Online Appendix to Optimal Labor-Market Policy in Recessions*

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February 14, 2014

Abstract

This document collects the proofs and steady-state results for the paper captioned above.

Appendix A Planner's problem

We state the social planner's problem in recursive form. The planner enters the period facing the following state variables: aggregate productivity state a, e^p employed workers and a promised utility difference Δ^p. Here as in the following, a superscript ^{p} marks the allocation in the planner problem. The planner maximizes a utilitarian welfare function, choosing state-contingent promised utility $\Delta^{p\prime}_{a'}$ levels, [26] consumption choices c^p_e and c^p_u, separation decisions ξ^p and market tightness θ^p. Let $\Psi_x(x)$ denote the option value of having a choice with $\Psi_x(x) := -\psi_x[(1-x)\log(1-x) + x\log(x)]$, and with μ_ϵ the average cost.[27]

The planner's problem can be written as

$$\mathtt{W}(a, e^p, \Delta^p) = \max_{\xi^p,\theta^p,c^p_e,c^p_u,\left\{\Delta^{p\prime}_{a'}\right\}} e^p\mathtt{u}(c^p_e)+(1-e^p)\mathtt{u}(c^p_u)+[\xi^p e^p+(1-e^p)](\Psi_s(s^p)+\overline{h})+\beta\mathbb{E}_a\mathtt{W}(a', e^{p\prime}, \Delta^{p\prime})$$

subject to the law of motion of the aggregate productivity shock, the budget constraint

$$e^p(1-\xi^p)\exp\{a\} = c^p_e e^p+(1-e^p)c^p_u+\mu_\epsilon(1-\xi^p)e^p-e^p\Psi_\xi(\xi^p)+\kappa_v[\xi^p e^p+(1-e^p)]s^p\theta^p, \quad (36)$$

the participation constraint

$$s^p = \frac{1}{1+e^{\frac{-f^p\beta\mathbb{E}_a\Delta^{p\prime}}{\psi_s}}} \quad (37)$$

the promise-keeping constraint

$$\Delta^p = \mathtt{u}(c^p_e) - \overline{h}(1-\xi^p) - \mathtt{u}(c^p_u) + \beta E_a\Delta^{p\prime}(1-\xi^p) + (1-\xi^p)\psi_s\log(1-s^p), \quad (38)$$

and the constraint on the law of motion for employment

$$e^{p\prime} = e^p(1-\xi^p) + [\xi^p e^p + (1-e^p)]s^p\chi\left(\theta^p\right)^\gamma. \quad (39)$$

[26] In terms of notation, subindex $_{a'}$ here indicates that the level of future promised utility is chosen in a state-contingent way.

[27] In the following, we sketch the derivations skipping over a number of intermediate steps. A technical appendix that goes through all of the derivations in much more detail is available from the authors upon request.

In the last expression, the job-finding rate is defined as $f \equiv \chi(\theta^p)^\gamma$. The planner also takes as given the law of motion of the aggregate productivity shock. Denoting by λ^p_Δ the Lagrange multiplier on the promise-keeping constraint and by λ^p_c the Lagrange multiplier on the budget constraint, the first-order conditions with respect to the two consumption levels c_e and c_u deliver:

$$\lambda^p_\Delta = \frac{[\mathtt{u}'(c^p_e) - \mathtt{u}'(c^p_u)]e^p(1-e^p)}{\mathtt{u}'(c^p_e)(1-e^p) + e^p\mathtt{u}'(c^p_u)}, \tag{40}$$

$$\lambda^p_c = \frac{\mathtt{u}'(c^p_e)\mathtt{u}'(c^p_u)}{\mathtt{u}'(c^p_e)(1-e^p) + e^p\mathtt{u}'(c^p_u)} = \left[\frac{e^p}{\mathtt{u}'(c_e)} + \frac{1-e^p}{\mathtt{u}'(c_u)}\right]^{-1}. \tag{41}$$

The first-order condition regarding separations, ξ^p, is given by

$$\begin{aligned} 0 = \; & [\Psi_s(s) + \overline{h}] + \lambda^p_c[-\exp\{a\} + \mu_\epsilon + \Psi'_\xi(\xi^p) - \kappa_v s^p\theta^p] \\ & + \tfrac{\lambda^p_\Delta}{e^p}[\mathbb{E}_a\beta\Delta^{p\prime} - \overline{h}] + \tfrac{\lambda^p_\Delta}{e^p}\psi_s\log(1-s) + \beta\mathbb{E}_a\tfrac{\partial\mathtt{W}}{\partial e'}[-1 + sf^p]. \end{aligned} \tag{42}$$

The first-order condition for market tightness θ delivers:

$$\begin{aligned} 0 = & -\left[\kappa_v + \kappa_v\frac{\partial s^p}{\partial\theta^p}\frac{\theta^p}{s^p}\right] - \mathbb{E}_a\beta\frac{\Delta^{p\prime}}{\lambda^p_c}\frac{\partial s^p}{\partial\theta^p}\frac{f^p}{s^p} + \mathbb{E}_a\beta\frac{\lambda^{p\prime}_c}{\lambda^p_c}\frac{\frac{\partial\mathtt{W}}{\partial e^{p\prime}}}{\lambda^{p\prime}_c}\left[\gamma\frac{f^p}{\theta^p} + \frac{\partial s^p}{\partial\theta^p}\frac{f^p}{s^p}\right] \\ & + \frac{\lambda^p_\Delta}{\lambda^p_c}\psi_s\frac{1}{1-s^p}\frac{\partial s^p}{\partial\theta^p}\frac{1}{s^p}\frac{(1-\xi^p)}{[\xi^p e^p + (1-e^p)]}. \end{aligned} \tag{43}$$

The first-order conditions for state-contingent promised utility $\Delta^{p\prime}$ are:

$$\begin{aligned} \beta\frac{\lambda^p_\Delta}{\lambda^p_c}(1-\xi^p) = \; & \beta\frac{\lambda^{p\prime}_c}{\lambda^p_c}\frac{\frac{\partial\mathtt{W}}{\partial\Delta^{p\prime}}}{\lambda^{p\prime}_c} + \frac{\lambda^p_\Delta}{\lambda^p_c}\frac{\psi_s}{1-s^p}(1-\xi^p)\frac{\partial s^p}{\partial\Delta^{p\prime}} \\ & + [1-(1-\xi^p)e^p]\psi_s\log\left(\frac{1-s^p}{s^p}\right)\frac{\partial s^p}{\partial\Delta^{p\prime}} \\ & + \mathbb{E}_a\beta\frac{\lambda^{p\prime}_c}{\lambda^p_c}\frac{\frac{\partial\mathtt{W}}{\partial e^{p\prime}}}{\lambda^{p\prime}_c}f^p\frac{\partial s^p}{\partial\Delta^{p\prime}}[1-(1-\xi^p)e^p] - \kappa_v\frac{\partial s^p}{\partial\Delta^{p\prime}}\theta^p[1-(1-\xi^p)e^p]. \end{aligned} \tag{44}$$

The envelope conditions are:

$$\begin{aligned}
\frac{\frac{\partial \mathtt{W}}{\partial \Delta^p}}{\lambda_c^p} &= \frac{\lambda_\Delta^p}{\lambda_c^p}. && (45)\\
\frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} &= \frac{\left[\mathtt{u}(c_e^p) - \mathtt{u}(c_u^p)\right]}{\lambda_c^p} - \frac{(1-\xi^p)\left[\Psi(s^p) + \overline{h}\right]}{\lambda_c^p} + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} (1 - s^p f^p)(1-\xi^p) \\
&\quad + (1-\xi^p)\left[\exp\{a\} - \mu_\epsilon\right] - c_e^p + c_u^p + \Psi_\xi(\xi^p) \\
&\quad + \kappa_v s^p \theta^p (1-\xi^p). && (46)
\end{aligned}$$

After a sequence of manipulations one can derive a "bargaining equation of the planner":

$$\kappa_v \frac{\theta^p}{\gamma f^p} = \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} + (1+\varsigma^p)\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\mathtt{u}'(c_e^{p\prime})}, \tag{47}$$

where we have defined $J^p := \frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} - \frac{\Delta^p}{\lambda_c^p}$, and the wedge ς^p as

$$\varsigma^p = \frac{e^p(1-\xi^p)}{\left[\xi^p e^p + (1-e^p)\right] f^p s^p} \left[1 - \frac{\frac{\mathtt{u}'\left(c_e^{p\prime}\right)}{\lambda_c^{p\prime}}}{\frac{\mathtt{u}'\left(c_e^{p}\right)}{\lambda_c^{p}}}\right]. \tag{48}$$

Similarly, one can derive the "planner's free-entry condition":

$$\kappa_v \frac{\theta^p}{f^p} = E_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} - \zeta^p, \tag{49}$$

where we have defined the wedge ζ^p as

$$\begin{aligned}
\zeta^p &= \frac{\psi_s}{f^p(1-s^p)} \frac{1}{\left[\xi^p e^p + (1-e^p)\right]} \frac{1}{\lambda_c^p} \left[\frac{\lambda_c^p}{\mathtt{u}'(c_e^p)} - 1\right] \\
&\quad + \frac{\psi_s}{f^p(1-s^p)} \frac{1}{s^p f^p \left[\xi^p e^p + (1-e^p)\right]} \frac{1}{\lambda_c^p} e^{p\prime} \left[\frac{\lambda_c^{p\prime}}{\mathtt{u}'(c_e^{p\prime})} - \frac{\lambda_c^p}{\mathtt{u}'(c_e^p)}\right]. && (50)
\end{aligned}$$

An extensive derivation is available upon request.

Appendix A.1 The ratio of marginal utilities next period is measurable this period

This section shows that the planner promises marginal utilities of consumption in the next period such that the ratio of these, $\mathtt{u}'(c_u')/(\mathtt{u}'(c_e'))$, is measurable in this period. That is, the next period's ratio of marginal utilities is known today. As a special case, for CRRA utility, under the planner's allocations, the "replacement rate" in the next period, $b^{p\prime} := c_u^{p\prime}/c_e^{p\prime}$ is therefore known in this period.

The promise-keeping constraint, equation (44), can be rewritten as

$$\begin{aligned}\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) \quad = \quad & \beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial\Delta^{p\prime}}}{\lambda_c^{p\prime}} + \frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\psi_s}{1-s^p}(1-\xi^p)\frac{\partial s^p}{\partial\Delta^{p\prime}} \\ & -[1-(1-\xi^p)e^p]f^p\beta\mathbb{E}_a\left\{\frac{\Delta^{p\prime}}{\lambda_c^p}\right\}\frac{\partial s^p}{\partial\Delta^{p\prime}} \\ & +\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}f^p\frac{\partial s^p}{\partial\Delta^{p\prime}}[1-(1-\xi^p)e^p] - \kappa_v\frac{\partial s^p}{\partial\Delta^{p\prime}}\theta^p[1-(1-\xi^p)e^p].\end{aligned}$$

Observe that, by envelope condition (45),

$$\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial\Delta^{p\prime}}}{\lambda_c^{p\prime}} = \frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\lambda_\Delta'}{\lambda_c^{p\prime}} = \frac{\lambda_\Delta'}{\lambda_c^p.}$$

Then observe that $\partial s^p/(\partial\Delta^{p\prime})$ is measurable this period. As a result, all terms in the promise-keeping constraint apart from λ_Δ' are measurable this period. Therefore, λ_Δ' also needs to be measurable this period. λ_Δ' therefore is independent of the realization of the future shock.

Now, use the first-order conditions for consumption, which imply equation (40). Rearranging this, and moving it one period forward,

$$\lambda_\Delta^{p\prime} = \frac{\left[1-\frac{\mathtt{u}'(c_u^{p\prime})}{\mathtt{u}'(c_e^{p\prime})}\right]e^{p\prime}(1-e^{p\prime})}{(1-e^{p\prime})+e^{p\prime}\frac{\mathtt{u}'(c_u^{p\prime})}{\mathtt{u}'(c_e^{p\prime})}}.$$

Employment at the beginning of the next period is known as of this period; compare employment-flow equation (39). Therefore, if $\lambda_\Delta^{p\prime}$ is measurable this period, so must the ratio of the next period's marginal utilities $\frac{\mathtt{u}'(c_u^{p\prime})}{\mathtt{u}'(c_e^{p\prime})}$. The claim regarding the replacement rate follows from the fact that for CRRA utility

$$\frac{\mathtt{u}'(c_u^{p\prime})}{\mathtt{u}'(c_e^{p\prime})} = \left(\frac{c_u^{p\prime}}{c_e^{p\prime}}\right)^{-\sigma},$$

where $\sigma > 0$ is the coefficient of relative risk aversion.

Appendix B Decentralization

This section decentralizes the equilibrium allocation in the planner's problem by means of a set of benefit and tax rules. First, we define a decentralized equilibrium and collect all equilibrium conditions in the decentralized economy. Then, we sketch the proof of Proposition 2.[28] In order to show the equivalence and save on notation, this section uses t-notation throughout.

Appendix B.1 Definition: decentralized equilibrium

A decentralized equilibrium is a sequence of job-finding rates f_t, vacancy-filling rates q_t, separation rates and separation cutoff levels ξ_t and ϵ_t^{ξ}, search intensities s_t, labor market tightness θ_t, matches m_t, vacancies v_t, consumption levels $c_{e,t}$, $c_{0,t}$, and $c_{u,t}$, aggregate levels of output y_t, and dividends Π_t, a discount factor $Q_{t,t+1}$, wage rates w_t, severance payments $w_{eu,t}$, employment rates e_t, firm values J_t, and surpluses of the worker Δ_t, and a sequence of government policies (a profit tax rate, $\tau_{J,t}$, a vacancy subsidy, $\tau_{v,t}$, a layoff tax $\tau_{\xi,t}$ and unemployment benefits B_t) such that the following are true:

1. The value of the firm is given by (13) and can be rearranged to:

$$\begin{aligned} J_t \quad = \quad & [\exp\{a_t\} - \mu_\epsilon - w_t - \tau_{J,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1}] \\ & -\psi_\epsilon \log(1-\xi_t) + \xi_t \left[\frac{\beta \mathbb{E}_t \Delta^e_{u,t+1} + \psi_s \log(1-s_t) - \overline{h}}{\mathtt{u}'(c_{e,t})}\right]. \end{aligned} \tag{51}$$

[28] A detailed derivation is available from the authors upon request.

2. The surplus of the worker is given by (3) and (6) and can be rearranged to:

$$\Delta^e_{u,t} = \mathtt{u}(c_{e,t}) - \overline{h}(1-\xi_{,t}) - \mathtt{u}(c_{u,t}) + \beta E_a \Delta^e_{u,t+1}(1-\xi_t) + (1-\xi_t)\psi_s \log(1-s_t). \quad (52)$$

3. The search intensity is chosen optimally and is given by:

$$s_t = \frac{1}{1 + e^{\frac{-f_t \beta \mathbb{E}_t \Delta^e_{u,t+1}}{\psi_s}}}. \quad (53)$$

4. Firms choose the number of vacancies optimally, the free-entry condition equation (14) is repeated below:

$$\kappa_v \frac{\theta_t}{f_t}(1-\tau_{v,t}) = \mathbb{E}_t Q_{t,t+1} J_{t+1}. \quad (54)$$

5. Wages w_t, severance payments, $w_{eu,t}$, and separation cutoffs, ϵ^ξ_t, are bargained according to the Nash-bargaining protocol (15). The resulting first-order conditions are:

   (a) For the wage, equation (16) is repeated here:

$$(1-\eta_t)J_t = \eta_t \frac{\Delta^e_{u,t}}{\mathtt{u}'(c_{e,t})}. \quad (55)$$

   (b) For the severance payments

$$w_{eu,t} = w_{e,t}. \quad (56)$$

   (c) For the separation cutoff, equation (17) is repeated here:

$$\begin{aligned}\epsilon^\xi_t \quad = \quad & [\exp\{a_t\} - \tau_{J,t} + \tau_{\xi,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1} \\ & + \frac{\beta \mathbb{E}_t \Delta^e_{u,t+1} + \psi_s \log(1-s_t) - \overline{h}}{\mathtt{u}'(c_{e,t})}.\end{aligned} \quad (57)$$

6. The separation cutoff implies a share of separations, ξ_t, that is in line with the logistic distribution. The corresponding equation (7) is repeated here:

$$\xi_t = 1/(1 + \exp\{(\epsilon^\xi_t - \mu_\epsilon)/\psi_\epsilon\}. \quad (58)$$

7. Matches link vacancies and workers who search for a job according to matching function (2), repeated here:

$$m_t = s_t f_t[\xi_t e_t + u_t]. \tag{59}$$

8. The job-finding rate is defined as

$$f_t = m_t/(s_t[\xi_t e_t + u_t]). \tag{60}$$

9. The vacancy-filling rate is defined as

$$q_t = m_t/v_t. \tag{61}$$

10. Labor-market tightness is defined as

$$\theta_t = v_t/(s_t[\xi_t e_t + u_t]). \tag{62}$$

11. Employment evolves according to (1), or alternatively:

$$e_{t+1} = (1 - \xi_t)e_t + s_t f_t[\xi_t e_t + u_t]. \tag{63}$$

12. Firms price future cash flow using discount factor $Q_{t,t+1} := \beta\frac{\lambda_{t+1}}{\lambda_t}$, where λ_t is given by equation (12):

$$\lambda_t = \frac{\mathtt{u}'(c_{e,t})\mathtt{u}'(c_{u,t})}{\mathtt{u}'(c_{e,t})(1 - e_t) + e_t\mathtt{u}'(c_{u,t})}. \tag{64}$$

13. Dividends are given by equation (19), which can be rewritten as

$$\Pi_t = e_t(1 - \xi_t)[\exp\{a_t\} - \mu_\epsilon - \tau_{J,t}] - e_t w_t - e_t\xi_t\tau_{\xi,t} + e_t\Psi(\xi_t) - \kappa_v(1 - \tau_{v,t})v_t. \tag{65}$$

14. Consumption is given by (11), namely

    (a) Consumption when employed during the period:

$$c_{e,t} = w_t + \Pi_t. \tag{66}$$

(b) Consumption when laid off at the beginning of the period:

$$c_{0,t} = w_t + \Pi_t. \tag{67}$$

(c) Consumption when unemployed at the beginning of the period:

$$c_{u,t} = B_t + \Pi_t. \tag{68}$$

15. Production equals demand (goods markets clear, Section I.C.6 in the text)

$$y_t = e_t(1 - \xi_t)\exp\{a_t\}. \tag{69}$$

$$y_t = e_t c_{e,t} + u_t c_{u,t} + \mu_\epsilon(1 - \xi_t)e_t - e_t\Psi_\epsilon(\xi_t) + \kappa_v v_t \tag{70}$$

16. The government budget constraint, equation (18), holds:

$$e_t(1 - \xi_t)\tau_{J,t} + e_t\xi_t\tau_{\xi,t} = u_t B_t + \kappa_v\tau_v v_t. \tag{71}$$

Appendix B.2 Proof of Proposition 2

A detailed proof is available from the authors as part of the longer technical appendix. In the interest of space, here we limit ourselves to sketching the steps. The planner's allocation is characterized by five first-order conditions:

1. With respect to separations: equation (42),
2. With respect to market tightness (hiring): equation (49),
3. With respect to future promised utility: equation (47),
4. With respect to consumption when employed: equation (41),
5. With respect to consumption when unemployed: equation (40),

and four constraints

1. The budget constraint: equation (36),

2. The participation constraint: equation (37),
3. The promise-keeping constraint: equation (38),
4. The law of motion for employment: equation (39).

The proof proceeds by guessing that the allocations in the planner's problem satisfy the equilibrium conditions of the decentralized economy if the tax and benefit rules stated in Proposition 2 are used. This guess is then verified. Next, the proof shows that with these allocations and the resulting values for taxes and benefits, the government's budget is balanced. Then, the proof argues that taking the steps sketched above in reverse order, the opposite would be true as well: the equilibrium allocations that result when the tax rules described in Proposition 2 are used in the decentralized economy satisfy the equilibrium conditions of the planner's problem.

Appendix C Relation to the Baily-Chetty formula

In this appendix, we show how our formula for optimal UI benefits relates to the Baily-Chetty formula, compare Baily (1978) and Chetty (2006). We do so in the steady state and for the limiting case of $\beta \to 1$.

The average duration of unemployment is given by $D := 1/(sf)$. The average duration over which the government pays unemployment benefits is given by $D_2 := D-1$. Note that D_2 is one period shorter than D. The reason is that in the first period of an unemployment spell the worker still consumes c_e (financed by a severance payment by the firm), not a state-financed c_u.

Define the elasticity of the duration of unemployment with respect to an increase in the next period's consumption of an unemployed worker as

$$\epsilon_D := \left.\frac{\partial \log D}{\partial \log c'_u}\right|_{f,\Delta''}.$$

This elasticity treats the continuation value next period, Δ'', and aggregate variables (such as the aggregate job-finding rate) now and in the future as constant. Evaluating for $\beta \to 1$, we have that

$$\epsilon_D = \frac{f}{\psi_s}(1-s)\mathrm{u}'(c'_u)c'_u. \tag{72}$$

Evaluating this at steady-state values gives

$$\epsilon_D = \frac{f}{\psi_s}(1-s)\mathtt{u}'(c_u)c_u.$$

Also define the elasticity of the duration of unemployment benefit payments with respect to an increase in the next period's consumption of an unemployed worker as

$$\epsilon_{D_2} := \left.\frac{\partial \log D_2}{\partial \log c'_u}\right|_{f,\Delta''} = \frac{D}{D_2}\,\epsilon_D.$$

Next, from equations (35) and (41), observe that

$$\zeta \quad = \qquad \frac{\psi_s}{f(1-s)}\frac{1-e}{\xi e+(1-e)}\frac{\mathtt{u}'(c_u)-\mathtt{u}'(c_e)}{\mathtt{u}'(c_e)\mathtt{u}'(c_u)} = \frac{1-e}{\xi e+(1-e)}\,c_u\frac{\frac{\mathtt{u}'(c_u)-\mathtt{u}'(c_e)}{\mathtt{u}'(c_e)}}{\epsilon_D}.$$

From equation (22), we have that as $\beta \to 1$:

$$c_u = \zeta s f.$$

Substitute for ζ from above to obtain

$$c_u = sf\frac{1-e}{\xi e+(1-e)}\,c_u\frac{\frac{\mathtt{u}'(c_u)-\mathtt{u}'(c_e)}{\mathtt{u}'(c_e)}}{\epsilon_D},$$

or

$$\mathtt{u}'(c_u) = \mathtt{u}'(c_e)\left[1+\epsilon_D D\frac{\xi e+(1-e)}{1-e}\right]. \tag{73}$$

Express this in terms of ϵ_{D_2}:

$$\mathtt{u}'(c_u) = \mathtt{u}'(c_e)\left[1+\epsilon_{D_2} D_2\frac{\xi e+(1-e)}{1-e}\right].$$

To simplify this further, note that from the employment-flow equation we have that

$$sf = \frac{\xi e}{\xi e+1-e},$$

and that

$$1 - sf = \frac{1-e}{\xi e + 1 - e}.$$

Also, $D_2 = (1 - sf)/sf = D(1 - sf)$. So

$$D_2 \frac{\xi e + (1-e)}{1-e} = D(1-sf)\frac{\xi e + (1-e)}{1-e} = D.$$

We can therefore rewrite the optimal UI benefit formula (73) as

$$\mathrm{u}'(c_u) = \mathrm{u}'(c_e)\left[1 + D\,\epsilon_{D_2}\right]. \tag{74}$$

An increase in UI payments for an unemployed worker reduces the consumption of an employed worker one for one. It also changes the duration of unemployment by ϵ_{D_2}. Since unemployment spells last on average for D periods, we have that the total indirect effect on consumption of an employed worker is $D\,\epsilon_{D_2}$.

Appendix D Steady-state properties under the optimal policy mix

This appendix presents the steady states in the calibrated baseline, the constrained-efficient economy and the first-best; see Table 5. This complements the analysis in Sections III and IV.A. We discuss these briefly. If the planner could directly control the search effort (and thus attain the first-best), the planner would perfectly insure the consumption of all workers regardless of their labor-market state. At the same time, the vast majority of unemployed workers would have to search for a job. More vacancies would be posted so that the steady-state job-finding rate would be higher than in the baseline economy. Out of efficiency considerations, the rate of separation would be higher as well, so that only the lowest-cost would matches produce. Output would rise 0.7 percent above the steady-state level of output in the baseline economy. Note that due to perfect consumption insurance in the first-best, the tensions summarized by ζ_t are zero.

This is a major difference to the constrained-efficient allocation. If search effort cannot be commanded or observed, the planner has to trade off the utility benefits from insurance and the distortions in behavior that insurance generates. The planner does this by

Table 5: Steady state – baseline, constrained-efficient planner, first-best

	planner	baseline	first-best		planner	baseline	first-best
B	0.427	[0.424]	–	u	0.021	[0.057]	(0.046)
b	0.453	[0.451]	–	$urate$	0.026	[0.064]	(0.064)
c_e	0.945	[0.945]	(0.924)	e	0.978	[0.943]	(0.953)
c_0	0.945	[0.945]	(0.924)	s	0.845	[0.843]	(0.947)
c_u	0.428	[0.426]	(0.924)	f	0.369	[0.282]	(0.329)
Δ_u^e	0.996	[1.291]	–	ξ	0.010	[0.018]	(0.022)
w	0.943	[0.943]	–	q	0.180	[0.338]	(0.235)
w_{eu}	0.943	[0.943]	–	θ	2.049	[0.834]	(1.405)
J	0.403	[0.523]	–	v	0.053	[0.052]	(0.089)
Π	.0015	[0.002]	–	y	0.968	[0.925]	(0.932)
τ_ξ	1.499	[0.680]	–	τ_J	0.000	[0.013]	–
τ_v	0.588	[0.000]	–	ζ	1.343	[1.929]	(0.000)

Notes: The table compares the steady-state values of the planner's problem (constrained-efficient) with those of the baseline calibration of Section III (in brackets), and the first-best allocation (in parentheses).

providing some – but at a 45 percent replacement rate less than perfect – unemployment insurance. In addition, the planner makes sure that those workers who do search have a reasonably high chance of first finding a job and then a much lower probability of losing the job than in the baseline calibration. As a result of this – and with the search intensity affected little – the unemployment rate falls from 6.4 percent in the baseline to 2.6 percent, and steady-state employment rises. Note that the increase in output and employment is stronger than in the first-best. This is due to the fact that, for reasons of incentive-compatibility, the planner seeks to provide insurance primarily through job security and opportunities to work rather than by means of unemployment benefits.[29]

As suggested by Proposition 1, the tax, subsidy and benefit system that supports the constrained-efficient allocation features positive unemployment benefits. The government would use positive vacancy subsidies and positive layoff taxes to ameliorate the resulting distortions. The size of both of these instruments is quantitatively significant. The vacancy subsidy is worth about 59 percent of the cost of posting a vacancy. Layoff taxes

[29] Our paper restricts the set of contracts that firms and workers can write to one-period contracts. In some sense, the result of "insurance through the job" on behalf of the government is related to the results that the implicit-contract literature stresses for privately optimal contracts in the absence of social insurance; see, for example, Azariadis (1975), and Akerlof and Miyazaki (1980).

amount to about 1.6 monthly wages. The production tax is close to zero in the steady state.

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