May 2009

Online Appendixes for "A Theory of Military Dictatorships"

By Daron Acemoglu, Davide Ticchi and Andrea Vindigni

Appendix B: Key Notation for Section I

$\beta \in (0,1)$: discount factor.

$\chi_{j,t} \in \{0,1\}$: indicator function denoting the occupational choice of the agent and determining whether or not the individual benefits from the public good.

n: size of high-skill agents ($1-n$ is size of low-skill agents).

A^H: income of high-skill producers.

A^L: income of low-skill producers.

$Y \equiv (1-n)\,A^L + nA^H$: potential output of the economy.

$\theta \in (n,1)$: parameter of income inequality (higher θ corresponds to greater inequality).

$x_t \in \{0,\bar{x}\}$: size of the military. $\bar{x}$: size of the military necessary for repression.

$a_t \in \{0,1\}$: decision regarding the size of the military at time t. $a_t = 1$ corresponds to $x_t = \bar{x}$, $a_t = 0$ corresponds to $x_t = 0$.

τ_t: tax rate on the income of the producers.

$C\,(\tau_t)$: fraction of the output lost due to tax distortions.

G_t: level of public good provision.

w_t: wage for soldiers.

ϕ: fraction of production lost due to the disruption created by the coup.

$\varphi_t \in \{1-\phi, 1\}$: fraction of output that remains after a coup attempt $(1-\phi)$. $\varphi_t = 1$ when there is no coup.

$s_t \in \{D, E, M, TD\}$: political regime at time t. $s_t = D$: democracy. $s_t = E$: oligarchy. $s_t = M$: military dictatorship. $s_t = TD$: transitional democracy.

$\psi_t \in \{0,1\}$: decision of the military whether or not to undertake a coup against the regime in power, with $\psi_t = 1$ corresponding to a coup attempt.

$\rho_t \in \{0,1\}$: decision of the military whether or not to repress the citizens in oligarchy, with $\rho_t = 1$ corresponding to repression.

$\pi \in [0,1]$: probability that repression fails.

$\gamma \in [0,1]$: probability that the coup attempt is successful.

τ^D: tax rate in democracy.

G^D: level of public good in democracy.

$a^L \equiv \left(1 - \tau^D\right) A^L + G^D$: net per period return to a low-skill producer in democracy.

$\hat{\tau}$: tax rate maximizing government revenues (peak of the Laffer curve).

w^M: soldiers' wage in a military dictatorship.

S, N, P: strategies of the elite corresponding to "smooth transition", "non-prevention" and "prevention," respectively.

Appendix C: Additional Material and Proofs

Details of the Model of Subsection III.A

It can be verified that Lemma 1 applies without any change in this modified model. In particular, in any MPE starting in a subgame with $s_t = E$ and $\eta_t = \eta^{NI}$, the military never chooses $\psi_t = 0$ and $\rho_t = 0$.

As in the baseline model, coups against oligarchy can be prevented only if the appropriate no-coup constraint is satisfied. This constraint is binding only when $\eta_t = \eta^{NI}$ because in state $\eta_t = \eta^I$ the elite are insulated from coups and transitions to democracy and it is still given by (34), that is, by $V^M(E \mid \text{repression}) \geq V^M(E \mid \text{coup})$, where $V^M(E \mid \text{coup})$ is still defined in (32) and $V^M(E \mid \text{repression})$ is the value of a soldier when the state variable η_t takes the value η^{NI} and the military repress. This is a consequence of the fact that coups are only possible when $\eta_t = \eta^{NI}$ and conditional on this event, they succeed with probability γ as in the baseline model. The value of a typical soldier when $\eta_t = \eta^{NI}$ and the military repress is

$$V^M(E \mid \text{repression}) = w^P + \beta\left[\pi V^M(TD) + (1-\pi)V^M(E,P)\right] \tag{C1}$$

where $V^M(TD)$ is still given by (25) and $V^M(E,P)$ is the value of soldiers under prevention before they know the realization of η_t. This value is defined recursively as

$$V^M(E,P) = w^P + \beta\left\{(1-\mu)V^M(E,P) + \mu\left[\pi V^M(TD) + (1-\pi)V^M(E,P)\right]\right\}$$

and it is therefore given by

$$V^M(E,P) = \frac{w^P + \beta\mu\pi V^M(TD)}{1-\beta(1-\mu\pi)}. \tag{C2}$$

Combining (C1) and (C2), we obtain

$$V^M(E \mid \text{repression}) = \frac{(1-\beta\pi(1-\mu))\,w^P + \beta\pi(1-\beta(1-\mu))\,V^M(TD)}{1-\beta(1-\mu\pi)}. \tag{C3}$$

From (C3), (32) and the no-coup constraint (34),[1] the military wage consistent with coup prevention has the same expression of the efficiency wage w^P necessary to prevent coups against oligarchy as in the baseline model, (35), with the only difference that,

[1] Recall that $V^M(TD) = V^M(E \mid \text{coup}) = \beta\gamma V^M(M) + \beta(1-\gamma)V^L(D)$ (see footnote 17 for details).

because of the change in the fiscal technology, the expressions for w^M and a^L are now given by (43). With these changes, the tax rate for the oligarchy to be able to finance the military wages necessary to prevent coups becomes:[2]

$$\tau^P = \beta\gamma\hat{\tau} + \beta(1-\gamma)\frac{a^L\bar{x}}{Y - \bar{x}A^L}. \tag{C4}$$

The net present discounted value of the elite from prevention, starting with $s_t = E$, is recursively defined as

$$V^H(E,P) = \left(1-\tau^P\right)A^H + \beta\left[(1-\mu\pi)V^H(E,P) + \mu\pi V^H(TD)\right],$$

and can then be rewritten as

$$V^H(E,P) = \frac{\left(1-\tau^P\right)A^H + \beta\mu\pi V^H(TD)}{1-\beta(1-\mu\pi)} \tag{C5}$$

where $V^H(TD)$ is given by (42).

The net present discounted value of the elite from non-prevention, starting with $s_t = E$, is then given by

$$V^H(E,N) = \hat{a}^H + \beta\left[(1-\mu)V^H(E,N) + \mu V^H(\text{coup})\right].$$

The first term in this expression, $\hat{a}^H \equiv (1-\mu\phi)A^H$, is the expected flow payoff to the elite, which takes into account that with non-prevention there will be coups when possible and thus income disruption. The probability that a coup will take place is μ, because a coup can take place only when $\eta_t = \eta^{NI}$. In addition, $V^H(\text{coup}) \equiv (1-\gamma)V^H(D) + \gamma V^H(M)$ denotes the expected *future* value to the elite in case there is a coup. From (41), we have that $V^H(\text{coup}) = (1-\hat{\tau})A^H/(1-\beta)$. In addition, with probability $(1-\mu)$, the elite are insulated from political change today and the same political state recurs tomorrow, i.e. $s_{t+1} = E$. Therefore,

$$V^H(E,N) = \frac{\hat{a}^H + \beta\mu V^H(\text{coup})}{1-\beta(1-\mu)}. \tag{C6}$$

The following conditions on the set of parameters are useful for the characterization of the equilibrium of the model.

[2]The participation constraint of soldiers under prevention is $V^M(E,P) \geq V^L(E,S)$ and it is always satisfied. This follows by combining the no-coup constraint, (34), $V^M(E \mid \text{repression}) \geq V^M(E \mid \text{coup})$, which implies $w^P \geq (1-\beta)V^M(E \mid \text{coup})$, and Assumption 1, which implies that $V^M(TD) \geq V^L(D)$.

Condition 2 $\mu < \overline{\mu} \equiv \beta\hat{\tau}/\left(\phi + \beta\hat{\tau}\right)$.

Condition 2 ensures that the elite strictly prefer non-prevention to smooth transition, that is, $V^H(E,N) > V^H(E,S)$, where $V^H(E,S)$ is defined in (28), with $V^H(D)$ now given by (41). If this condition did not hold, the elite would prefer S to N for any value of π (or would be indifferent between them when $\mu = \overline{\mu}$). This follows since both $V^H(E,N)$ and $V^H(E,S)$ are independent of π. Therefore, when Condition 2 fails to hold, the MPE in any subgame starting in $s = E$ would be identical to that in Proposition 4 in the previous section and would not feature coups against oligarchy.[3]

The participation constraint of soldiers under non-prevention is

$$V^M(E,N) \geq V^L(E,S), \tag{C7}$$

where $V^M(E,N)$ is the value of soldiers under non-prevention before the realization of the variable η_t. This value is defined recursively as

$$V^M(E,N) = (1-\mu)\,\beta V^M(E,N) + \mu V^M(E \mid \text{coup})$$

where $V^M(E \mid \text{coup})$ is still defined in (32), and it is equal to

$$V^M(E,N) = \frac{\mu}{1-\beta(1-\mu)} V^M(E \mid \text{coup}). \tag{C8}$$

Using (C8) and $V^L(E,S) = A^L + \beta V^L(D)$ in (C7), we obtain that the participation constraint of soldiers under non-prevention is satisfied if and only if the following condition is satisfied.

Condition 3

$$\mu \geq \underline{\mu} \equiv \frac{1-\beta}{\beta} \frac{(1-\beta)A^L + \beta a^L}{\gamma\left(w^M - a^L\right) + (1-\beta)\left(a^L - A^L\right)}.$$

If this condition does not hold, the participation constraint of soldiers under non-prevention is violated and this means that this strategy is not feasible for the elite. Moreover, notice that this condition is always satisfied when β is high enough, and it can be easily verified that the set of parameters where Conditions 2 and 3 both hold is not empty.

[3] Notice that $\overline{\mu} < 1$ and also that, except the simplification in the fiscal technology, the baseline framework is a special case of this extended model with $\mu = 1$.

Let us next define $\bar{\pi} \in [0,1]$ in a similar fashion to $\hat{\pi}$ in subsection F. In particular, let $\bar{\pi}$ be the solution to the equation

$$V^H(E, P \mid \pi = \bar{\pi}) = V^H(E, N), \tag{C9}$$

when such a solution exists, with the value of prevention for the elite defined as in (C5). By the same argument as in the proof of Proposition 4, $V^H(E, P \mid \pi)$ is strictly decreasing in π and $V^H(E, N)$ is independent of π, so that when a solution $\bar{\pi} \in (0,1)$ to (C9) exists, it is uniquely defined and $V^H(E, P \mid \pi) \gtreqless V^H(E, N)$ for any $\pi \lesseqgtr \bar{\pi}$. When (C9) does not have any solution $\bar{\pi} \in [0,1]$, then we set $\bar{\pi} = 0$ (when $V^H(E, P \mid \pi = 0) < V^H(E, N)$) or $\bar{\pi} = 1$ ($V^H(E, P \mid \pi = 1) > V^H(E, N)$).

Finally, notice also that the MPE in transitional democracy is still given by Proposition 3 (again with the only difference that the threshold $\hat{\gamma}$ in (21) now features w^M and a^L defined by (43)). Using these observations, we obtain a more complete version of Proposition 6.

Proposition 10 *Consider the extended model presented in subsection A and suppose that $\mu \neq \overline{\mu}$, where $\overline{\mu}$ is defined in Condition 2. If Condition 2 or 3 does not hold, the MPE is identical to that in Proposition 4. If Conditions 2 and 3 hold and $\pi \neq \bar{\pi}$, where $\bar{\pi}$ is defined above, then there exists a unique MPE as follows:*

1. *If $\pi \in [0, \bar{\pi})$, then whenever $s = E$, the elite build an army for repression (i.e., $a = 1$), set $\tau = \tau^P$ and $w = w^P$, and prevent military coups. The military chooses $\psi = 0$ and $\rho = 1$ (no coup and repression). Transitional democracy arises with probability $p(TD \mid E) = \mu\pi$, while oligarchy persists with probability $p(E \mid E) = 1 - \mu\pi$. Proposition 3 characterizes the unique MPE starting in any subgame $s = TD$, so that $q(D) = 1$ when $\gamma \in [\bar{\gamma}, \hat{\gamma}]$, and $q(D) = 1 - \gamma$ and $q(M) = \gamma$ when $\gamma \in (\hat{\gamma}, 1]$.*

2. *If $\pi \in (\bar{\pi}, 1]$, the elite build an army for repression (i.e., $a = 1$), set $\tau = 0$ and $w = 0$, and do not prevent coups. The military chooses $\psi = 1$ (coup) in state μ^{NI}. Military dictatorship arises with probability $p(M \mid E) = \mu\gamma$, consolidated democracy arises with probability $p(D \mid E) = \mu(1 - \gamma)$, while oligarchy persists with probability $p(E \mid E) = 1 - \mu$. Consequently, the long-run likelihood of regimes are given by $q(D) = (1 - \gamma)$ and $q(M) = \gamma$.*

Proof. First, note that, because both $V^H(E,S)$ and $V^H(E,N)$, given in (28) and in (C6) respectively, are independent of π, either $V^H(E,S) > V^H(E,N)$ or $V^H(E,S) < V^H(E,N)$ for any value of π (the case where $V^H(E,S) = V^H(E,N)$ is ruled out by the assumption that $\mu \neq \overline{\mu}$). If Condition 2 does not hold, then $V^H(E,S) > V^H(E,N)$ and non-prevention is never chosen by the elite. In this case, the equilibrium is the same as in Proposition 4. When Condition 3 does not hold, the participation constraint of soldiers under non-prevention cannot be satisfied, thus this strategy is not feasible and again the equilibrium from Proposition 4 applies.

Let us then focus on the case where both Conditions 2 and 3 hold. In this case, $V^H(E,S) < V^H(E,N)$ and non-prevention is feasible and preferred by the elite to smooth transition. By the argument in the text and the definition of $\bar{\pi}$, we have that $V^H(E,P \mid \pi) \gtrless V^H(E,N)$ for any $\pi \lessgtr \bar{\pi}$, which establishes the result. ■

Remark 2 The requirement that $\pi \neq \bar{\pi}$ plays an identical role to the assumption that $\pi \neq \hat{\pi}$ in Proposition 4. When π is equal to $\bar{\pi}$, then the elite will have two best responses, so that the equilibrium is not unique, but its nature is unchanged from that described in Proposition 10. Also, the case where $V^H(E,N \mid \pi = 0) = V^H(E,P \mid \pi = 0)$ is not covered in Proposition 10 since it emerges when $\pi = \bar{\pi} = 0$, which is ruled out by the restriction $\pi \neq \bar{\pi}$. Finally, the requirement that $\mu \neq \overline{\mu}$ rules out the case where the elite obtain the same value from non-prevention and smooth transition.

Proof of Proposition 7

We begin by showing that the threshold $\tilde{\gamma}(R)$ defined in (48) is strictly decreasing in R. Straightforward differentiation of $\tilde{\gamma}(R)$ gives

$$\tilde{\gamma}'(R) = \frac{1-\beta}{\beta} \frac{\bar{x}\tilde{w}^M - \tilde{a}^L}{\bar{x}\left(\tilde{w}^M - \tilde{a}^L\right)^2},$$

where $\tilde{w}^M$ and $\tilde{a}^L$ are defined in (44) and in (45). Next, observe that taking into account the expressions of w^M and a^L defined in (43), we obtain

$$\bar{x}\tilde{w}^M - \tilde{a}^L = \bar{x}w^M - a^L = -\hat{\tau}\bar{x}A^L - (1-\hat{\tau})A^L \leq 0.$$

Therefore, $\tilde{\gamma}'(R) \leq 0$, with equality if and only if $A^L = 0$.

Next consider the decision of the elite. First, define R^* as the level of natural resources such that

$$\tilde{w}^P(R^*)\,\bar{x} = \hat{\tau}\left(Y - \bar{x}A^L\right). \tag{C10}$$

In other words, R^* is the level of natural resources such that when $R = R^*$, total military wages necessary for coup prevention can be financed by taxing production income *only* at the maximum possible rate $\hat{\tau}$. By substituting for $\tilde{w}^M$ and $\tilde{a}^L$ in (52), we obtain

$$\tilde{w}^P(R) = w^P + \beta\frac{\gamma + (1-\gamma)\,\bar{x}}{\bar{x}}R. \tag{C11}$$

Combining this expression with (C10), we have

$$R^* = \bar{x}\frac{w^M - w^P}{\beta\left(\gamma + (1-\gamma)\,\bar{x}\right)}. \tag{C12}$$

Claim 1 *Suppose that R^* is given by (C12) and $R > R^*$. Then in any MPE with coup prevention, the elite set $\tilde{\tau}^P = \hat{\tau}$ and choose $\zeta^P \geq 0$ to balance the government budget constraint, which implies*

$$\zeta^P = \beta\left(\gamma + (1-\gamma)\,\bar{x}\right) - \bar{x}\frac{w^M - w^P}{R}. \tag{C13}$$

Proof. The expression of the government budget constraint provided by (51) implies that

$$\tilde{\tau}^P = \frac{\tilde{w}^P(R)\,\bar{x} - \zeta R}{\left(Y - \bar{x}A^L\right)}.$$

Using this expression, the per period utility of the elite in oligarchy can be written as

$$\left(1 - \frac{\tilde{w}^P(R)\,\bar{x} - \zeta R}{Y - \bar{x}A^L}\right)A^H + (1-\zeta)\,R/n.$$

This expression is everywhere decreasing in ζ provided that $nA^H + \bar{x}A^L < Y$, which is always the case, since $\bar{x} < (1-n)$ by assumption, and since $Y \equiv nA^H + (1-n)\,A^L$. Therefore, $\tilde{\tau}^P$ will be set at the maximum possible level $\hat{\tau}$, and ζ will be determined to satisfy the government budget constraint, that is, ζ^P as given in (C13). ■

Using the fact that in equilibrium $\tilde{\tau}^P = \hat{\tau}$, and that ζ^P is given by (C13), we have that (50) can be written as

$$\tilde{V}^H(E, P \mid \pi) = \frac{(1-\beta)\left((1-\hat{\tau})\,A^H + \left(1-\zeta^P\right)R/n\right) + \beta\,(1-\beta)\,\tilde{V}^H(TD)\,\pi}{(1-\beta)\left(1-\beta\,(1-\pi)\right)}.$$

We also have

$$\tilde{V}^H(E,S) = \frac{(1-\beta)\left(A^H + R/n\right) + \beta(1-\hat{\tau})A^H}{1-\beta}.$$

Moreover, using (49), the threshold $\tilde{\pi}(R)$ at which $\tilde{V}^H(E,S) = \tilde{V}^H(E,P \mid \pi)$ is given by

$$\tilde{\pi}(R) = \frac{1-\beta}{\beta} \frac{\left(1-\zeta^P\right)R/n - \left(\Delta - (1-\hat{\tau})A^H\right)}{\Delta - (1-\beta)\tilde{V}^H(TD)},$$

where $\Delta \equiv (1-\beta)\left(A^H + R/n\right) + \beta(1-\hat{\tau})A^H$. Now since $\partial\left(\left(1-\zeta^P\right)R\right)/\partial R = 1-\beta(\gamma + (1-\gamma)\bar{x})$, we have

$$\begin{aligned} \tilde{\pi}'(R) &= \frac{1-\beta}{\beta} \frac{[1-\beta(\gamma+(1-\gamma)\bar{x}) - (1-\beta)]\left[\Delta - (1-\beta)\tilde{V}^H(TD)\right]}{n\left[\Delta - (1-\beta)\tilde{V}^H(TD)\right]^2} \\ &\quad - \frac{1-\beta}{\beta} \frac{\left[\left(1-\zeta^P\right)R/n - \left(\Delta - (1-\hat{\tau})A^H\right)\right](1-\beta)}{n\left[\Delta - (1-\beta)\tilde{V}^H(TD)\right]^2}. \end{aligned}$$

The numerator of this expression is decreasing in $\tilde{V}^H(TD)$ and $\tilde{V}^H(TD) \leq (1-\hat{\tau})A^H/(1-\beta)$. Therefore,

$$[1-\beta(\gamma+(1-\gamma)\bar{x})]\left(\Delta - (1-\hat{\tau})A^H\right) > (1-\beta)\left(1-\zeta^P\right)R/n \tag{C14}$$

is sufficient for $\tilde{\pi}'(R) > 0$. Using the fact that $\Delta - (1-\hat{\tau})A^H = (1-\beta)\left(\hat{\tau}A^H + R/n\right)$, substituting for ζ^P and rearranging terms, (C14) is equivalent to

$$n[1-\beta(\gamma+(1-\gamma)\bar{x})]\hat{\tau}A^H > \bar{x}\left(w^M - w^P\right),$$

which in turn is the same as the following condition:

$$\bar{x} > \frac{\hat{\tau}(1-\beta\gamma)(1-n)}{\hat{\tau}(1-\beta\gamma) + \beta(1-\gamma)(1-n\hat{\tau})} \equiv \hat{x}.$$

This establishes that when $R > R^*$ and $\bar{x} > \hat{x}$, $\tilde{\pi}'(R) > 0$ and thus higher resource rents make repression more likely. This completes the proof of the proposition. ■

Proof of Proposition 8

The analysis of the MPE in this case is very similar to that in Section I, except that whether the foreign threat is still active is now an additional state variable. Let us start in a subgame with $s = TD$ and with the foreign threat active (the analysis of the case where there is no foreign threat is identical to that in Section I and is omitted). The value to the military from attempting a coup is now given by[4]

$$V^M(TD \mid \text{coup}) = \beta \left\{ \gamma V^M(M) + (1-\gamma)\left[\lambda V^L(D) + (1-\lambda) V^M(TD \mid \text{coup})\right] \right\}.$$

This expression differs from the version in the baseline model, (18), because when the coup is not successful (with probability $1-\gamma$) the military can now be reformed only with probability λ, while with probability $1-\lambda$ the external threat does not disappear, it is not optimal to reform the military and the political system remains in transitional democracy. This value can also be rewritten as

$$V^M(TD \mid \text{coup}) = \frac{\beta\gamma w^M + \beta\lambda(1-\gamma) a^L}{(1-\beta)(1-\beta(1-\gamma)(1-\lambda))}.$$

The return to the military from not attempting a coup is

$$V^M(TD \mid \text{no coup}) = w^{TP} + \beta\left[\lambda V^L(D) + (1-\lambda) V^M(TD \mid \text{no coup})\right],$$

where we again use w^{TP} to denote the military wage in transitional democracy when there is coup prevention. Note, however, that the expression for this wage will be slightly different than the one in Section II (see below). This value function also takes into account that the same state will recur with probability $1-\lambda$ (when the foreign threat remains active and there is no opportunity to reform the military). Rearranging this expression, we obtain

$$V^M(TD \mid \text{no coup}) = \frac{(1-\beta) w^{TP} + \beta\lambda a^L}{(1-\beta)(1-\beta(1-\lambda))},$$

where a^L is now defined in (43). The expression for w^{TP} in this extended environment can be obtained by solving the incentive compatibility equation, $V^M(TD \mid \text{coup}) = V^M(TD \mid \text{no coup})$, as

$$w^{TP} = \frac{1-\beta(1-\lambda)}{(1-\beta)[1-\beta(1-\gamma)(1-\lambda)]}\beta\gamma w^M - \frac{\lambda}{(1-\beta)[1-\beta(1-\gamma)(1-\lambda)]}\beta\gamma a^L, \tag{C15}$$

where w^M is the soldiers' wage in a military dictatorship given by (43).

[4]It is again straightforward to verify that Assumption 1 ensures that the military participation constraint is satisfied.

As in our analysis in Section II, transitional democracies will prevent coups if two conditions are satisfied: first, low-skill producers should prefer to prevent coups; second, they should be able to pay high enough wages to the military to achieve this. Let us start with the second requirement. The necessary condition for the transitional democracy to pay high enough wages to the military again takes the form $w^{TP} \leq w^{M}$. Using the expressions for these two wage levels, the condition for the prevention of coups in transitional democracy leads to (53), that we rewrite

$$\gamma \leq \frac{(1-\beta)(1-\beta(1-\lambda))}{\beta\lambda}\frac{w^{M}}{w^{M}-a^{L}} \equiv \hat{\gamma}(\lambda).$$

Condition (53) is the generalization of condition (21) and shows that transitional democracies can prevent coups as long as the probability that coup attempts will be successful is not too high. Moreover, it can be verified that $\hat{\gamma}(\lambda)$ is a strictly decreasing function of λ and that $\hat{\gamma}(\lambda) \rightarrow \hat{\gamma}$ as $\lambda \rightarrow 1$. This implies the interesting result that condition (53) becomes more difficult to satisfy as λ increases (in the limit as $\lambda \rightarrow 1$, this condition coincides with (21)).

We next verify that low-skill producers prefer to prevent coups when this is feasible. If they prevent coups, their value in transitional democracy is

$$V^{L}(TD \mid \text{no coup}) = (1-\hat{\tau})A^{L} + G^{TP} + \beta\left[\lambda V^{L}(D) + (1-\lambda)V^{L}(TD,P)\right], \quad \text{(C16)}$$

where $V^{L}(TD,P) = V^{L}(TD \mid \text{no coup})$ and $G^{TP} = \hat{\tau}\left(Y - \bar{x}A^{L}\right) - w^{TP}\bar{x}$ incorporates the fact that taxes will be equal to $\hat{\tau}$ (given the tax distortion technology adopted at the beginning of Section III), and whatever is left over from paying soldiers the efficiency wage goes into public good expenditures.[5] Using the fact that $V^{L}(D) = a^{L}/(1-\beta)$, $V^{L}(TD,P) = V^{L}(TD \mid \text{no coup})$, that $G^{TP} = \hat{\tau}\left(Y - \bar{x}A^{L}\right) - w^{TP}\bar{x} \geq 0$, and that w^{TP} is given by (C15), the value to low-skill producers when they prevent coups $V^{L}(TD \mid \text{no coup})$, (C16), can be rewritten as

$$\begin{aligned} V^{L}(TD \mid \text{no coup}) &= \frac{(1-\hat{\tau})A^{L} + \hat{\tau}\left(Y - \bar{x}A^{L}\right)}{1-\beta(1-\lambda)} \\ &\quad + \frac{\beta\lambda a^{L}}{(1-\beta)(1-\beta(1-\lambda))} \\ &\quad + \frac{\beta\gamma\lambda\bar{x}a^{L}}{(1-\beta)(1-\beta(1-\lambda))(1-\beta(1-\gamma)(1-\lambda))} \end{aligned} \quad \text{(C17)}$$

[5] The hypothesis that coup prevention is possible, that is, $w^{TP} \leq w^{M}$, ensures that $G^{TP} \geq 0$.

$$-\frac{\beta\gamma\bar{x}w^M}{(1-\beta)(1-\beta(1-\gamma)(1-\lambda))}.$$

Alternatively, without prevention, the value to low-skill producers is

$$\begin{aligned} V^L(TD \mid \text{coup}) &= (1-\hat{\tau})(1-\phi)A^L + G^{TN} + \\ &+\beta\left\{\gamma V^L(M) + (1-\gamma)\left[\lambda V^L(D) + (1-\lambda)V^L(TD \mid \text{coup})\right]\right\}, \end{aligned} \tag{C18}$$

where $G^{TN} \equiv \hat{\tau}(1-\phi)\left(Y - \bar{x}A^L\right)$, since in this case zero wages are paid to soldiers (i.e., $w^{TN} = 0$). This expression also takes into account that, as before, when a coup attempt fails, the military can be reformed, and therefore there is a transition to a fully consolidated democracy, only with probability λ. Using the expressions of $V^L(D) = a^L/(1-\beta)$, of $V^L(M)$ in (15), and the fact that $G^{TN} \equiv \hat{\tau}(1-\phi)\left(Y - \bar{x}A^L\right)$, the value to low-skill producers when they do not prevent coups $V^L(TD \mid \text{coup})$, given by (C18), can be rewritten as

$$V^L(TD \mid \text{coup}) = \frac{\begin{array}{c}(1-\beta)\left[(1-\hat{\tau})(1-\phi)A^L + \hat{\tau}(1-\phi)\left(Y-\bar{x}A^L\right)\right] \\ +\beta\gamma(1-\hat{\tau})A^L + \beta\lambda(1-\gamma)a^L\end{array}}{(1-\beta)(1-\beta(1-\gamma)(1-\lambda))}. \tag{C19}$$

Assuming that coup prevention is a feasible strategy, namely that condition (53) holds and therefore the wage w^{TP} defined in (C15) can be offered to the military, low-skill producers prefer to prevent coups if $V^L(TD \mid \text{no coup}) \geq V^L(TD \mid \text{coup})$. Combining (C17) and (C19), and taking into account that w^M and a^L are given by (43), this condition is equivalent to

$$\beta\gamma\lambda\hat{\tau}\bar{x}A^L + \beta\gamma\lambda\bar{x}a^L + \phi(1-\beta)(1-\beta(1-\lambda))\left[(1-\hat{\tau})A^L + \hat{\tau}\left(Y-\bar{x}A^L\right)\right] > 0. \tag{C20}$$

The left-hand-side of this inequality is always positive and, therefore $V^L(TD \mid \text{no coup})$ is always greater than $V^L(TD \mid \text{coup})$, which means that the low-skill producers always prefer to prevent coups.

The rest of the proposition, including the fact that $\hat{\gamma}(\lambda)$ is strictly decreasing in λ, follows immediately from the arguments in the text. ■

Proof of Proposition 9

Using (54) and (55), the threshold $\breve{\pi}$ (defined as $V^H(E,S) = V^H(E,P \mid \breve{\pi})$) can be written as

$$\breve{\pi} = \frac{1-\beta}{\beta(1-\alpha)} \frac{\left(\tau^P - \beta\hat{\tau} + \alpha\beta\left(\hat{\tau} - \tau^P\right)\right) A^H}{\left[(1-\beta) V^H(TD) - (1-\beta\hat{\tau}) A^H\right] + \alpha\beta\left[(1-\hat{\tau}) A^H - (1-\beta) V^H(TD)\right]}.$$

If there is no coup in transitional democracy, (42) implies $V^H(TD) = (1-\hat{\tau}) A^H / (1-\beta)$ and

$$\breve{\pi} = 1 - \frac{(1-\alpha\beta)\tau^P}{(1-\alpha)\beta\hat{\tau}},$$

which is strictly decreasing in α.

If coups take place along the equilibrium path in transitional democracy, then (42) yields

$$V^H(TD) = (1-\hat{\tau})(1-\phi) A^H + \frac{\beta}{1-\beta}(1-\hat{\tau}) A^H.$$

Using this expression, the threshold $\breve{\pi}$ becomes

$$\breve{\pi} = \frac{\tau^P - \beta\hat{\tau} + \alpha\beta\left(\hat{\tau} - \tau^P\right)}{\beta(1-\alpha)\left[\alpha\beta\phi(1-\hat{\tau}) - \hat{\tau} - \phi(1-\hat{\tau})\right]}.$$

Since τ^P does not depend on α, we obtain that the derivative of this expression $d\breve{\pi}/d\alpha$ is proportional to

$$\begin{aligned} B(\phi) \equiv & \left[\alpha(1-\beta)\tau^P - (1-\alpha)\left(\alpha\beta\left(\hat{\tau}-\tau^P\right) - \left(\beta\hat{\tau} - \tau^P\right)\right)\right](1-\hat{\tau})\beta^2\phi \\ & - \left(\hat{\tau} + (1-\hat{\tau})\phi\right)(1-\beta)\beta\tau^P, \end{aligned}$$

where $B(\phi)$ is the numerator of $d\breve{\pi}/d\alpha$. This expression is linear in ϕ, and is negative when $\phi = 0$. Therefore, it has at most one root $\phi = \phi^*$ over the interval $[0,1]$. This implies that for any $\phi < \phi^*$, $B(\phi) < 0$ (so that $\breve{\pi}$ is strictly decreasing in α) and for any $\phi > \phi^*$, $B(\phi) > 0$ (so that $\breve{\pi}$ is strictly increasing in α). Moreover, if $B(\phi)$ has no root in $[0,1]$, then $B(\phi) < 0$ for all $\phi \in [0,1]$ and we set $\phi^* = 1$. This establishes all the claims in the proposition. ■