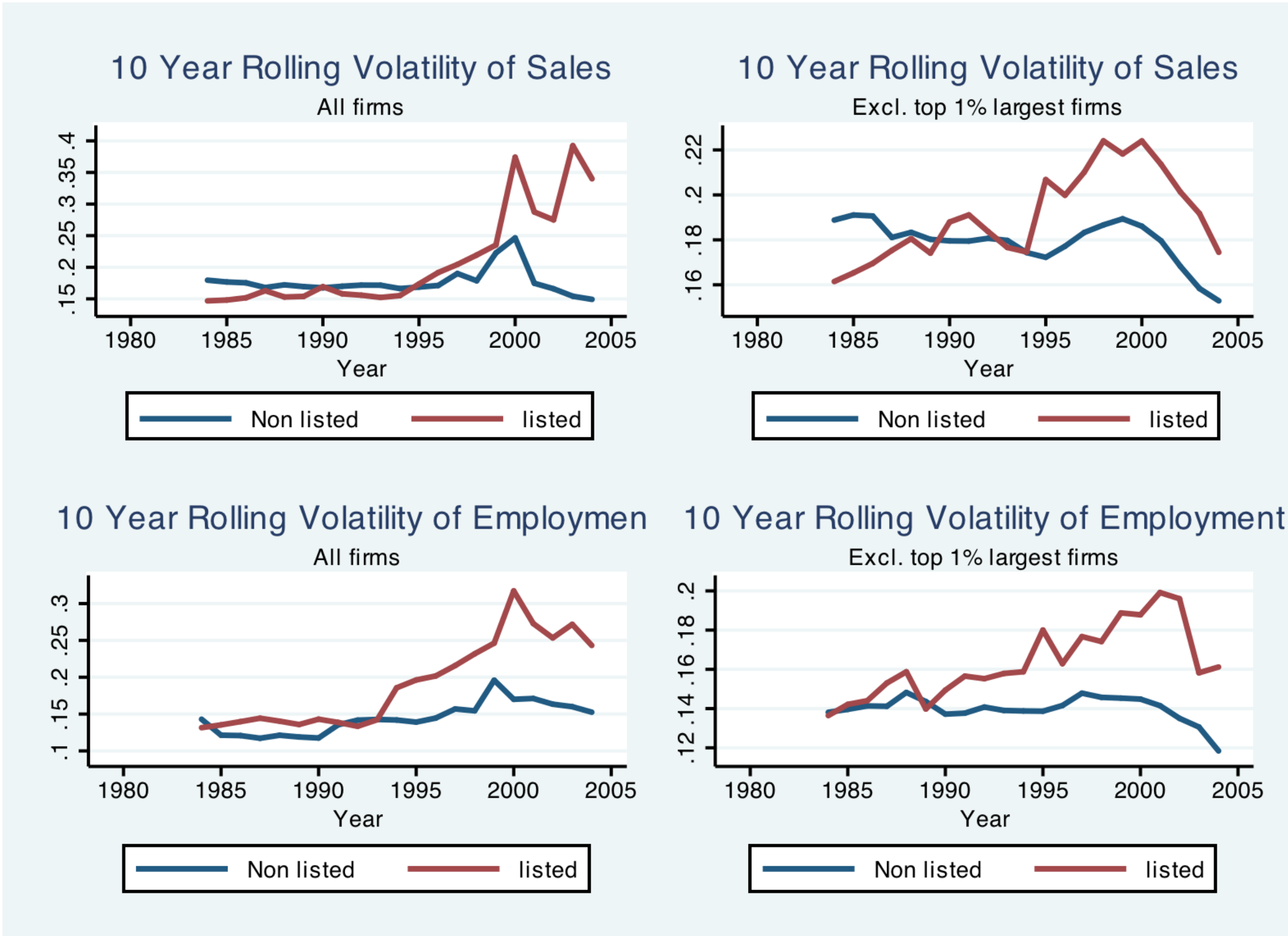
1.4 References

Pagano, Marco. 1993. "The Flotation of Companies on The Stock Market: A Coordination Failure Model", *European Economic Review*, 37(5)

Thesmar, David, and Mathias Thoenig. 2004. "Financial Market Development and the Rise in Firm Level Uncertainty". CEPR WP 4761.

2 Additional Descriptive Statistics




10 Year Rolling Volatility of Sales
All firms
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1980 1985 1990 1995 2000 2005
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10 Year Rolling Volatility of Sales
Excl. top 1% largest firms
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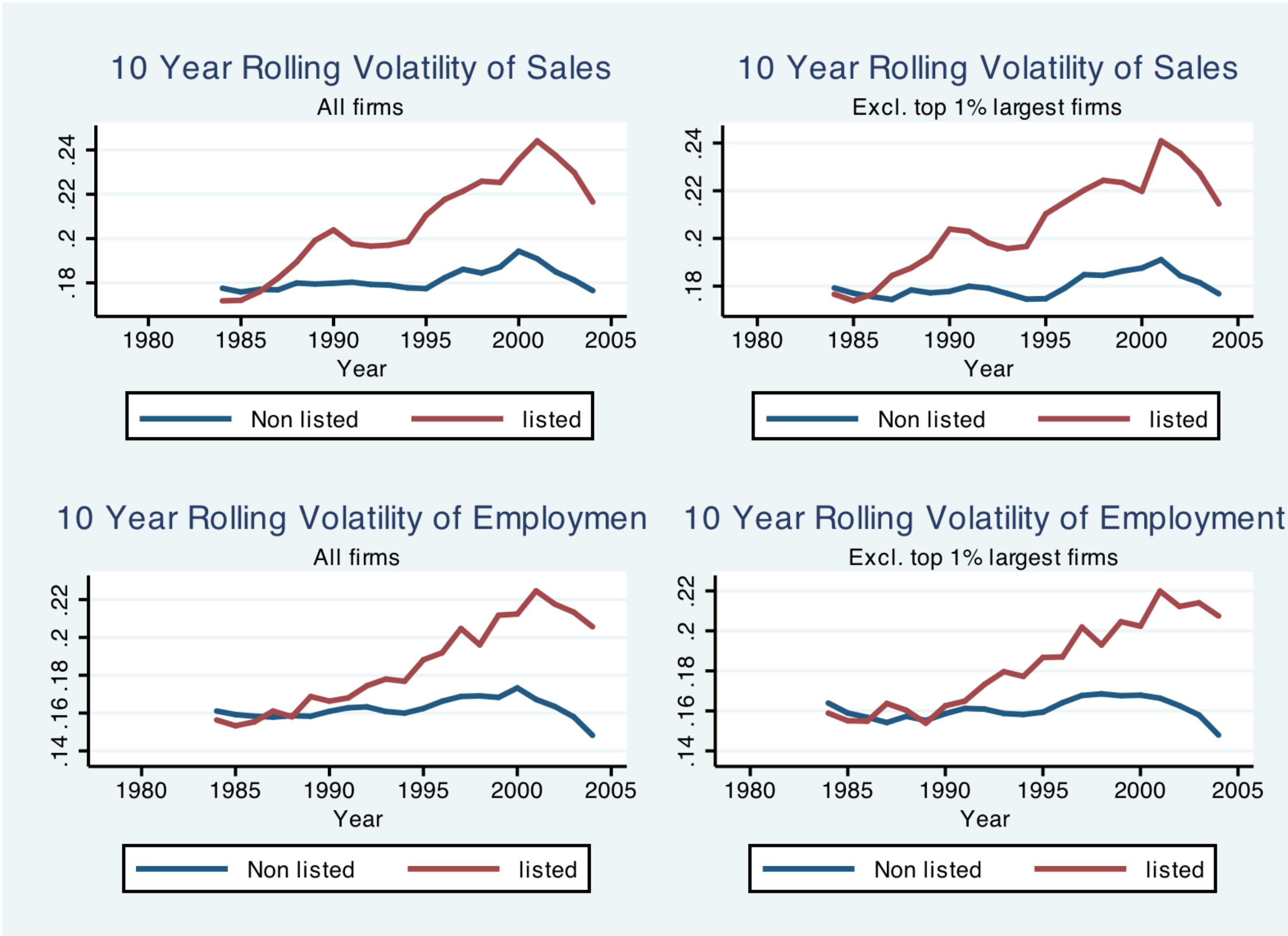
10 Year Rolling Volatility of Sales
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10 Year Rolling Volatility of Employment
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