Confucianism and the East Asian Miracle

By **MING-YIH LIANG**

Appendix

This appendix contains proofs and derivations of **Propositions 1-3** *in the main text.*

The aggregate dynamics of the economy can be conveniently described by a system of deterministic differential equations involving the variables C_1, C_2, N_{11} and N_{12}. Maximization of utility by the two representative households results in the familiar Euler equations (11) and (21). As in the text, we denote the two constant growth rates of C_1 and C_2 as γ_{C1} and γ_{C2} where $\gamma_{C2} > \gamma_{C1}$ because $\rho_2 < \rho_1$. The striking aspect of equations (11) and (21) is that consumption growths do not depend on the number of intermediates, N s.

To study the dynamic behavior of N_{11} and N_{12}, we must solve the system of simultaneous differential equations (14) and (20). The equality between w_1 and the marginal product of labor implies

(A1) $$w_1 = (1-\alpha)\cdot(Y_1 / L_1).$$

After some manipulation, the interest rate, given by equations (7) and (8) can be written as

(A2) $$r_1 = (1/\eta)\cdot(1-\alpha)\cdot\alpha\cdot(Y_1 / N_1).$$

Hence, aggregate income, $w_1 L_1 + r_1 \eta_1 N_1$, equals $Y_1 - \alpha^2 Y_1$. It follows that the Western household's budget constraint in equation (14) becomes

(A3) $$\begin{aligned}\eta \dot{N}_{11} &= Y_1 - \alpha^2 Y_1 - C_1 - r_1 \eta N_{12} \\ &= (1-\alpha^2)\, Y_1 - C_1 - r_1 \eta N_{12} \\ &= (1+\alpha)\,(1-\alpha)\, Y_1 - C_1 - r_1 \eta N_{12}.\end{aligned}$$

If we substitute for Y_1 and r_1 from equations (5) and (8) into (A3) and also use equation (7), we get a formula for the Western household's budget constraint

(A4) $$\eta \dot{N}_{11} = [\pi_1 \cdot (1+\alpha)\,/\,\alpha] \cdot N_{11} + (\pi_1/\alpha) \cdot N_{12} - C_1 \,.$$

By a similar process of substitution into equation (20), using equations (5), (6), (7), (8) and (18), we can also get a formula for the Eastern household's budget constraint

(A5) $$(\nu\,\lambda + \eta)\,\dot{N}_{12} = [\lambda\,\pi_2 \cdot (1+\alpha)\,/\,\alpha + \pi_1] \cdot N_{12}$$
$$+\ [\lambda\,\pi_2 \cdot (1+\alpha)\,/\,\alpha\,] \cdot N_{11} - \nu\,\lambda\,\dot{N}_{11} - C_2 \,.$$

We now must solve the system of simultaneous differential equations (A4) and (A5). We can substitute $(\lambda\pi_2/\pi_1)\cdot$(A4) into equation (A5) to get

(A6) $$\dot{N}_{12} = r_1\,N_{12} + a \cdot C_1 - b \cdot C_2 \,,$$

where a and b are defined as

$$a = \nu\,\lambda\,/\,[\,(\nu\,\lambda + \eta)\cdot\eta\,]\,,$$

$$b = 1\,/\,(\,\nu\,\lambda + \eta\,)\,.$$

(A5) is a first-order, linear differential equation in N_{12}. The general solution of this equation is

(A7) $$N_{12}(t) = (\text{ constant }) \cdot e^{r_1 t} - [a\,/\,(r_1 - \gamma_{C1})] \cdot C_1(t) + [b\,/\,(r_1 - \gamma_{C2})\,] \cdot C_2(t)\,.$$

We assume that the production function is sufficiently productive to ensure growth in consumption, but not so productive as to yield unbounded utility

(A8) $$r_1 > \rho_2 > r_1 \cdot (1-\theta)\,.$$

The first part of this condition guarantees that $\gamma_{C2} > 0$. The second part ensures that the attainable utility is bounded and implies that $r_1 - \gamma_{C2} > 0$. The transversality condition for the dynamic optimization by the Eastern household implies

(A9) $$\lim_{t\to\infty} \{N_{12}(t) \cdot e^{-r_1 t}\} = 0\,.$$

If we substitute for $N_{12}(t)$ from equation (A7) into the transversality condition in equation (A9), we get

(A10)
$$\lim_{t\to\infty} \{\text{constant} - [a / (r_1 - \gamma_{C1})] \cdot C_1(T_0) \cdot e^{-(r_1-\gamma_{C1})t} + [b / (r_1 - \gamma_{C2})] \cdot C_2(T_0) \cdot e^{-(r_1-\gamma_{C2})t}\} = 0 .$$

Since $C_1(T_0)$ and $C_2(T_0)$ are finite and $r_1 - \gamma_{C2} > 0$, $r_1 - \gamma_{C1} > 0$, the second and the third terms in the braces converge toward zero. Hence, the transversality condition requires the constant to be zero. The solution of N_{12} becomes

(A11)
$$N_{12}(t) = -[a / (r_1 - \gamma_{C1})] \cdot C_1(t) + [b / (r_1 - \gamma_{C2})] \cdot C_2(t) .$$

If we substitute $N_{12}(t)$ from equation (A11) into equation (A4), we get

(A12)
$$\dot{N}_{11} = [r_1 \cdot (1+\alpha) / \alpha] \cdot N_{11} - f \cdot C_1 + g \cdot C_2 ,$$

where f and g are defined as

$$f = a \cdot r_1 / [\alpha \cdot (r_1 - \gamma_{C1})] + 1/\eta ,$$

$$g = b \cdot r_1 / [\alpha \cdot (r_1 - \gamma_{C2})] .$$

(A12) is a first-order, linear differential equation in N_{11}. By repeating the same procedure as above, we can solve for N_{11} as

(A13)
$$N_{11}(t) = \{f / [r_1 \cdot (1+\alpha) / \alpha - \gamma_{C1})]\} \cdot C_1(t) - \{g / [r_1 (1+\alpha) / \alpha - \gamma_{C2})]\} \cdot C_2(t)$$

Finally, if we add (A11) and (A13) together, we get

(A14)
$$N_1(t) = N_{11}(t) + N_{12}(t) = m \cdot C_1(t) + n \cdot C_2(t)$$

where m and n are defined as

$$m = b / [r_1 \cdot (1+\alpha) / \alpha - \gamma_{C1}] ,$$

$$n = b / [r_1 \cdot (1+\alpha) / \alpha - \gamma_{C2}] .$$

The transversality conditions imply that $r_1 - \gamma_{C1} > 0$ and $r_1 - \gamma_{C2} > 0$, and hence $[r_1 \cdot (1+\alpha) / \alpha] - \gamma_{C1} > 0$, $[r_1 \cdot (1+\alpha) / \alpha] - \gamma_{C2} > 0$. Since a, b, f, g, m, n > 0, the dynamic analysis of N_{12}, N_{11} and N_1 as described in the text can be derived from the following three theorems of real numbers:

Let A, B, C be real numbers and functions of time, t. $A(T_0), B(T_0), C(T_0) > 0$ at $t = T_0$.

***Theorem 1**:* If $A = B - C$, and both B and C grow at positive constant rates with $\dot{B}/B > \dot{C}/C$, then $\dot{A}/A > \dot{B}/B$ initially and $\dot{A}/A$ declines monotonically toward $\dot{B}/B$ as $t \to \infty$.

Theorem 2: If $A = B - C$, and both B and C grow at positive constant rates with $\dot{B}/B < \dot{C}/C$, then $\dot{A}/A < \dot{B}/B$ initially and $\dot{A}/A$ declines monotonically. Let $\dot{A}/A > 0$ initially, then $\dot{A}/A$ declines until at the some point, $t = T_1 > T_0$, $\dot{A}/A = 0$ and A begins to decline from then on for $t > T_1$; A will eventually decline until at $t = T_2 > T_1$, when $B = C$, $A = 0$. Immediately after $t > T_2$, $\dot{A}/A > \dot{C}/C$ and then declines monotonically toward $\dot{C}/C$ as $t \to \infty$.

Theorem 3: If $A = B + C$, and both B and C grow at positive constant rates with $\dot{B}/B > \dot{C}/C$, then $\dot{B}/B > \dot{A}/A > \dot{C}/C$ initially and $\dot{A}/A$ rise monotonically toward $\dot{B}/B$ as $t \to \infty$.